Introduction to Non-Linear Optics and Its Applications
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Lecture - 40
Doubly Resonant Oscillator (DRO) (Contd.)

So, welcome students to the next class of Introduction to Non-linear Optics and Its Application. So, in the last class we started the evolution process inside the DRO. So, this is lecture number 40.

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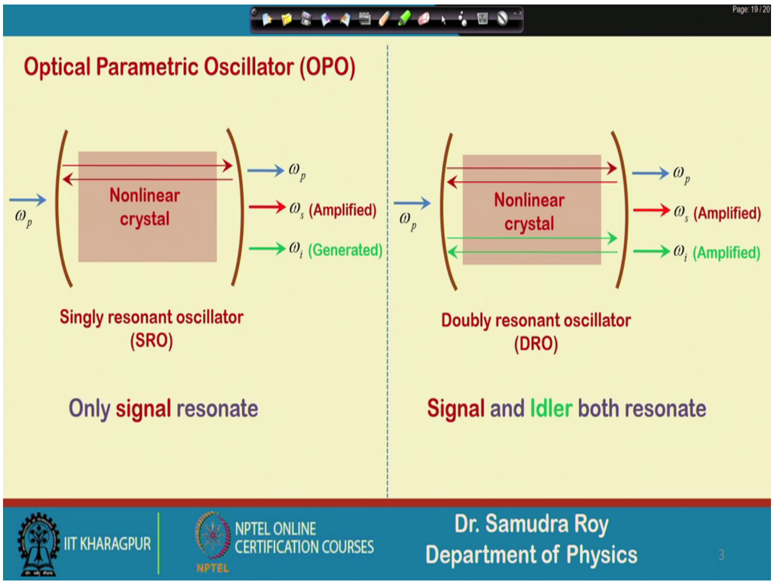

So, we will continue this topic; so DRO the process of the DRO. So, again go back to the slides. And we will find this relationship, which is the schematic figure of optical parametric oscillator. And this optical parametric oscillator schematic figure in the left hand side is shown how the singly resonant oscillator is working; that one wave is one wave with the frequency omega s is oscillating inside the cavity, and as a result will get one amplified wave which is omega s. And in doubly resonate oscillator two different frequency signal and idler both resonates.

So, these two are simultaneously resonate. So, since this is resonate simultaneously, we need to find out what is the threshold condition. So, today our goal is to find out the threshold condition and in the singly resonate oscillator we have the gain like this.

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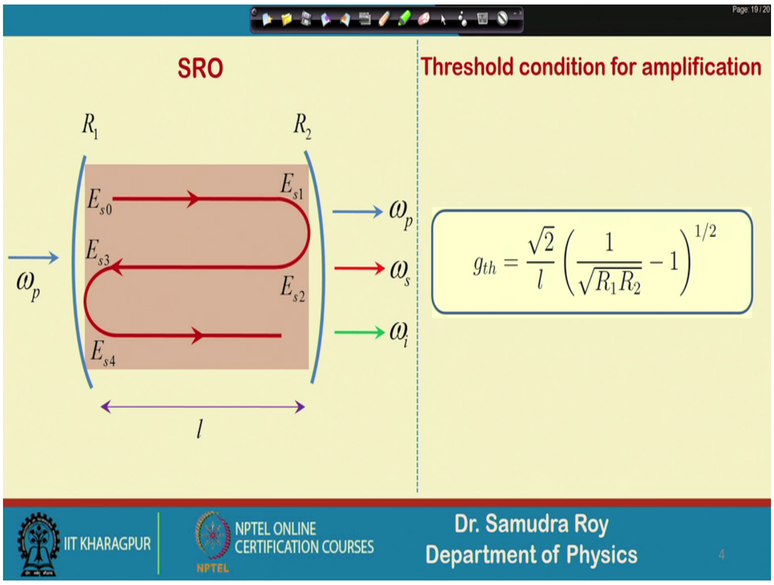

So, the threshold gain we figure out in terms of the cavity length and the reflectivity of the two mirrors and we have this expression in our hand.

