Introduction to Non- Linear Optics and Its Applications

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Lecture - 37
OPA Under non - phase matching condition, Expression of gain

So, welcome student to the new class of Introduction to Non-Linear Optics and its Application. So, today we have lecture number 37. So, in the previous lecture we studied a parametric, optical parametric amplification and under optical parametric amplification we studied the some frequency generation and find that the pump which is the some frequency of signal and idler will going to evolve periodically.

And the total energy can be exchanged

to idler, if I consider that the signal is constant, and there is a periodic exchange of power between them. And we can suitably find out the length of the crystal or length of the non-linear material at which we will get the maximum power and that will be the distance for which we will getting the maximum efficiency of some frequency generation.

But in all the calculation in the previous classes we consider one very important assumption that the phase matching delta k is equal to 0. But today we will going to do more general calculation with considering delta k is not equal to 0, and tried to find out if delta k is not equal to 0; that means, there is no absolute phase match then how the optical parametric amplification will behave and how the field will going to evolve under that condition.

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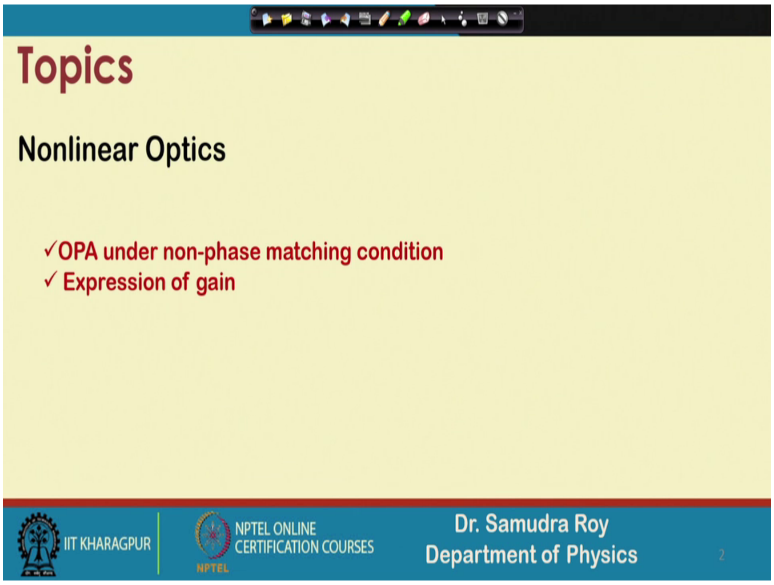

So, let us see what we have today. So, we have optical parametric amplification under non-phase matching condition as I mentioned. Non-phase matching condition means delta k is not equal to 0. So, all the calculation or all the differential coupled differential equation that will going to use will be delta k naught equal to 0 and expression of the gain.

The expression of the gain means when I launch a signal along with a pump then ideal will going to idler wave will going to evolve. But signal is also going to increase that we have seen in the previous classes, but we never mention anything about the gain because it is amplifying the signal is amplifying. So, if something is amplifying so; obviously, some kind of gain is associated with that. So, today we will find how this expression of gain one can derive from this condition when delta k is not equal to 0, and this expression will give you how this how the gain structure is there and how to tune this gain structure.

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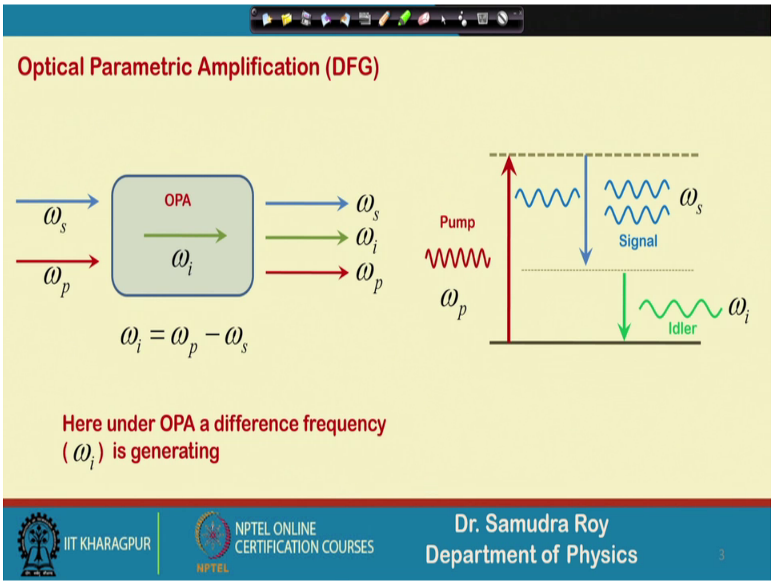

Well, let us start with our old concept that optical parametric amplification for different frequency generation will launch omega p and omega s, two frequencies and we find that omega s omega i and omega p are generated So, omega s will amplify the signal will amplify omega i will generate and omega p which is pump here we considered this is strong pump; that means, constant will remain constant. So, this is the corresponding energy diagram, and it suggest that if small amount of signal are there in the input. So, signal will going to in evolve.

