Introduction To Non-Linear Optics And Its Applications
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Lecture – 04
Basic Linear Optics (Contd.)

So, welcome student to this Non-linear Introduction to non-linear optics course. So, before going to the non-linear optics first we learn few basic things we called it the basic optics or Basic Linear Optics. So, today is lecture number 4.

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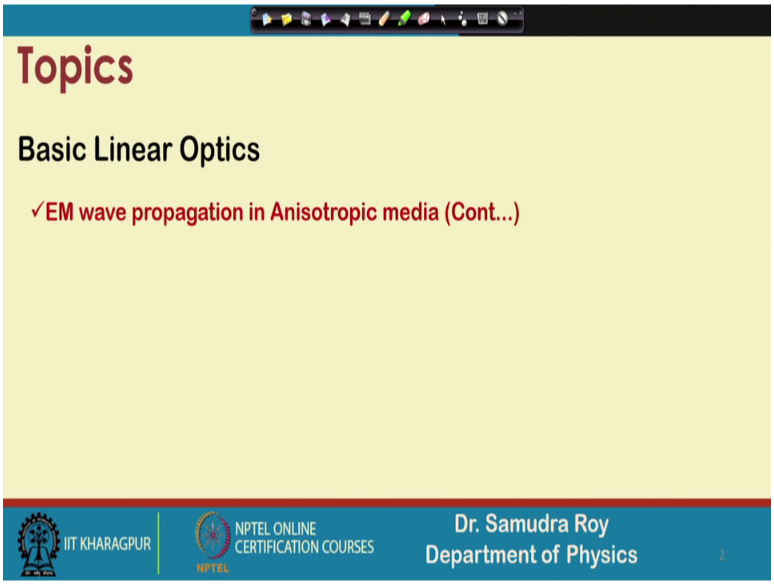

So, we continue with our basic linear optics; in the last class we have learnt something about anisotropic medium.

So, today we will continue with the of anisotropic media and, learn few more thing on anisotropic medium, how the electric field and the propagation vector E are related to each other, how D and E these 2 vectors are related to each other; that we will going to learn with all mathematical derivations ok.

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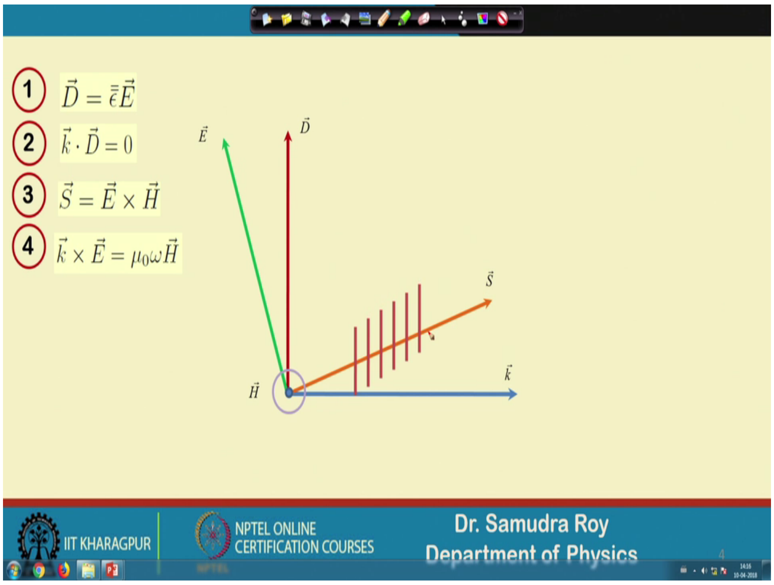

So, let us start with this old picture; this picture is not a very new thing because, in the last class we have used this picture which, suggest that in anisotropic medium how the electric field D, the propagation constant vector k, the magnetic field H and the pointing vector S are related to each other.

So, quite interesting things are there thus picture is quite simple, but let me remind you how these things are coming; first we learn this equation number 1 whatever is given here, that D is equal to epsilon bar bar E; that means, D and E are not parallel that is the most important thing here. Then we learned k and D are related with k dot D is equal to 0; that means k and D are perpendicular to each other. If you look this figure whatever the figure is given in this slide, you will find that k and D are perpendicular no problem in that, D and E are not parallel to each other; that means, D and E are not collinear; that means they have different direction.

