Introduction to Non-Linear Optics and its Applications
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Lecture – 33
Manley – Rowe Relation (3 wave mixing), Parametric down conversion

So, welcome student to the next class of Introduction to Non-Linear Optics and its Application. So, today we will have lecture number 33. So, today in the lecture we have two very important topic.

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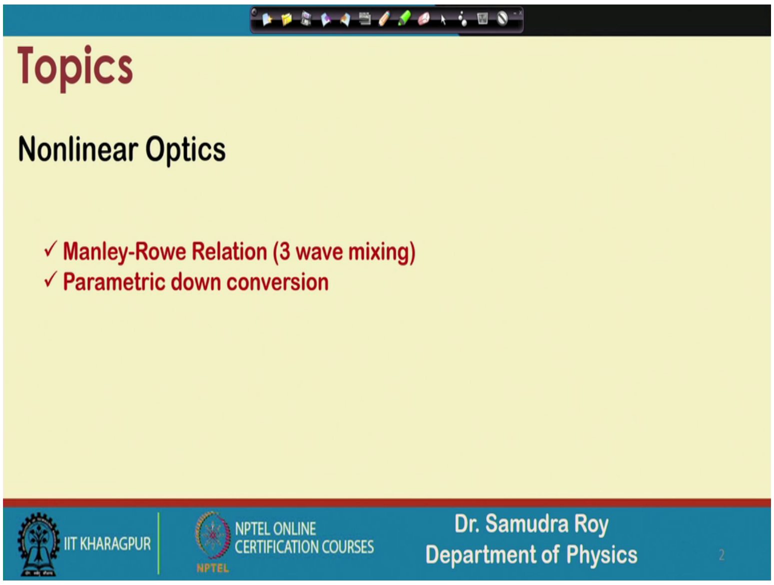

One is the Manley-Rowe relation for 3 wave mixing. This relation we have already discussed for the second harmonic generation when we are discussing about the second harmonic generation process, we have already discuss in detail. So, again this equation will come into the picture when we deal with this 3 wave mixing. This is the general form of the second harmonic generation then a we start a concept call Parametric down conversion.

So, what is parametric down conversion we will try to find? So, the name suggest that we will try to down conversion the there is some sort of down conversion in frequency. So, in second harmonic generation we are launching a frequency omega and getting a frequency 2 omega. But for parametric down conversion what happened that we like to launch a frequency of 2 omega and try to find out whether we are getting to getting a frequency sub harmonic like omega naught.

So, how this process one can initiate and what is the condition to generate this kind of sub harmonic generation we will try to find out. So, let us go back to our slides.

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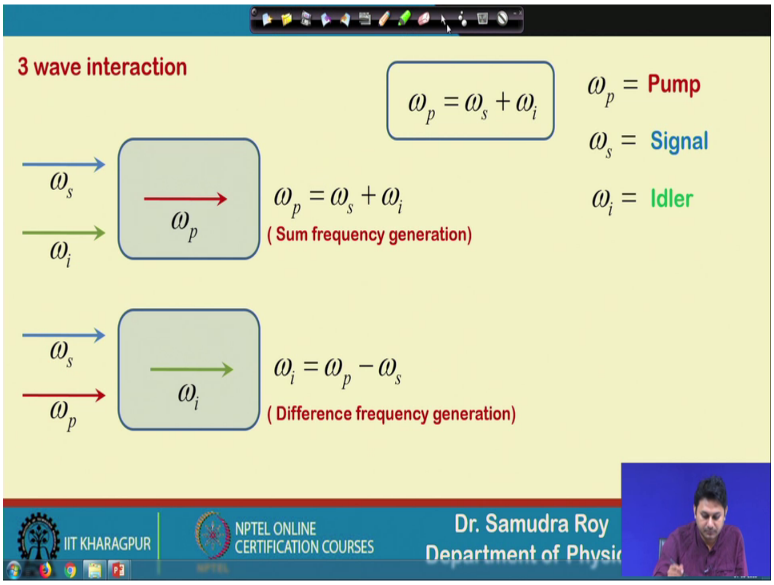

So, in 3 wave interaction. So, we introduced some kind of nomenclature. So, this nomenclature are very important that we called the omega p as pump the frequency of the pump, omega s as the frequency of the signal and omega i is a idler. So omega p, omega s and omega i they are related to a very simple equation omega p is equal to omega s plus omega i.

So, when I try to find out the sum frequency then we can say omega s plus omega i is equal to omega p. So, omega p is basically the sum frequency and the different frequency we can generate through omega p and omega s and omega p minus; omega s is our different frequency. And omega i is represented by omega p and omega s is given the corresponding different frequency.

In all the cases what happened that we should launch the signal both the cases. So, omega s is the signal that we will launch both the cases. For some frequency with omega s we launch omega i, for different frequency for omega s we launch omega p that is the difference well.