And it will also amplify, but at the same term idler wave is also generated. So, idler wave is something for which having the frequency for which we have omega p minus omega s; that means, this is nothing, but the difference frequency. So, signal is amplifying difference frequency is generating, so this condition we will continue which we have already done in our previous class. But today we will do with the condition that there is no phase match here.

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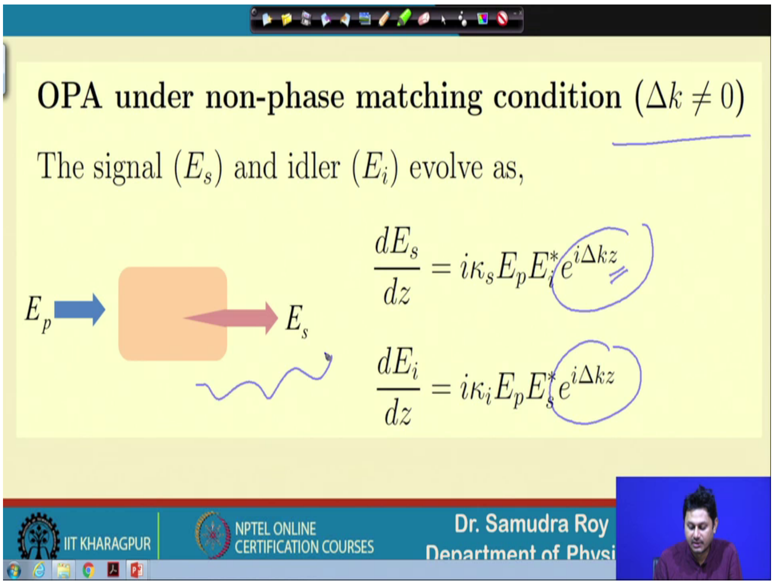

So, the condition that is imposed here is delta k not equal to 0. So, delta k not equal to 0 essentially means the phase matching the phase matching is absolute phase matching absent.

So, once the absolute phase matching when we say there is no absolute phase matching then this E to the power of i delta k and E to the power of i delta k term will be here. Previous it was not there, so that is why we have a simple differential equation. Here now we need to deal with this term because whenever we try to decouple this two equation then this term will make some kind of effect because this z z dependencies. So, when we make a derivative of this quantity we need to take care of this term also, which is the function of z.

Here in this figure we just pictorially show that how using a pump we can generate or we can amplify this E s. So, E s is something, E s is s signal that is amplifying. So, that is why the gain term is there that we are mentioning in the first slide. So, we will also calculate that part.

Using that two equation, now we our scheme is to decouple this, this is the two equations our aim is to decouple this. So, in order to decouple what we do we make a derivative of this term E p is constant. So, we will have a derivative of this quantity and this quantity. Derivative of this quantity will be replaced by this and also this term will be there. So, this term again will going to replace in some way. So, let us see what is the treatment, how we can do that.

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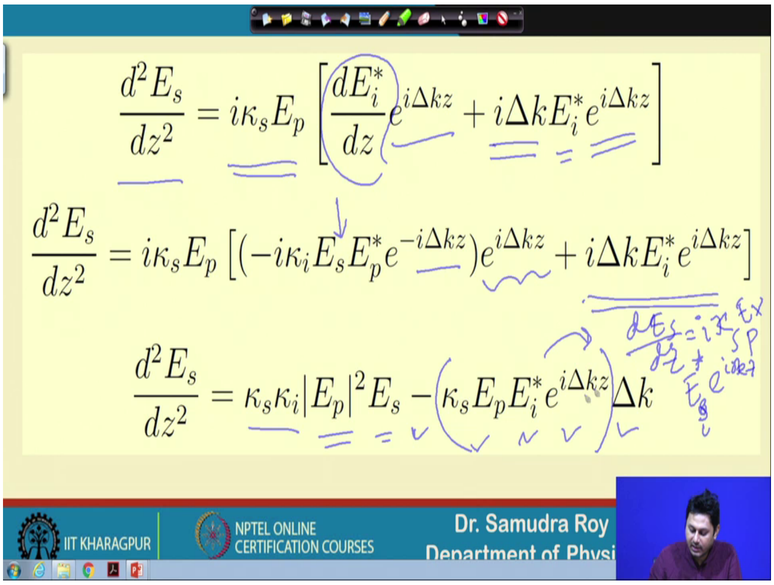

