Introduction to Non- linear Optics and its Applications
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Lecture - 35
Optical Parametric Amplification (OPA),
Difference Frequency Generation Under OPA

So, welcome student to the next class of Introduction to Non-Linear Optics and its Application. So, in the previous class we have started important concept called parametric process or parametric down or up conversion. We took a specific example, and we try to find out if we launch an electric field with frequency 2 omega. Whether it is possible to find out a field having a frequency omega; that means, is it really possible to find out some electric field having sub harmonics or the sub harmonics generation is really possible or not.

In order to find the sub harmonic, we find there is a necessary condition, classically at least there is a necessary condition that we should launch some kind of sub harmonic wave in the input. If there is no sub harmonic wave in the input will never get any kind of sub harmonic classically, then quantum noise is a process through which we can generate sub harmonic.

But also we find there is a important phase relationship, and if this phase relationship is such a way that exponentially we can generate sub harmonic, then sub harmonic will generate. And there is a possibility also that it will attenuate because we will find some exponential decay in term is there because of certain phase conditions. And in that case even though in the input we have sub harmonic waves which is the necessary boundary condition, but that sub harmonic wave will not going to sustained throughout the distance and it will decay exponentially.

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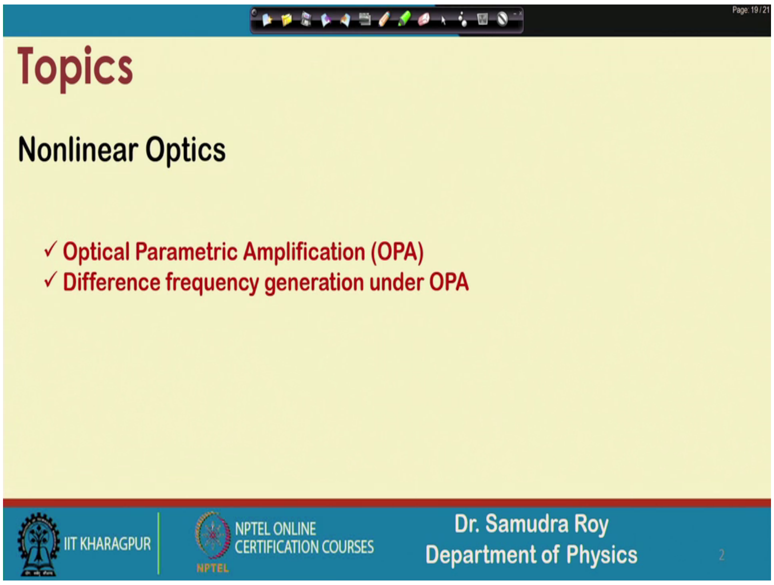

So, we will start from that point. So, today it is class number 35. And today's topic is optical parametric amplification we will continue with that. And we will study special case difference frequency generation under optical parametric amplification.

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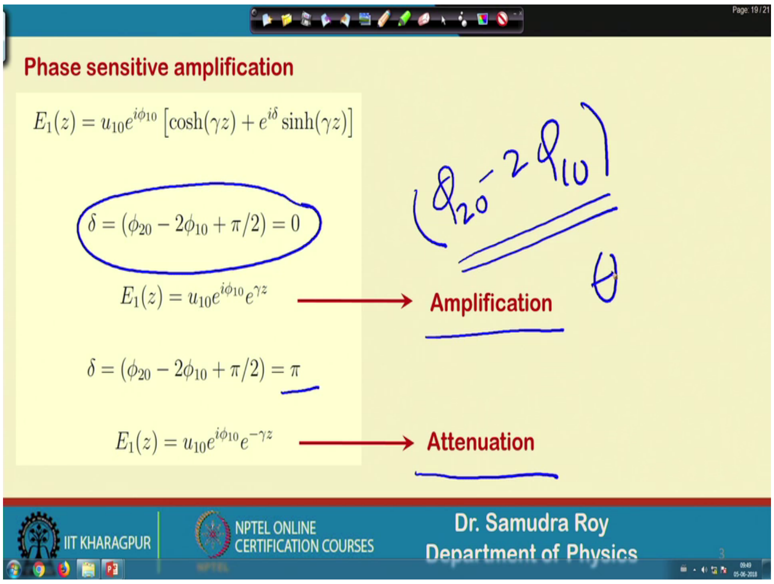

So, this is the slide that we have used in the last class, and we find that for sub harmonics delta is a essential parameter, which is the phase relationship between the two field E1 and E2. And this phase relationship suggest that if delta equal to 0, we have amplification and if delta equal to pi then we get attenuation.

