Introduction to Non-Linear Optics and Its Applications

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Lecture - 36
Sum Frequency Generation Under OPA

So, welcome student to the next class of Introduction to Non-linear Optics and its Application. Today we have lecture number 36. In the previous few classes we have studied this parametric amplification process which is generated due to the presence of non-linearity of the medium that we have learnt.

So, today we will going to extend that thing. So, for we have learned how if I launch a field with frequency say omega p which is called the pump, it can split into two different frequencies omega s and omega i which is namely signal and idler and oh when they are propagating together there is a process so that they can amplified. So, this amplification process is generally called parametric amplification process.

So, today we will going to see what happened if two frequency are adding up and then generate a new frequency, which is a summation of that two frequency and how this things we going to amplify and what are the consequences of that. So, in the previous case and this case also, the process is essentially the parametric process. So, let us start with today's topic and today we have a one important topic sum frequency generation under OPA.

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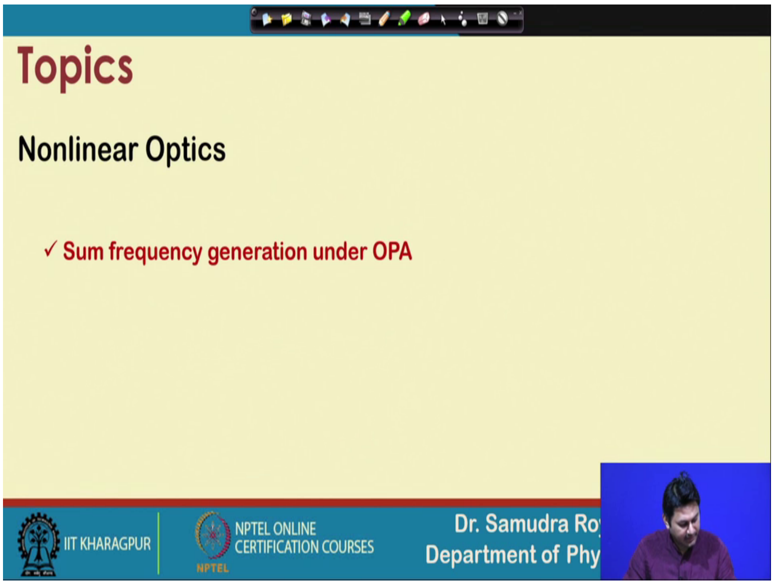

So, sum frequency generation we have already started studied in the previous classes. So, the basic concept is known, that if two frequency is a lunch together. So, there is a possibility that we will get a frequency which is sum over that, and this is entirely a non-linear process the non-linear frequency mixing is there, but the field that we will going to generate this sum frequency is going to evolve.

And this evolution process is important and as I mentioned there is a parametric process and it will related to some kind of application. So, it is important to study. Because in many cases we required a frequency and frequency mixing can generate a such kind of frequency, and we can from this frequency we can make some sort of laser with that particular wavelength.

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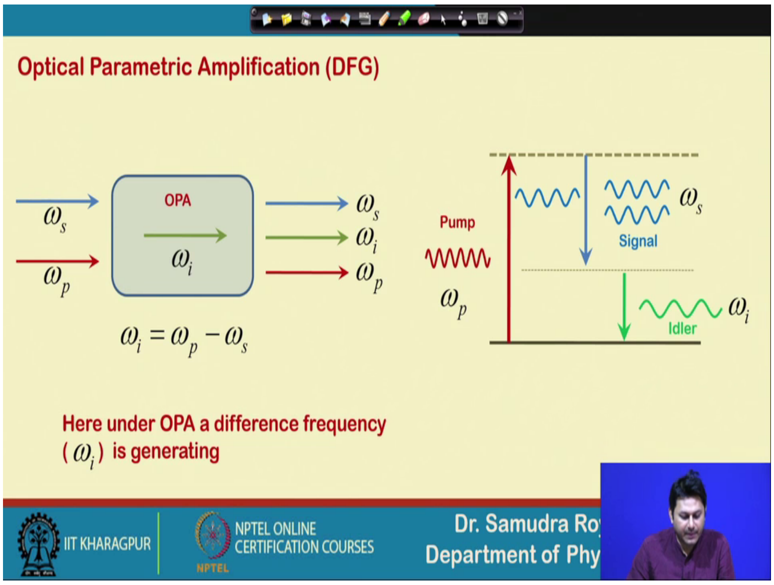