So, we will make a derivative of this term when you make a derivative of this term I will have this term i kappa s E p constant. So, we will not going to affect. So, next is derivative of this quantity E to the power i k z and then the next is the derivative of i k z. So, I have I delta k out E i will be E i star will be there and E to the power i delta k z. d E i start d z, I can directly replace from the expression that we have. So, it is i kappa i E s star E p, but now it is star of that. So, that is why negative I will be there negative sign and E s will be star less and E p will be star and also exponential term will have a negative sign.

So, I replace this d i star d z here and here I will whatever the term we have we just write it this way when I replace this I will now have two term this plus this. So, i kappa E p if I now multiply the first term I will get this i minus i equal to 1. So, kappa is kappa i, i will have E p E p star gives me E p mod square and E s exponential i delta k z will be cancelling out with a E to the power minus i delta k z term which are reciprocal to each other. And if I multiply this term with this we will have a negative sign because i multiplied by i will have a minus sign here delta k will be there E to the power i k is it will be there E i star and E p is there.

Now, if I carefully this term then we can see this is nothing, but this term is nothing, but d E s d z because d s d z is i. So, one i will be there kappa s E p multiplied by E s E p multiplied by E i star and E to the power of i delta k z. So, this term is nothing, but 1 by i of this, now, if I replace this thing, so entire equation will be in terms of s.

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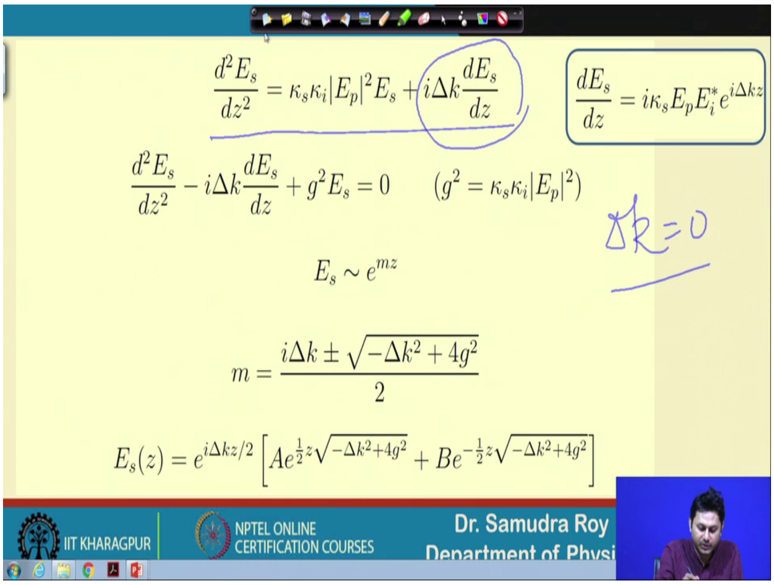

So, if I replace this as I mentioned. So, entire term is now entire equation in terms of E s we can decouple the entire equation.

Now, if you note very carefully you can see that because of this delta k term we have an extra derivative here, extra term here. It was not there when delta k equal to 0 and we have a simple second order differential equation, which we can solve very easily, but here we will have an extra first order derivative term. But even if we have the first derivative term there is no problem you can even having this term we can solve this.

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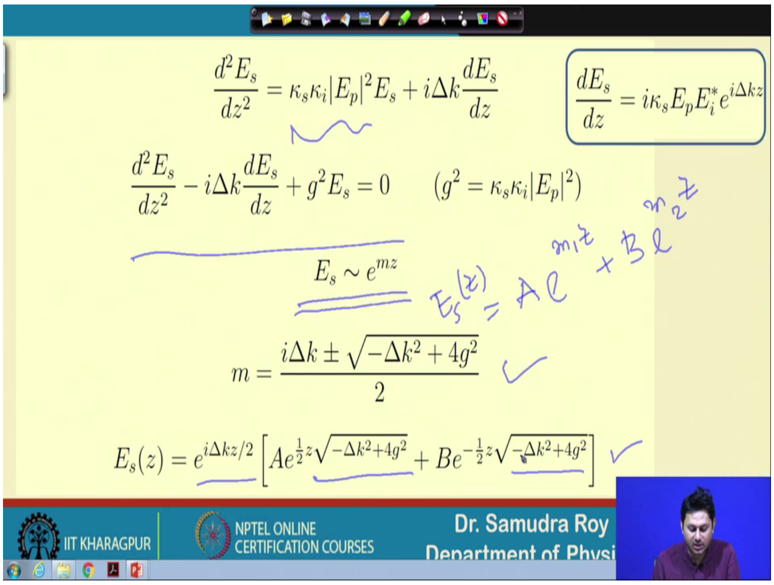