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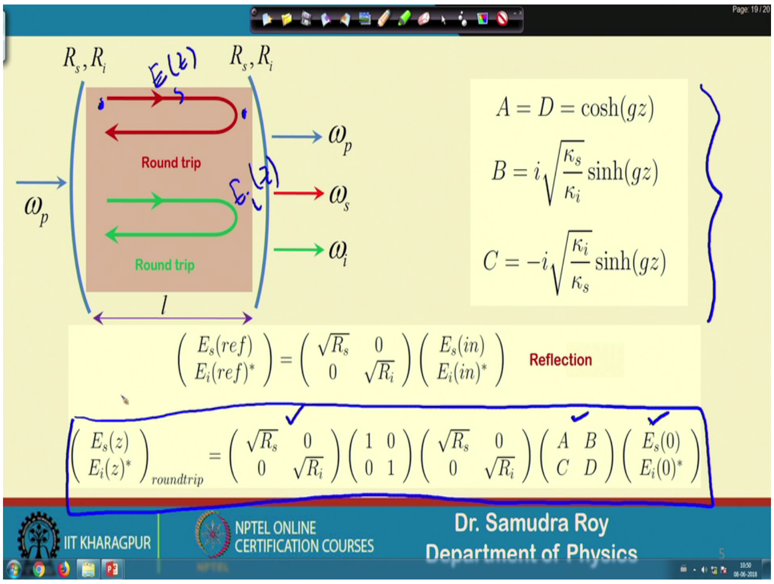

And also in the previous class we find what is the round trip after round trip, what is the expression of the field and with respect to the input field for two fields E s and E i. So, this is the expression. This is the expression we derived last step with the reflection matrix and the corresponding propagation matrix is propagation matrix by ABC is represented in this term.

So, in order to do that what we need to calculate, what we need what we have calculated that the we solve the differential equation of E s and E i and the field is not going from here to here, and we find out what is the general form of the E for E s and E i.

So, just solve the differential equation so, that it becomes a function of Z. After reflection it goes back and when it is goes back, there is no change of the field because the phase matching is not there. So, indeed matrix is associated and the reflection matrix is associated because there is a reflection. And when it comes to this point again it will be reflected for the next mirror two to start the next trip before starting the next a round trip. So, this reflection basically gives basically given by this matrix.

So, we have 4 matrix together. And once we have 4 matrix together then this 4 matrix can combined and we can generate the relationship between the input and output field and then we find; what is the threshold condition to amplify the process.

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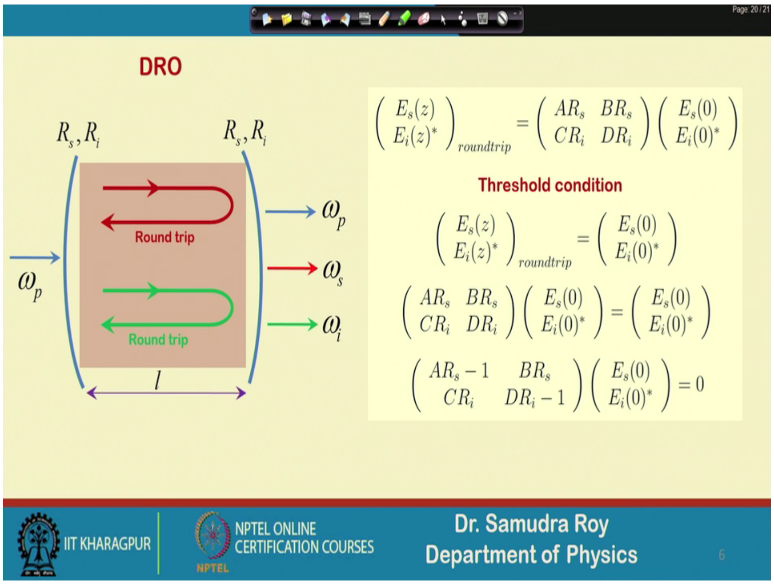

We will for DRO if I go back and multiply all this mattresses to make it more concise, then if you do we will find this matrix form. This is the compact matrix for round trip.