So, these are the vectors and they should have different direction. Here in the figure we find the D in one direction and E is another direction. So, there is a angle between D and E we will come to this point in the later part. Then S is usually E cross H; all this expression is derived in anisotropic system. So, E and D relation is the most important one which is not linear here. So, I mean linear means they are co-linear so they are not parallel, which is the main important thing here.

Then S is equal to E cross H which is the usual thing and that is why S is moving in one direction and which is parallel to E perpendicular to E whatever the E vector, we have and H which is perpendicular to this particular plane. So, it is perpendicular to both so that means, it is perpendicular to the plane containing the vector E and H. And then finally, we have one important equation which is k cross E is equal to sum constant into H all this equation is coming from Maxwell's equation.

So that means, k is also perpendicular to I mean k H is perpendicular to k and E. So, again H is perpendicular to the plane containing E D S and k so; obviously, H will be perpendicular to k and E, which is in the same plane ok. This is we have already done. And one important thing is here is that k and S will be not in the same direction; that means energy flow and the direction of the propagation energy flow and the direction of the propagation means here, the wave vector is moving in one direction and the energy is moving in another direction.

If I try to understand this figure, what if you try to understand what is going on, then we need to look, we need to see this figure, this figure suggest that if you look this figure this figure suggest that the same figure, but I have some line this is the phase bound or the wave front. So, wave front we know that it is parallel to the perpendicular to the k vector so; that means, wave front are moving in S direction, but the wave fronts are parallel to k vector.

So, energy is flowing along S direction, but it is perpendicular to the k vector or the propagation constant vector. So, some sort of angle will be here between k and S. So, this angle is important, this angle is important; if I called this angles say phi. So, phi is a angle between S and k. And we will learn that how this phi is important and how to determine, this phi in this particular system, where we have the information of the refractive index.

So, that thing we will come slowly. So, here the important thing that we learn that k and S are not in the same direction E and D are not in the same direction, but E and S are perpendicular to each other and D and k are perpendicular to each other; that means, the angle between S and k this phi and the angle between the vectors D and E this angle will be the same angle ok. So, let us go back to the next slide ok.

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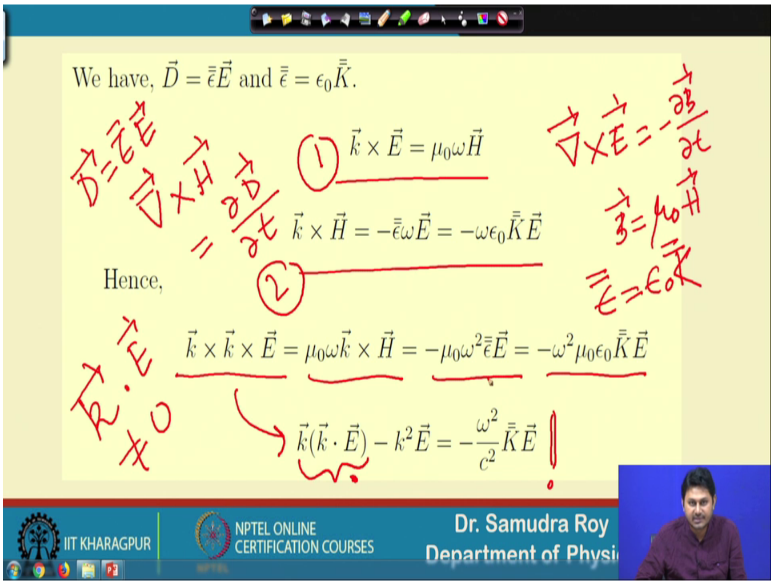

So, here now we try to understand the entire picture by using the Maxwell's equations. So, these are the Maxwell's equation two Maxwell's equation is written here; one is this and, another is this. So, as I mentioned that these things are not very new because, this equation has already been derived in the previous classes, but k cross E again I am doing this. So, k cross E is nothing, but this curl cross E, I just replaced this curl operator to k that we can do and, then it is minus of del B del t that was the original Maxwell's equations.

So, this curl cross E is replaced by k and then del del t we know that is replaced by minus omega. So, there omega is there and B is replaced because, we know that B is mu 0 H. This is another relationship between B mu and H; I use this relation and these leads to the first equation. Second equation in the similar way if I write the second equation the second equation was curl of H was equal to del D of del t this is another Maxwell's equation, this is the forth Maxwell's equation.