So, this is the expression that we have right now; this term is replaced by g square. So, this is the differential equation we have a standard second order differential equation having the first order differential term. So, when once when we have this kind of equation this is some sort of homogeneous equation we do not have any kind of source term here. So, the life is easy here.

So, we just try to find out the solution of the form E to the power m z. So, if I put this term here then we have solution of m like this the standard procedure. When you have a solution like this the next thing we can put all these values here. So, my solution standard solution will be A E to the power of m 1 z plus B E to the power of m 2 z that will be our solution for E s z and if I replace this I readily get E s is E to the power i delta k z by 2 that we are getting from this A E to the power half z delta k square plus 4 g square B plus E to the power half z delta k square plus 4 g square.

So, we have an expression the complete expression of E s and if I put delta k equal to 0 again it will convert to our old expression that we had in the previous calculation it will be either sake it seems to be a sake hyperbolic kind of solution when I put delta k equal to 0. But when delta k not equal to 0 the solution is slightly modifying and now this modified solution we want to see few things.

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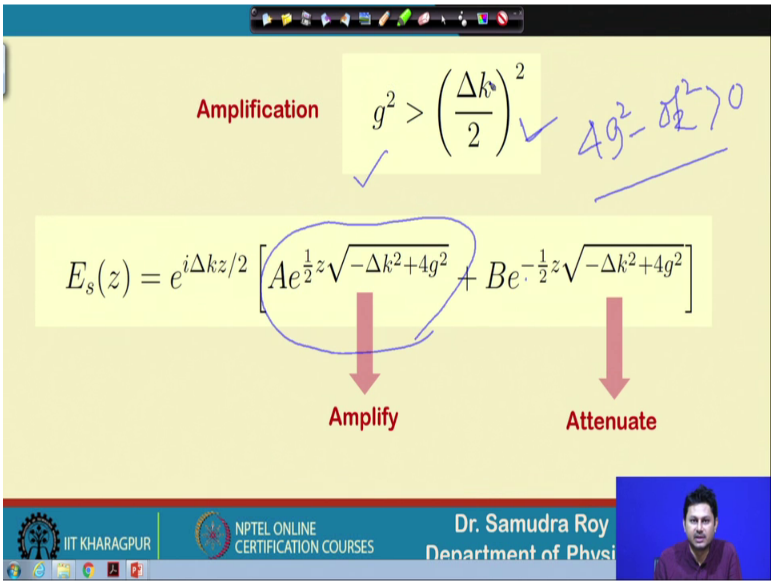

So, our aim here is to find out the amplification of E s. So, we have an expression of E s containing two term both the term are exponential. Now, you can if you look carefully, you can see in the root over this term in the exponential root over term we have a quantity delta k square plus 4 g square. So, now, if g square is greater than this quantity if g square is greater than this quantity or eventually I am trying to say 4 g square minus delta k square is greater than 0, that means, if I make this quantity is greater than 0. So, that the entire quantity is positive or real.

In that case if 4 g square is greater than delta k's square then we can say this is a this is a positive term, root over a positive means this a positive term So, this positive term makes this quantity amplify. So, this is a amplifying term of E s this term however, because of this negative sign we will going to die out. So, this will attenuate, but I am not going to bother about this term which is attenuating. Important thing is some portion of the field is amplifying. So, the condition of amplification under delta k not equal to 0 is this which is important.

Now, if I say delta k equal to 0 then the condition is simply that g has to be greater than 0 which is always the case, because g is a positive quantity, so g has to always be greater than 0. So, we will have readily we will have the amplification of E s, but the delta k not equal to 0 put kind of restriction or put kind of threshold condition for amplification.