So, let us see what we have. So, optical parametric amplification for different frequency generation here in the slide DFG suggesting that it is difference frequency generation. The process is understood omega s and omega p are 2 input frequencies that we have launch to the system. And because of the optical parametric amplification 3 different frequencies can evolve with the output omega s omega i and omega p. Where omega i is a difference frequency namely it is called the idler frequency. And the process the energy diagram or the energy process is also shown in the right hand side that if a pump is launched into the system, then two different frequencies can generate one is omega s and omega one is omega i, but the energy conservation are there and the signal can be amplified because of this process.

And this is this amplification process is basically due to this optical parametric amplification and also the idler will going to generate. So, in difference frequency the evolution of the idler is important, because we need to find out what is this difference what is that frequency that is the difference between omega p and omega s that is launch in the input.

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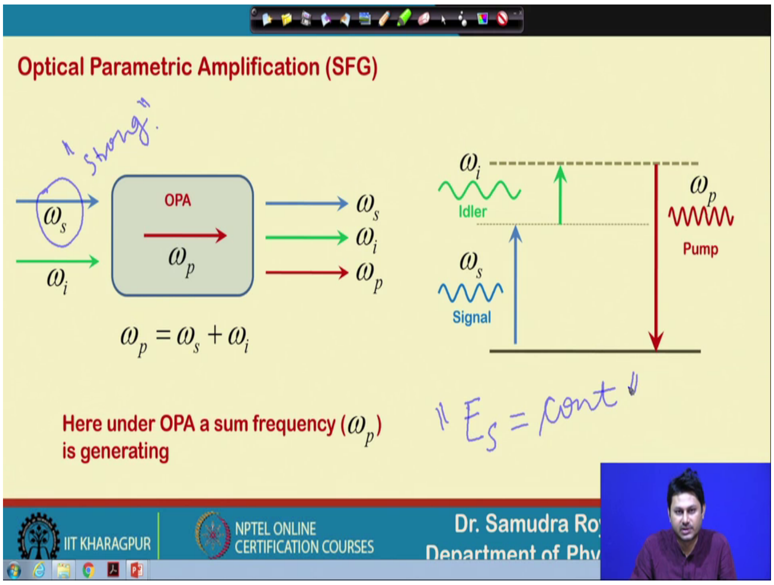

Well, the scenario is slightly different for sum frequency generation as shown in this figure. In the left hand side the figure is almost same this is a schematic figure of how sum frequencies are generated, that if I launch omega s and omega i the frequencies of 2 fields containing omega s and omega i. Then due to this optical parametric amplification process one new frequencies, one can expect to generate and that new frequency is omega p which is summation over these two frequencies.

So, now the question is how this new frequency is going to evolve, and also the idler is there in the system. So, how the idler wave is there that we need to find out. One thing you should note here that here since I am generating the sum frequency, the launched wave frequency omega s which we called the signal is strong. When you say the pump is strong, then that essentially means that there is no change over distance the amplitude of this wave will not going to change over distance. Here, we say the signal is strong and the word strong means there is no change of amplitude throughout the distance, does not the field associated with omega s if I write this field is Es we will consider this as a constant.

So, we have seen before that whenever we make one parameter or one variable constant in the coupled equation then it is easier for us to solve that particular equation. So, here also we will consider that the efficiency of conversion of this pump to signal is relatively low, when the efficiency small, then what happened that a small amount of energy can pass from this signal to pump and you can considered the signal to be constant. So, it basically help us to solve the differential equation quite easily, otherwise it will be a combustion job to tackle with all the three differential equation, which are couple to each other. So, all these 3 differential equation by the way is already derived and these equations are something like this.

