

**Introduction to Non-Linear Optics and its Applications**  
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**Lecture - 38**  
**Optical Parametric Oscillator ( OPO ), Singly Resonant Oscillator**

So, welcome student to the next class of Introduction to Non-Linear Optics and its Application.

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**Optical Parametric Oscillator (OPO)**

An optical parametric oscillator (OPO) is a light source similar to a LASER, also using a kind of LASER-resonator, but based on optical gain from parametric amplification in a nonlinear crystal rather than from stimulated emission.

**LASER**: Shows a transition from a metastable state (represented by three blue circles) to a ground state, emitting coherent radiation with energy  $\hbar\omega$ .

**OPO**: Shows a transition from a metastable state to a ground state, emitting a signal at frequency  $\omega_s$  and an idler at frequency  $\omega_i$ , with a pump energy  $\omega_p$ .

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So, in this class we will go to study one important concept which is optical parametric oscillator. So, so far we are dealing with optical parametric amplification. So, we find that some kind of amplification is there under this non-linear process. So, we will try to find out that how to use this kind of amplification in real practical fields.

So, optical parametric oscillator is a light source similar to the laser and also like laser we have a resonator here that means, the light is bouncing inside this cavity. But here it is not based on optical gain rather parametric amplification process or non-linear effects are involved in such kind of oscillation oscillator are such kind of amplification process.

So, in the laser we know that we have a metastable state and when we excite the system which has a metastable state. So, we get some kind of gain. So, we have coherent radiation under that. But for OPO we will get something different. So, we will have

pump and this pump will be split into two parts and these two parts essentially give you two different frequencies with the energy conservation condition that  $\omega_p$  is equal to  $\omega_s$  and  $\omega_i$ . Now,  $\omega_i$  is a signal that will go to amplify and  $\omega_s$  is the idler that will go to generate. Now, this amplification of signal we discussed in the last class and now we will go to use this amplification and make an oscillator. So, that I can amplify and use this as a laser source.

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**Optical Parametric Oscillator (OPO): Features**

- Wavelength Versatility: Ultraviolet → Mid IR/THz
- Temporal Versatility: CW → Femtosecond
- Operation: > Room Temperature
- High Power/Pulse Energies: 30 W, 200 mJ
- High Efficiency: 50-90%
- Compact, Solid-State design

**Stringent material requirement:**  
Optical nonlinearity, transparency, Phase-matching, Damage threshold

**Stringent pump laser requirement:**  
High spectral and spatial coherence, Sufficiently high intensity

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Well, optical parametric oscillator has some important features that let me describe very briefly that it has the wavelength versatility. So, you can generate ultraviolet to mid IR range wavelengths in terms of frequency it is terahertz limits if I generate in mid IR. So, what I mean is a wide range of wavelength one can generate and it depends on the material.

Also the temporal versatility it is there you can generate CW to femtosecond kind of pulses out of this resonator. And in operation domain is normally room temperature and greater than that. And also we have high power and pulse energies out of this optical parametric oscillators and it will give a legitimate amount of power and that limit or range 20 watt to 200 milli Joule in terms of energies.

And also it is a very high efficiency material. So, 50 to 90 percent efficiency will have which is very very large compared to any kind of laser. It is compact and solid state design. So, we have a compact material or we have a compact solid state system through

which we can generate this kind of amplification and the processes entirely done by the material the nonlinearity of the material, and it will generate different frequencies and we will amplify one can amplify this frequencies by making a cavity.

So, we will discuss that, but also there are few drawbacks are there I assumes a drawbacks, but the this things no one need to careful enough. First of all the stringent material requirement, so if I make an optical parametric oscillator then obviously, we need to choose proper optical material having high amount of optical nonlinearity.

And also the transparency should be there because at the end of the day I need the light to come out from the system and phase matching condition is very important here because in the previous classes we find that if the phase matching is there that means, if  $\Delta k$  equal to 0 then we have a good efficiency otherwise the efficiency dropdown radically. And then damage threshold is something that we need to take care of because we need to pump from the outside.

So, if the material is such that it will not going to be it is it going to damage for this launching pump power then it will be difficult. So, the damage threshold is something that I we need to take care of this is under material requirement. Also the stringent pump laser requirement because in this case also we need to excite the system by launching some kind of pump and then because of the nonlinearity the vibration of the dipole inside and some kind of frequency mixing is there we have the splitting from one photon, pump photon it will going to split to two different photon of frequency  $\omega_s$  and  $\omega_i$  by making energy and momentum conservation.

