Introduction to Non-linear Optics and it is Applications
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Lecture- 34
Parametric down conversion (Contd.), Optical Parametric Amplification (OPA)

So welcome student, to the next class of Introduction to Non-linear Optics and it is Application. So today, we have lecture number 34.

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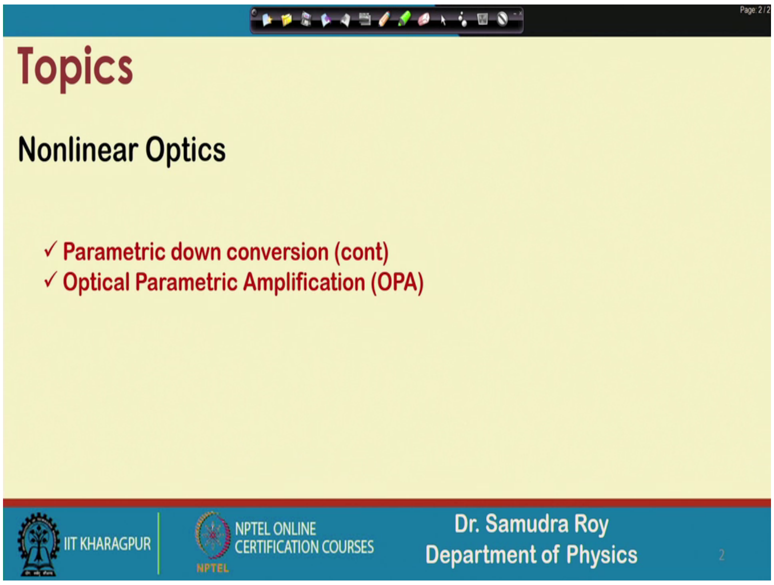

And today, we continue the discussion parametric down conversion, that we have started in our last class and then we will start a new topic optical parametric amplification, which is some sort of similar of this parametric down conversion and this is a special case, but optical parametric amplification in general. We study how this second order nonlinearity, not only generate the frequency at 2 omega. If I launch a frequency at omega, but there is a possibility that, it can generate a 2 different frequency namely signal and idler omega s and omega I or omega p or omega I.

In one case, we called it is a some frequency and another case we call is a difference frequency. When they generate there is a possibility that, they can also amplify based on their initial phase condition. So initial phase because, all these waves should have some initial phase; if they have the initial phase, how this initial phase is important in the amplification process that we going to study.

So, the thing is in second harmonic process we find that the second harmonic waves are generating, but in general parametric optical amplification, we find that ah, any 2 frequencies can be generated and how they are going to amplify that we will study in detail.

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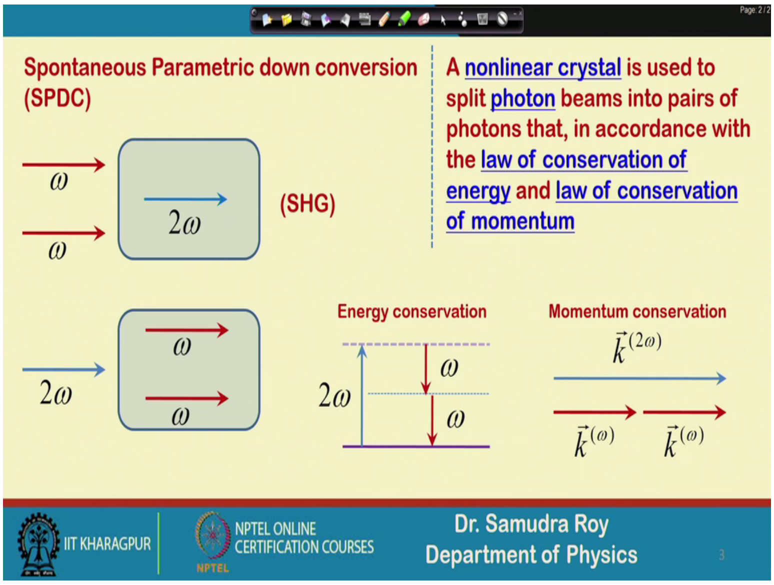

So, let us go back to the old slide, the spontaneous parametric down conversion or SPDC. So, where we was studying one condition that, if I launch a frequency 2 omega then, is there a possibility to generate 2 photon of frequency omega and omega.

Considering that the energy conservation and momentum conservation is valid so; that means, there is a special case. I am launching a frequency 2 omega and I want to generate E2 frequencies omega and omega. That is coming from this 2 omega frequency and they are matching the energy and momentum conservation are still valid.

So the question was, is it really possible to generate such kind of sub harmonics in the previous case. We have already shown that a second harmonic generation process is valid; that means, if I able to launch a frequency omega in the input, which we called the fundamental wave. Then there is a possibility that, we can generate a frequency 2 omega double of that, and we call it the second harmonic generation.

So, this is some sort of opposite process or the rivers process, where we launch 2omega and expect to generate a frequency which is half of that; that means, omega; however, the momentum and energy conservation is still valid as shown in the figure.