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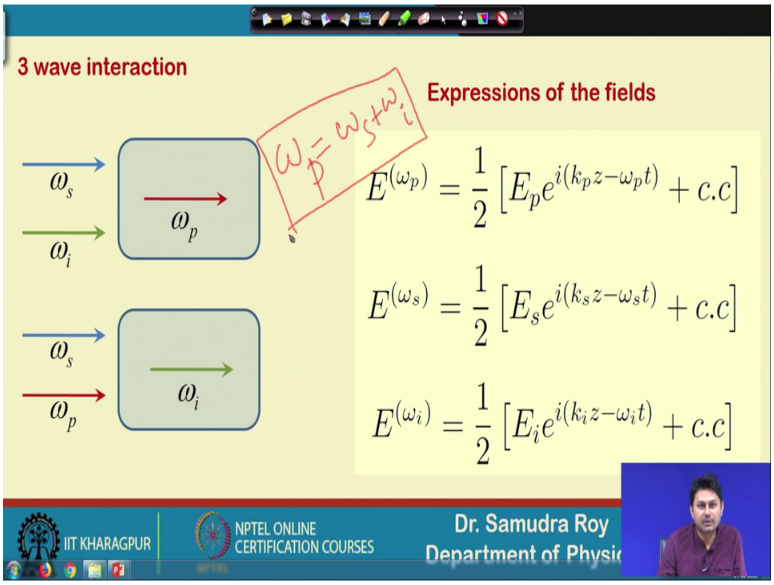

So, in the next slide we will have the expression of the electric field that we defined in our last class. So, this expression is straight forward that we have an amplitude of pump, signal and idler as E p E s and E i. The phase is associated with all the expression and inside the phase we have a propagation constant k p k s and k i for three cases and also the frequency omega p omega s and omega i. So, 2 frequency 3 frequencies are there omega p omega s and omega i and they are related to omega p is equal to omega s plus omega i as usual.

So, this equation always valid, we need to consider that omega p is equal to omega s plus omega i this equation is always valid. So, this is our fundamental some sort of fundamental equation. So, based on this equation whatever the treatment we will do that all this treatment.

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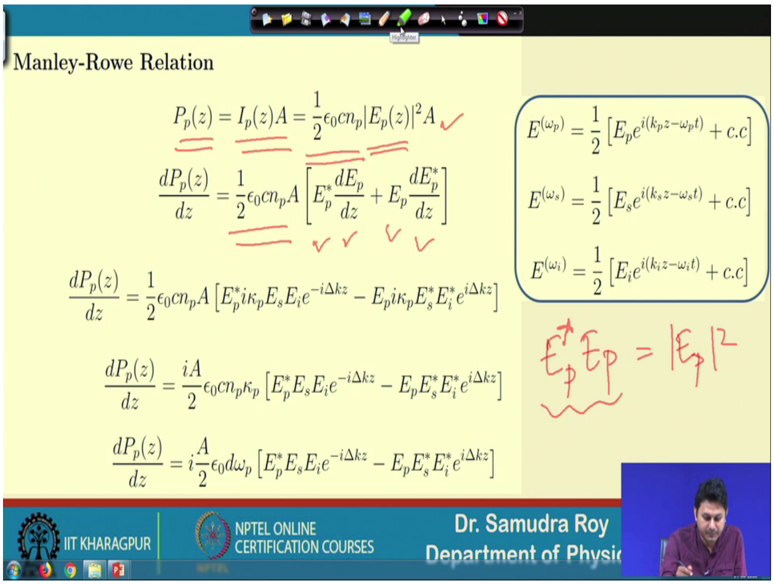

So, today we will try to find out the Manley-Rowe equation. So, the Manley-Rowe equation is something which gives us the conservation of energy or the conservation of 4 numbers.

So, here if I if we see we will find one important thing, pump is represented by I p multiplied by A, A is a area. So, we know that the intensity is half of epsilon 0 cn p and mode of E p square for pump multiplied by A. If I make a derivative with respect to z of this pump then we will have this term half epsilon 0 cn p A which is constant.

And then the derivative of this quantity which is not squares, so two term will appear because E p star E p is our mode of E p square. So, when you make a derivative there are two functions so, that is why we will first case we have E p then d dz of E p then E p d dz of E p star. So, once we have these two terms then the next thing is that we will going to replace this quantity, we will going to replace this quantity. Already in the previous class here all the E p E s E w term is there, but in the previous class we have already find out d E p dz was i d omega p n pc E s E i e to the power of i delta k. This was the term; this was the expression that we had derived in our previous class.

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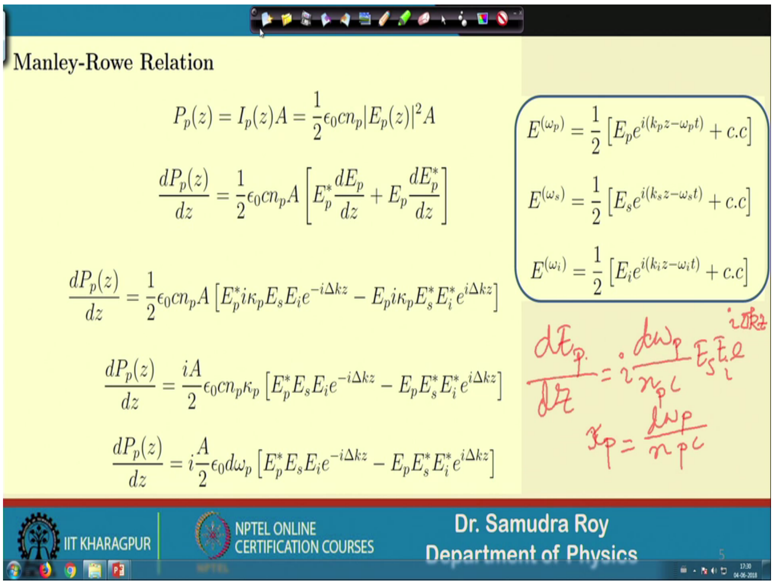

So, now what we will introduce one term called kappa p which is d omega p divided by n p into c then this expression can be simplified.