So, strong pump laser is still require to excite such kind of system. So, high spectral and spatial coherence is required and also sufficient high energy of the pump is required because in order to excite the nonlinearity the second order nonlinearity non by the second order coefficient is small. So, on excite the second order nonlinearity a prime consideration is that I should excite the material by external light and this excitation should be sufficient to generate second harmonics or are the second order effect. So, that is why the pump intense intensity has to be very high.

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Well, after this thing let me briefly describe the optical parametric oscillator. So, you can see in this figure we have a non-linear crystal here and we are launching some kind of pump outside the system. And this pump is now enter into the system these two things are mirrors and this mirrors should have some kind of reflectivity and if when we puts this kind of pump inside a non-linear crystal. So, what happened? This pump will split and generate two different frequency omega s and omega i that we know.

So, now omega s is now generating here and what happened for this particular mirror the reflectivity of the frequency at omega is very high. So, it will start vibrating. So, start making an round trip oscillation the field associated with omega s will make an round trip oscillation and we have some amplified omega s and omega i will generate, but this omega i will come out because the mirror is made in such a way that it only make a round trip oscillation for the frequency omega s.

Since only one frequency is making a round trip we call this singly resonate oscillator. So, singly resonant oscillator can amplify only one frequency and in this case it is omega s. Now, also when it is vibrating, so the mode of frequencies are there so, we know that in oscillator if I consider this is oscillator. So, l is the length of the oscillator n in n s into l is the length of optical length of the system. So, the modes that will be inside the system has to be multiplication this quantity optical length has to be multiplication of

$\lambda_s$  by 2 multiplied by  $m$ . So, these are the modes that should be oscillating inside the system.

So, now, if I consider these this term then we can find  $\lambda_s$  oscillating modes which is function of  $m$  can be replace by  $2 \lambda_s$  divided by  $m$ . From this also we can find out what is the frequency corresponding frequency from  $\lambda_s$  frequency conversion and finally, we can from the frequency we can find out what is in terms of angular frequency  $\omega_s$ . So, angular frequency  $\omega_s$  suggest that  $\frac{2\pi c}{\lambda_s}$  this is the quantity multiplied by  $m$  that are the corresponding modes that we have inside the system. So, these are the modes that will going to vibrate inside this system.

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**Optical Parametric Oscillator (OPO)**

**Singly resonant oscillator**

Diagram: A pump wave with angular frequency  $\omega_p$  enters a nonlinear crystal from the left. The crystal is labeled "Nonlinear crystal". Two waves exit to the right: a signal wave with angular frequency  $\omega_s$  and an idler wave with angular frequency  $\omega_i$ .

Handwritten notes on the slide:

- $\Delta k = 0$
- $\frac{dE_s}{dz} = i\kappa_s E_p E_i^*$
- $\frac{dE_i}{dz} = i\kappa_i E_p E_s^*$
- $\frac{d^2 E_s}{dz^2} = i\kappa_s E_p (-i\kappa_i E_p^* E_s) = \kappa_s \kappa_i |E_p|^2 E_s$
- $g^2 = \kappa_s \kappa_i |E_p|^2$
- $\frac{d^2 E_s}{dz^2} = g^2 E_s$

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Well, for optical parametric oscillator under this system for singly resonant case we need to find out what is the condition or what is the pump condition for which we can have amplification. This is the old problem we are doing, but in a new way because here the fields are moving a making an round trip. So, in order to find out what is the threshold value for generating any kind of amplification that is a coming out from this optical parametric oscillator we need to solve the differential equation once again. And here we explicitly consider  $\Delta k = 0$  that means, there is a phase matching.

So, under phase matching we have two equations of quickly we solve this, this is the difference this two differential equation is solve several time here the pump is constant. So, under this thing we can have a double derivative of this quantity and we make have a

double derivative of E I we just replace this, when I replace we will have an expression like this. So, we will have a differential equation for the signal which is going to amplify like this  $g^2 E_s$ . So, this is the differential equation we know we already had this in our previous treatments where not going to discuss this in detail.