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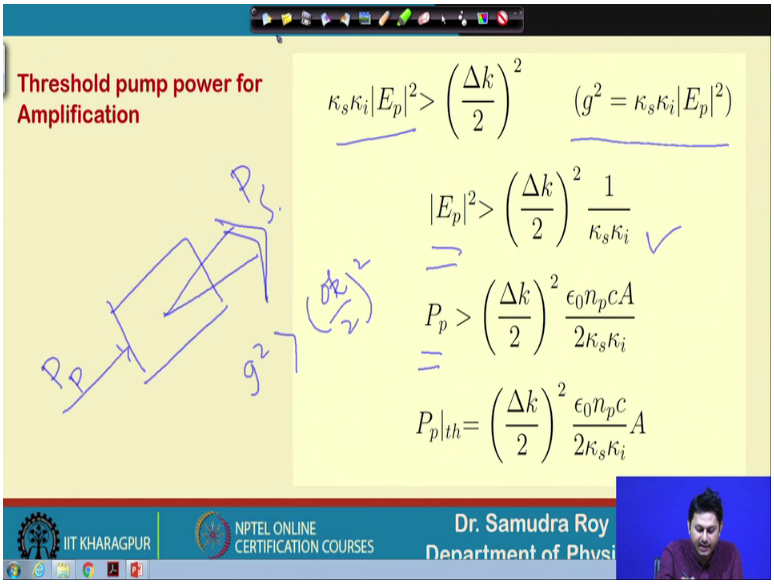

So, this threshold condition we can find out quite easily because g inside g we have the information of pump inside the g we have the information of the pump which is this.

So, this information of pump I can use to find out if this is my system, I am launching a pump power and try to amplify the signal which is P s in order to amplify this quantity I need to put some kind of threshold pump here. If I puts any pump power then that does not ensure that P s will going to generate or P s will going to amplify. In order to amplify we find the condition and the condition was simply g square is greater than delta k divided by 2 square of that. So, this is the condition.

So, after putting this condition here we can see that E p has to be greater than this quantity and P p which is related to E p as our normal expression that P p is equal to half epsilon 0 n p c mod of E p square A. So, this is really like this. So, I just replace E p in terms of P p and will have an another term here.

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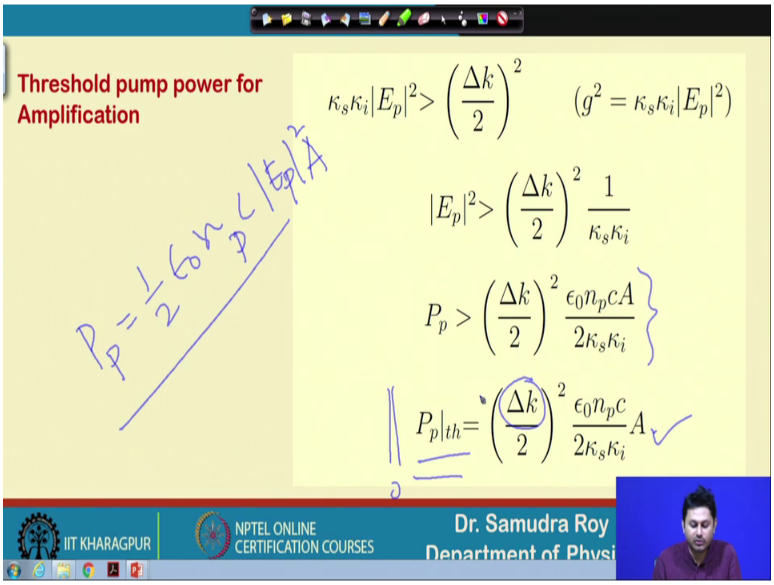

So, finally, we get a threshold value of pump for which I am getting some kind of amplification of signal. So, the signal will going to amplify when my pump will have some threshold value and this threshold value of pump is directly proportional to the square of delta k. That means, if the phase mismatch is much then we need to put more and more pump to get the amplification of the signal power, signal power will going to amplify for this. So, some kind of restriction is here to delta k, so that we need to take care of.