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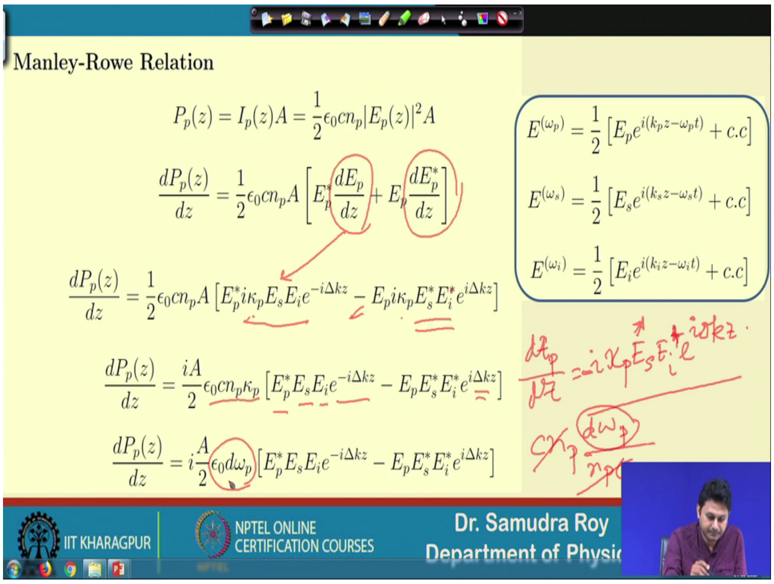

And this expression is now d of E p dz is equal to i of kappa p E of s E of i e to the power of i delta k z, I just replace this thing here. So, E p star multiplied by this quantity i kappa kappa p E s E i and e to the power of i minus of delta k ok, in this case there was a minus sign if you remember the previous class. Also, I replace this which is nothing, but the star of this things when I make a star so, one negative sign will have and this term will plus.

So, we will have a negative sign and plus here and also E s will become E s star and E i become non-star because E i so, here we have star. So, this then E both the cases there was no star. So, it will be E s star E i star ok, this is right. Now, what we will do we will just take kappa p common then i A by 2 epsilon 0 cn p kappa p.

And inside this bracket we have E p star E s E i multiplied by e to the power of minus i delta kz and E p star E s E i star e to the power i delta kz; further I can simplify. So, this epsilon 0 cn p kappa p can be simplified to as epsilon 0 d omega p. Why? Because epsilon 0 cn p multiplied by kappa p and what was my kappa p, kappa p was simply d of omega p so, divided by n p of c. So, this n p c n p c will cancel out we will have d of omega p so, which is here.

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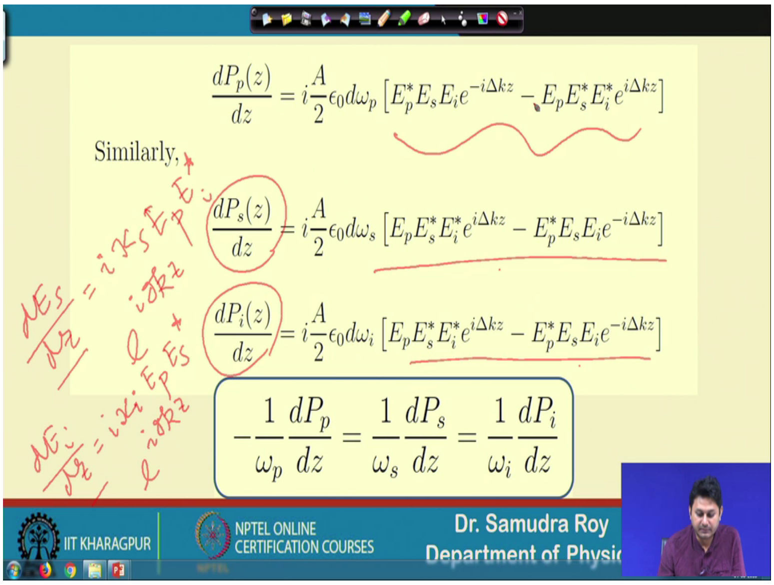

Next we will have this form. So, now, in the exactly in the similar way just using other 2 equation, just using other 2 equation d E s dz is equal to i kappa s E p E i star e to the power of i delta k z; this was the expression of E s d E s dz. And in the similar way we have another equation that is this i kappa p kappa i E p E s star e to the power i delta k z. Using these 2 equation again we can find out what is the value of the term dP s d z and dP i dz.

And if you calculate I again I ask the student to do this calculation by yourself and you will get a result something like this and this. And if you compare these three result you will find that whatever you are getting here exactly a similar term you will get here and here; in case of signal and idler. Only difference between these and this is one negative sign and if you have a negative sign here and here, because it is E p E s E i it is E p E p star E s E i, but here you have a E p E s star E i star which is this term and this term is here.

So, one negative sign is related to this and once you have a negative sign for all these cases then these 3 equation can be represented in terms of a more general equation if I divide this 1 by omega p here than the right-hand side will be 1 by A 2 and epsilon z and this in the bracket.