Now, if we look very carefully about this term, phi 2 0 minus 2 phi 1 0. This term is not a very new term because this phase we have already found when we were calculating something related to this theta, we find one very important constant if you remember that u 2 u 1 square cos theta was constant.

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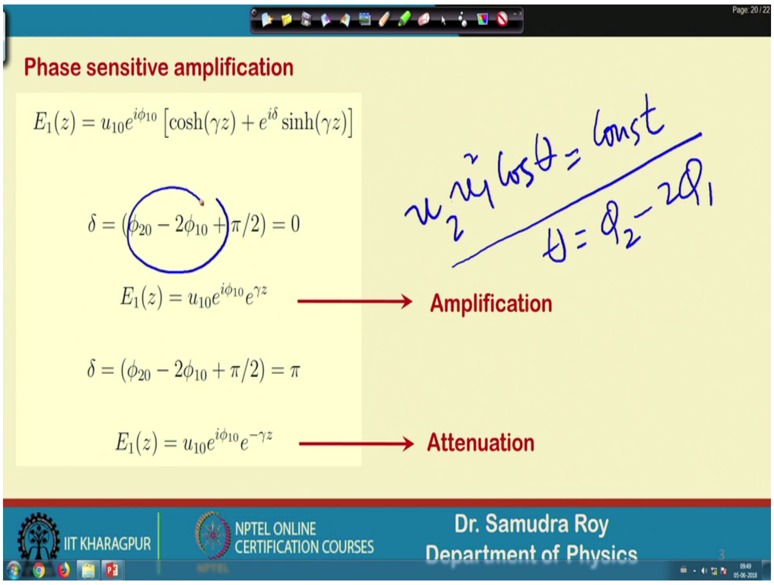

So, this relation we find that from this relation that theta was phi 2 minus 2 phi 1. So, exactly this quantity is here. So, it is coming in same way. So, once we have this delta including this term then readily we find that this is some sort of phase relationship that we already got in the previous calculation. If I extend this thing I think this will be clear.

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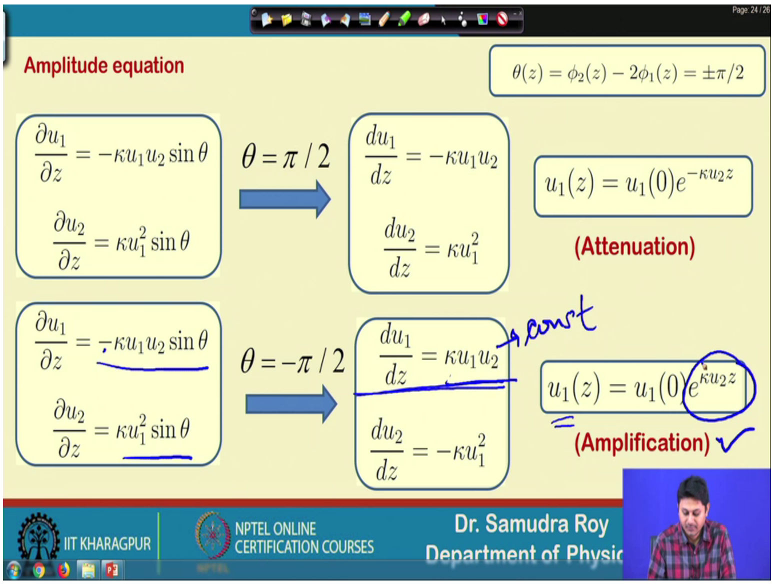

So, we have already this expression for amplitude. And this amplitude expression for u 1 and u 2 gives me that it is related to sin theta and sin theta. That time in the corner in the sticky note you can find that delta theta was phi 2 minus 2 phi 1, and we mentioned that if this value is plus minus pi by 2 then we will going to get this value was plus minus pi by 2. And for this plus minus pi by 2 we can have our expression u 1 and u 2 slightly modified.