So, this del D del t is replaced by epsilon omega epsilon bar omega E because, D and E is related to for for anisotropic system, we know that D vector and E vector is related to these epsilon which is a tensor quantity. So, I use this so, D t is replaced by minus omega. So, this that is why minus is here and then this is the form. Now, here it is extended epsilon we again we write in the previous case, we write that epsilon is epsilon 0 into big K.

So, big K is nothing but the refractive index square of this refractive index. So, I just replace all these things and I have two equation in my hand which is essentially the two Maxwell's equation. Now, the way we derive the wave equation here, we will do the same thing, but the treatment is slightly different the treatment will be in that case we just operate curl over curl here, we will do the same thing, we will operate k over k. So, if I make a cross product of this equation, this equation both the side. This is suppose this is my equation 1, if I make a cross product of equation 1, then I will have these things.

So, mu 0 omega will be as usual, H will be there and k cross H term will come because I am making a cross product entire equation with k. Then k cross H I will use the equation 2 information from equation 2, I will replace and I will have that k cross H can be replaced by minus epsilon 0 omega k double bar E. So, I will replace this 1 omega and mu is there so, you will have this omega square and mu term.

So finally, this terms leads to this omega square mu 0 epsilon 0 k bar bar E. After having that we can expand this k cross k cross E like we expand a cross b cross c. So, we know the rule is a a cross b cross c is b c a minus c a b, if I do this straight forward vector rule, then we have these things will be simple equal to k and then k dot E minus k square, which is nothing, but the k dot k and then E.

So, in the right hand side we have omega square divided by c square because, mu 0 epsilon 0 is nothing, but 1 by c square. So, we have omega square c square k bar E. So, we have an equation in our hand this equation is nothing but the straight forward wave equation, but in different form in the form of k in that case it was not with the operator, but as if we are doing the same thing in furious phase so, we have k in our hands instant of instead of having the del square or del operator.

Here one interesting thing is also there that k dot E in anisotropic system is not equal to 0 in isotropic system k and E is 0, but k dot E in anisotropic system is not equal to 0, so obviously, that produce some extra effect that is interesting that k dot E, if k dot E is equal to 0, then this term was not there, this term was not there and we will have an equation equivalent to the wave equation that we have in our previous case, but here we find that k dot E is not equal to 0 so; that means, some extra will come here ok.

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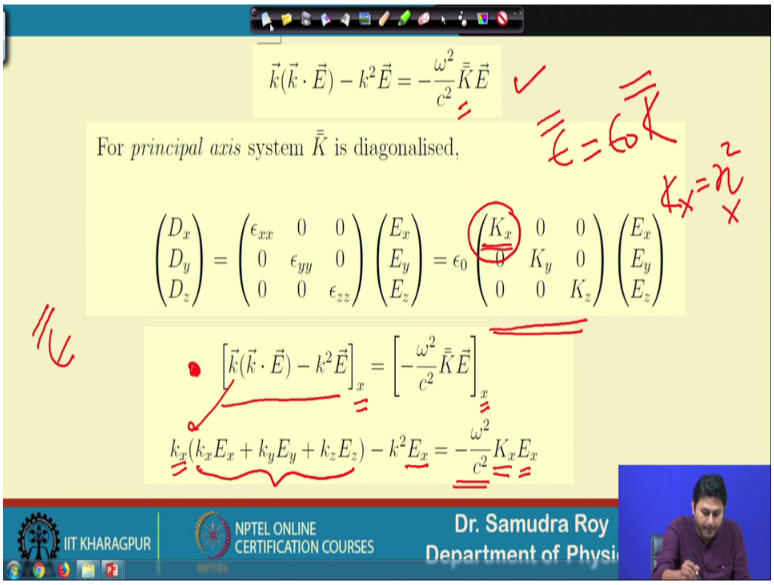

So, we finally, have one equation in the last slide that is my equation that k multiplied by k dot E minus k square E is equal to omega square c square with the negative sign K bar bar and then vector E. Now, this equation in the vector equation and should have 3 components so, we need to figure out which are the 3 components because, that is important in component wise if I divide then there will be 3 equation in our hand x y and z.

But before doing that we need to find out what is the x component, what is the meaning of x component of k. So, now, we know what is the relationship between D and E, this is very old equation in the principal axis system, we know that epsilon this epsilon bar this quantity this epsilon bar distance of quantity can be diagonalize, then the diagonal elements are epsilon x x epsilon y y and epsilon z z, these things can be replaced by K because, epsilon bar bar is equal to epsilon 0; K double bar, this is the relationship between epsilon bar bar and K bar bar we defined.