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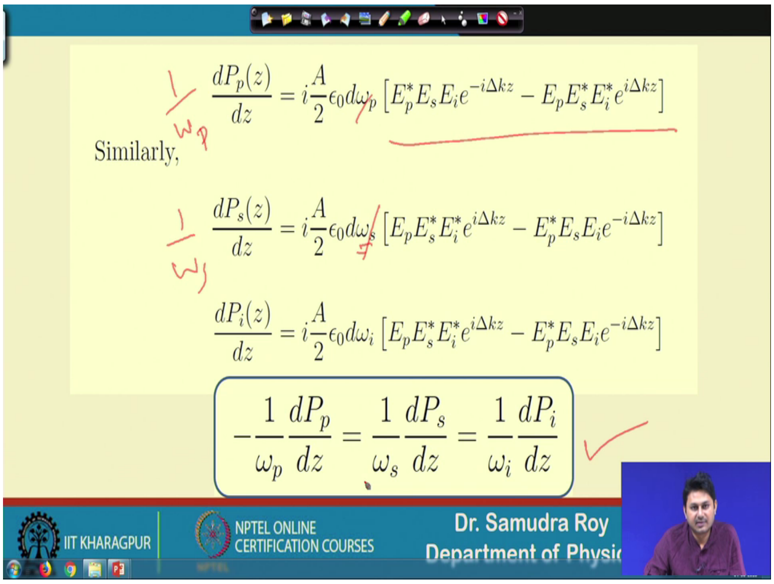

The similar way if I write 1 by omega s here so, this omega s will not be there and we have these something in bracket. So, these and this term 1 by omega p, this term and this term are negative to each other. So, this is equal to minus of this, in the similar way this is this term are same. So, I can write one expression that 1 by p with the negative term dP dz is equal to 1 by omega s dP dz is equal to 1 by omega i dP i dz. So, this equation which is in the bracket or which is in inside this box is called the Manley-Rowe relation.

This is the same equation that we have derived in case of second harmonic generation. We are eventually getting the similar kind of equation, but since we are dealing with 3 waves this equation looks slightly different. And instead of having 12 equation 2 relation we have 3 expression because of the signal idler and pump. Now, if the signal or idler are same then we have a two term here and these two term basically suggest that we are generating second harmonic.

But here since, we are not considering explicitly the second harmonic is generated because second harmonic is a special case. So, if any frequency omega s and omega i are generated from a pump omega p and then I can write this Manley-Rowe equation. And this Manley-Rowe equation suggest that they should follow this identity.

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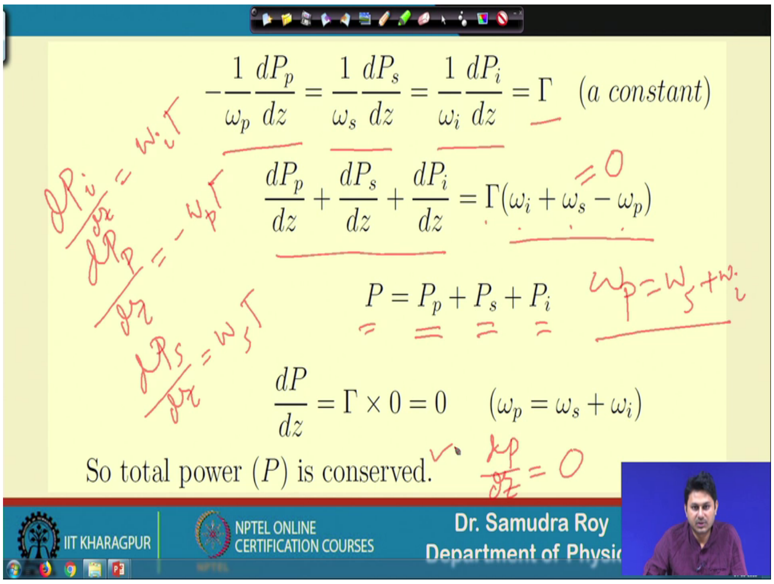

From this Manley-Rowe equation we can further derive important outcomes. And one important outcome is this is our Manley-Rowe equation that 1 by omega p P s is equal to 1 by omega s 1 by omega i P i dz. If this is a constant gamma then I can write dP dz dP dz. So, I can write this right dP dz is equal to minus of omega p gamma d of P s dz I can write omega s of gamma and d of P i dz I can write omega i of gamma.

Now, if I add these three things together like we have done here so, gamma will be common and we can write omega i plus omega s is minus omega p this. Now, we know that omega p is equal to omega s plus omega i that is true in all cases. So, from this equation I can write that this term is 0. So, total power if I write P p plus P s plus P i so, P this equation is nothing, but d of d z of P; gamma is here which is multiplied by this quantity which is 0. So, eventually we have dP dz where P is a total power is 0 or the total power is conserved.

So, Manley-Rowe equation is nothing, but the conservation of total power or the total energy. So, this is a another representation to show that from Manley-Rowe relation I can or we can derive a important thing which is the conservation of total energy. So, whatever the three expression we derived so, this three expression are consistent with the conservation of energy.

And that is important and we to show that this is really conserved the total energy. So, even though the energies are exchanged between pump and signal and idler, but every time the energy governs that every time the total energies are remain conserved. Here we show that the Manley-Rowe equation for these three expressions are in such a way that they can conserve the total power or the total energy. So, conservation of energy is valid here for this 3 equation that we have derived for pump signal and the other and idler ok.

