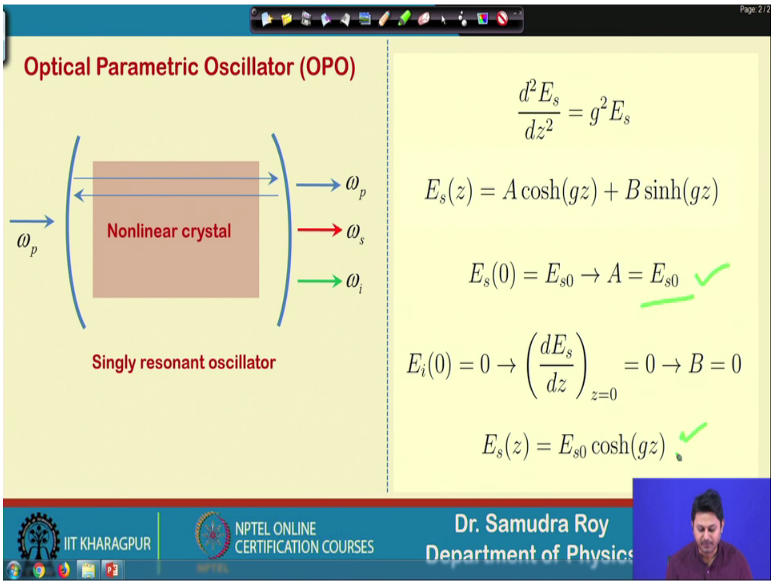
Introduction to Non - Linear Optics and its Applications
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture – 39
Doubly Resonant Oscillator (DRO)

So, welcome student to the next class of Introduction to Non-Linear Optics and your Application course. So, in the previous class we have started a one important concept and that is the optical parametric oscillator. And this is basically a device through which you we can amplify a light show some sort of laser kind of things, but the principal is slightly different that we have discussed.

But this amplification process is mainly generated due to the nonlinearity of the crystal that is used as a active material here, and then the light is bounce back from two mirrors and then we increase the intensity of a signal or idler. The signal or idler are mainly generated due to the non-linear process this non-linear frequency mixing from a given pump. But before starting to today's class I will like to mention one important thing that we have done in the last class there was one typing mistake I like to mention. So, please see this equation.

(Refer Slide Time: 01:20)

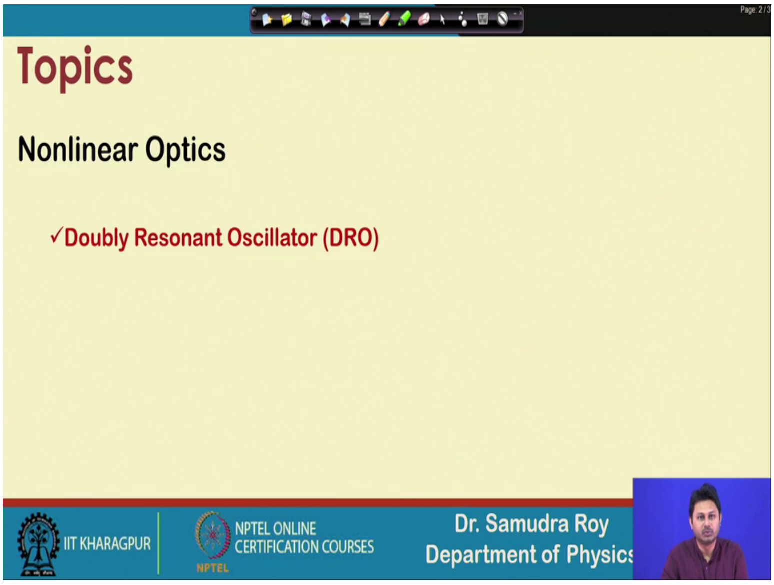


So, which is written here E z at the solution of we are trying to discuss about how the signal we will going to evolve, and then I noticed that there is a slight typing mistake and this typing mistake I like to mention here please not carefully. That we wrote the solution here a sin hyperbolic B cos hyperbolic which is these and this.

So, this is not the incorrect solution this is in fact, the correct solution, but what about the boundary condition I put here that at E s equal to 0, A is equal to this thing this is wrong, this is wrong. So, that is why you should please note that the exact solution will be this, in place of sin we should have A cos hyperbolic g z plus B sin hyperbolic g z then this things this solutions and the boundary conditions are correct.

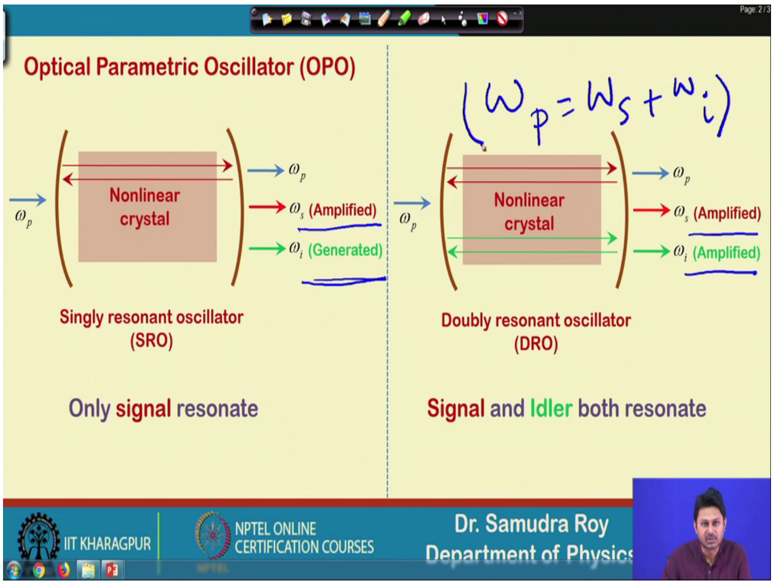
So, just make this light changes when you study. So, please note that there is a slight mistake here. So, I will I noticed that that is why I thought I should explain once again. Apart from that the other boundary conditions are same, ok.