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The slide is titled "Optical Parametric Oscillator (OPO)". On the left, a diagram shows a "Nonlinear crystal" inside a "Singly resonant oscillator". An input signal at frequency  $\omega_p$  enters from the left. Two output signals,  $\omega_s$  and  $\omega_i$ , exit to the right. On the right side of the slide, the following mathematical derivation is shown:

$$\frac{d^2 E_s}{dz^2} = g^2 E_s$$

$$E_s(z) = A \sinh(gz) + B \cosh(gz)$$

$$E_s(0) = E_{s0} \rightarrow A = E_{s0}$$

$$E_i(0) = 0 \rightarrow \left( \frac{dE_s}{dz} \right)_{z=0} = 0 \rightarrow B = 0$$

$$E_s(z) = E_{s0} \cosh(gz)$$

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The solution is also well known. So, we when we have this expression in our hand we will have a solution like cos hyperbolic and combination of cos hyperbolic and sin hyperbolic.

So, once we have the solution of cos hyperbolic and sin hyperbolic then the next thing is to put the boundary condition to find out what is going on. So, when we have a boundary condition then readily the first boundary conditions suggest that A should be  $E_{s0}$  because  $E_s$  at 0 is 0 because we have something here to start with. So, sum amount of  $\omega_s$  is here and then it will going to generate from this quantum noise actually we can generate this things and then in order to evaluate B we make a derivative of the quantity at  $z$  equal to 0 it is 0. So, that suggest we have B equal to 0.

So, we will finally, have our old expression that we have been using for last few classes that the signal we will going to increase and the growth is something like that the evolution of the signal is something like that it will be changing in the form of cos hyperbolic  $gz$ .

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**Singly resonant oscillator**

Diagram showing a cavity of length  $l$  between mirrors  $R_1$  and  $R_2$ . The electric field components are labeled  $E_{s0}$ ,  $E_{s1}$ ,  $E_{s2}$ ,  $E_{s3}$ , and  $E_{s4}$  at different points. The pump frequency is  $\omega_p$ , and the signal frequencies are  $\omega_s$  and  $\omega_i$ .

Equations:

$$E_s(z) = E_{s0} \cosh(gz)$$

$$E_{s1} = E_{s0} \cosh(gl)$$

$$E_{s2} = \sqrt{R_2} E_{s0} \cosh(gl)$$

$$E_{s3} = \sqrt{R_2} E_{s0} \cosh(gl) \quad [\Delta k \neq 0]$$

$$E_{s4} = \sqrt{R_1 R_2} E_{s0} \cosh(gl)$$

Handwritten note:  $E_s(z=0) = E_{s0}$

Handwritten note:  $E_{s4}/E_{s0}$

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Well, once we have the expression then the next thing is this is very important slide, next thing in order to find are the threshold we need to find out what should be the value of these at different points inside this non-linear crystal. So, this is the cavity, inside the cavity we have the length of this cavity cell inside the cavity we have the non-linear material and now  $\nu$  point. So, this is the pump that we are launching.

This is the pump we are launching and we considered the reflectivity is  $R$  here and reflectivity  $R_1$  and reflectivity of this mirror is  $R_2$  here. We know our electric field for signal is evolving with this expression. So,  $E_{s0} \cosh(gz)$  this is expression for signal and now at this point we consider at this point the signal is  $E_{s0}$ , that we know. So,  $E_s$  at  $z$  equal to 0 is  $E_{s0}$  this is our boundary condition and we use this boundary condition. So, once it is known the next thing is that I need to find out when it make a round trip this is the process of round trip at each point what should be the value of our electric field. I write if this is electric field the value electric field at  $z$  equal to 0.

$E_{s1}$  is electric field of at  $z$  equal to  $l$  points. So, from here to here we find that there is a change of electric field. So, this electric field value I now note as  $E_{s1}$ . Here we have a reflection. So, we need to take account of that also and after reflection of from this mirror the field may change and I designate this field at  $E_{s2}$ . And now again it come back to this part of the mirror I write it as  $E_{s3}$  because it is coming from here to here and then again it reflects and when it reflects I write it as  $E_{s4}$ . So,  $E_{s4}$ ,  $E_{s1}$  I start with

$E_{s0}$  it goes here reflect goes here reflect and this is the final field I have. So, now, the condition is, if we need some kind of gain then  $E_{s4}$  has to be greater than  $E_{s0}$  the amplitude part has to be greater than  $E_{s0}$  because if  $E_{s0}$  is greater than  $E_{s4}$  is greater than  $E_{s0}$  then only we can say that this resonator is amplifying the signal.