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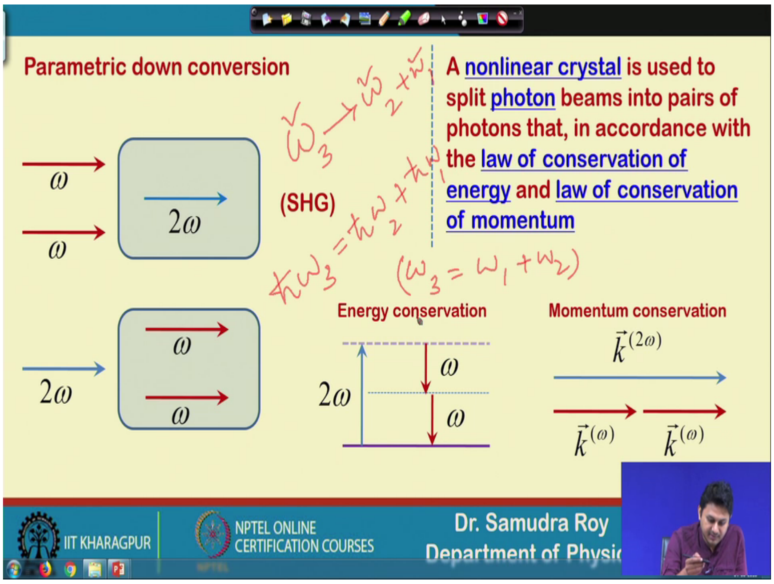

So, now, we will go to the another thing that is parametric down conversion. So, here what we are doing that in nonlinear crystal used to split photon beams into pairs of photon that, in accordance with the laws of conservation of the energy and conservation of momentum; that means, we are launching 1 photon and this photon can splits and generate to other photons.

So, in general if I write in general way. So, omega is a photon. So, omega 3 and it is divided to 2 photon omega 2 plus omega 1. So, from 1 photon I can generate 2 photons, but the generation in such a way the generation should be in such a way that the loss of energy and momentum is conserved. So, what is the energy here total energy if h so, h cross omega 3 this is the energy of the omega 3. It is from this we are generating 2 photons.

So, the energy of the 2 photons is h cross omega 1 and h cross omega 2 and h cross omega 1. So that means, the energy according to the conservation of energy my omega 3 has to be equal to omega 1 plus omega 2 that is my 1 equation; is a conservation of energy equation.

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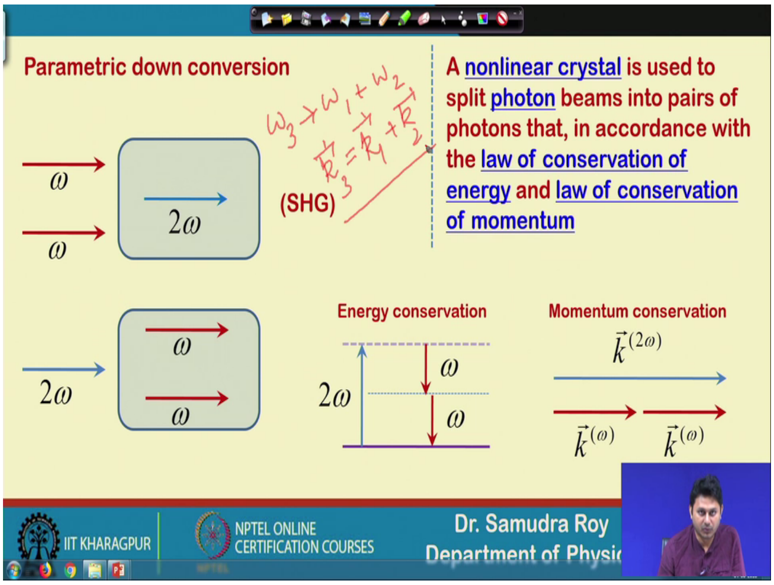

Now, for this same treatment from where were getting omega 3 is split into 2 photon omega 1 and omega 2; the momentum has to be conserved also. So, k 3 which is the momentum or the propagation vector has to be equal to k 1 plus k 2. This k 3 equal to k 1 plus k 2 is nothing, but the phase matching condition; this is another way to write the phase matching condition. So, this momentum and energy conservation is valid. What happened in under that condition, if we split 2 photons that or from 1 photon 2 photons are generated or 2 photon are merging to generate 1 photon that we try to understand.

So, here in the second harmonic generation process we find that for 2 photon is merging to generate 1 photon of frequency 2 omega and readily we can see that the energy and momentum conservation are valid. Here in the next case what we try to do that we try to generate 2 photon of frequency omega, but we are generating that from 1 photon of frequency 2 omega. This is some sort of frequency down conversion that we try to generate sub harmonics. So, energy conservation again this is a degenerate case omega 1.

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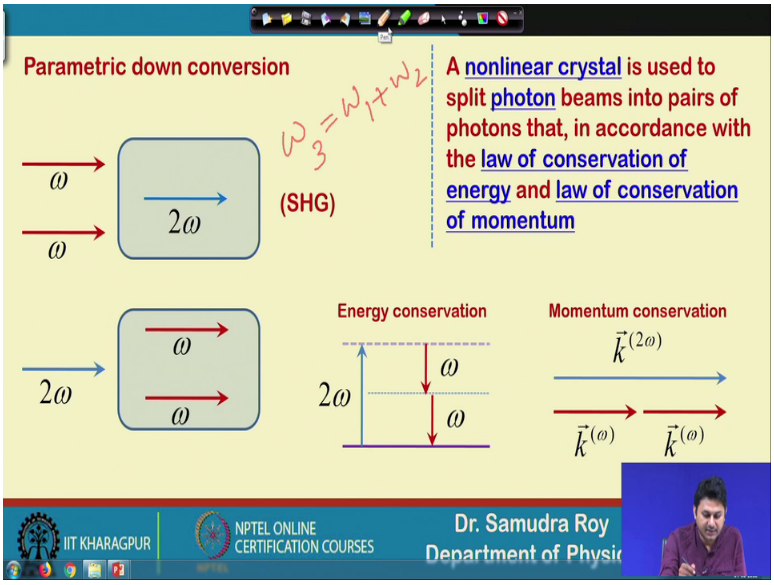

So, previously what I say that omega 3 was equal to omega 1 plus omega 2. So, if omega 1 and omega 2 are same then it is nothing, but the second harmonic generation. In the similar way, I can write that for degenerate case I can generate 2 omega from 2 omega I generate omega 1 omega and omega because, omega 1 and omega 2 same right now. And a momentum conservation suggest that this is something like this which is nothing, but the collinear phase matching kind of stuff.