So, K bar bar is replaced by this. So, the x component K in the principal axis system the x component of K is simply K x; which is nothing but the refractive index square along x direction, because K X is equal to n square, or n X square that is an important identity you will going to use. So, once we know the value of k in terms of x y z. The next thing is to find out the x component of this entire equation, when you find out the x component of the entire equation. So, this equation I put a suffix x for this equation again I put a suffix x.

So, x component of the left hand side this is a vector equation the vector quantity. So, whatever, we have the x component will be equivalent to the right hand side x component again right hand side is a vector quantity. So, we will have only the x component and then check what is the value. So, if I do the x component if I try to extract the x component from this equation, then x component of the first part is simply k x and the dot product of this; please mind that k dot E is a scalar quantity.

So, the only vector quantity here is k. So, I will like to take the x component of the vectors. So, I will take only the k x from here and this party is a scalar so I need to write k x E x plus k y E y plus k z E z. This portion the second part of the left hand side containing k square which is again a scalar quantity, but E is a vector quantity, once I take the x component I should write E x as simple as that. In the right hand side also minus omega square c square will remain same because, this is a scalar quantity and then we have the x component of the entire stuff.

So, K bar bar x component as shown here, in the previous case that it is just K x x. So, I will write it as K x and the x component of the E will be E x. So, these equation which is the equation that is coming from the Maxwell's equation directly, in terms of K and E, I now divide this equation entire vector equation into three part, first part is x component and obviously, the second two part is y and z components, then all the x y z component of k and E will be now our hand and then, we find that what will happen if I launch the electric field in a particular direction; that means, if the k values given to you with all the components then what will happen ok.

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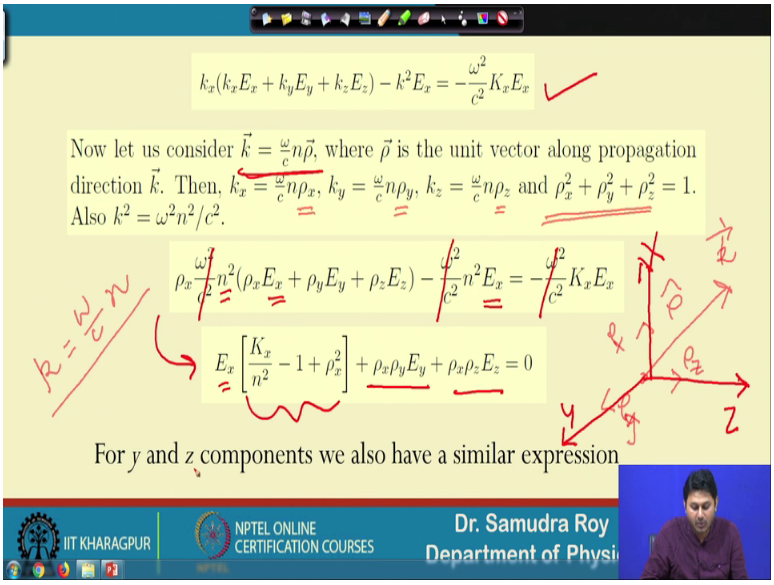

So, the x component as I mentioned the x component of the previous equation is this, this is x component of the previous equation. Now, this x component of the previous equation I can slightly modify and the modification is the k vector can be in any direction, since the k vector has any direction. So, k vector can be represented by this amplitude and some unit rho. So, if I draw a simple figure here; so this is my x y z component right x y z coordinate, say this is my z coordinate, this is my x coordinate and, this is my y coordinates or y axis.

So, now what we will do? We will launch an k and k will be some specific direction; suppose this is the direction of k, this is the direction of k. So, x y z is our reference or the coordinate system and I launch a vector k in this particular system. And rho is unit vector, rho is unit vector along that particular direction, rho is a unit vector along particular direction if that is case, then k can be represented in terms of rho x y rho, x rho y and rho z, where rho x rho y rho z is nothing, but here the vectors this is y this is rho z and this is rho x. What will be the amplitude of the k?