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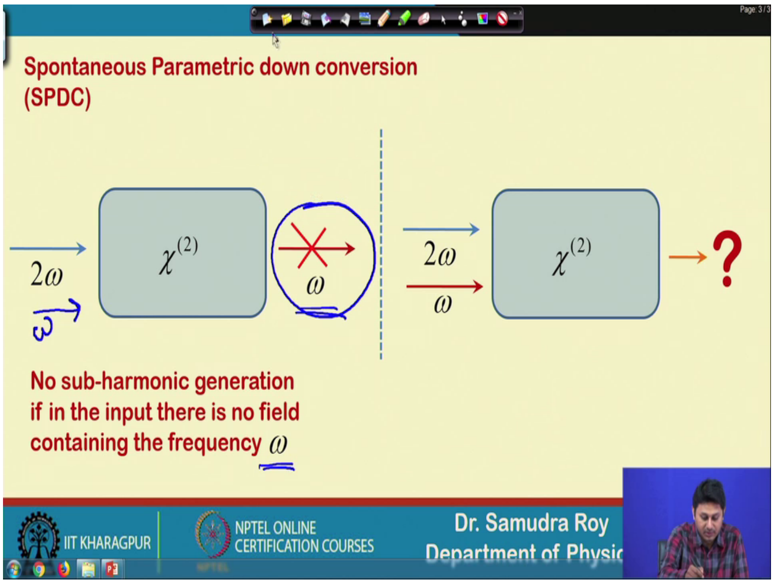

So, this is old slide I am using as a recap and the question was a 2 omega. If I launch 2 mega last day we show with the help of mathematical calculation. That if I launch a frequency 2 omega the omega; that means, the sub harmonic will not going to generate unless, we launch some kind of initial wave here.

So, the boundary condition suggest that, even if I launch 2 omega if there is no omega frequency at the input; that means, there is no field containing omega frequency. Then, this omega that means, sub harmonic will never going to be generating ; that means, no sub harmonic will going to generate, if in the input there is no field containing frequency omega as written clearly here.

However, is it still possible to generate omega? If I frequency a field with frequency omega, if I launch a input if a wave with 2 omega and try to generate the corresponding sub harmonic, but at the same time I launch another wave with frequency omega. Because, in the previous case we find that it is not going to be ah, I mean omega frequency it not going to be generated unless we launch some kind of input.

So, here the second case we will going to study that, in a second order system we launch 2 omega and along with that we launch omega frequency and try to find out whether we can generate any kind of sub harmonic in the output or not.

So, the mathematics will exactly be the same only the boundary conditions is now changed. So, this change boundary condition maybe generate because of this ah, new boundary condition, there is a possibility that it should generate some kind of sub harmonic. So, let us see that.

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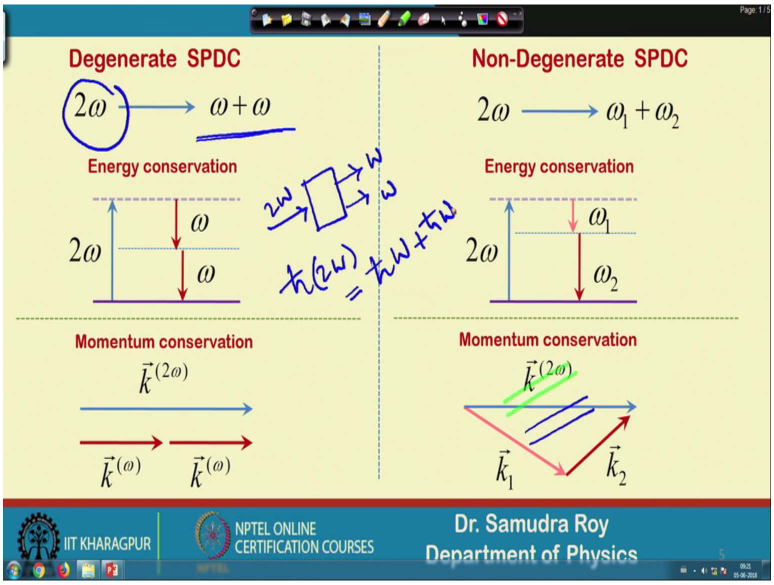

So, before going to that process we should clear about the degenerates spontaneous parametric down conversion and non-degenerate spontaneous parametric down conversion. So, these 2 things are important because right now we are dealing with a frequency omega and sub harmonic frequency ah, a frequency 2 omega and it is sub harmonic frequency omega.

So, why it is degenerate? Because here, we can see that 2 omega is split it to generate to same frequency omega and omega. So, the structure is something like that I am launching a frequency 2 omega and 1 omega and 1 omega is generating.

So, the energy conservation is valid here. Because here, in the diagram there is a old diagram the energy conservation shows that ah, if I write this h cross of 2 omega this is the energy of the photon of 2 omega frequency is equal to h cross omega plus h cross omega and also the momentum conservation is there. So, this is the degenerate process because, we are going to generate exactly the same frequency component omega and omega out of 2 omega frequency.

But this is not the case for non-degenerate spontaneous frequency. Parametric down conversion in non-degenerate parametric down conversion we have 2 omega frequency, but it can split to any arbitrary 2 frequencies omega1 and omega 2.