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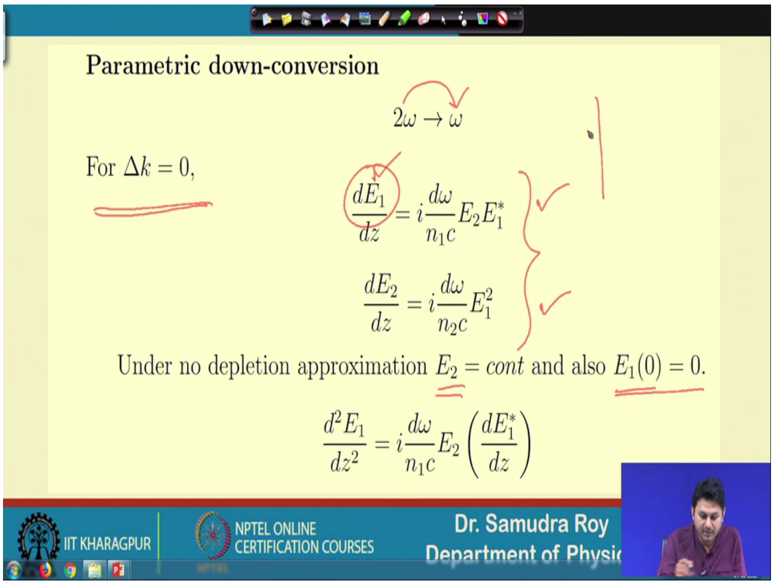

Now, the question is really it is possible to generate or not? It is really possible to generate. So, in parametric down conversion what we are trying to do from 2 omega to omega I want to generate; 2 equations should be in our hand and these 2 equation is this. This 2 equations the governing equation of the fundamental wave and second harmonics so, now I generate second harmonic two fundamental. So that means, I am generating E 1 in our case delta k is a phase matching, we considered the phase matching is already there; that means, the momentum conservation is already valid.

So, momentum conservation is valid, energy conservation is valid. So, now, our aim is to find out whether we can generate some kind of sub harmonic under this kind of condition or not. So, sub harmonic means try to find out the evolution of E 1. Under no depletion approximation E 2 is constant and E 1 is 0. So, what is going on here if I write, if I draw a picture here this picture should be something like this.

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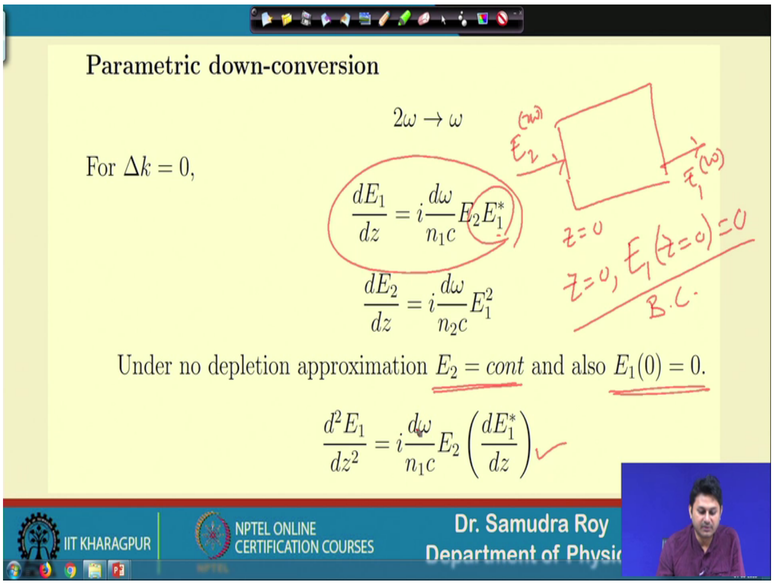

This is E 2 with frequency 2 omega and try to generate here E 1 with frequency omega. The question is really it is possible to generate? This is at z equal to 0 so, at z equal to 0 what happened there is no wave of frequency omega. So, this quantity has to be 0. This is our boundary condition fine.

So, two condition we consider one is E 2 is constant and another is E 1 0 is 0. We have this expression in our hand so, what we do you make a second derivative to solve this equation. When you make a second derivative one equation will come as dE 1 star dz, here E 2 is constant so, there will be no derivative of this term..

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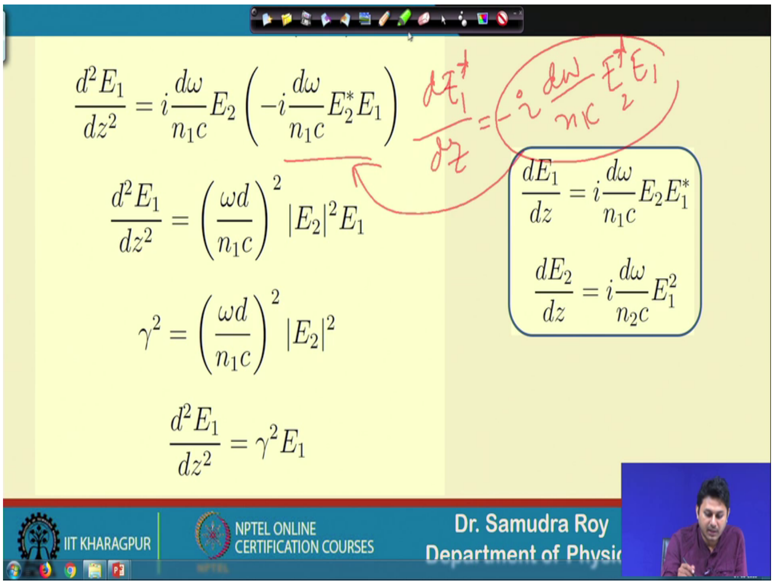