So now, here we can see that if I put dealt equal to pi by 2 or delta. So, let me write first in terms of delta what we had in the previous so, delta was equal to phi 2 0 minus phi 1 0 with 2 then plus pi by 2 is equal to 0. And we say that when this equal to 0 we will get some kind of amplification.

So, if it is 0 then from here we can from here we can say that phi 2 0 minus 2 phi 1 0 is equal to minus pi by 2 this pi by 2 it will go to this side and I will get pi by 2. So, once we have this pi by 2 term then please note that delta equal to minus of pi by 2 if I put this, then this quantity this equation and this equation if I look carefully. When put delta theta equal to minus pi by 2 sin pi by 2 is kappa u 1 u 2. So, kappa u 1 u 2 will be there and sin pi by 2 with the negative sign become negative. So, this negative sign will going to absorb.

So, we will have plus kappa u 1 u 2 for u 1 now if I only concentrate to this equation we can see that u 1 if we say u 2 is a constant. Then I can solve this differential equation.

And if I solve this differential equation you can readily see that u 1 is going to be amplify because we have an exponential term with the positive argument and; that means, it will exponentially going to increase. If u t is constant that was our assumption from the beginning and this is happening because our phase matching condition whatever the phase condition we have. So, this phase conditions suggest that delta become minus pi by 2 theta become minus pi by 2 or delta in terms of phi 2 and phi 1 phi 2 and phi 1 become minus pi by 2.

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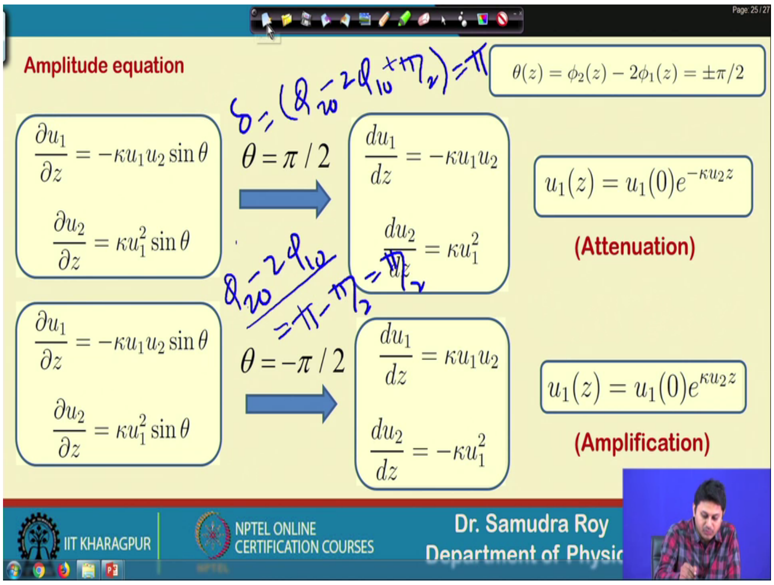

So, this is one condition, the second condition let me write is once again delta was phi 2 0 minus 2 phi 1 0, and then plus pi by 2 and we say, if it is pi. If it is pi, then what of the what value we will get from phi 2 0 minus 2 phi 1 0 this value is how much pi minus pi by 2 so it is pi by 2. This quantity is our theta. So, eventually we will have theta equal to pi by 2; that means, this term when we have this term then we can see that this equation and this equation, become same only negative sign is appearing here because of this negative sign if I do the similar process that we have done for this case we will find that exponential term will appear here as usual taking u 2 as a constant.

But one negative sign will also be there. This negative sign this negative sign suggest that u 1 will going to attenuate so; that means, this phase which is related to phi 2 and phi 1, is very important excite or attenuate a particular field in this case we are dealing with the sub harmonic fields or u 1 and you find that u 1 can be attenuated or amplified already we have seen in the previous class, but this is the equation that you had derived in 2 or 3 class ago. The in previous class this equation was there and using that 2 equation also we can come to the same conclusion. And you can cross verify that whatever the thing we have done is exactly the same thing we using this two important equations ok.