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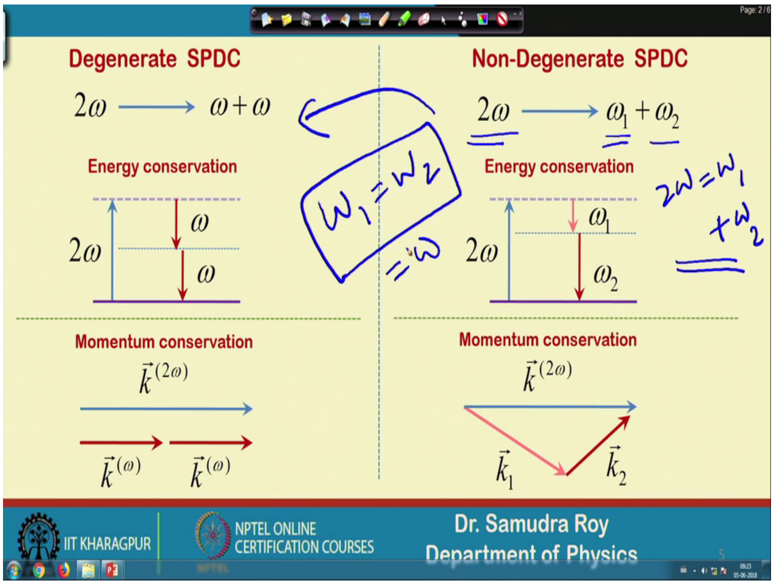

In such a way that, 2 omega has to be equal to omega1 plus omega 2. So, there should be infinite number of combination of omega1 and omega 2 that you can add to generate 2 omega. So this is non degenerate because, omega1 and omega 2 are now distinct, they are not same.

So, this is the energy conservation or at the energy diagram, where we can see that 2 omega photon is split to omega1 and omega 2 photon and readily you can find that, the energy conservation is valid with this simple expression and also for momentum conservation it not necessarily be the collinear kind of phase matching.

So, this is a non collinear kind of phase matching where, this is k 2 omega which is the momentum vector and k 1 and k 2 are the corresponding momentum vector of the frequency omega1 and omega 2 field. So, they are not necessarily be in the same direction, but the phase matching can still be possible with this non collinear kind of structure.

So, degenerate and non-degenerate are very important degenerate is the special case of non-degenerate spontaneous parametric down conversion. When omega1 is equal to omega 2, omega1 is equal to omega 2 then this non degenerate become a degenerate because, they are same which is equal to omega. So, this is the more general case non degenerate spontaneous parametric down conversion is more general case.

But right now, we are studying the degenerate case. This is a special case and the reason is that, we have already studied the second harmonic generation process. So, as a consequence of second harmonic generation because, we have already studied that process now we extend our study and try to find out what happened under a spontaneous frequency down conversion of frequency up conversion process. What is what is happening in case of in special case of second harmonic generation?

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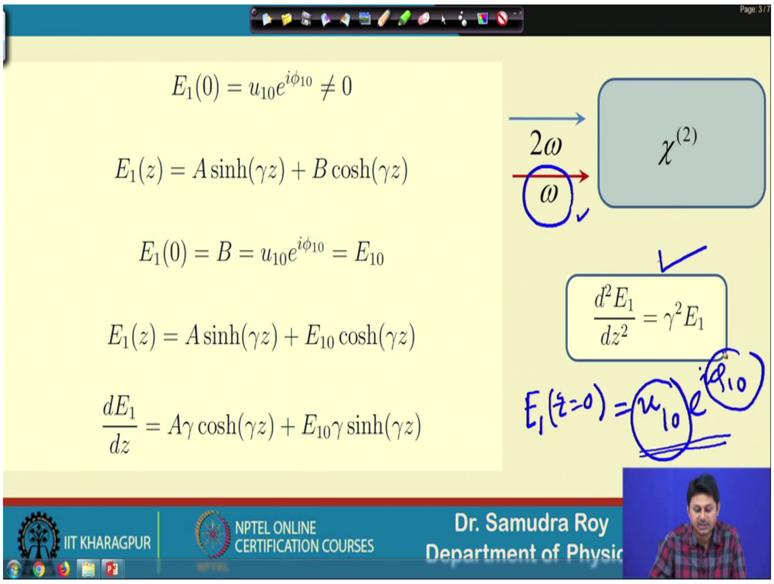

So, let us now go back to the next slide, which is the calculation part and now using this calculation part, we try to understand really sub harmonic will going to generate or not. So, if you remember the expression of d 2 E1 d z square was this. The expression was same that we have used in the previous calculation only. As I mentioned only the boundary condition will going to be modified.

So, in the previous case we had the same expression here ah, the differential equation that we derived. So, the differential equation is shown here only the thing you should note that extra boundary condition is added here. The boundary condition is in the previous case.

It was E1 at z equal to 0 was 0, but now we modify this boundary condition and saying that E1 at z equal to 0 is equal to sum value and we write this sum value as u1 0 E to the power of I phi 1 0. This is a non 0 value and this value is at z equal to 0 that is the electric field containing the frequency omega; that means, this at z equal to 0 point.

So, we have a amplitude part and a phase part together in this input and the reason is ah, if I start with the amplitude and phase, then we can understand what is the role of the phase not only the amplitude evolution was of the amplitude is important, but the evolution of these a phase is equally important here. So, that is why these 2 term are taken in the input.