(Refer Slide Time: 02:48)



So, let me go back to the today’s topic. Today we will going to understand what is doubly resonating oscillator or doubly resonate oscillator DRO.

(Refer Slide Time: 02:57)

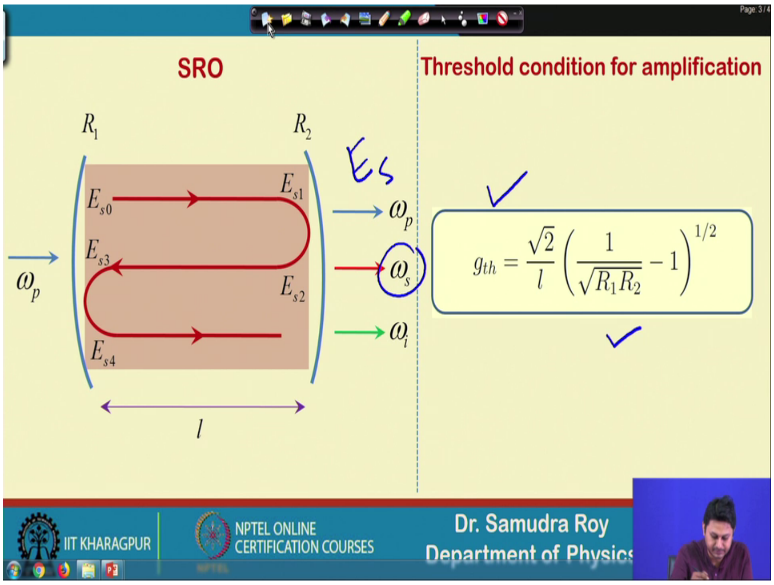


So, in optical parametric oscillator we have already discussed SRO. SRO is a system where we can amplify, so we can amplify a particular frequency in this case the frequency is omega s and as a result as a parametric process omega i will going to generate, omega i is a idler wave. So, only signal is going to resonate, but the mirror the reflectivity mirror is fix in such a way that this omega s will make a round trip, the field associated with omega s is make a round trip and it will going to amplify accordingly. But for doubly resonate oscillator what happened is both the field associate with frequency omega s and omega i will going to amplify.

So, here in state of vibrating one particular wavelength or frequency we have two wavelengths that will going to resonate together. So, signal and idler both will going to resonate. So, since both are going to resonate or both are going to make an oscillation. So, we call it a doubly resonate oscillator or DRO. So, they sincerely the difference between these two in single resonate oscillator only one wavelength will going to oscillate and we called this wavelength as or the frequency as omega s or signal frequency. But in doubly generate resonate oscillator in state of amplifying one frequency we can amplify two different frequencies that we will going to generate from the pump.

But for both the cases the condition the primary condition omega p is equal to omega s plus omega i that will remain same. So, omega s and omega i together generate omega p that is the pump. So, omega s and omega i both will going to be feeded by the within pump and in both the processes this equation remain conserved this equation is true.

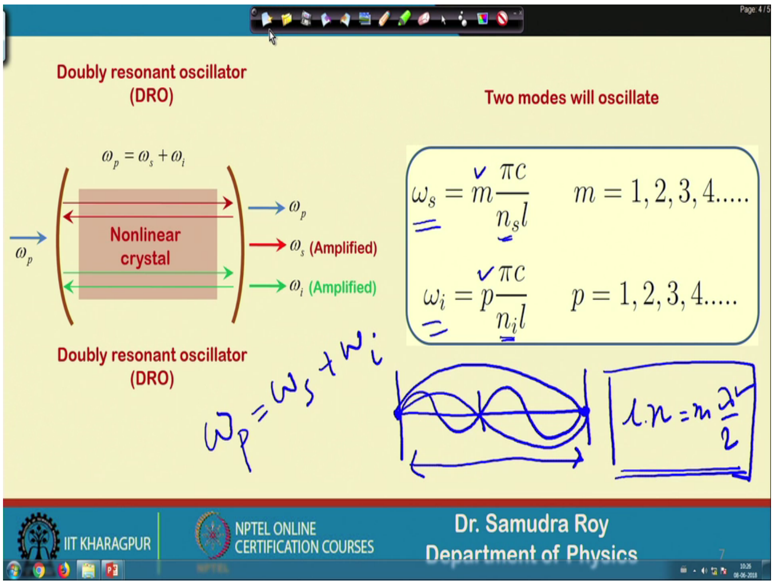
(Refer Slide Time: 05:30)



Well, let me remind you what we have in SRO. In SRO we have one particular field that is going to vibrate as shown here in this figure if the cavity length is l if I say this is a cavity length is l and the reflectivity of the two mirror is R 1 and R 2. Then we can able to find out what is the condition what is the condition for generating, this amplifying this particular frequency omega s.