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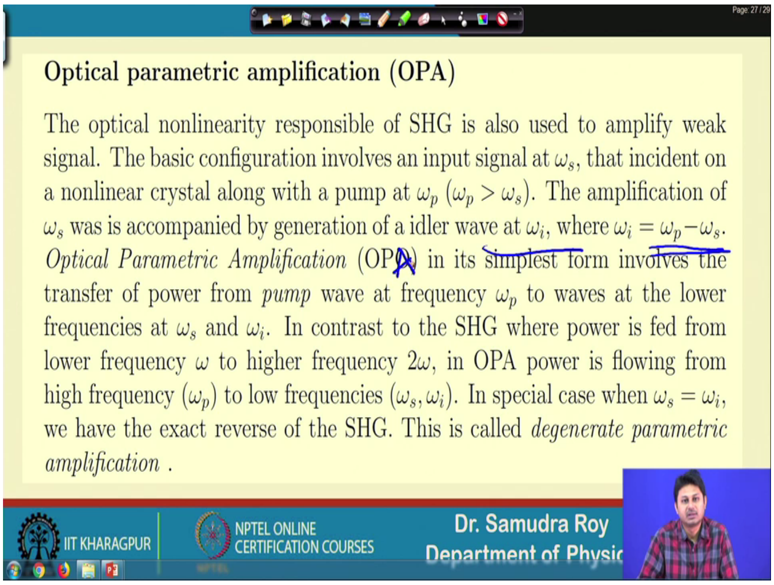

So, let me go back to the next important topic. And this is optical parametric amplification. So far we are dealing with a very specific case as I mentioned, and that is the sub harmonic generation and for sub harmonic generation we are going to generate from 2 omega, we are going to generate omega. But in general optical parametric amplification is defined in this way as shown in your screen. The optical nonlinearity responsible for second harmonic generation is also used to amplify weak signals.

So, this is this optical nonlinearities not only generates second harmonic generation, it also generate also leads to some kind of amplification of weak signal. The basic configuration involves an input signal at omega s that incident on a non-linear crystal along with the pump.

So that means, if I insert a input signal omega s and a pump omega p in the input normally the pump frequency is greater than the signal. Then we find the some sort of amplification is there for omega; that means, I am launching two fields in the input, one is omega p which we called pump is strong field and with that we also launch another field we called omega s, which is signal. And pump and signal are now together inside the system and now inside the system we have a non-linearity because of this nonlinearity, what happened? So, in degenerate case we find second harmonic generation may be there.

But here we are not looking for the second harmonic generation, in general we use omega s and omega p so; that means, this is non-degenerate case. So, omega s will going to amplify. Now omega s will going to amplify not only that there will be another wave omega i we called the idler wave. We called this as idler wave. So, this idler wave omega i which is the difference between these 2 will also get amplified so; that means, optical parametric amplification there should be a here. So, in it is simplest form involves the transfer of power from pump wave to frequency omega p, pump wave pump wave at frequency omega p to waves at lower frequency omega s and omega i.

So; that means, I launch omega p and omega s, omega i is generating inside the system and the pump will going to amplify. Pump basically gives the additional energy to feed the corresponding field related to frequency omega s and omega i. So, omega s and omega i both will get amplified by the launching strong pump. So, for the lower frequency for second harmonic generation we find that the lower power omega s to is feed by higher frequency omega 2 omega. That we have already seen in the previous calculation that sub harmonic is really possible to generate sub harmonic if we put suitable phase condition.

But in optical parametric amplification in general, we can also amplify lower frequency which are non-degenerate in general, but when omega s and omega i are same then it is basically the degenerate case we called degenerate parametric amplification. So, degenerate parametric amplification we have already studied. The point is we have already studied degenerate parametric amplification, and in the previous class and this class also we deal with the generation of omega frequency from 2 omega frequency. And this omega and 2 omega are basically they are the degenerate frequencies 2 2 omega frequencies are generated. So, that is why they are degenerate.

So now, we will remove this degeneracy and try to understand what happened in general if omega s and omega i are not same; that means, in non-degenerate case ok.