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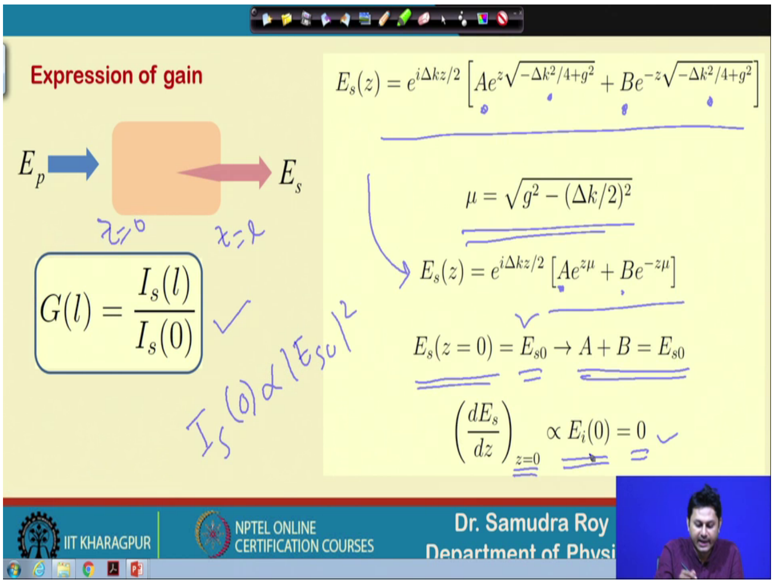

Well, after having the expression of this pump now, we try to find out what should be the expression of the gain that we are talking about. So, we are launching some kind of E p and generating E s. So, the gain means whatever the value we have at the initial point at z equal to 0, for the corresponding intensity or power.

So, this ratio and this is z equal to l that means, it is amplifying, so how much it is amplifying that is the ratio that can be considered as a ratio or that is quantized by the ratio of this quantity. So, what is the value of the intensity at z equal to l point, and what is the value of the intensity of the signal and z equal to 0 this two ratio basically gives us what is our G l.

So, in order to do this things what we need to find out because E s is in our hand , but I do not know what is my A and B. So, in order to find an expression of g and G l means this is at l distance. So, we need to find out A and B put in suitable boundary condition, and it is not that hard to do. So, what we have to do this things what we can do is just changed our variable.

So, I am not going to write every term root over of this big term. So, that is why I could another variable mu here. So, mu is root over of g square minus delta k divided by two and square of that. And if I do the E s can be represented in more compact form and that is this.

Now, we know E s at z equal to 0 point is E s 0 because at the output at the input I should have some sort of signal otherwise I will not never going to generate any kind of amplification of that that classical we find that some amount should be there. But quantum mechanically from quantum vacuum or quantum fluctuation one can generate, one can considered that there should be some value of field at z equal to 0. So, this value I considered E s 0. So, I s 0 is directly proportional to mod of E s 0. So, this value is our input. When I put z equal to 0 then I can readily see that this leads to one equation one relation A plus B which is E s 0.

Now, the next thing again we know that when we have two variable like A and B, one is to find out one variable one can find out by putting z equal to 0 and another condition we can find out by making the derivative of the left hand side and that is equal to 0 because there is no idler here at z equal to 0 point. So, d is d z at z equal to 0 that should be proportional to the E i that means, the idler field at z equal to 0 which is 0 there is no idler at z equal to 0. So, when it is 0 to we have another expression and this another expression give me another relationship between A and B and we can solve and find exactly what A and B we have.

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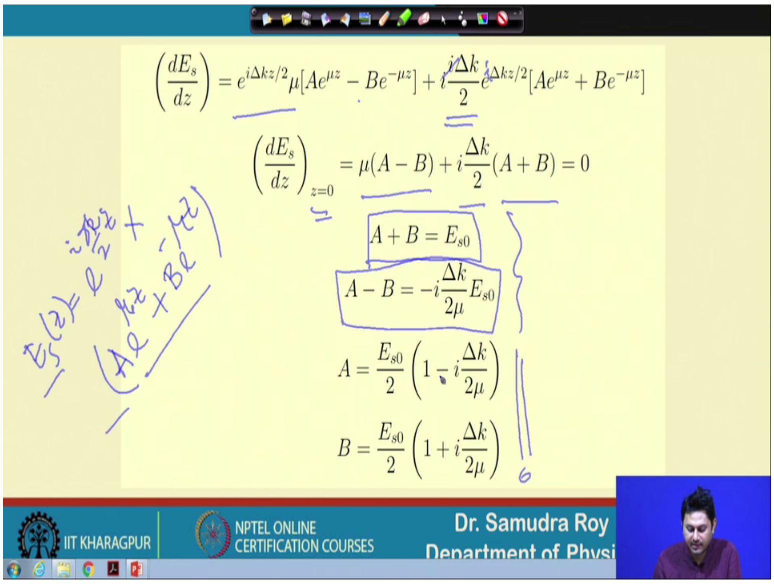

So, once we have this total expression in our hand which is E s if I write it should be something like E to the power of i delta k divided by 2 z multiplied by A e to the power of mu z plus B e to the power of minus mu z that was our term.