And the g value which is related to the pump amplitude or the pump intensity, it should have a threshold value and this threshold value should be equal to this root over of 2 divided by l 1 divided by root over R 1 R 2 minus 1 root over of half. So, once we have this value then we can from here calculate what is the corresponding to threshold value of the pump intensity; so, that I can amplify this signal field E s, signal field E s.

(Refer Slide Time: 07:11)



In doubly resonate oscillator in DRO as I mention two field which is associated with frequency omega s and omega i signal and idler both will going to amplify; So, omega s and omega i this two field E s or E i in terms of field notation.

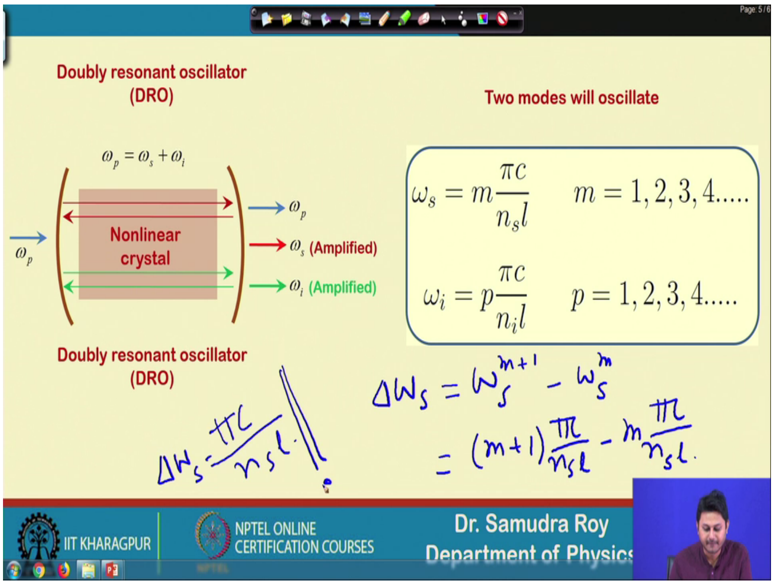
So, there should be few modes that will going to vibrate. So, we know that this modes can be calculated by the boundary conditions or the conditions that we imposes and in a cavity we always do that that if this is a cavity there are few modes that will going to vibrate. And there is some boundary condition and this is for example, this optical length l multiplied n this length has to be the length of the integer lambda by 2 because if you see that here we have mode wire these two points are fix points.

So, this is the frequency that will going to vibrate inside this cavity and that is why we will going to generate this kind of equation. And from this kind of equation one can have the value of lambda for which it will going to vibrate. There are other modes also possible for example, this is n mode that will also going to vibrate, this is another mode that also going to vibrate. So, these are the typical figures of the mode in a cavity where these two points of fixed point and for these two fix points we can have the amplitude 0 and this is the condition to have the modes for which it will going to vibrate.

Now, if I apply this condition then we will going to find out the explicit value of the frequency omega s and omega i. This frequency omega s and omega i now depends on some integers m and p. So, for different values of m we will have different modes that will going to vibrate or in principle that are allowed modes that will going to oscillate. For omega i also I can find out similar kind of expression and only change if you notice is here in the refractive index pi is a constant c is a constant l is a cavity length which is also constant, only thing that will going to change into this two cases and the corresponding refractive index because refractive index of the frequency omega s and the refractive index of the frequency omega i there not same there different.

So, all cases in this two cases these are the allowed mode that will going to vibrate inside the cavity, but this equation has to be maintained each and every time that is one thing. And also one can from this expression one can find out one thing that is what is the frequency spacing between two modes.

(Refer Slide Time: 10:28)

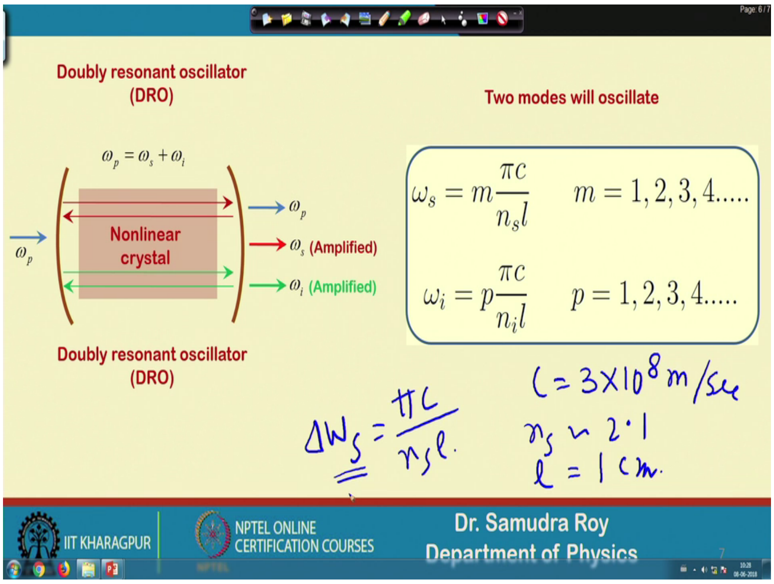


Because two modes will going to vibrate and if I want to find out what is the corresponding frequency spacing, then we can write it as omega delta omega s is basically omega s m plus 1 minus omega s m which is m plus one pi c n s l minus m pi c n s l. And from this two we can have pi c n s l.