We know that k is nothing but omega divided by c multiplied by N; that is the amplitude of k. If this is the amplitude of k, then amplitude will remain same. So, k x amplitude multiplied by this amount and this amount and this amount, where these are the unit vector along three directions. Now rho x, rho y, rho z since this is the unit vector so the component square rho x, rho y, and rho z, this square will be 1 that is another identity we have.

Now, if I replace the amplitude of k with their unit vectors, then I will replace k x as rho x omega square by c square n square and rho x E x rho y E y and rho z E z, this omega square c square is coming because all the cases we have k x, k y and k z. So, k x, k y, k z all have say their corresponding value omega c n. So, omega c n is common and this k x is something which is omega square c square and n because, another omega square c n sitting here and we come on that. So, that is why the square term is here and rho x is sitting because, here we have k x.

Here we have k square so k square is nothing, but these quantity because, k square is a amplitude of k and square of that. So, this is the amplitude of k and square of that and then E x as usual and omega square c square b K x E x. So, now, if I simplify this thing by taking E x common here, then I will have omega square c square n square one term and if I take all this term. So, here one thing omega square c square, omega square c square and omega square c square all are sitting here. So, omega square c square, omega square c square and omega square c square eventually will be cancel out.

Now, if I take E x then it will be E x E x common, then it will be E x rho square and from here, we have 1 minus n square and from here, we have 1 K x and here also n square is sitting here. So, when we divide the interacting will n square, then this n square this there will be 1 n square by this and minus 1. So, we will have one equation here and when you multiply rho x, rho y, rho x, rho z I will have two equation for E x and E z.

So, it will be the equation we I just write this equation in terms of E x some component, E y some component and E z some component equal to 0. In the similar way we can do for y and z component, the same equation because this is nearly the one component I am writing in this particular form, but I can do that with other components also. So, if we do the other component so, let us see how it looks.

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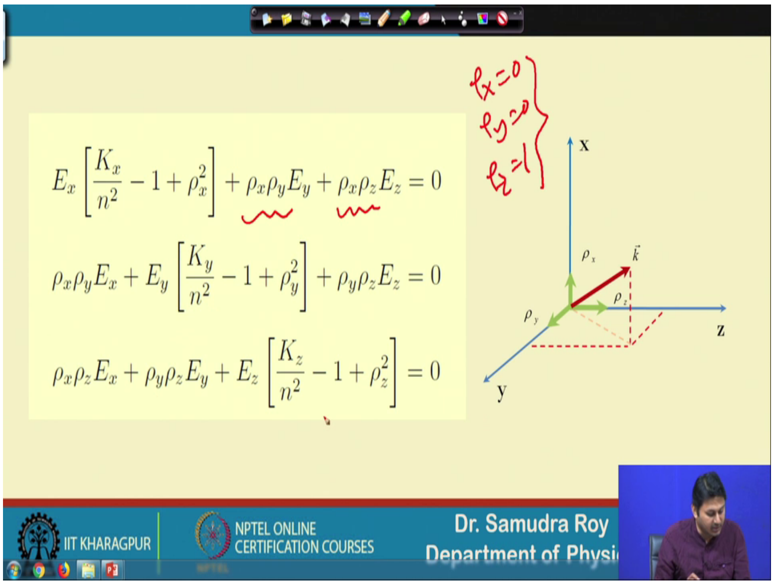

So, this is the previous equation and now this is the next equation as I mentioned, if I write in total what should be the form of this equation, the form of this equation which is a vector equation, but which is the vector equation, but here this equation is divided into three parts x part, y part and z part. So, when I find the x part I will have one equation which is this, when I divide to y part I have one equation which is this and one once divide the entire equation to z part z components so I have another equation like this.

These are thus it looks very symmetry kind of equation, if you do by your own hand then you will find this is quite simple and they are coming like a very similar kind of things will come from y and z only just replacing need to replace the x to y and rho x to rho y and rho y to rho z something like this.

So, this is the equation this is the form of k, that we have already drawn in the previous slide by hand, but here the figure is given that in, if this is k rho x. rho y. rho z is the corresponding unit vectors along that the direction cosine rather in this directions. So if this 3 equation in your hand now if I launch the k in specific direction I should have the value of rho x, rho y and all these things, then we have the knowledge that how E x will be there ok.

So, this is my equation. So, let us go back to the next equation which is the solution. So, what we are doing here.