So, our aim is to find out if I start with this expression or if I consider this is my goal then what should be the value of the threshold pump to get this, because when we going to evolve we know the  $g$  term is sitting here and inside the  $g$  the information of the pump is embedded. So, whatever the value whatever the condition we have this condition we can write in terms of  $g$  and then we can find out what is our threshold condition. So, that is the m.

Well, if we understand properly that this  $E_{s0}$ ,  $E_{s1}$  these values are here then we can write what is the value of  $E_{s1}$ ,  $E_{s2}$ ,  $E_{s3}$ ,  $E_{s4}$  one by one first value  $E_{s0}$  is  $E_{s0}$  because we start with that. So, if I put  $z$  equal to 0 in this case. So, we will have  $E_s$  at  $z$  equal to 0 is  $E_{s0}$ .

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**Singly resonant oscillator**

Diagram showing a resonator of length  $l$  between mirrors  $R_1$  and  $R_2$ . The field amplitudes at different points are  $E_{s0}$ ,  $E_{s1}$ ,  $E_{s2}$ ,  $E_{s3}$ , and  $E_{s4}$ . The frequencies are  $\omega_p$ ,  $\omega_s$ , and  $\omega_i$ .

Mathematical expressions for the field amplitudes:

$$E_s(z) = E_{s0} \cosh(gz)$$

$$E_{s1} = E_{s0} \cosh(gl)$$

$$E_{s2} = \sqrt{R_2} E_{s0} \cosh(gl)$$

$$E_{s3} = \sqrt{R_2} E_{s0} \cosh(gl) \quad [\Delta k \neq 0]$$

$$E_{s4} = \sqrt{R_1 R_2} E_{s0} \cosh(gl)$$

Handwritten notes on the slide:

$$E_{s2} = \sqrt{R_2} E_{s1}$$

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So, first term is ok. What about  $E_{s1}$  that is the next important thing. So, we have the expression here. So, if I want to find out what is  $E_{s1}$  I just need to do that at  $z$  equal to  $l$  you will find this things. So, this will be at  $z$  equal to  $l$ . So that means,  $E_{s1}$  is nothing, but the field at  $z$  equal to  $l$  point and when I put this we will find  $E_{s0} \cos$  hyperbolic of



simply  $g l$  because I need to find out the field at  $z$  point. So, I also figure out what is the field at this point.

Now, this field I have a reflection here at that point. So, when have a reflection we have  $E_{s2}$ . So,  $E_{s2}$  can be represented in terms of the reflectivity of the mirror, and if I do the  $E_{s0}$  is simply root over of  $R_2$  and whatever the field we have because  $E_{s0}$ ,  $E_{s2}$  is simply the reflectivity multiplied by  $E_{s1}$ . So, once we use this then  $E_{s1}$  is this quantity. So, I just replace this quantity and I get also  $E_{s2}$ . So, my  $E_{s1}$  is generated,  $E_{s2}$  is generated, the next thing is if the field goes from this to this what should be the value, so  $E_{s3}$  the next to find out  $E_{s3}$ .

Now, this is quite interesting because normally our tendency that if I want to find out  $E_{s3}$ , so just put another value of whatever the value we have that multiplied by this at  $z$  equal to  $l$  point will give you my  $E_{s3}$  but that is not the case. So, when the field is going from here to here you can you need to consider that at that path from here to here it is going to opposite direction  $\Delta k$  is not equal to 0.

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**Singly resonant oscillator**

Diagram showing a cavity of length  $l$  between mirrors  $R_1$  and  $R_2$ . The field components are  $E_{s0}$ ,  $E_{s1}$ ,  $E_{s2}$ ,  $E_{s3}$ , and  $E_{s4}$ . The frequencies are  $\omega_p$ ,  $\omega_s$ , and  $\omega_i$ . A handwritten note indicates  $\Delta k \neq 0$ .