This is the frequency spacing between two oscillatory mode two oscillatory mode that is allowed. So, there are many oscillatory modes that would be there in this cavity and for this modes there is a corresponding frequency spacing and this frequency spacing is represented by pi c divided by n l s. So, this is the frequency spacing between two corresponding modes.

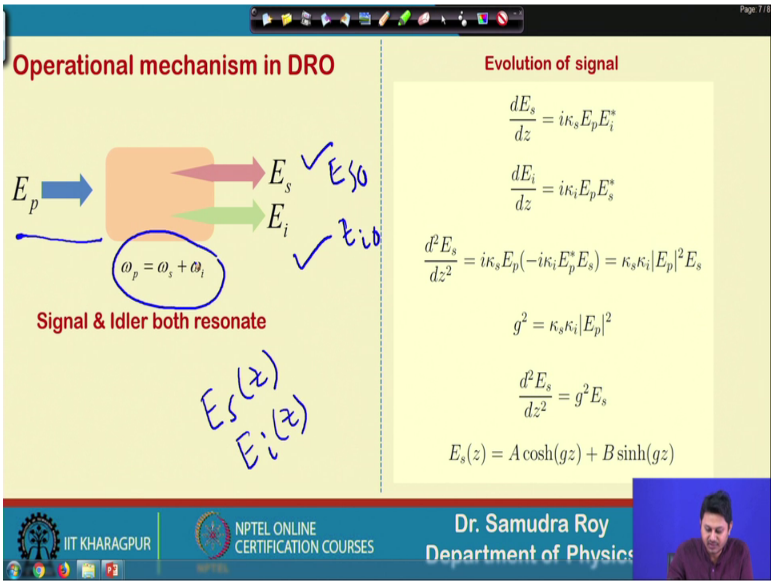
So, now c value is constant which we know 3 into 10 to the power 8 meter per second pi is also constant 3.14, n is a refractive index of the material. So, if it is a non-linear crystal a different kind of non-linear crystal as different refractive index. So, suppose it is 2.1 or 2.3 and l is a value say few centimeters then putting all this value one can understand what should be the frequencies spacing between these two modes that are allowed to vibrate. So, this calculation is easy and one can understand that what should be the order of this frequency spacing by just putting a suitable value here, l is order of centimeter as I mentioned.

(Refer Slide Time: 12:35)



So, delta omega s is equal to pi c n s l c is 3 into 10 to the power 8 meter per second, n s is of the order of say 2.1 l is a length which is a few centimeters say one centimeter order and then if I put and pi is 3.14. If I put this things here then we can readily find out what should be the frequency spacing. And unit wise also you can see that c is a in it a meter per second pi n n is a dimensionless l is a unit of meter. So, meter will cancel out we will have in the unit in terms of Hertz.

(Refer Slide Time: 13:28)

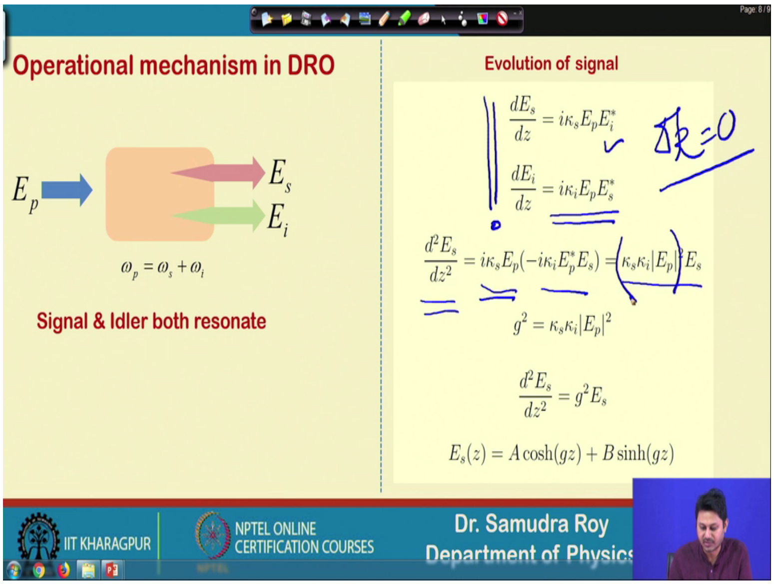


So, the next that we will going to find is operational mechanism in DRO. So, let us understand the left hand figure what we are showing, that once the pump is launched here the signal and idler, the signal and idler are generated pump is launched and signal and idler are generated. When the signal and idler are generated that means, it should grow from a value input value and both are amplified. So, that is the thing. So, signal and idler both are resonate and they are going to generate inside the system and they will generate from a given initial value.

If the given initial value is E s 0 for this case that is at z equal to 0 and E i 0 that is at z equal to 0. So, both of these things are going to evolve. So, E s is a function of z and E i is also a function of z and both will going to evolve. So, this evolution we need to find out.