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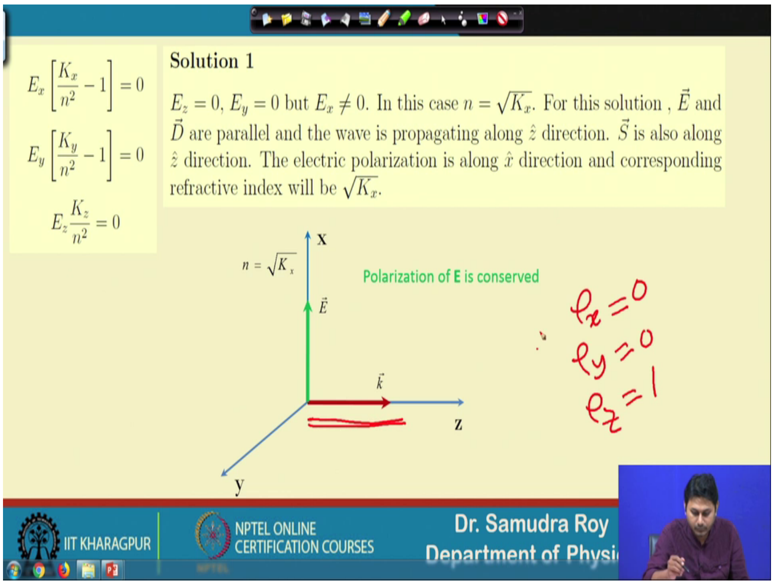

Let me go back and say this is three there are three equations are here. So, my goal here is to find out the solution of this equation, here the solution means I want to find out E x, E y, E z. So, in order to find out E x, E y, E z I need to know the values of rho x, rho y, rho z; that means I need to know one which direction I am basically launching.

So, if I know in which direction I am launching, then I can put the value of rho x, rho y, rho z and then we find out what is my E x and E y. So, let us try to find out the solution we call this solution 1. So, the solution 1 here is something, where we put ok. So, here let me now put so, here what we are doing now, if you see very carefully this figure we will find that my k is lying here, along z direction; my k is lying along z direction. If my k is lying along z direction so; that means, if I write my rho x, rho y, and rho z I will have rho x is equal to 0 and rho y is equal to 0 and rho z equal to 1 because, I am launching my k vector along z direction, I am launching my k vector along z direction.

Now, if this is the case if this is the case, then go back to my previous equation here, in this equation I need to put rho x and rho y 0, rho x and rho z that will also be 0 because, my rho x is 0, rho y is 0 and rho z is 1; why it is that? Because I am launching mu k in a preferred direction and then that direction is z. So, once I launch my k in my Z directions, I will have the rho x, rho y, rho z value like this.

So rho x, rho y, rho z value when I start putting this so I will find that this will 0, this will 0, this will 0 so I will have one equation, this will 0, this will 0, this will also 0 I will have one equation and another equation. This three equation if I write from these three equation, when I put the rho x, rho y, rho z value, if I do then I will have these equation in my hand, I will have these equation in my hand.

So, this equation three equation is now very much simplified equation and, this is suggesting that E z has to be 0; E z so solution 1; so solution 1 first I am looking for solution 1. So, solution 1 suggest that if I say E z is equal to 0, E y is equal to 0, so 1 existing equation is in my hand.

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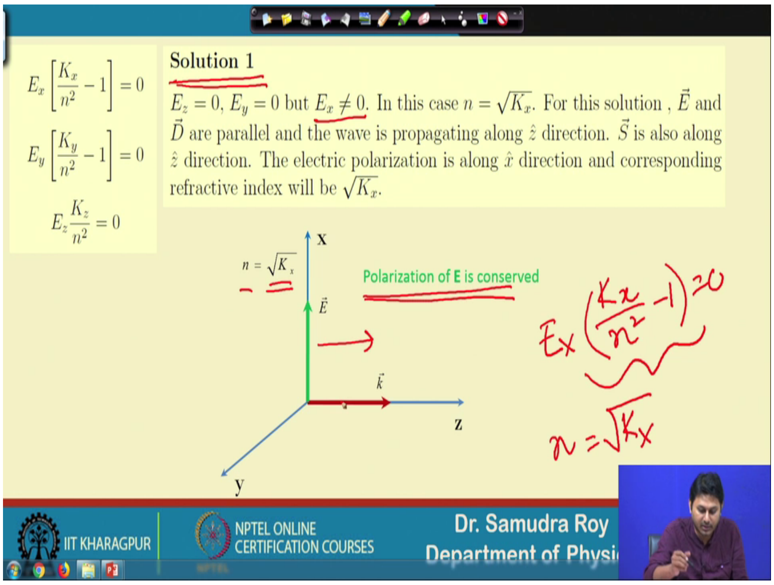

That means E x multiplied by K x n square minus 1 equal to 0. So, mind it k is launched along z direction. E is along this direction, E is now try to actually E I am trying to find out. So, solution 1 suggest that E y is equal to 0, E z is equal to 0 if that is case, but E x is not equal to 0 that is the condition I am putting here, in this equation.