So now, under this new input condition, this boundary condition rather we have may have a new kind of solution. So here, the general solution is written. This is the general solution of the expression that we have derived. So, d 2 E1 d z square is equal to gamma square E1. This is our differential equation and we have a general solution A sin hyperbolic of gamma z B cos hyperbolic of gamma z.

So once, we have this general solution, next we put the boundary condition. If I put z equal to 0 here, then the left hand side we have E1 0; that means, at z equal to 0, what value we will going to expect. If I put z equal to 0 the sin hyperbolic term will not be there and cos hyperbolic term tends to 1, and then we will have only B set E1 0 is equal to B. And this B should be equal to u1 0 E to the power I phi 1 0, which is nothing in general I can write.

So, this is the amplitude and phase part written which is our boundary condition as shown here, which we can write in short E1 0. So, E1 0 is nothing but the electric field.

The complex amplitude of the electric field at z equal to 0 and when I divide this complex amplitude part into amplitude and phase 1 can divide this into this. So, in a single equation we can write everything.

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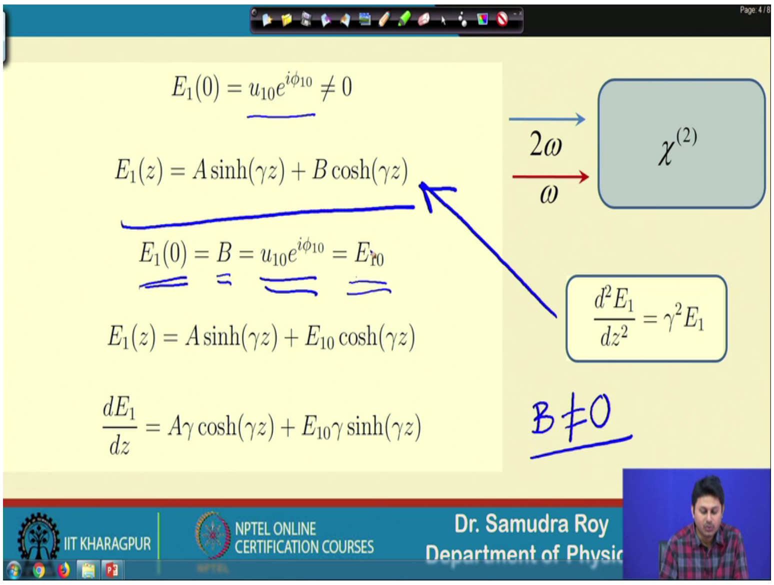

So, the coefficient B is interestingly now not equal to 0, the coefficient d is not equal to 0. Previously, if you remember the previous calculation B was 0 because, in the input we considered that there is no fundamental or E1 field. So, that is why this quantity was 0. So, B was 0.

Now, in the similar way we try to find out what is our E. So, in order to do that the next thing that we need to do is this is our, now the total field instead of d I now write E1 0 because, B is equal to E1 0 and that we make a derivative of this quantity.

Once we make a derivative of this quantity, we will have the derivative of this and the derivative of the first quantity gives me a gamma cos hyperbolic gamma z and derivative of the second quantity gives me E1 0 gamma sin hyperbolic of gamma z. So, why are making derivative because I am going to use the next boundary condition and there is a derivative form of that here.

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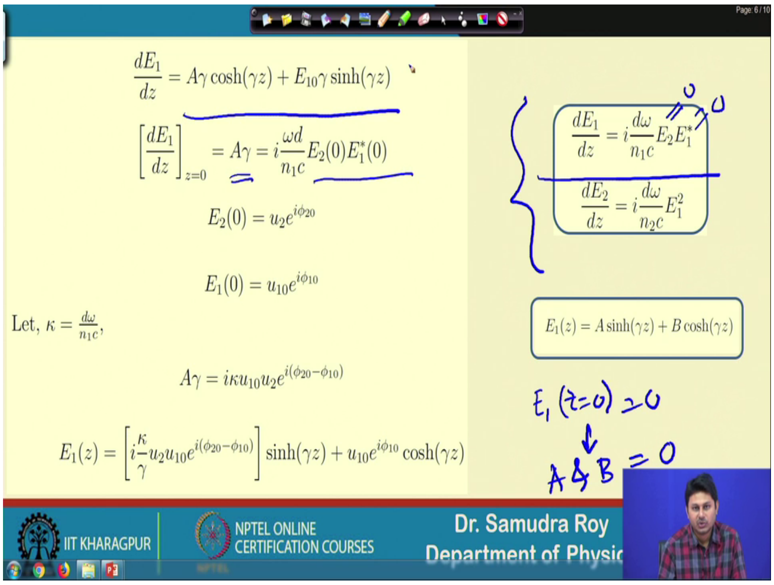

So, this was our main equation and from this main equation once we have it. So, this is our derivative term. So, in this main equation we can see that d1. D1 d z d E1 d z is this quantity. If I want to find out what is the value of d1 d z at z equal to 0 we need to put the value of this is equal to 0 and this is equal to 0 at z equal to 0 point what is the value. So, exactly that we are doing here.