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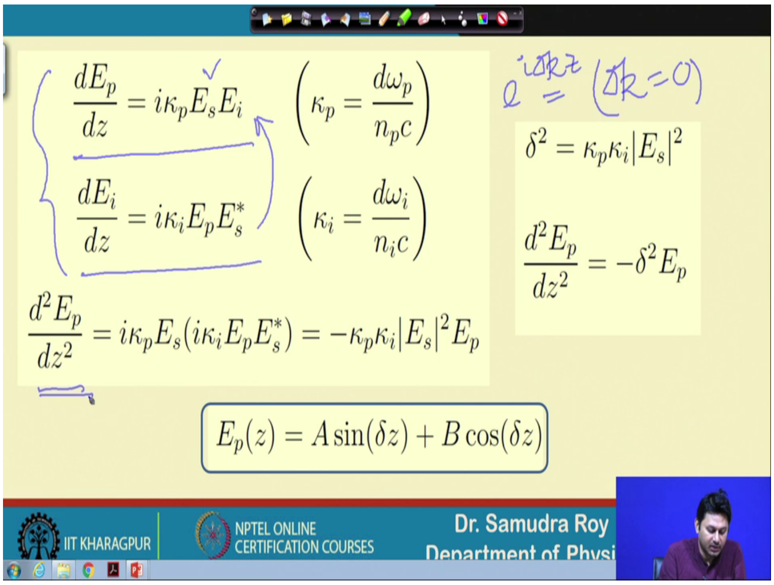

So, I am talking about the coupled differential equations. So, these are the equations I am talking about, but here we are you using only 2 equations; since pump is constant. So, I am not going to since signal is constant here. That means Es is constant.

So, I am not going to take the differential equation related to E Es. So, the first equation is the evolution of the pump and second equation is evolution of the idler. We know that this equation should contain another term which is e to the power of i delta k Z, where delta k is a phase mismatch, but for this particular treatment we just consider that there a absolute phase matching. That means, delta k is equal to 0. That is the condition for which I can write this kind of equation by just eliminating the exponential term. So, we are using this equation for last few classes. So, the process is exactly same. So, what we need to do to solve this equation when Es is constant. That we just replace E i from this equation and in order to replace E i from this equation or what we need to do? We need to make a derivative of this quantity.

So, if we make the derivative of the first equation once again, then we have a second order derivative d 2 E p d z square, this quantity i kappa Es is constant because Es is constant in our case. So, this term will be here, and we have a derivative of this quantity which is d E i d z. Now d E i d z this term is in our hand this, and we just to replace this things here in place of d E i d z. So, when we replace this term we will have a term like kappa p kappa i with the negative sign because this i and this i will give a negative sign, and also we have Es mod square which is also a constant and E p.

Now if I consider this term as delta square because all these things are positive. So, delta has to be a positive quantity, then we will have an expression like this. So, this is the well-known expression. So, one thing we should mention here that when we are dealing with the difference frequency, then there was a plus sign and the nature of the solution was different. So, in the previous class we try to find out how the difference frequencies evolving under optical parametric amplification and that time we notice that this equation this sign was plus.

So, we have a sine hyperbolic and cos hyperbolic kind of solution, the combination of these two as a solution we get. But here what we are getting is a sign solution because of this negative sign. So that means, somehow the E p; that means, the pump will going to evolve, but this evolution is seems to be periodic in nature. Whatever the boundary condition if I put then we find either it should be a sin cos or combination of both, but there should be some kind of sinusoidal variation.

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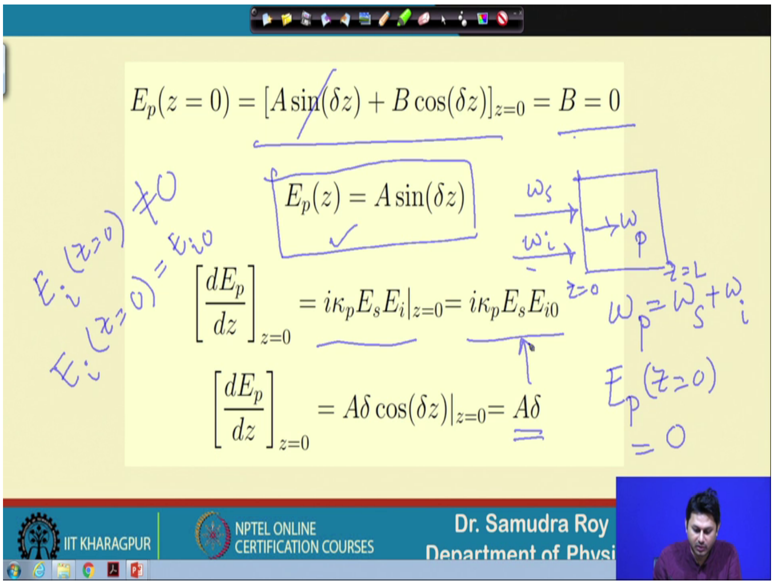

So, once we have this expression, the next thing is to evaluate A and B as we have done in our previous class. So, first we try to find out what is my E p. So, E p at z equal to 0, that we need to consider to find out the boundary condition and the structure is something like this if you remember. That I launch a signal with omega s I launch a idler with omega i and omega p is generating with the condition that omega p is equal to omega s plus omega i that was the condition. If this is the condition then we can see that this is z equal to 0 point and this is say z equal to some L point.