So, we will have one field here at this point and this point after making a round trip; what should be the value of the fields what should be the functional form of the field of E s and E i together can be represented by these matrix these master matrix. So, now, once we have this matrix in our hand.

So, we can have a relationship between E s and E 0 and E i E i z E i z and E i 0. So, E s z E s 0 this from this relation we can have that the condition of gain for E s, in the similar way I can have the condition for gain of E i also and what is the condition of threshold condition for gain? The threshold condition for the gain is after making a round trip the value of this field at least should be equal to what about the value we have at the starting point.

So, if the loss is introduce then; obviously, because of the loss this field will having a reduce amplitude, we will have amplitude which is reduced, but right now the loss is not there. Even if the loss is not there after making one round trip it is essential to find out what should be the value of the field at Z should be equal to the value at input.

So, after making one round trip if these two fields are at least same, we can say this is at least the condition where we should have some kind of amplification. So, greater than these things will be amplification equal to these things is the threshold condition. So, now we know this quantity E s z E i z star, round trip is nothing but this matrix form this is the condition or this is the expression that we have derived that should be equal to this value E s 0 E i 0 star.

Now, E s 0 E i 0 star and from this expression I can now have here AR s minus 1 B. So, if I make it this side and then make the related algebra I will have an expression something like this. So, these matrix expression suggest that if I need to have a non trivial solution, then the determinant of this quantity has to be 0.

So, what we are doing we have a round trip expression in our hand? From this round trip expression we can try to find out the relationship between the field that is at Z equal to 0 and the field at round trip. So, round trip expression in our hand and once we put this condition same that is our threshold condition for amplification of the input field and this threshold condition is represented by this.

So, if I solve this equation then I will get one information but without solving even solving we have a condition to have the non trivial solution.

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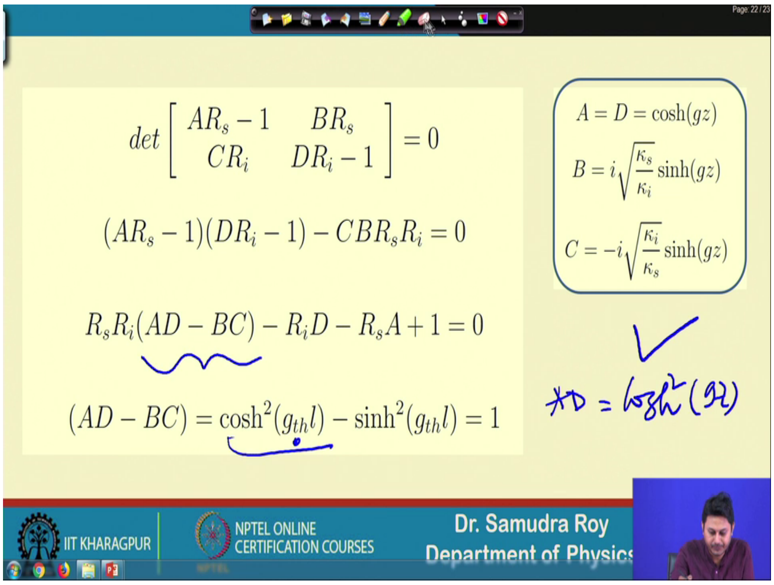

The condition of the non trivial solution as I mentioned if these two are non zero then this equation will satisfy once the determinant of this quantity is 0.

So, determinant of this matrix basically, determinant of this matrix is 0. So, once the determinant of matrix is 0 now is the time to calculate what is going on. So, we calculate the determinant simply. So, AR this multiplied by this cross multiplication minus this cross multiplication is equal to 0 AR s minus 1 DR i minus 1 CB R s R 1. But you should remember what is the value of ABC and D these are the matrix elements and these values of this matrix elements are already calculated. So, these are the matrix element that is in our hand. So, what we do right now. So, we make R i R s common from this expression to this expression.