So, d E 1 z at z equal to 0 from this equation we can say this is a gamma. Because, when I put 0 from this equation z is 0. So, this term become a gamma and here z equal to 0 sin hyperbolic of 0 is 0. So, this term will not be there. So, a gamma should be equal to from this equation. I can find that it is I omega d divided by n1 c multiplied by E 2 at z equal to 0 and E1 star at z equal to 0.

So now, one thing you should note that again we find something which is non zero. Previously, in the previous calculation when we put input there was no input. So, u1 z equal to 0 was 0.

Under this condition the consequence was A and B both are 0. That is why entire thing was vanished. So, the conclusion was, we will not going to get any kind of electric field any kind of electric field having the frequency component omega or in other word sub harmonic will not going to generated.

So now, the sub harmonic here we find that with small amount of input value with sub harmonic frequencies. There is a possibility that, we can amplify or we can generate something out of that at least the boundary condition using that boundary condition we find A and B is not equal to 0; that means, there should be some kind of evolution of that sub harmonic field. So now, we put the boundary condition here and then we find that a gamma should be equal to E 2 at 0 and E 1 star at 0. So, E 2 at 0 again we put one quantity and this is amplitude and phase term.

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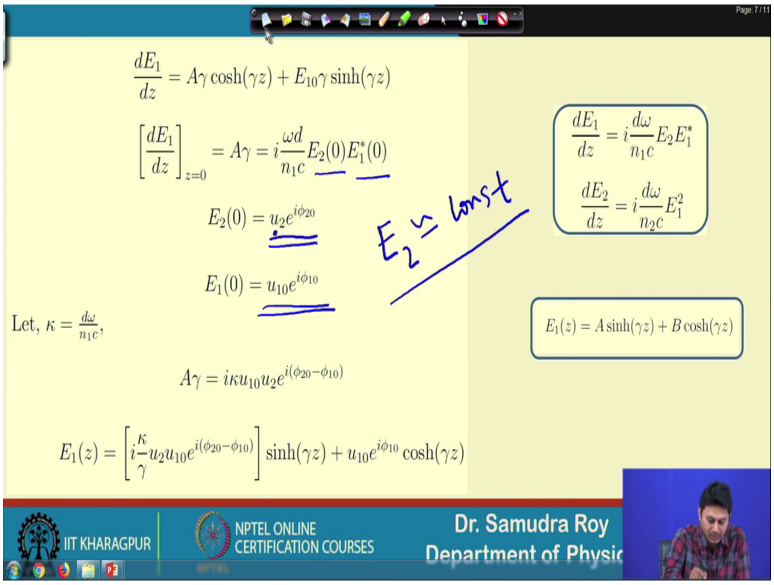

So, u 2 E to the power I phi 2 0, I put it u 2 to make sure that E2 our valid condition was our approximation was u 2 is nearly equal to constant. That means there is no change of amplitude of E 2 E 2 so; that means, u 2 is some sort of constant here.

So, it is not going to change over distance mode of u 2 is not going to change over distance. So, the power of u 2 is not going to change over distance and u one 0 was already defined it is u1 0 E to the power I phi 1 0 when I put these E 2 thing together.

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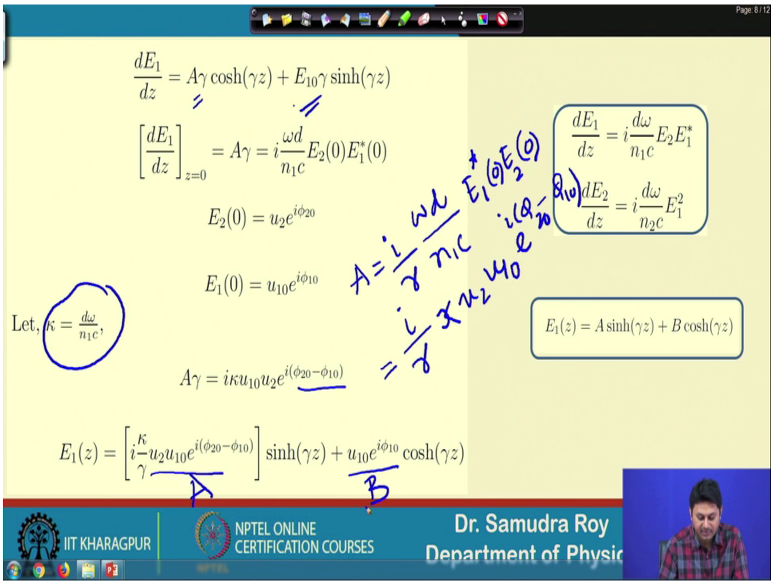

Then we find, what should be the value here in for a gamma. For a gamma, it should be A equal to I divided by gamma omega d n1 c E1 0 star and E2 0, it should be the value now. If we remember that, it was kappa. D omega n1 c was kappa.

So, if I put it is 1 divided by gamma into kappa and if I use the phase amplitude term here. The phase amplitude term gives me u 2 u1 0 E to the power of 1 phi 2 0 minus phi 1 0 which is these things. So, we have a phase associated with A and once we have the A and B together, then I can put this A value here and here we have already E 0 which is our B. So, when I put this.