So, the boundary conditions such as that at z equal to 0, there is no field containing the frequency omega p. That means E p at Z equal to 0 is 0 that is the mathematical description of the boundary condition that we have mentioned. Here if I put this boundary condition because E p at any Z point or the solution of the E p is known to us.

So, now if I put Z equal to 0 in this equation, then we find that this term will not be there and only we have B and this B is 0. That means, one part is readily eliminated when I put the boundary condition and the boundary conditions suggest that there is no pump field or the sum frequency field at Z equal to 0. So, our equation or the solution simply becomes E p Z is equal to A sin delta z that is the solution we have.But still we need to find out what should be our a. So, again in order to find the A the next boundary condition that we are going to use and this is the same procedure that we have been using for last few classes. That therefore, one condition we just put what is the value of the field at z equal to 0. And in other case we try to find out the derivative because the derivatives is proportional to the field of other frequencies and we know what is the value of the field of other frequency at z equal to 0 point. So, d E p d z at z equal to 0, which is essentially i kappa p E s E i. So, this value at z equal to 0 is simply i kappa p E s E i 0.

So, the condition here that we have taken is the field idler field at z equal to 0 is not equal to 0 as I mentioned, because we need to launch some idler at the input to generate the frequency at omega p, which is a sum over to frequencies. So, this value we have taken as since this is nonzero value. So, we have taken E i Z equal to 0 is simply E i 0 this is the case where we use this things. Now there e you know what is the value of E p at Z point. So, we make a derivative of that. So, when we make a derivative of that you we will get A delta and once we get A delta that should be equal to that quantity; that means, A delta has to be equal to this.

So, we will finally, get what should be the value of A.

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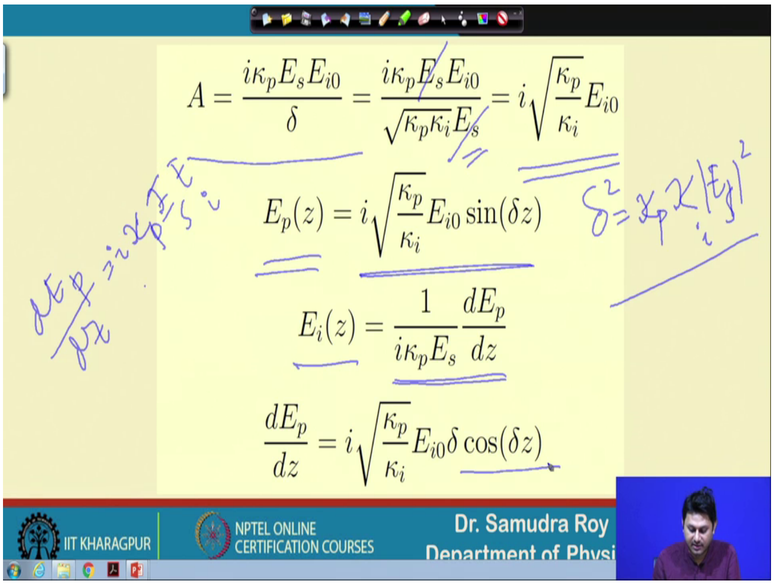

So, A finally, we find A should be something like this i kappa Es E i 0 divided by delta; now next I put the value of delta because if you remember delta square was kappa p kappa i mod of Es square. So, root over of delta is root over of kappa p kappa i and Es mod of Es square can be considered as a completely real quantity. So, that is why when I take a root over of that. So, I can say this is simply Es. So that means, you are not considered any kind of phase at the input or the if even if there is a phase we can call this phase as 0, and respect to that phase when measuring other phases.