Equations for field amplitudes:

$$E_s(z) = E_{s0} \cosh(gz)$$

$$E_{s1} = E_{s0} \cosh(gl)$$

$$E_{s2} = \sqrt{R_2} E_{s0} \cosh(gl)$$

$$E_{s3} = \sqrt{R_2} E_{s0} \cosh(gl) \quad [\Delta k \neq 0]$$

$$E_{s4} = \sqrt{R_1 R_2} E_{s0} \cosh(gl)$$

Handwritten note:  $\vec{k} = \vec{k}_p - \vec{k}$

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So, let me I clearly. So, at this part  $\Delta k$  is not equal to 0 why  $\Delta k$  is not equal to 0 because our phase matching condition suggest that  $\Delta k$  if I write in vectorial form because this is better  $\Delta k$  is equal to  $k_p$  minus. So, let me at once again  $\Delta k$  is  $k_p$  minus  $k_s$  minus  $k_i$ .

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**Singly resonant oscillator**

Diagram showing a cavity with mirrors  $R_1$  and  $R_2$  and length  $l$ . Fields  $E_{s0}$ ,  $E_{s1}$ ,  $E_{s2}$ ,  $E_{s3}$ , and  $E_{s4}$  are shown. Frequencies  $\omega_p$ ,  $\omega_s$ , and  $\omega_i$  are indicated. Handwritten notes include  $\Delta k = 0$  and  $\Delta k \neq 0$ .

$$E_s(z) = E_{s0} \cosh(gz)$$

$$E_{s1} = E_{s0} \cosh(gl)$$

$$E_{s2} = \sqrt{R_2} E_{s0} \cosh(gl)$$

$$E_{s3} = \sqrt{R_2} E_{s0} \cosh(gl) \quad [\Delta k \neq 0]$$

$$E_{s4} = \sqrt{R_1 R_2} E_{s0} \cosh(gl)$$

Handwritten vector equation:  $\vec{\Delta k} = \vec{k}_p + \vec{k}_s - \vec{k}_i$

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In vectorial form this is my delta k. So, when delta k equal to 0 that means,  $k_p$  is equal to  $k_s$ ,  $k_i$ . But here since the wave is moving in opposite direction, so this expression in this expression field pump is moving in this direction, idler moving in this direction, but here in this if in this case, I consider delta k equal to 0 for this path from here to here the field is moving in opposite direction.

So, as shown here as a note delta k is not equal to 0 because since the field is moving in opposite direction for this expression I need to put 1 plus here because it is moving in opposite direction. As soon I push put a plus sin here we should not write delta k equal to 0 rather delta k is not equal to 0. Since there is no phase match in this case so there will be no amplification, and also if I consider there is no loss into the system. So, what eventually we find in  $E_s$  is the same value that we have here.

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The slide is titled "Singly resonant oscillator". On the left, a diagram shows a cavity of length  $l$  between mirrors  $R_1$  and  $R_2$ . An input field  $E_{s0}$  at frequency  $\omega_p$  enters from the left. Inside the cavity, four field components are shown:  $E_{s1}$  (top right),  $E_{s2}$  (bottom right),  $E_{s3}$  (top left), and  $E_{s4}$  (bottom left). Output fields are shown at frequencies  $\omega_s$  and  $\omega_i$ . On the right, the general solution is given as  $E_s(z) = E_{s0} \cosh(gz)$ . Below this, the following equations are listed:

$$E_{s1} = E_{s0} \cosh(gl)$$

$$E_{s2} = \sqrt{R_2} E_{s0} \cosh(gl)$$

$$E_{s3} = \sqrt{R_2} E_{s0} \cosh(gl) \quad [\Delta k \neq 0]$$

$$E_{s4} = \sqrt{R_1 R_2} E_{s0} \cosh(gl)$$

Handwritten in blue ink below the equations is  $E_{s3} = E_{s2}$ . The slide footer includes the IIT Kharagpur logo, NPTEL Online Certification Courses logo, and the name "Dr. Samudra Roy, Department of Physi".

So,  $E_{s3}$  is eventually,  $E_{s2}$  the same value we will have because there is no phase matching. So, there is no amplification because of that  $\Delta k \neq 0$  condition and we will eventually have the same field if I consider there is no loss, if it is lost then obviously, there will be an exponential term associated with that. But for the time in we considered there is no linear loss.

Well, once we have this then again it is reflecting. So, when it is reflecting, so we will have  $E_{s4}$ . So,  $E_{s4}$  is root over of  $R_1 R_2$  whatever the value we have and root over of that root over of  $r$  multiplied by that. So, we will eventually have  $E_{s4}$  like this. So, starting from  $E_{s0}$  we now come to  $E_{s4}$  in this particular structure. So, once we have this in our hand then the next thing is to find out the threshold condition.