So, we will get simply A. This is our A that we have just derived and this is our B, which we have derived in the previous slides. So, we have a total electric field E1 in this particular form. It looks quite complicated right now, but we will see it is not that complicated it looks. It can be simplified and when it is simplified, then we have a very nice a physical implication out of this expression.

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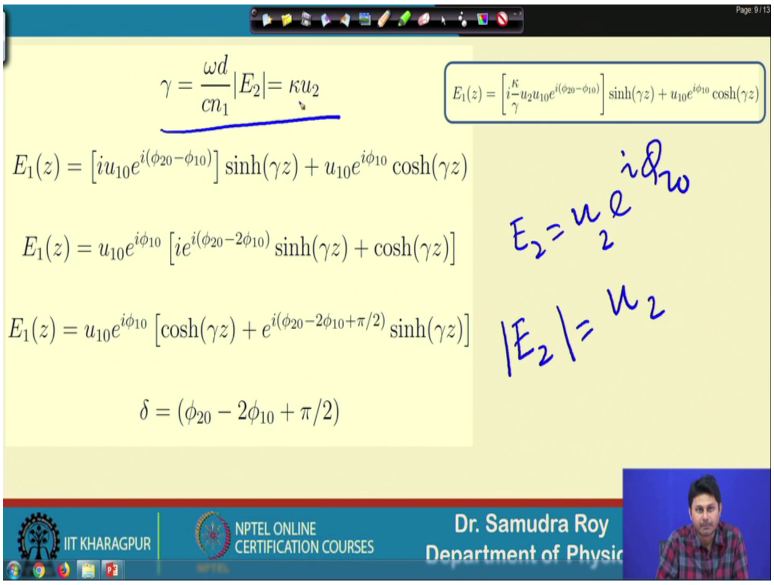

So, here we will see that gamma is equal to this value by definition we have already defined this gamma before. So, gamma is omega d divided by c n 1 mode of u 2 and mode of u 2 is. So, u 2 was defined as E2 is defined as u 2 E to the power of I phi of 2 0.

So, mode of E2 is simply u 2; that means, the only the amplitude part. So, an omega d c n 1 is nothing but kappa. So, it should be kappa. So, gamma becomes simply kappa u 2 and this is the term that we have derived in the last class. So, u E1 is equal to that quantity.

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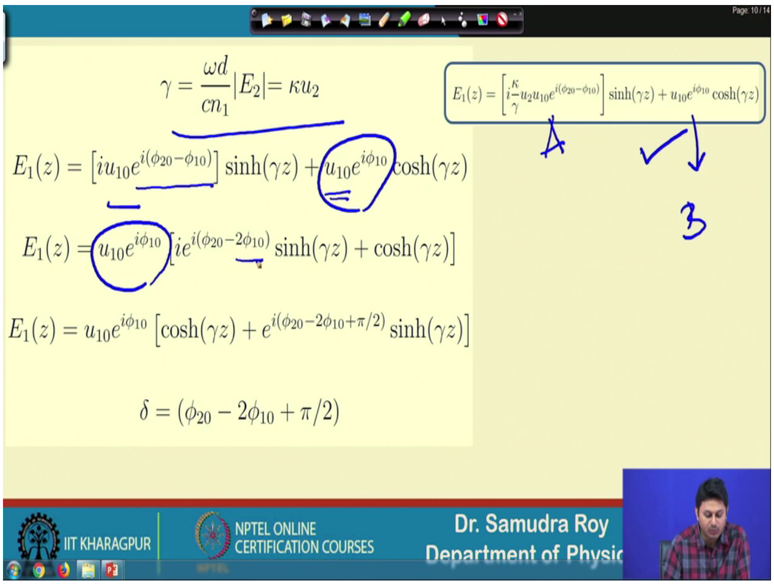

Which is A and this quantity which is B, then sin hyperbolic gamma z and cos hyperbolic gamma z which is the z dependent part. So now, in this kappa gamma u 2 we can replace this as 1 because, here in this expression we find that gamma is equal to kappa u 2, if I put gamma is equal to kappa u 2. So, this term this 2 term k u 2 will going to cancel out and we will have simply I u 1 0 and E to the power of I phi 2 0 minus phi 1 0 which is a phase term then, sin hyperbolic of gamma z and u 1 0 E to the power of I phi 1 0 cos hyperbolic of gamma z.

So, here we have E1 term u 1 0 here also we havE1 term u 1 0. So, the next thing that we will going to do, we as we are just going to take the common u 1 0 E to the power of I phi 10.

So, this term we take common from both the side. When we take this term common you can see that, there we have E to the power of I phi 1 0 with a negative sign. So, I am taking common u to the power of I phi 1 0. So, that is why A 2 term is appearing to compensate that quantity.

So eventually, we will have u1 0 E to the power I phi 1 0 common then inside the bracket we have I E to the power of I phi 2 0 minus 2 phi 1 0 sin hyperbolic gamma z plus cos hyperbolic of gamma z. Just slight modification, just write cos hyperbolic gamma z plus this quantity when I write now exponential term you can see that, this I is now absorb inside the exponential term.