So, I need to make a derivative of this. So, I make a derivative the first term derivative of this will be mu multiplied by this with the negative sign because we have a negative sign here and next term of the derivative we have i kappa i delta k by 2 term, yeah so on i will not be here. So, I making 2 i here so please note. So, and I am here one i should be there I just my mistake this i is coming here, anyway.

So, now we are making z equal to 0 point. So, at z equal to 0 point if I put this., so mu A minus B plus I delta k to a plus B will be 0. So, A plus B is E s 0 that we have already know and now if I put E s 0 here then I also know what is the value of A minus B, so A minus B is this quantity. So, once we know A plus B and A minus B, then I can solve this and I will find A and B like E s 0, 1 minus i delta k by 2 mu and E s 0 to 1 plus i delta k B 2 mu A and B value I can figure out.

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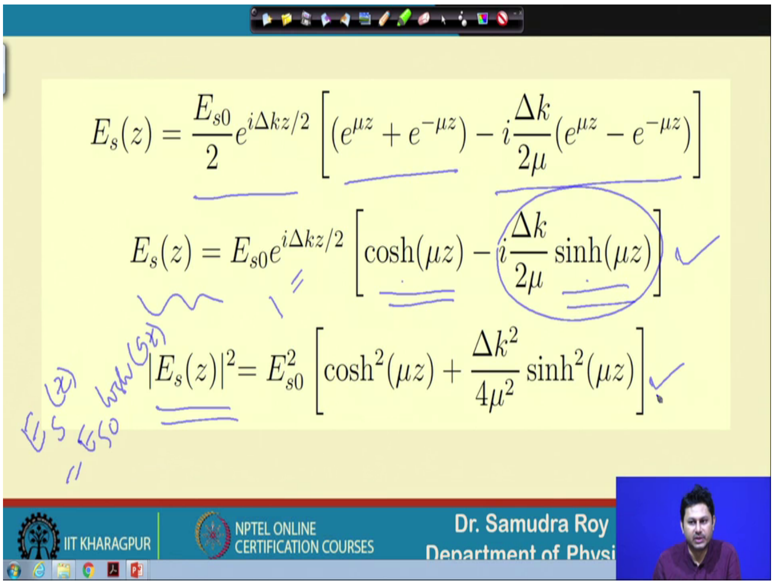

Once I figure out A and B the next things to put this value here and if I put this value in the total electric field E s the total signal field. So, it will simply comes out to be exponential mu plus exponential minus mu and this.

If I take common if I put these two inside then it will be cos hyperbolic of mu z first contribution, and the second contribution will be sin hyperbolic by mu z and this two term I put inside. So, eventually I have E s z is this quantity once cos hyperbolic and one sin hyperbolic.

Again if you note carefully that if I put delta k equal to 0 this term will not be there this term will not be there and this term will be one. So, we will E s 0 cos hyperbolic mu z inside the mu, again delta k term will 0. So, we will have g z, which is exactly the same expression that we have derived to by considering delta k equal to 0. So, if you remember E s at z was derived as E s 0 cos hyperbolic of g z, exactly the same expression one can achieve by just putting delta k equal to 0.

Now, I need to find out what is the corresponding intensity. So, E s z mod square is the mod of this quantity, when I have a mod of this quantity I have this square plus this square term. So, we have E s 0 square and this. So, we will get this.

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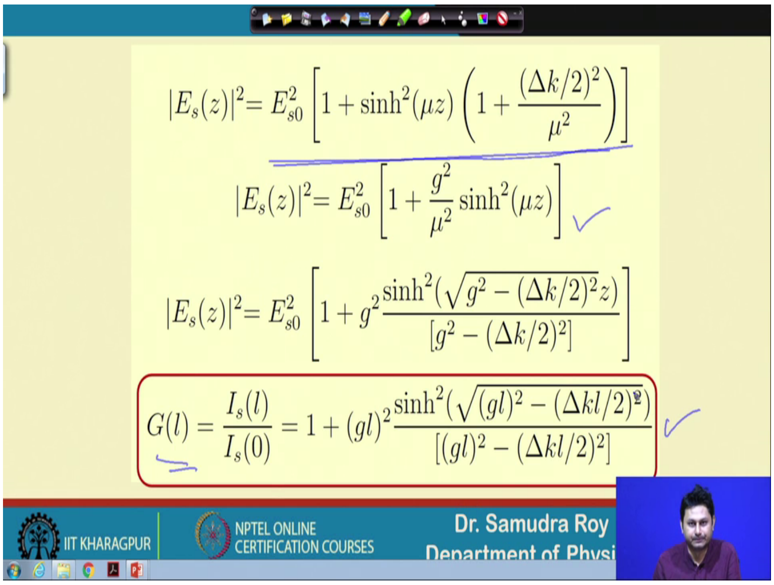

So, once we have this expression then I can again simplify we are almost there actually. So, E s square is E s 0 because cos s we have a cos hyperbolic term and sin hyperbolic term, we use the identity between cos hyperbolic and sin hyperbolic. And when we use the identity between cos hyperbolic and sin hyperbolic we can have this kind of term.