Once we have this, then the next thing is we just replace in terms of d E 1 star dz; we replace because d d E 1 dz already we know this is the value. So, I need to replace this minus i because complex conjugate of this will be d 1 star dz. So, it will be d omega n 1 c E 2 star E 1; here should be star and this should be E 1 because I making a complex conjugate of that. So, I just replace this things here as I do and once I replace this thing here; then I can find the this i i will remain 1 d omega n 1 c d omega n 1 c E 2 E 2 star.

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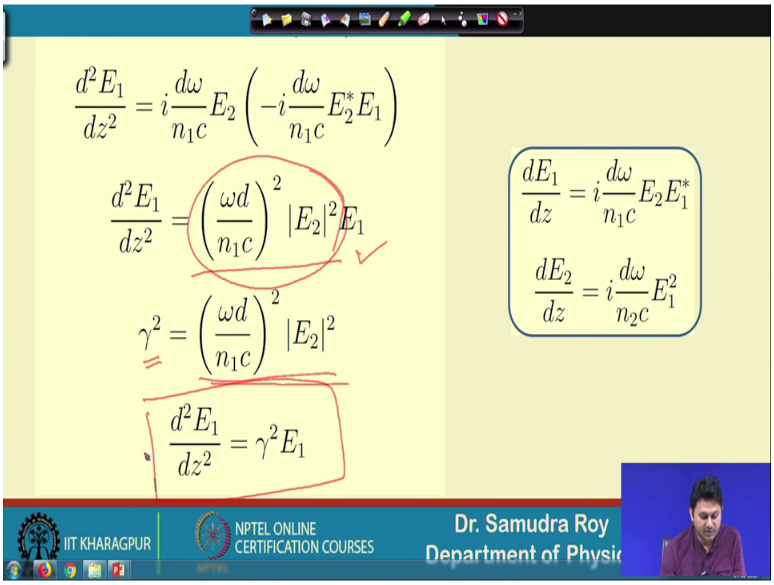

So, I can write it in this way omega d divided by n 1 c whole square of that and mode of E 2 square and E 1. The next thing one can do is make this entire thing as a constant because omega d n 1 c these are constant. And E 2 is a is a pulse here or E 2 is a field associated with the second harmonic or the frequency 2 omega and from the beginning we considered this is very strong. So, we can consider this has a constant also.

So, that is why we write a constant here gamma which is this. So, gamma is omega d n 1 c square and mode of E 2 square we write it as gamma square. So, the total equation become simplified and we have in our hand is second order differential equation with the form d 2 E 1 divided by dz square is equal to gamma square E 1. This is a second order differential equation and also we know what is the solution of that. So, what kind of solution we will have?

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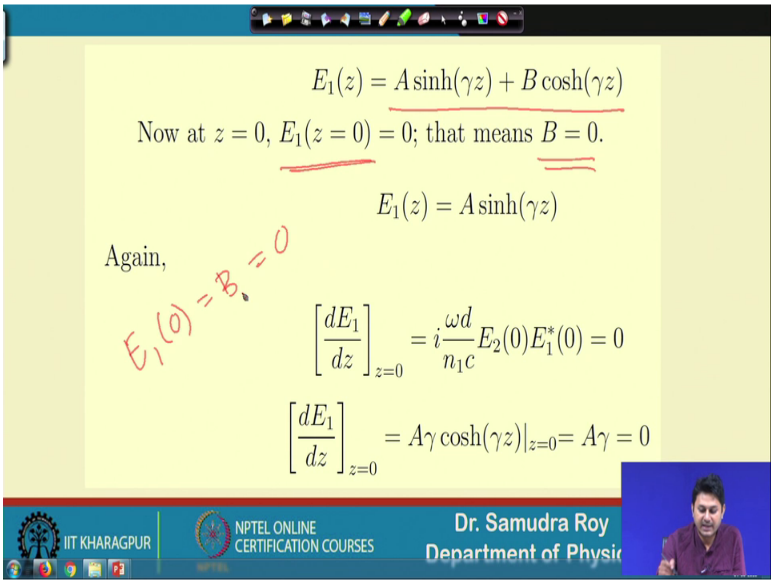

So, we will have a sin hyperbolic kind of solution or cos hyperbolic of solution. Eventually we have a exponential hyperbolic solution and this exponential hyperbolic solution one can write very easily with this sin and cos hyperbolic. So, this is a general solution. Now, if I put the boundary condition so, the boundary condition. suggest that E 1 is z equal to 0 because at the beginning there is no sub harmonic waves.

So, if I do then readily at if I put z equal to 0 then I readily can see that this is 0 and this is 1. So, we have B, but the entire thing is 0 means B equal to 0. So, E of 1 0 which is B and this quantity is 0 ; that means, B is equal to 0. Once B is equal to 0 I can eliminate this term.