If that is case if it this is not 0 E z is not 0, then this has to be 0 and from here I find the refractive index value is nothing, but root over of K x that means, along y axis my refractive index will be K x and the electric field will be in this particular direction in y direction, in x direction in this case. So, if I launch the electric field, if I if the direction of my electric field will be in z direction, the electric field polarization will be perpendicular to this; in this case it is in x direction.

So, now it will start propagating over this and the polarization of E will remains conserved, there will be no change in the polarization, E will be E that is one issue, second thing very interesting that even in anisotropic system, if I launch k vector in my z direction, then E and k remain main perpendicular, which was not there in the previous general case which was not there; that E and k was not perpendicular to each other, but here, we find that if I launch my E vector along z direction the E vector can be perpendicular to the k.

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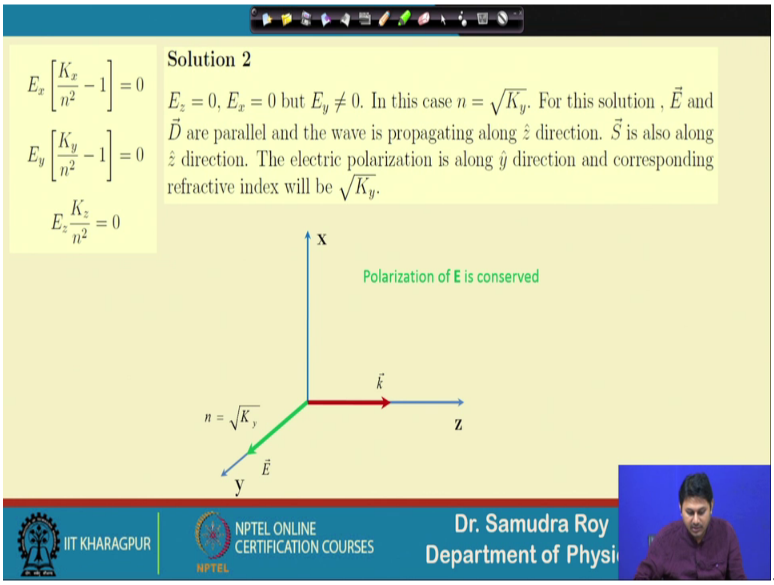

So, this is solution 1. So, now, try to find out what is solution 2? If the solution one is there so it seems that there is another solution and this another solution is solution 2. The solution 2 is again there is nothing new only I say that E z is equal to 0 because, this equation is consistent with that and before I put E y 0, now I am putting E x is equal to 0, there is no x component of the E. So, since there is no x component of the E we have only non-zero component along y directions.

So; that means, electric field is now along y direction, if electric field is now along y direction. So, what happened; the polarization again remain conserved E will be propagating that the polarization of the E will be an this direction and it is perpendicular to the k and they are propagating without any kind of change of this polarization. So, these are the two solution I find out. So, here I like to stop because solution 3 still possible and we will find out what is the solution 3, in solutions 3 what we will do in the next class that we will take E x and E y both non zero.

If we take this thing both E x and E y non-zero, we will find some interesting fact. So, we will do that in the next class. So, here we will like to stop. So, today we learned in anisotropic system how the E x and E y this components, if I launch the electric field along Z direction, then what happened the polarization of electric field along along one direction; that means, perpendicular to the perpendicular to z direction; that means, x and y the polarization remain intact. So, these are the two solutions, these are the two solutions we are getting, but as I mentioned we will get from this equation we can get more solutions. So, solution three is still possible to we will find out what is the solution 3.

So, thank you for today's I mean your cooperation and your attentions. So, let me stop here. So, next day we will start next class we will start the light propagation problem exactly where we stop here. So, solution one is there, solution two is there. So, I will start from solution three with this note.

Thank you very much and see you in the next class.