What should be the equation of this evolution? Both the cases again how these two frequency will generate will be governed by this master equation omega p is equal to omega s plus omega i. In order to understand how this two fields will going to evolve inside the material non-linear material which will going to use as active material in DRO we need to just solve; This two differential equation that we have been solving for last few classes. This two differential equation is under the condition delta k is equal to 0.

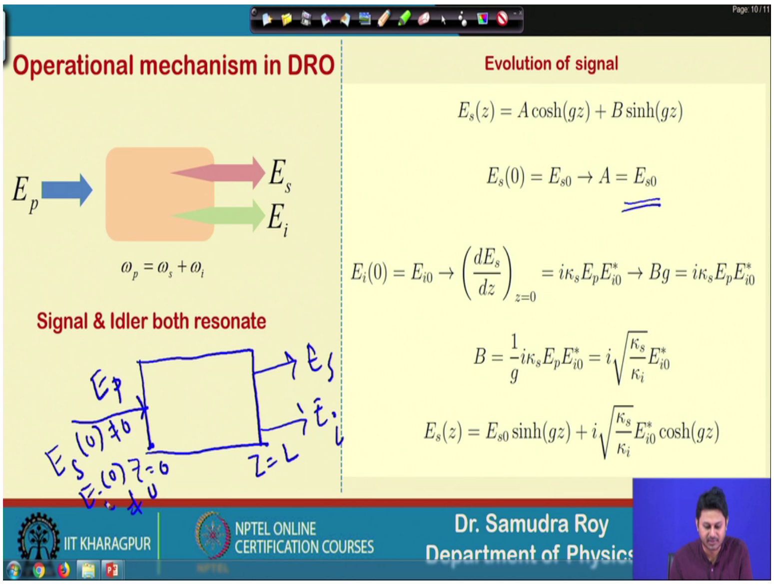
(Refer Slide Time: 15:21)



So, delta k is equal to 0 is the condition for which I can write these two equations and now d 2 E s d z square is i k s E p multiplied by the derivative of this quantity. And this derivative of this quantity can be represented here with a star that is why minus i k i E p star E s one can write this treatment we have done in the previous class also. And then we will find kappa s kappa i E p square E s which is a constant this portion is a constant and finally, we come to this equation that we have shown in the previous class also. So, this is evolution equation of E s.

Now, in this particular situation we need to put some boundary condition and this boundary condition basically, give us the idea how these two fields are generated. So, if I write the solution please note the solution as I mention the beginning of the class the solution A cos hyperbolic plus B sin hyperbolic g z this is the general solution. So, if we consider this is a general solution then in the next we will like to put the boundary condition to figure out what is A and B.

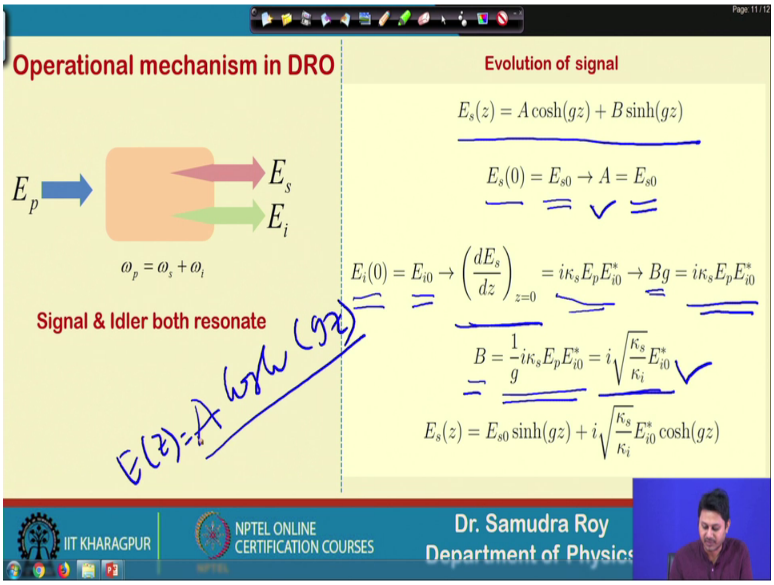
(Refer Slide Time: 16:48)



The boundary condition is this problem is slightly different because if you carefully note what is going on then we can see this is a structure E p is generated and E s and E i, E p is launched and E s and E i is generated and this is z equal to 0 point and z equal to l point at z equal to 0 what happened E s 0 is non-zero and E i 0 is also non-zero. So, they should have some value.

This value I called E s 0 at z equal to 0 and E i 0 at z equal to 0. So, both the cases we find the input value is non 0. Since E s and E i both are 0 at z equal to 0 point the solution the form of the solution is slightly different that we have achieved in our the previous class.