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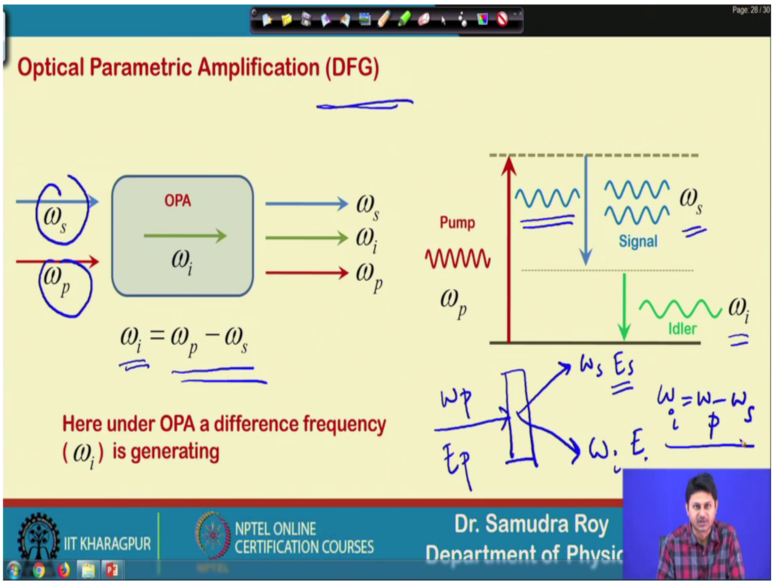

So, let us understand this process in a more systematic way. So, whatever is shown in the previous slide, now this is pictorially it is shown here that if I launch 2 waves which is a strong pump shown here, omega p and with a signal omega s. So, omega pump omega p is a frequency of pump which is the strong wave and omega s is a signal.

So, inside the system under the OPA process or optical parametric amplification process, we can generate another two wave of frequency omega i, which is a difference of omega p and omega s. So, since it is a difference between omega p and omega s. So, this is nothing but the different frequency generation. So, this is something different frequency generation, but how the different frequency will going to amplify the field, associated with the frequency which is difference between 2 frequency pump and signal. How this field will going to amplify, or how the original signal that was there in the input will going to amplify will going to we will going to study these things in this particular lecture.

So, under optical parametric amplification as I mentioned omega i is generating so; that means, the field that is a generating is difference frequency field, in this energy diagram things are more clear that I am launching one pump omega p. This wave is already there which is signal in the blue line. So, this signal get amplified so we will get two blue lines here blue waves, which is at frequency omega s. And also there is a difference frequency these and these there is a difference frequency which we called idler that is also generating.

So, pump the system absorb pump, and this pump see if I draw this figure like this is pump; with frequency omega or in terms of field E p it will it will going to generate two different waves, one is omega s or in field s and another is omega i, in terms of field it is E i these 2 fields are generate. And these two fields now will going to find out how these 2 fields are amplified valid they are amplifying or not. And there is a frequency relationship and omega i is equal to omega p minus omega s.

This is the energy conservation this is coming from energy conservation. So, this is our structure this our structure.

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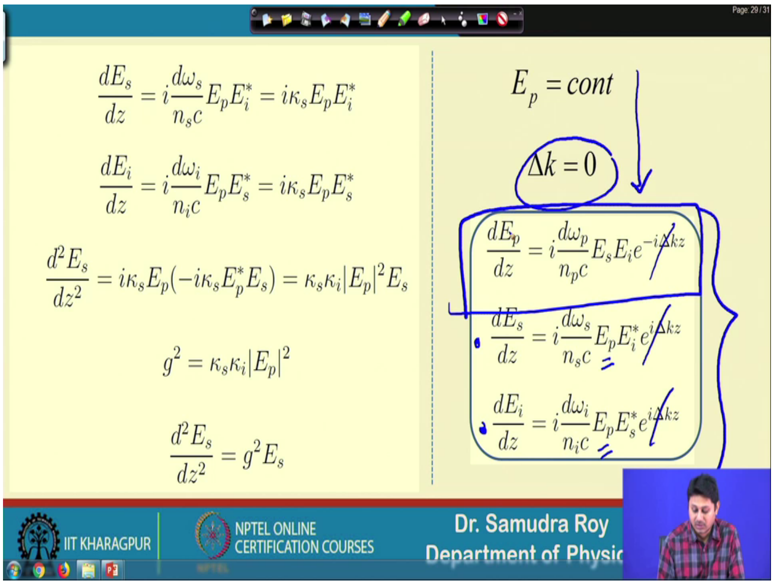

So, in the next slide we will do all the mathematics and try to find out really these 2 things will going to amplify or not. So, our aim has to find out the previous slide that omega p and omega omega s and omega i will going to generate from omega p. And whether these two things will going to amplify or not that we will going to study mathematically.