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**Threshold condition for amplification**

The diagram shows a laser cavity of length  $l$  between mirrors  $R_1$  and  $R_2$ . Three energy levels are shown:  $E_{s0}$ ,  $E_{s1}$ , and  $E_{s2}$ . The light path starts at  $E_{s0}$ , goes to  $E_{s1}$ , then to  $E_{s2}$ , and back to  $E_{s0}$ . The threshold condition is given by:

$$E_{s4} = E_{s0}$$

$$\sqrt{R_1 R_2} E_{s0} \cosh(g_{th} l) = E_{s0}$$

$$\cosh(g_{th} l) = \frac{1}{\sqrt{R_1 R_2}}$$

$$\sqrt{R_1 R_2} \approx 1 \rightarrow \cosh(g_{th} l) \approx 1 + \frac{g_{th}^2 l^2}{2}$$

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So, the threshold condition as I mention when  $E_{s1}$  is at least equal to  $E_{s4}$ ,  $E_{s4}$  is equal to  $E_{s0}$  that means, these value and one after one round trip this value if these two are same then that is the threshold condition. But in order to amplify as I mentioned  $E_{s4}$  has to be greater than  $E_{s0}$  but equal to is a condition which we called the threshold condition. So, when I put this value because  $E_{s4}$ , I derived and it is this and when I equate these things I write  $g_{th}$  because  $g$  is now replaced by  $g_{th}$ . So, this is equal to  $E_{s0}$ . So, now,  $E_{s0} E_{s0}$  will cancel out. So, we have cos hyperbolic of  $g_{th} l$  is equal to 1 divided by root over of  $R_1$  and  $R_2$ .

Now, root over of  $R_1$  and  $R_2$  these quantities reflectivity and reflectivity is very high and very near to 1. So, these quantities very near to 1 this root over of  $R_1$  and  $R_2$ . So, we have if these quantities one, if I expand this cos hyperbolic term then we have 1 plus under this approximation we have 1 plus  $g_{th}^2 l^2$  divided by 2. This is gives the expansion with this condition that  $R_1$  and  $R_2$  are very high.

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**Threshold pump power**

$$1 + \frac{g_{th}^2 l^2}{2} = \frac{1}{\sqrt{R_1 R_2}}$$

$$g_{th} = \frac{\sqrt{2}}{l} \left( \frac{1}{\sqrt{R_1 R_2}} - 1 \right)^{1/2}$$

$$g_{th}^2 = \kappa_s \kappa_i |E_p|_{th}^2 = \frac{d^2}{c^2} \left( \frac{\omega_s \omega_i}{n_s n_i} \right) |E_p|_{th}^2$$

$$P_{pth} = I_{pth} A = \frac{1}{2} \epsilon_0 c n_p A |E_p|_{th}^2$$

$$P_{pth} = \frac{1}{2} \epsilon_0 c n_p A \frac{c^2}{d^2} \left( \frac{n_s n_i}{\omega_s \omega_i} \right) g_{th}^2$$

$$P_{pth} = \frac{1}{2} \epsilon_0 c n_p A \frac{c^2}{d^2} \left( \frac{n_s n_i}{\omega_s \omega_i} \right) \frac{2}{l^2} \left( \frac{1}{\sqrt{R_1 R_2}} - 1 \right)$$

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So, g threshold is equal to this root over of 2 l and g threshold square which is related to the if P threshold square which is nothing, but the pump of and the pump power is this. So, pump power threshold is replaced by E p mode square threshold which now have g th. So, I just replace this g th and finally, come to this expression.

I want the student to please do this calculation once again by your own hand, so that you can understand what is going on, but all the steps are given here. I am not explaining each and every step because this is simple algebra and I believe you are quite capable to do that. But important thing is that I can now find a threshold power this threshold power is really required to amplify the signal inside the optical parametric oscillator.

So, today we find. So, we will now going to conclude today's class. So, today we studied in detail about optical parametric oscillator, and given example that for singly resonant oscillator how the threshold power can be achieved. And in the next class, we will extend this concept and try to find out in state of vibrating one wave which is at signal if I want to find out the vibration of two wave which is called doubly resonating oscillator.

So, what are the consequences of that and how to find out for two resonating wave, how to find out the threshold condition etcetera that we will do in the next class. So, with this note let me conclude here.

Thank you for your attention and see you in the next class.