And I will have AD minus BC. So, A D minus B C if I see carefully A D is this quantity. So, A D is nothing but cos hyperbolic of g Z which we have here, and since it is a threshold condition because I am making a threshold condition I equate the equation under threshold conditions. So, I will write g threshold here because whatever the value of g we are going to impose is basically the threshold condition of that. Now if I try to find out what is my B C?

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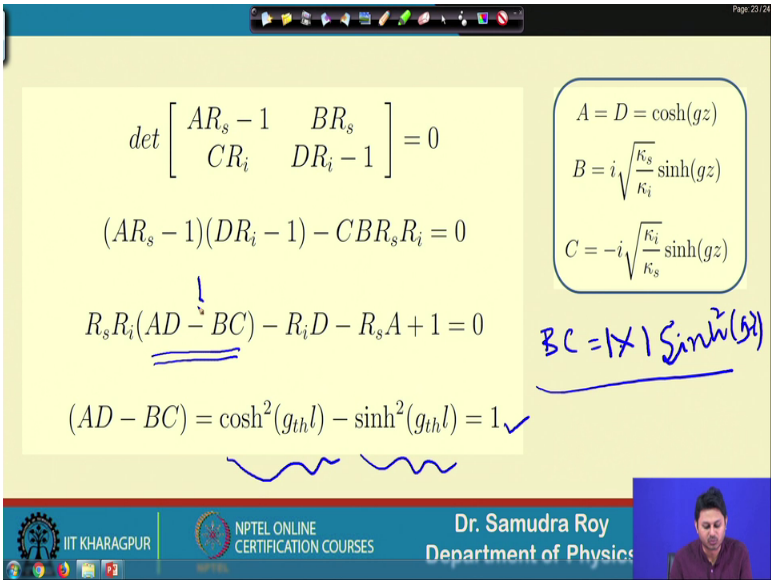

So, B C is i multiplied by i minus i it will give me 1 root over of kappa s kappa i multiplied by root over of kappa s kappa i. So, it will once again give you 1 sin hyperbolic multiplied by sin hyperbolic it will give me sin hyperbolic of square g Z. So, B C is simply sin hyperbolic square of g Z. So, A D minus B C is simply cos hyperbolic square g threshold l minus sin hyperbolic square g threshold l.

So, cos hyperbolic square minus sin cos hyperbolic square we know cos hyperbolic square x minus sin hyperbolic square x is nothing but 1. So, we will have a more simplified expression by just putting this is equal to 1. So, expression is now simplified, it looks cumbersome between I calculate all this relationship is there so, we will have this expression.

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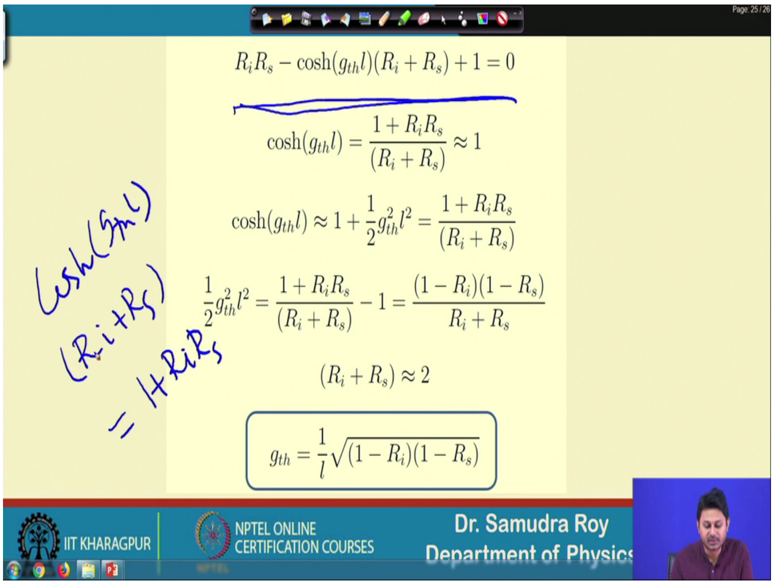