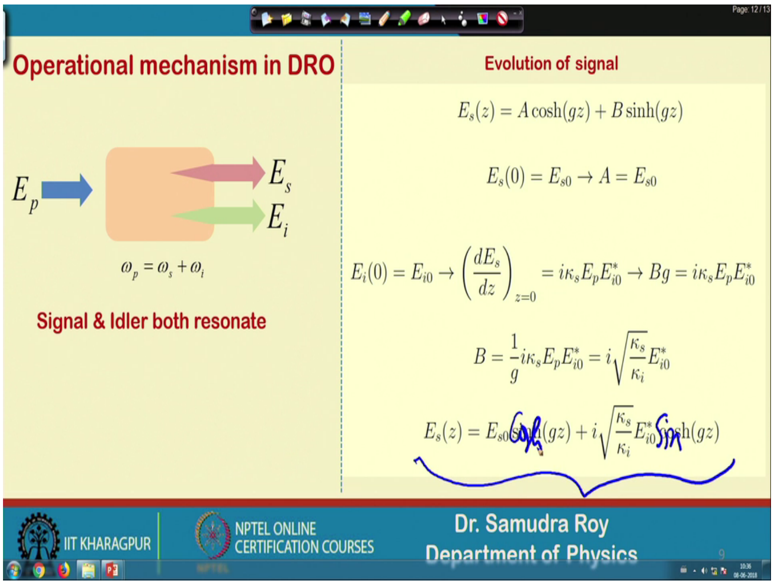
(Refer Slide Time: 18:08)



So, here we will calculate in detail. So, this is my solution boundary condition is E s at z equal to 0 suggest that it is equal to E s 0 which is equal to a and this is my E s 0 this is first. Second thing is E i 0 is E i 0 that means, at z equal to 0 the value of idler field is E i 0. If I want to find B then what we should do we should make a derivative and find out what is the value at z equal to 0 of the derivative the standard procedure when you do then we put this value and this value basically give me one expression B g. Because when I make derivative of this quantity and put z equal to 0 then the term associated with cos will cancel out.

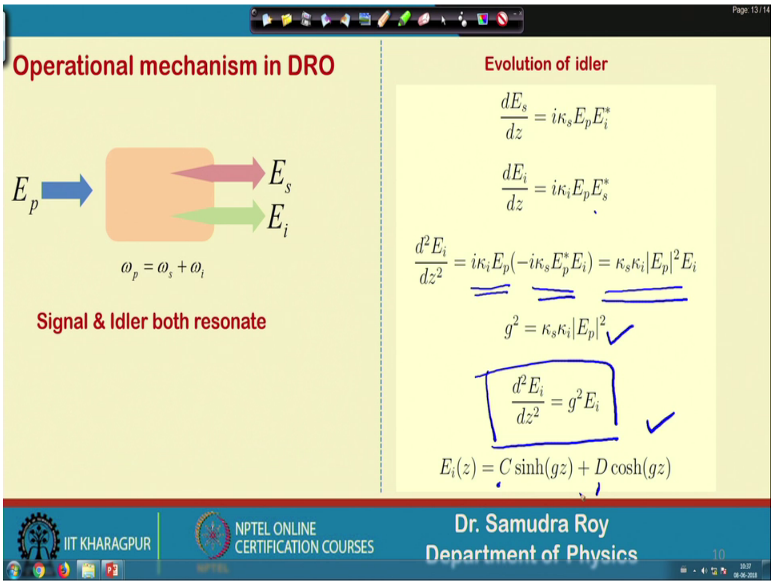
The term associated with B gives me B g and when I put g equal to z equal to 0 the cos here which is coming after derivative we will cancel out and we will get B g is equal to i kappa E p i and E i 0 star. So, B is basically this quantity and this quantity. So, 1 divided by g multiplied by i kappa s E p E i 0 star since E i is E i at z equal to 0 at non-zero we will have B is a non-zero term. If you remember the previous calculation that how E s will going to evolve in this kind of parametric process then it is simply A cos hyperbolic of g z it is just this kind of expression we derived. But in that case we considered that E i was not there at z equal to 0 point, but since E i is there so our solution will be simply this E s 0 sin hyperbolic g z plus i root over of kappa s kappa i this.

(Refer Slide Time: 20:08)



So, A and B are evaluate and you find a most general solution which is this again I am making a mistake this has to be cos hyperbolic and this has to be sin hyperbolic according to the solution.

(Refer Slide Time: 20:45)

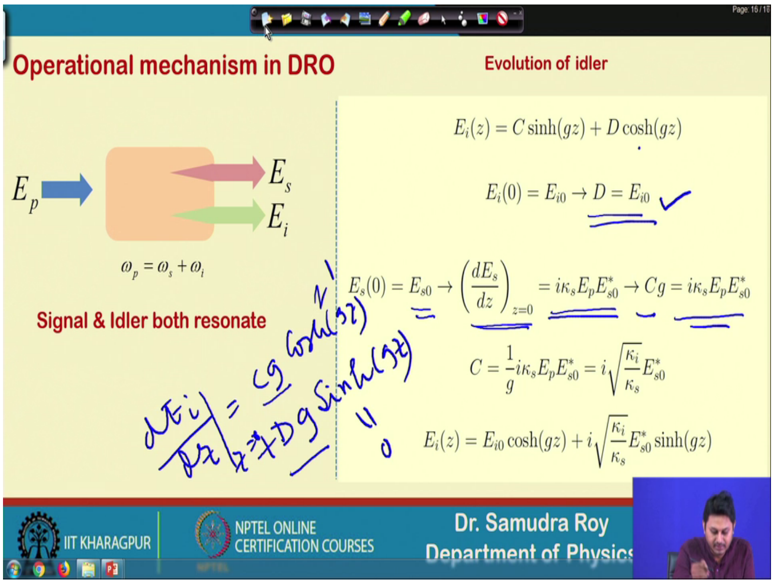


So, E s 0 as a general solution and in the similar way, in the similar way we can also find the solution for E s, for E i. After having the solution for E s we can readily find out what is the solution of E i and there are two way to find out, one way is to directly solve this equation because E s are at z point is known I just put E s and then make a integration and I find E i that is one procedure to do that. And another procedure is that we have just followed to calculate E s.