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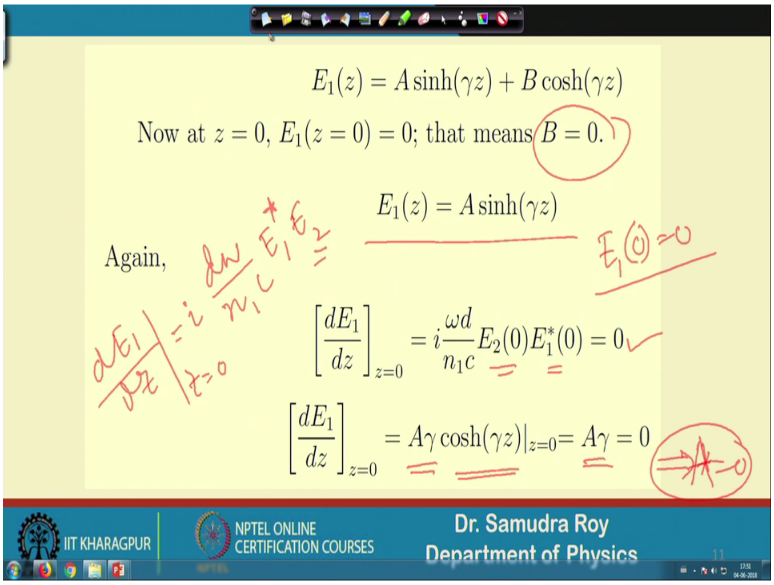

So, we will have simply A 1 sin hyperbolic gamma z as a solution of E 1 z. Next thing is to find out A and in order to do that what we will do, we just make a derivative of this term. If I make a derivative we know that d E 1 dz is i d of omega n 1 c E 1 star E 2 that was our equation. So, this quantity at z equal to 0 if I want to find then it will be i omega d n 1 c E 2 at z equal to 0 and E 1 star at z equal to 0.

Now, E 1 star at z equal 0 is 0 because I consider we considered there is no field. So, E 1 at z equal to 0 and E 1 star at z equal to 0 at the same thing here amplitude is not there or vanishing so, this quantity is 0. From here also we can find out d E 1 dz, if I do then I will have A gamma cos hyperbolic of gamma z at z equal to 0. So, at z equal to 0 cos hyperbolic of gamma become 1. So, we have A gamma equal to 0 or from here we can see gamma is a constant so, A equal to 0. Already we get B equal to 0 now; the next thing I get is A equal to 0 A equal to 0. So, once we have A equal to 0; that means, the entire E is vanished there is no E 1.

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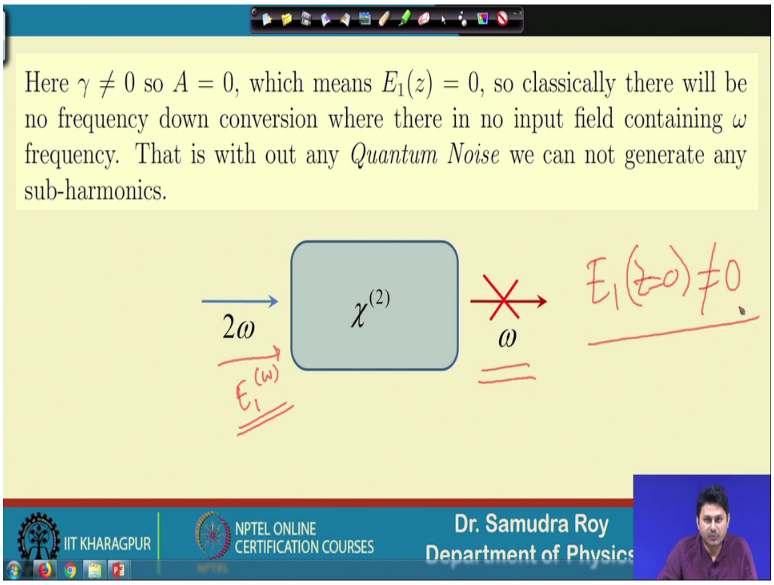

And that is interesting that here gamma is not equal to 0 as I mentioned. So, A is equal to 0 which means E 1 z equal to 0; that means, there should not be any so, the thing is that we will have 2 omega here which is launched. This is the medium where chi 2 is not equal to 0, but this medium suggest that there should not be any kind of omega field generations. So, omega field will not going to generate so that means, there will be no sub harmonic generation.

But in generally we find that the sub harmonic are still generating; that means, if I launch 0 omega, omega can be generated. So, classically we find that there is sub harmonic generation is not possible unless, we put some kind of varies tiny amount of electric field here with field omega. So, I need some amount of omega. So, the boundary condition E 1 at z equal to 0 should not be 0 that is boundary condition we now, put to generate some kind of sub harmonic classically.

But quantum mechanically we can see that some kind of quantum noise should be there and because of this quantum noise there is a possibility that we can generate sub harmonic. So, without any quantum noise it is not possible to generate. So, in the next class so, let me conclude here today. So, so far we are dealing with the sub harmonic generation.

So, this is a parametric process; that means, I am launching 2 omega and try to generate omega. And this process classically it not possible in this process, we find that classically it is not possible to find out any kind of sub harmonic waves. In order to generate sub harmonic waves what we need to do, there we need to put some kind of input of this sub harmonic field. And in the next class you will find that even if I put the input that may not be amplified.

So, there is a possibility that even if I put some kind of input value in terms of E 1 or the fundamental wave; so, it will not going to increase and it may be decrease depending on the phase, initial phase of the system. So, in the next class we will find out how the phase will be important here and this phase sensitive amplification and phase sensitive attenuation we will discuss in detail in the context of optical parametric amplification. So, with this note let me conclude here. So, in the so, see you in the next class and,

Thank you for your attention.