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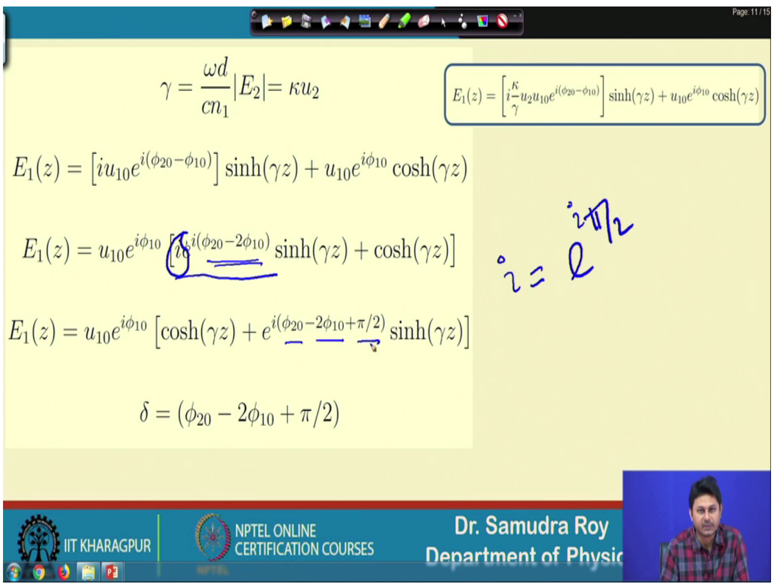

Because, we know I can be written in terms of exponential and it can written as I pi by 2 E to the power I pi by 2 is I. So, we just write this I in terms of exponential and put this term inside this total phase. So now, this term become phi 2 0 2 phi 1 0 and plus pi by 2 here.

Once we have this then what we will do, we will put a name here called delta. So, delta is something which is related to the phase of E2. E2 field and the phase of E1 field, but phi 2 0 minus 2 phi 1 0 plus pi by 2 is delta. So now, if I put this things here.

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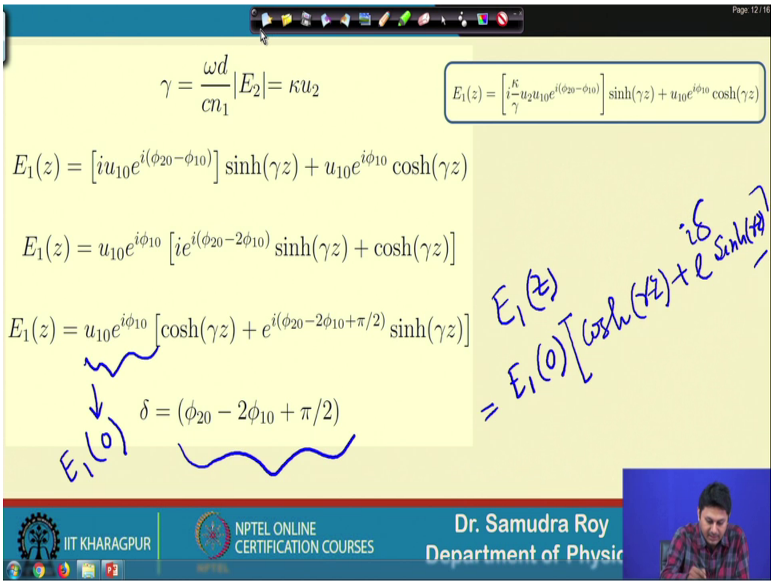

So, we will going to havE1 expression. So, E1 0 E1 z is nothing but E of 1 0 mind it. This quantity is nothing but E of 1 at z equal to 0 that is the input, multiplied by cos of hyperbolic of gamma z plus E to the power of I delta sin of hyperbolic of gamma z. So, we will have a term like this let us see, what is the implication of this term?

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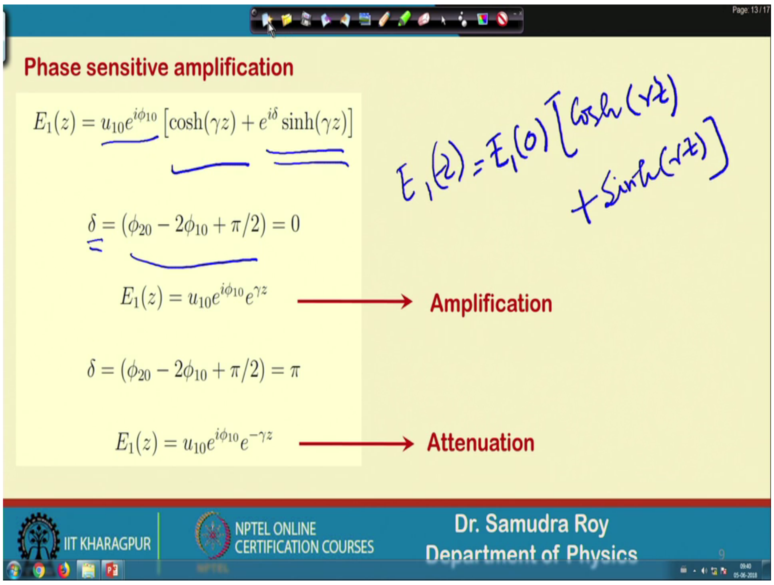

As I mention we havE1 term here, which is E to the power of I 0, E to the power of I phi 0 and then cos hyperbolic of gamma z. E to the power I delta sin hyperbolic of gamma z and then I say my delta is this quantity. Now, I put delta equal to 2 different values in the first case I put delta equal to 0.