So, Es whatever the field we have at the input is having some kind of phase, which we considered the reference. So, we consider these as a 0. So, then this becomes simply a real quantity. So, when we do that then this E s E s will cancel out, by the way this assumption or whatever we are doing here is we can say without any loss of generality. So, there is no harm to take this kind of consideration. So, we all we can always do that. So, A finally, after doing this A become i root over of kappa p kappa kappa p divided by kappa i E i 0.

So, E p finally, I find that E p is something like this, root over of kappa p kappa i E i 0 sin delta z. So that means, it is evolving and it is evolving as a sin function and at z equal to 0 point this quantity is 0 all the boundary condition is valid. But at the same point we need to find out how the idler is evolving because pump is constants. So, pump is not going to change, but this signal is constant here, to signal is not going to change. So, signal is feeding these two fields pump and idler.

So, once pump and idler are feeded by this signal field. So, both this things will going to evolve. So, we need to find out how E i is also evolving. So, in order to find out the evolution of E i will not going to solve again everything, rather we just use the relationship between E i and E p because we know that d E p d z this is again our fundamental equation or the mother equation is i kappa p then E of s and E of i, that was our expression.

So, E i from here I can write it is one by i kappa p Es which is this quantity and d E p d z now d E p d z is one can figure out because E p the functional form an E p is known. And if we do that d d E p d d Z can I find d E p it is a simply i divided i root over of kappa i kappa kappa p kappa i E i 0 then delta cos of deltas z, thus derivative of this quantity with respect to z.

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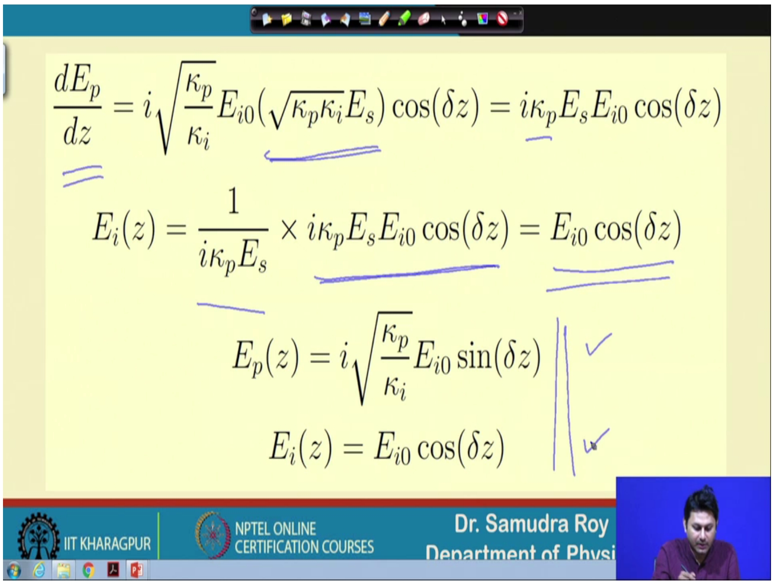

Once we have these quantity d E p d z, then we can just put this things and we can figure out what should be the value of p i. So, d E p d z is this quantity. So, i kappa p kappa i E 0 and I put this quantity, which is nothing, but delta. So, when I put this delta. So, this delta this quantity kappa i kappa i will cancel out, and kappa p kappa p become i kappa p and Es will be there Es Es E i 0 will be there and cos of delta z is there. So, from here we can find out what is my E i, because E i is this quantity multiplied by whatever we have this thing, this thing I now replace with i kappa p is i 0.

So, finally, we come with this solution which is E i 0 cos of delta there. You can see that again the boundary condition is satisfying because we know that p i at z equal to 0 it E i 0 and if you put this z equal to 0 then cos of 0 become 0. So, we have E i at z equal to 0 is simply E i 0. So, these two expression we have in our hand which is E p and E i. So, e p evolve in this way E i will evolve in this fashion. So, now, after having this expression in our hand; so we now calculate. So, these are the 2 expressions we have these are the 2 expressions we have, we know calculate; what is the value of the corresponding powers because we always present the thing in terms of powers.