Also here we have D and A; D and a are same. So, I can write R s R i d and a ifI take common then it should be cos it should be cos hyperbolic, it should be cos hyperbolic if i take common d and a. So, R s R i multiplied by cos hyperbolic plus 1 that will be our expression here. R s R i minus cos hyperbolic R i R s plus 1 is equal to 0 is our expression. So, from this expression we can readily see that I can have a value of g threshold. So, I can have a value of g threshold in terms of R i and R s; however, cos hyperbolic g threshold l is a series function.

So, we need to make some kind of approximation that eventually do and we will come to that point what should be the value of g threshold. So, cos hyperbolic g threshold from this expression I can write is 1 if I write this and try to solve what is cos hyperbolic, then I can by simple algebra I can find there is 1 plus R i R s divided by R i plus R s. Because if I write it should be cos hyperbolic g t h l multiplied by R i plus R s is equal to 1 plus R i R s and then cos hyperbolic l is R i R s divided by i plus R s.

Now, R i and R s are very close to 1 because this is the reflectivity of the mirror of two frequencies and it has to be very strong a very high value.

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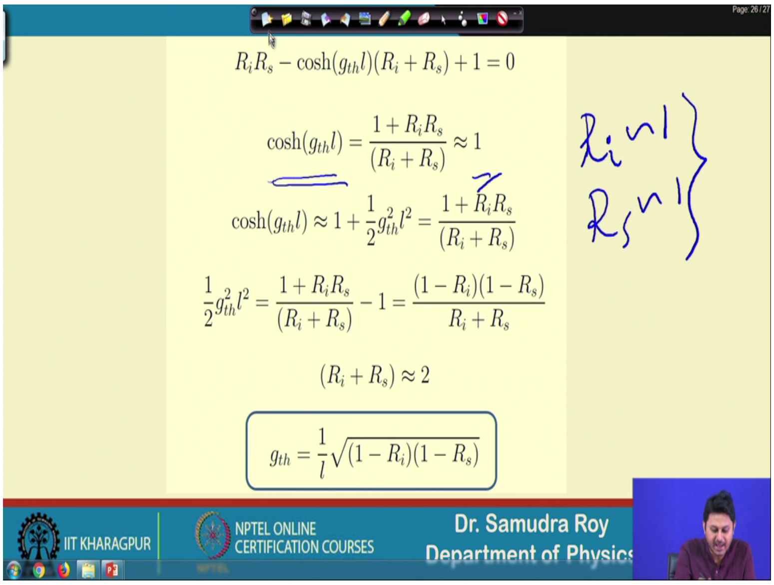

So, these are nearly equal to 1 since this is nearly equal to 1 R i should be nearly equal to 1 R s should be nearly equal to 1. So, R i multiplied by R s is nearly equal to one. So, this entire quantity should be nearly equal to 1.

So, cos hyperbolic l is nearly equal to 1 so; that means, I can make an expansion of cos hyperbolic. So, cos hyperbolic expansion I will go up to second term, because this is possible when there threshold value of the g your whatever the g is comparatively small and if I expand this we will have 1 plus half g threshold l square, which is expansion of this one. So, here we make another this is 1 plus g threshold square; so cos hyperbolic.

So, now from this expression what we try to do, we try to find out what is the value of g threshold. So, cos hyperbolic g threshold l is equal to this quantity it is difficult to find out what is the g threshold in a compact form. So, that is why we make this expansion.

So, after making the expansion we have half g threshold l square is equal to 1 minus R i R s divided by R i plus R s minus 1. So, that quantity basically give rise to one simple expression 1 minus R i multiplied by 1 minus R s divided by R i plus R s. If you make the simple algebra just put it here R i minus R s will be here and take common then you will get this result readily.