If I put delta equal to 0 then this quantity simply becomes E1 z will be E1 0 and then we will have cos hyperbolic of gamma z plus delta is 0 means, sin hyperbolic of gamma z. We know that cos hyperbolic of gamma z plus sin hyperbolic gamma z is nothing but exponential of gamma z.

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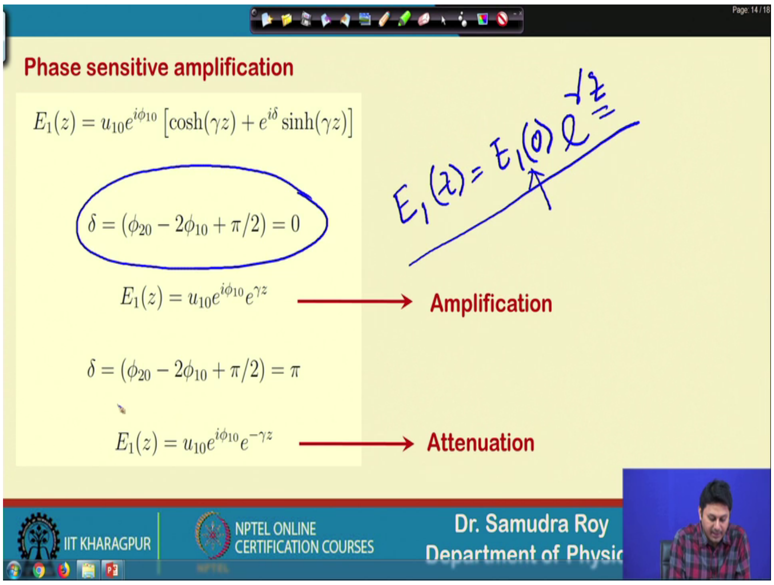

So, if I write in compact form E1 z become E1 0 E to the power of gamma z. So, this is the interesting term. So, when delta is 0 these kind of phase matching are there. So, we will see that because of this kind of phase relation we have an amplification. So, with z if z is increased then we will find that E1 z we will going to increase.

So, we will have the amplification of the total field E1. Whatever, the value we have at the input it will going to amplify that; that means, sub harmonic will going to generate on the other hand. If I put dealt equal to pi then E to the power I delta become have become e to the power I pi. So, we will have a negative sign here.

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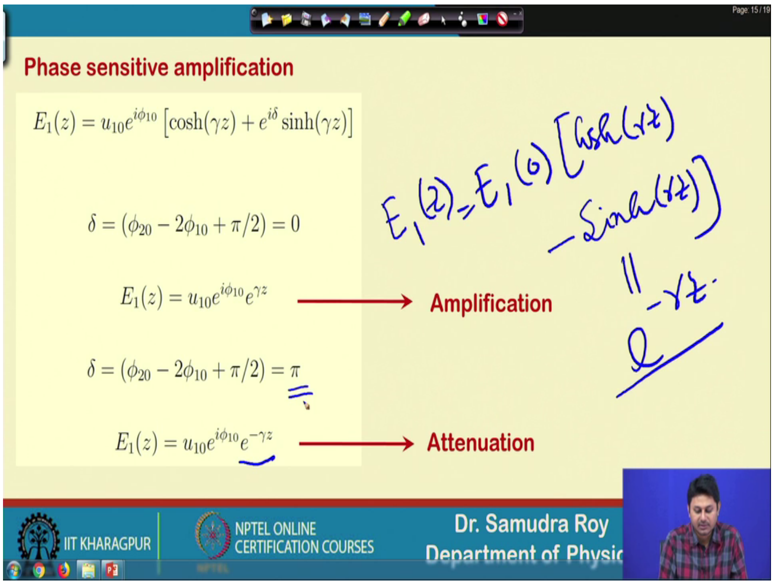

So, we will get this kind of expression E1 z is equal to E1 0 cos hyperbolic of gamma z minus sin hyperbolic of gamma z. This term is nothing but E to the power of minus gamma z. So, exponentially d k term will be involved here like it shows like it is shown here so; that means, if the phase relationship between u 2 E2 and E1 in such that the delta become pi, then we will get an amplification; that means, there the sub harmonic will not going to generate any more.

So, we find one interesting thing here that, sub harmonic can be generated an amplified. If there is a phase relationship between E2 and E1 and this specific phase relationship basically allows the sub harmonic to be generated. So, this is a phase sensitive kind of amplification on the other hand. If the phase is such a way that it can destroy or make this things exponentially d k then, it will not going to get any kind of sub harmonics.

So, this is some sort of parametric process where the phase is associated with that. So, we find that even sub harmonic can be generated, but it depends on the initial phase and if the initial phase is such a way that sub harmonic input is still there, but we will not going to generate any kind of sub harmonic field because, it will going to d k exponentially. So, with this note let me conclude here. So, in the next class we will start from this point and try to find out more about the parametric amplification and.

Thank you for your attention and see you in the next class.