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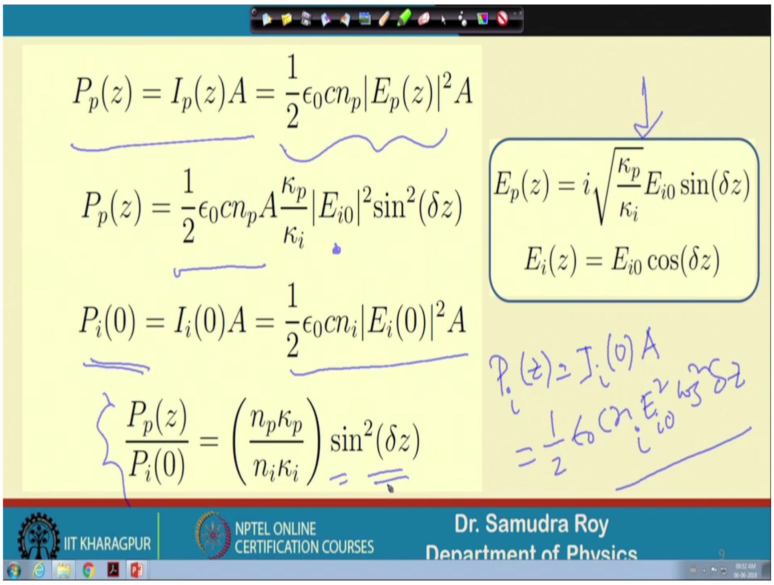

So, the power associated with the electric field E p can be simply written as this P p is equal to I p multiplied by A, I p is the intensity and A is a area. So, intensity multiplied by area is the corresponding power. So, intensity again can be represented in terms of mode of field square. So, which is this expression we are using this expression several time. So, by the time you are familiar with this. So, whenever we have intensity we can replace the intensity in terms of field or whenever we have field we can transform this field to intensity, because intensity is a measurable quantity so as the pump. So, we always try to find out in terms of intensity or power.

So, this quantity E p mode of z square we know because these are the 2 solution that we have generated through this coupled equation and when we put this thing in this expression, this becomes half epsilon 0 in n p c this quantity is as usual because this is already there, and mode of E p square means square of this quantity or this multiplied by the complex conjugate. When we make the complex conjugate this i term will not be there. So, we have kappa p k I mode of E i 0 and sin square delta z.

In the similar way I can also find out what is my p i; that means, the power of idler at z equal to 0 point. I am just taking what is the value of z equal to 0 point because when I this because I know that this is the value at z equal to 0 point. So, just to replace these things from this equation, I just want to find out what is the ratio of pump power at any point z to the idler power at z equal to 0 what is the ratio between these two.

So, we need do, we find that this is simply this now this is cos theta. So, if I want to find out what is my P i Z we can find it out. So, it is I 0 A which is half of epsilon 0 c n i and then E of i mode square. So, this term will be simply become E i 0 cos s square delta Z it will evolve the power will evolve like cos s square. So, the ratio between these two is having this, when you make a ratio these divided by these. So, half epsilon 0 in this term will cancel out, n p kappa p will be there in the denominator we have n i kappa I which is also there I will also be cancel out, this term this term are canceling out each other. So, we will have sin square delta z here. So, this is the ratio of these two things why I am taking the ratio I will show in the next slide.

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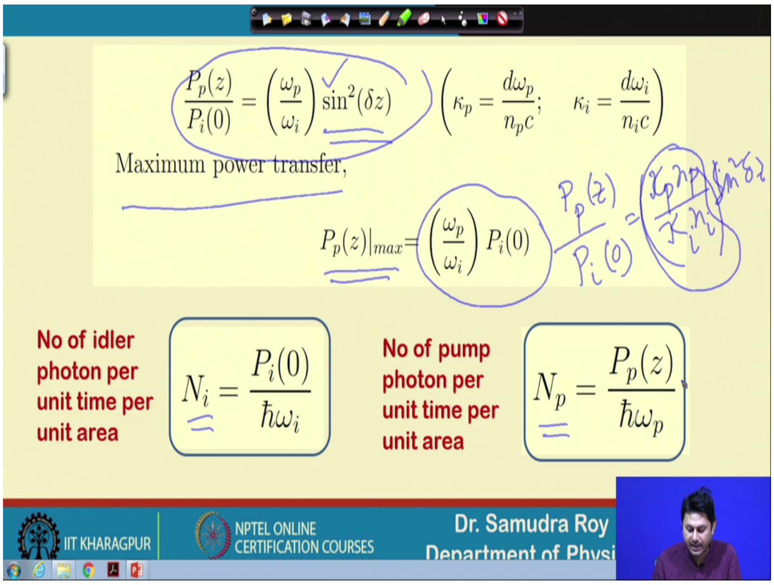

So, once we have the ratio of P p z and P i at 0 point, then this ratio can further be modified because kappa p whatever. So, let me. So, let us right what we had. So, in the previous slide P p divided by P i 0 Z this was kappa p n p divided by kappa i n i.