So, we will start with our old 3 equation, this is the equation of 3 different waves pump signal and idler. So, this 3 equation is a coupled equation. And in coupled equation we put 2 important approximation, and these 2 approximation is one the pump is constant;

that means, we are launching a strong pump. And absolute phase matching condition is achieved. We start with the two important approximations that the pump is constant and absolute phase matchings are there.

So, when delta k is equal to 0. So, we can say that this term this term and this term will not be there that is one assumption. And secondly, in E s and E i these 2 equation is important here E p is sitting both the cases and this E p will remain constant; that means, we can treat it as a constant. So, we will basically discard this equation, where the variation of E p was there. So, we will not going to take this equation we just take this 2 and 3 equation and considered E p as a constant.

So, using this 3 equation, and considering delta k equal to 0 and E p is constant we can find out the evolution of E s and E y. So, if I see here E s. So, E s z can be written as this form d E s z, d E s d z is equal to i. D omega s n s c which is this and E p E i star. We simplify this kappa s because kappa s can take care of d omega s n s c. And d i d z is this quantity which I write also in compact form with kappa. So, here we should write it is kappa i this is not kappa s both the cases it not shouldn't be kappa s it should be kappa i.

So, when I you write, when I differentiate this quantity once again. So, we will have d 2 E s d z d 2 E s d z. So, when I differentiate this quantity is constant. So, we will have a quantity t E i star d z d E i star d z. I can replace from here and when I replace I will have a negative sign here because I am making a star of that and E p star and E s. This quantity should be E i this is the typing mistake I write E s kappa s.

So now this 2 term can be written in this way. So, i minus i become one kappa s kappa i will be multiplied an E p E p star. So, I will write mode of E p square and E s. Now I consider one quantity g which is g square is equal to kappa s kappa i and mode of E p whole square. So, this is my g. So now, if I put this g we will have a differential equation that is having which is having this particular form.

So, this is not a very new kind of differential equation, the signal will going to have this kind of form that we know because we have already calculated this kind of things in the previous calculation. Once we have the differential equation of the signal field readily we can have the solution, readily we can have the solution.

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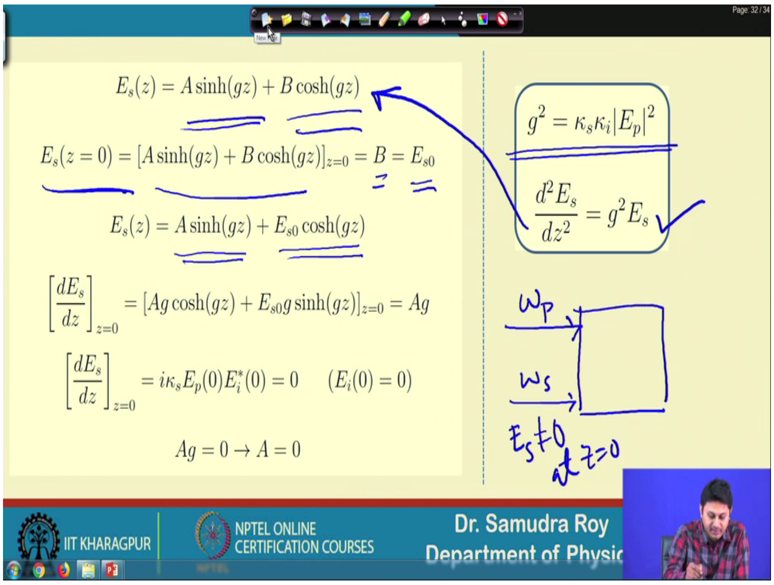

So, we have so, in the right hand sticky note, we can find the g square is defined in this way. Our differential equation is this a direct solution we know what is the form because we are been using this solution for long; that E s is equal to A sin hyperbolic of g z and B cos hyperbolic g z plus B cos hyperbolic of g z. This is the most general form or the most general solution of this given equation. The next thing is that the boundary condition and if you remember the structure it is important to remember the structure, what is that?