So, the procedure that will going to follow is same. Only thing is in state of making a derivative for E s I will make a derivative from E i. So, you make a derivative from E i. So, this two derivative gives me i k i E p and this derivative and this is the derivative for that. So, the second order derivative of E i will exactly give us the expression that we had for E s, with the same g E p is constant.

So, we can consider this as a constant this is a positive quantity g is a positive quantity and we will get this. Again the solution can be represent in terms of C cos hyperbolic g z plus D cos hyperbolic g z the general solution, only thing is the constants are now change C and D are the constants is previously used at A and B and is now changed.

(Refer Slide Time: 22:28)

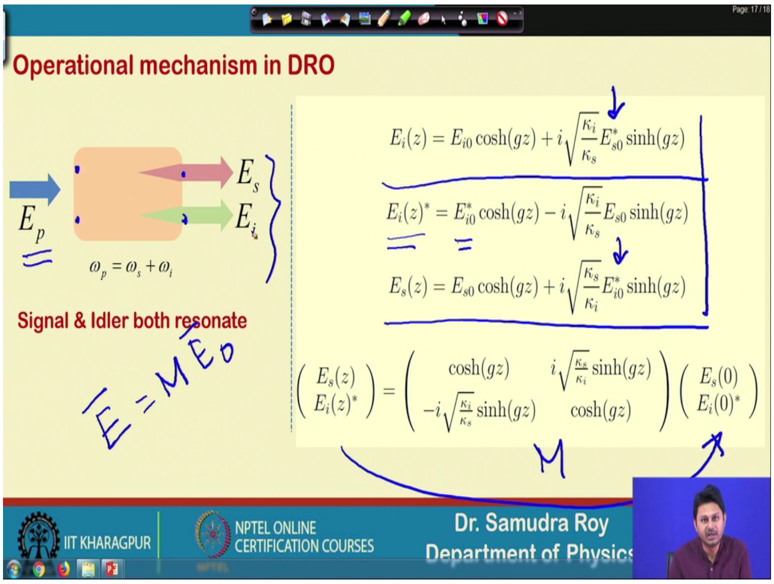


So, once we do this the next thing is to put the boundary condition and once I put the boundary condition exactly the similar process that we have done in the previous calculation. So, E i had comes out to be D because when I put z equal to 0 then this quantity not be there and cos hyperbolic will be 1 at z equal to 0. So, D will be the value at E i z equal to 0 point. So, this is a constant and I find D equal to E i 0 E s 0 is.

E s 0, E s at z equal to 0 is E s 0 that we also considered in the previous calculation and then d s d z at z equal to 0 that is the derivative when I make a derivative of this I will have i kappa s E p E s 0 star and C g is equal to this quantity because when I make a derivative of this quantity this quantity become d E i d z. If I make a derivative it will be C g cos hyperbolic of g z plus D g sin hyperbolic of g z So, cos hyperbolic and sin hyperbolic these two terms, these two terms at z equal to 0 if I put z equal to 0 will become C g and will become C g because this quantity will vanish. So, at z equal to 0 this term will 0, this term will 1 and that is why you get C g which is equal to this.

So, finally, if you do the all this calculation we will come to one expression similar to E s and this is our total expression of E i. So, E i z will going to evolve in this way. E i 0 cos hyperbolic g z plus i, this, see if all the boundary conditions and I will get. So, now E s and E i the explicit form of both these two fields and now known to us. So, let us now summarize whatever the calculation we have.

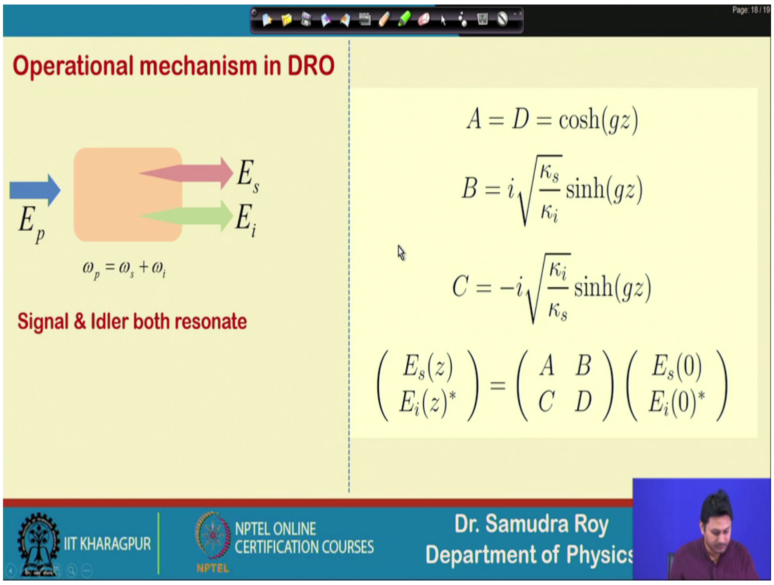
(Refer Slide Time: 25:06)



So, the summarize calculation suggest that if I launch an E p inside the system then what happened. So, if I launch an E p inside the system then what happened? We will have the evolution of E s and E i the E s and E i the solution is written in this form. So, if you if you see carefully then we will find that one case. So, this is my E i E i is related to E a star and E s is related to E i start, so I can make E i star. So, that I will get this E i star here. And from the solution I can write a matrix form of E s z E i z that means, this is what is whatever the field of E s and E i at z point and it is related to what is the field at z equal to 0.