Now, again R i and R s are very close to 1 because there are the reflectivity of the two mirrors of two different wavelengths and this reflectivity is very strong. So, very strong means here they are very nearly equal to 1. So, R i plus R s is nearly equal to 1; that means, this R i plus R s is equal to 2. So, I can replace this things is equal to 2.

So, if I make this approximation. Then g threshold these two and these two will cancel out. So, g threshold simply becomes 1 by l root over of whatever the quantity we have. So, finally, we have an expression of the g threshold for doubly resonate oscillator and for doubly generate oscillator we find the expression in terms of R i and R s these are the refractivity reflectivity of the mirror for the signal and idler.

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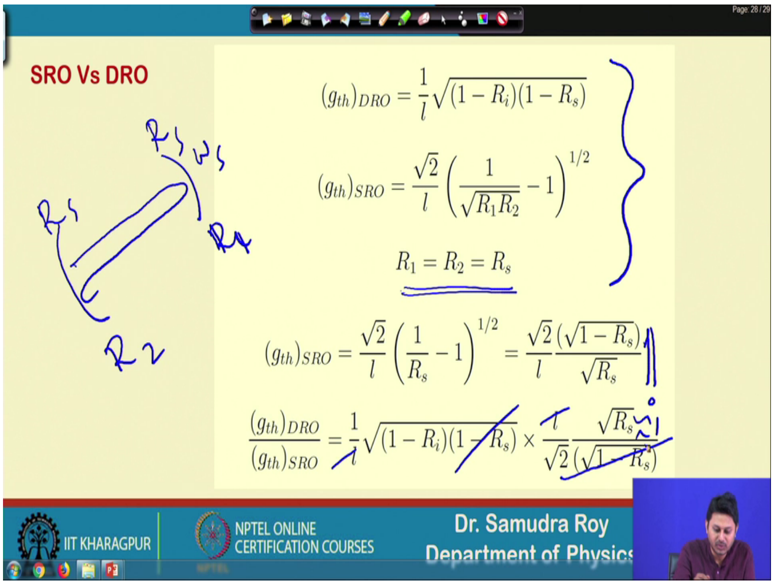

After having the expression next thing we will going to compare the expression that we have for SRO and if I write these two together. So, these two's are expression of the threshold of g and DRO, and the threshold of SRO and if I find these two expression then we can simplify a little bit, because in this case we can see that R i and R s is closely equal to 1. So, I can simplify, but our goal is to find out what is the ratio of the threshold value of DRO and SRO.

So, now we will going to simplify this as I mention I write R i and R s is equal to R i R 1 and R 2 is equal to R s for singly resonate oscillator, because in singly resonate oscillator only omega s will going to oscillate. So, I can write the reflectivity here as omega R s and R s in state of writing are R i and R i.

So, R i and R 2 R 1 and R 2 which is R 1 and R 2 in the previous notation now I just change because these two are very close to each other and same as R s. So, if I do then this expression becomes root over of 2 1 1 divided by R s minus 1 or root over of 2 divided by l and then R 1 minus R s make a root and root over of R s.

So, once we have the expression a little bit simpler expression, then we are now in a position to make a ratio of g threshold DRO and g threshold SRO, when we make a ratio then one divided by R root over of this things and 1 l divided by root over of this. So, these things one minus R s and 1 minus R s root over of these 2 things will cancel out, l will cancel out root over of R s will be there. So, root over of R s by 2 and these things will be there, but R s is nearly equal to once. So, I can write this is nearly equal to 1.