So, omega p was launched along with omega s. So, the field associated with omega s is not equal to 0, at z equal to 0. So, we should have some kind of signal field at z equal to 0. So, E s at z equal to 0 basically the signal at z equal to 0. So, this is the quantity at z equal to 0, i can put this value is b. So, b has to be equal to some input value and we defined this input value at E s 0, this is the input. If now I put this value B so E s become A sin hyperbolic of g z plus E s cos hyperbolic of g z i just replace B to E s 0.

Next the similar process we will going to use to find out the value of A. So, we will make a derivative. So, when we make A derivative, E s d z. So, we will have A g cos hyperbolic of g z and the derivative of this quantity is E s 0 g sin hyperbolic of g z cos become sin hyperbolic, sin hyperbolic become cos hyperbolic and when I put z equal to 0 the entire quantity become A g.

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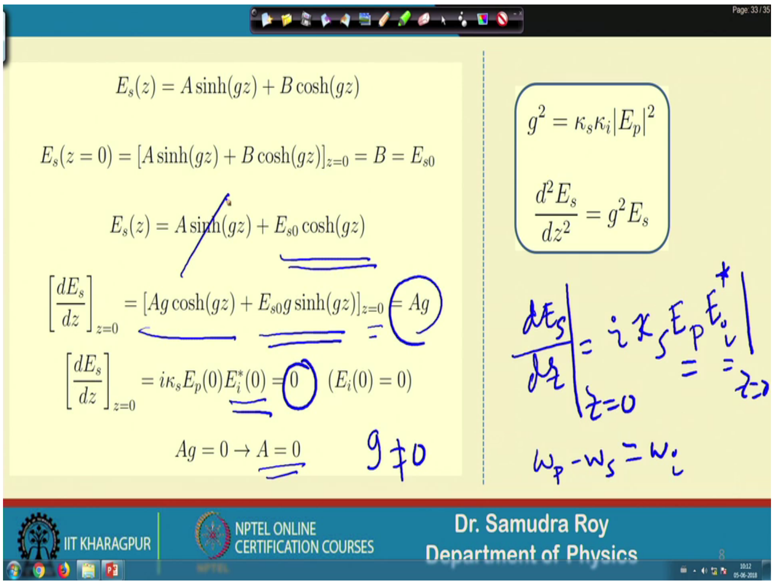

Now d s d z, in the previous slide if i go d s d z this quantity is how much this quantity is i kappa s E p E i star, when i find this quantity at z equal to 0. So, we need to find out this quantity at z equal to 0. So, E p at z equal to 0 and E i at z equal to 0, but E i is a field, is a different frequency field the frequency associated with E i is a difference frequency omega p minus omega s.

So, at the input this frequency was not there. The field of this frequency was not there. So, if it is not there so i can write E i at 0 equal to 0. So, when E i at z equal to 0 is 0 then the entire quantity has to be 0. So, A g is equal to 0 and this corresponds to a equal to 0 because g is a quantity which is not equal to 0. So, eventually we find that E s is equal to E s 0 cos hyperbolic g z because this quantity is not there applying all this boundary conditions ok.

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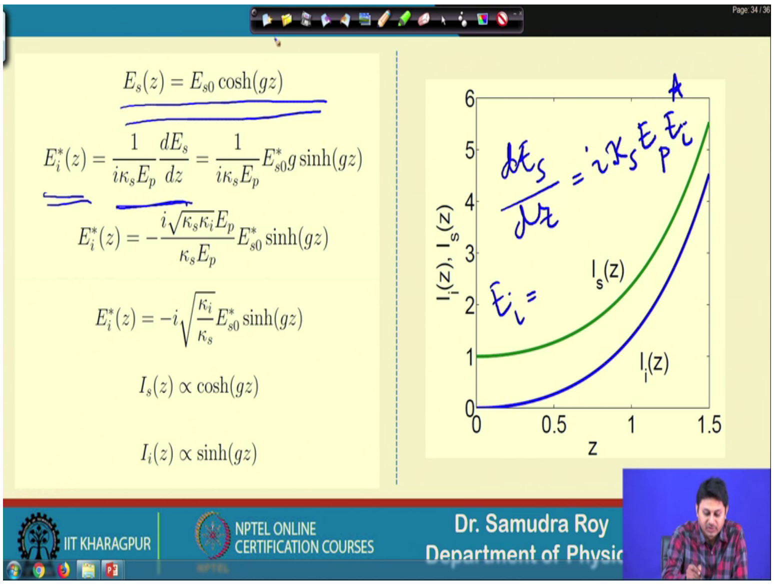