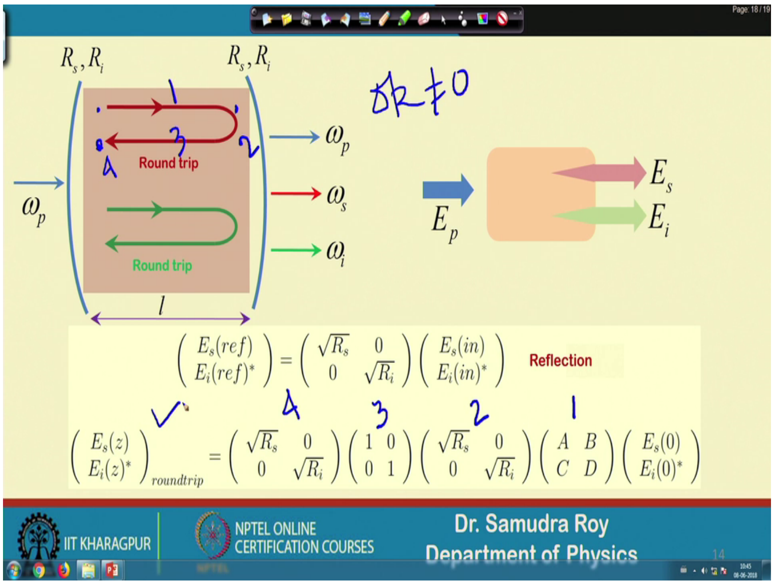
So, if I know the value of this point I can know the value of this point of E s and E i with this matrix form. We just simply need to, so if I make a vector form. So, E s and E i form your vector I write E bar which is equal to sum M which is this matrix E bar 0 that means, this is the relationship between the pair of field E s and E i that is generating into the system due to this non-linear process, and we can write this in terms of this matrix form.

(Refer Slide Time: 26:55)



Well, after having this matrix form the next thing is I can confined this matrix in this way. So, if I confined this matrix in this way just replace the name of the term say A equal to D equal to cos hyperbolic B is equal to sin hyperbolic and C is this quantity. So, I can write in this things in this compact matrix from. So, A B C D is my transformation matrix that can transform the field from 0 to z point.

(Refer Slide Time: 27:26)



Well finally, what is going on here is a very interesting that this field is generating here and make a round trip, this field is generating here and make a round trip. Since these two field is make a round trip if I want to find out what is my round trip field then we need to multiply the reflection matrix of the system also.

So, my goal here is to find out the field starting from here from this point to coming here. So, how this field will going to evolve can be can be understood by the matrix multiplication. So, how these things are happening, so let us try to understand. So, from here to this point, from this point to this point if I say this is process one in process one the field is associated with the initial field with this matrix one which we have already derived. Then there is a, then there is a reflection the field there is a reflection of the field I called is process 2.

So, this is the reflection matrix we know that the field is reflected then what should be the value of the new field can be represented in terms of reflection matrix. So, this is process 2, where the reflection matrix is shown. That if I multiply this whatever the value we have with the reflection matrix then I will get with the field after reflection. Then the field goes from here to here which is process 3.

We know that in process 3 since the phase matching condition delta k is not equal to 0. So that means, the phase matching condition is not valid. So, there will be no change of the field at all. So, whatever the field we have at this point the same field will going to have this point. So, if I make a transformation then there should be say unit matrix to transfer this two fields that means, whatever the field we have if I multiply the unit matrix I will have the same field here, the pair of same field.

And then finally, there is a reflection here also at the 4 point. So, it will reflect and now start for the next trip. So, this reflection I can do with 4. So, at the end of the day I have 1 2 3 4, this 1 2 3 4 are the 4 matrix and this 4 matrix are now multiplied together and when this 4 matrix are multiplied together I can have the field at exactly the g round 3 position.

So, today we will have this expression in our hand. So, tomorrow in the next class we will do the calculation with this matrix form and try to find out what is the condition for the threshold for doubling resonate oscillator. For today we just find out the operational condition and how the fields are evolving inside the doubling resonate oscillator. In the singly resonant oscillator we have single equation, but here we should have two equations.

And to handle with this two equation we use the matrix form which is more convenient and in this matrix form we have what is the field relationship between the fields that is at input and after making one round trip. So, 4 matrices are used and we have a compact form and this from you will be using in the next class to find what is going on in terms of the amplification of the signal and idler inside the arrow.

So, with this note let me conclude here. So, see you in the next class.

Thank you for your attention.