And then there was sin square delta z. So, this quantity if I replace with these then we find that kappa p n p is d omega p divided by c and kappa n i is d omega i divided by c. So, their ratio simply we can omega p divided by omega i sin square delta z term will be there as usual. So, the next thing is if I want to find out because these quantities now varying with sin square so; that means, the ratio of this quantity is varying sinusoidally now if I consider; what is the maximum power transfer. So, what is the maximum value of this quantity. So, readily we can see that the maximum of this value will be when this quantity is 1. So, sin square delta z is 1. So, when sin square delta z is one. So, we can say this value is maximum.

So; that means, the maximum power at pump, and this quantity in terms of the idler power at z equal to 0 is this. This multiplied by sum parameter omega p and omega i this is a frequency of pump and idler. Now the number of idler photon per unit time per unit area that is moving can be represented in terms of power and energy we know that. So, power at 0 point divided by z equal to 0 of idler power divided by the total energy can give you can give you the total number of idler photon, also we can find out what is the pump photon number of pump photon.

So, now, idler photon and pump photon are related to this expression, and now if we look carefully if I now make a ratio of these two things can we find interesting things.

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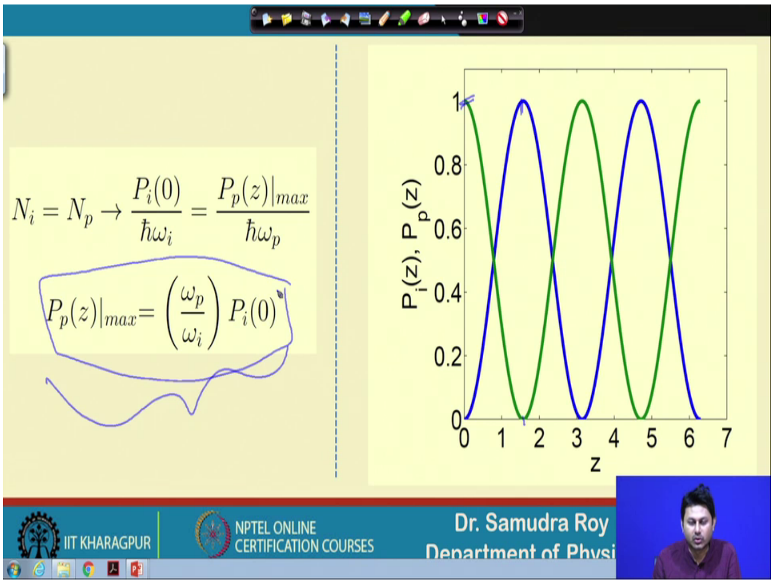

That if I say the number of idler photon that is generated, if this is equal to the number of pump photon. Then this condition is same as whatever the condition we figure out regarding the power transfer. So, that suggests that if I now plot these two things together, then we can find that the idler power and pump power can exchange the energy and when total amount of idler power. So, this is the idler input idler power and now it is reducing and reducing to 0 value and the pump idler power the pump power is now increasing to the maxima.

So, now, there is a power exchange between the idler and pump and if this ratio is this; that means, the total amount of idler photon is now totally converted to the total number of pump photon. So, that means, there is a power conversion, and this power conversion is periodic in nature.

So, today we will like to conclude here. So, we have learnt important thing today that sum frequency generation under optical parametric amplification is possible, but in sum frequency generation process we find that like a difference frequency generation, the signal is evolving as well as the is idler is evolving over z monotonically. But here we find that this monotonous evolution is not possible for pump.

So, pump means the sum of these two frequencies. So, that will going to evolve periodically and there is energy exchange between idler photon 2 idler wave 2 in the pump wave, and we can suitably have the value of z for where for which you we can get the maximum pump power; that means, the sum frequency power.

So, with this note let me conclude here. So, thank you for your attention and see you in the next class where we going to study more about the optical parametric amplification and optical parametric oscillation. So, see you in the next class.

Thanks.