So, as I mentioned we find out what is E s and E s is E s 0 cos hyperbolic of g z; that means, E s is going to evolve some it is it is non 0 at z equal to 0, but with z it can change it can increase. So, once we have E s then we can also find out E i because E i and E s are related to this equation. So, E s let me write it here. So, if we remember d E s d z was i kappa s E p E i star. So, if I try to find out E i star then E i star will be simply this 1 divided by i kappa s E p d s d z.

So; that means, if I know my E s then if I make a derivative, then this derivative gives me E s star g sin hyperbolic of g z. And i i kappa s E p and E i star, can be represented because now what I will do that I will just write g s in terms of kappa and so g g was kappa s kappa i mode of E p square.

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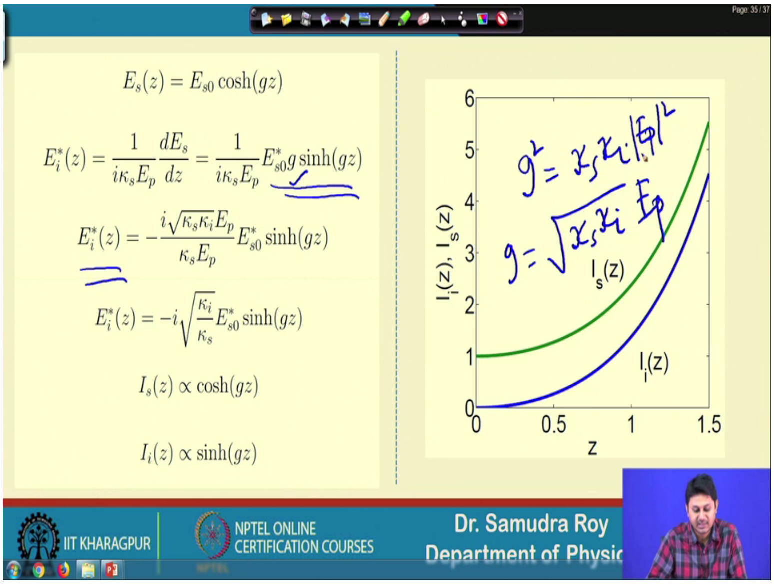

So, it was g square g was root over of kappa s kappa i and E p. We consider there is no phase associated with p this is the strong path and if it has a phase then we can consider this phase has to be 0. Then we can simply write mode of E p square is equal to E p.

So, if I write this E p it will cancel out and eventually will have one term minus i root over of kappa i is E s star 0 sin hyperbolic g. So, once we have this E i and E s then you can readily see that the corresponding intensity of signal and idler is proportional to sake hyperbolic square, there should be a square term and sin hyperbolic square. Again I making some mistake here please note because E is proportional to cos hyperbolic so i which is proportional to mode of E s square should be square of this.

So, when we have a square both squared terminal if i plot as a function of z E i and e E s i i s and i i then we will find that the curve is something like this so; that means, both of them are increasing, both of them are amplifying. And this is amplifying because they are getting some kind of power, pump that power from a some kind of energy from the pump power and as a result we will get some kind of amplification. This kind of amplification is a called the parametric amplification. So, we have studied the parametric amplification for different frequency generation; that means, for omega i.

So, in the next class we will do the similar kind of approach and try to find out what happened, I generate some frequency generation under optical parametric amplification; So, for optical parametric amplification how the some frequency going to generate we will study in the next class. So, thank you for your attention in this class so see you in the next class.

Thanks.