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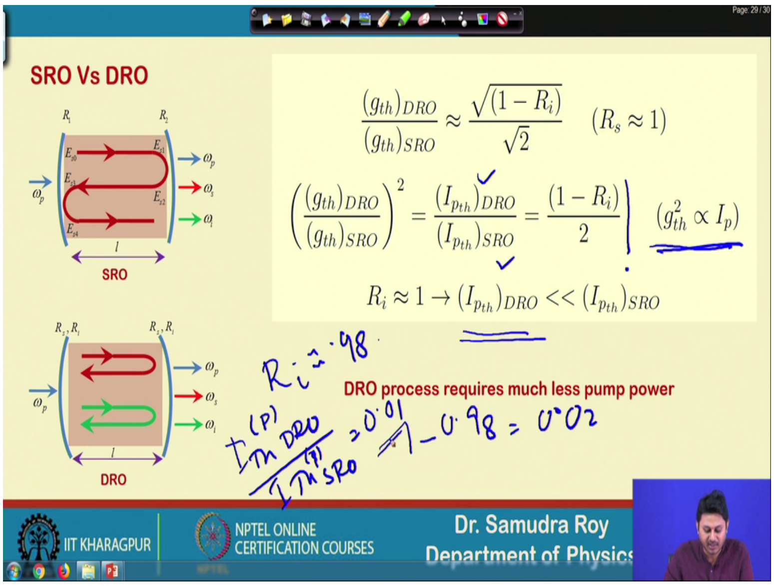

So, we will have one expression something like this in our hand this is expression making R s equal to 1. So, these are the ratios these are the two S R 1 DRO and we have the ratios here, and this ratio suggest that it will be 1 minus R s R i divided by root over of 2. So, now, g threshold is proportional to the intensity of the pump that we know; g threshold is proportional to do intensity of the pump.

Since, it is proportional to the intensity of the pump. So, what we will do that we make a square of that. So, that this ratio became the ratio of the power of DRO or the ratio of the intensity of the DRO versus SRO and this intensity is the threshold intensity to have the amplification.

So, we need to find out what should be the ratio of the threshold intensity of these two quantity, and the right hand side we find 1 minus R i by 2. So, now, you can see that R i is a reflective index R i is a reflectivity coefficient and the reflectivity coefficient for this field E i or the frequency omega i, it should be very close to 1.

So, if this quantity is very close to 1 the numerator is very small. If the numerator is very very small we can readily see that the threshold value of the DRO is much much less than the threshold value of SRO. That means, the power required to excite the amplification in SRO is much much greater than the power required to excite the amplification in DRO. That is an important conclusion that we have.

So, we can put the some value also say if R i is of the order of 0.98 which is very close to 1 then 1 minus R i is 1 minus 0.98 it is 0.02 and now it is divided by 2. So, i threshold for DRO and i threshold for SRO becomes 0.01.

So; that means, the power required to excite or the power required to amplify in SRO is much much less than power equal to DRO is much much less than the power required to SRO. But there are few drawbacks in SRO in DRO also even though the power required power amplification in DRO is much less, but there are some other issues.

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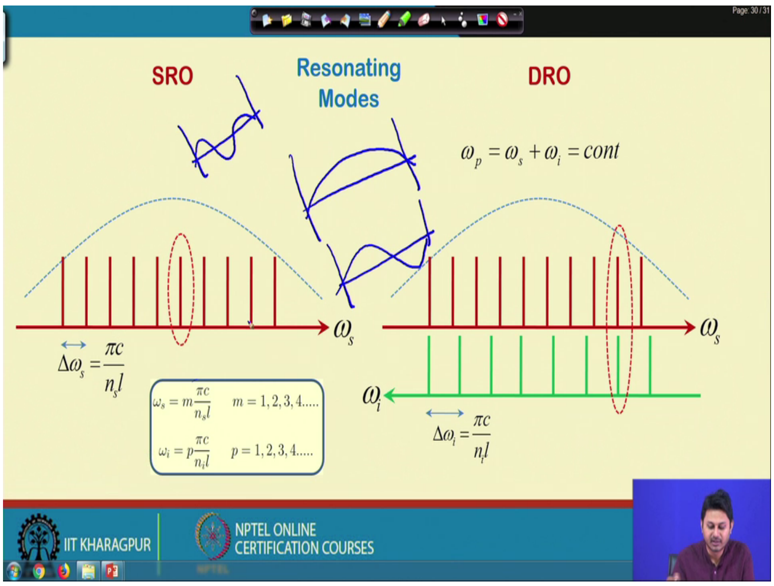

So, let me describe these issues before the conclusion of this class. So, in SRO what happened these are the modes the allowed modes this is interesting. So, in SRO we have these are the allowed modes represented by the vertical lines in omega axis, in the frequency axis. And these are the allowed modes and what are the allowed modes can be figure out with this expression? For this allowed mode the question is which should be really resonating there are many modes all these things are there so; that means, if I draw different modes.