This simple algebraic things I want the student to do by themselves all the expressions are shown here you just need to by your own hand to find out whether your calculation is matching with the given expression or not. This is a very straight forward calculation I am not going all the details in this slides because it will be very combustion with. I just give you the key steps. And once we have this expression then you can simplify with this mu square and delta k by 2 you can simplify and write in terms of g it will be like this and finally, let us go to the final expression and I expect the student will work out and find out whatever the expression is shown here is really matching with their calculation or not. However, all the key steps as I mention is already given.

So, the expression of the gain is shown here. We have an expression which is a complicated kind of form and this complicated kind of from suggest that the expression is is coming because of the fact that data k not equal to 0. So, let me summarize whatever we have.

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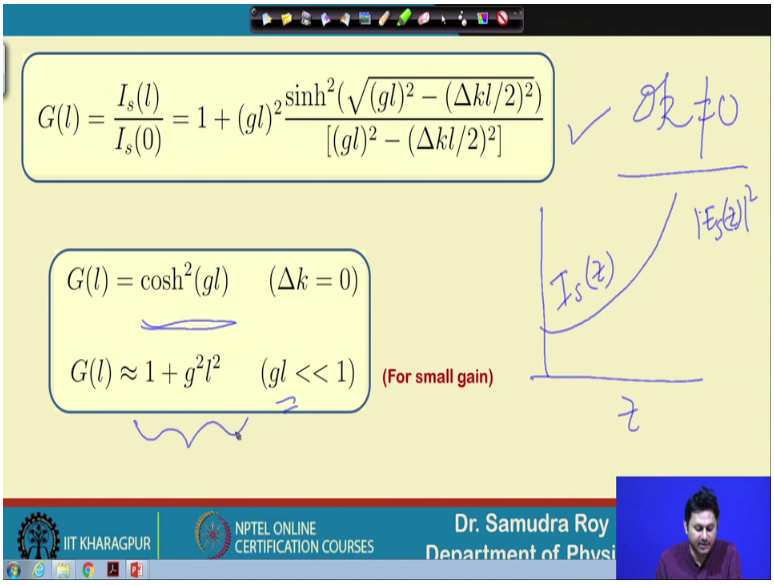

So, this is my gain, this is my gain under delta k not equal to 0. So, this is the most realistic case that we have and if I plot this things it will be interesting to show how this plot is there under delta k, and if delta k equal to 0 we know that it will going to increase and it will going to increase like this for signal this plot we have already shown that how mod of E square or the intensity of signal is increasing with respect to z. And it was the plot was something like this.

A similar kind of plot one can expect if someone plot this thing and he will find that because of the presence of delta k the amplification will be not that much, and it will going to restrict the condition.

Now, delta k equal to 0 if I make delta k equal to 0 that means, absolute phase matching then this equation simplify and it keeps cos square g l simply the expression of the gain is become cos square g l under delta k equal to 0. And one can also further simplify this when I say this gain is very small then G l is very very smaller than 1, then one can approximate that this gain can be written in 1 plus g square l square. This is the expression in many books you will find and this general expression also I believe in some books you will get.

So, with this note I should conclude here. So, today we have learnt two important things one is under delta k not equal to 0, what should be the threshold condition for amplification of signal and what is the expression of the gain because signal is amplifying so that means, some kind of gain associated with that. If I consider gain as a ratio between two quantities which are the intensity of the signal at output and intensity of the signal at the input if I could the ratio, then I find that the gain is greater than one because the simplification expression as shown here in the last slide is 1 plus g square l square that means, g square l square is greater than one. So, gain is greater than one means we have some kind of amplification. And this amplification the general expression is also shown here.

So, with this note I will like to conclude. So, in the next class we will a study more on parametric amplification and we will start a new thing which is para optical parametric oscillator. So, how we can use this amplification for our use that we will going to learn.

So, with this note, thank you for your attention. See you in the next class.