So, we have this is one mode we have, another modes we have another modes and so on. These are the different modes that we can have this is the schematic picture to show how this mode can look like; but the question is among this modes which mode is readily really resonate inside the system which model basically amplify or the which corresponding frequency is amplify because all cases we will have. So, omega p is divided to two frequency omega s and omega i and the relationship is omega p is constant. So, omega s plus omega i has to be constant.

But there infinite way that omega i and omega s can constants. So, omega s can have different values and this different values and nothing but this different modes as well as omega i, but there summation is constant. So, if I consider only one case in SRO for where we have one condition one resonating condition.

So, these dotted thing is the gain curve of the system, because this resonated should have some kind of gain spectrum it will not going to amplify all the frequencies depending on the value of delta k. If delta k is not equal to 0 then we have a gain figure if gain band with that we have already discuss in our previous classes. So, the mode which is close to the maxima of the gain curve will basically going to oscillate and this is shown by this red dotted line. Red dotted circle basically gives you the identification of the modes that will going to vibrate or that will going to resonate in SRO. Corresponding frequency spacing is also shown these are the modes that all are allowed modes in the SRO in DRO case.

On the other hand, in state of having one modes we instating of one frequency we have two frequency that going to resonate. So, omega s and omega i are the two frequencies that will going to resonate; and if it is resonating these two frequencies are resonating then it is not necessarily the modes that you are having a maxima will going to vibrate that is interesting point here.

So, now what we will do. So, omega s these are the allowed modes, and omega i these are the allowed modes because the spacing is different it will not for one by one. So, we can see that there is a mismatch between these two points these two frequencies gradually, but there is a possibility that they will again make a similar they will again coincide a similar point here. So, these are the two modes when these two values are coinciding to each other, in this case omega s is increasing and in this case omega i is increasing why it is the because if i increase the omega s, omega i has to be reduced so, that the summation of these two become a constant.

So, that is why if this is in forward direction it is backward direction it is reducing, and frequency spacing is also represented here which is pi c divided by n i l. But interesting thing is they will going to resonate where these two modes are overlapping to each other, it is something like a vernier scale kind of stuff. So, these two are the corresponding modes which are overlapping and these two are vibrating.

But please note that it may not be necessarily the modes that is at the heights gain value. So, gain value is less, but still it will going to oscillate at this particular points or this for these two modes it will going to oscillates. So, since this is a very critical condition. So, it is the stability of the DRO is very very critical. So, even though there is a advantage of DRO that we have less threshold power to amplify the signal, but the problem is here if I want to x amplify the signal and the corresponding idler.

So, the corresponding frequency modes will need to be coincide like this and they will only then only they will going to vibrate. So, this condition is very critical condition. So, the stability issue be always there in DRO. So, will students. So, let me conclude today's class here.

So, today we have learn and detail about the operation, the how the operation can done in the DRO system in DRO amplification system, and we find that in DRO we have less threshold power required to amplify compared to SRO, but there are some critical issues in DRO regarding the resonance frequency. So, the resonance frequency had to be coincide for omega s and omega i then only it will resonate; and not only that the resonant frequency the location of resonance frequency may not be in the peak point of the gain curve.

So, that is which that is why the efficiency eventually the efficiency is not be that much and also the criticality should be there. So, it is very difficult to handle with this DRO system; anywhere with this note let me conclude here. So, so far we are discussing about the chi two effects. So, the chi two effect phenomena is almost complete.

So, in the next class we will going to start a new brand new topic, which is related to chi three a higher order non-linearity different kind of phenomena will be there. So, with this note let me conclude.

Thank you for your attention. And see you in the next class.