Introduction to Non-Linear Optics and Its Applications
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Lecture - 32
3 Wave interaction, Equation for pump, Signal and idler wave, Non-collinear phase matching

So, welcome student to the next class of Introduction to Non-linear Optics and its Application course. So, today the lecture number is 32. So, we will going to learn few important topics today.

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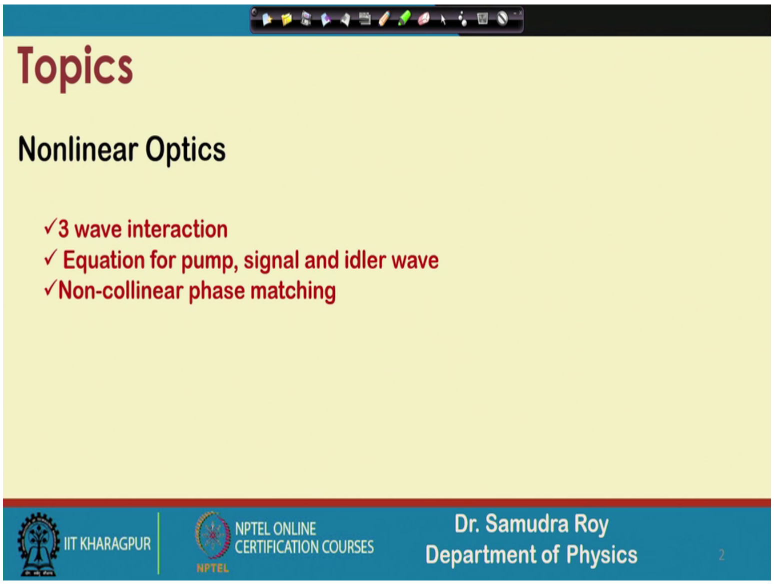

So, first topic that will going to learn is 3 wave interaction, in the last class we have already started the concept what is 3 wave interaction and what are the nomenclature for that. Then we try to find out the equation of pump signal and idler wave and finally, the non collinear phase matching.

So, far we are dealing with the phase matching and when the vector diagram of the phase matching was shown, it was normally collinear is collinear in nature. But now today we will going to show that if we generate some pump and signal and idler and pump, then it is not necessarily that they are same. So, that is why the wave vector may have a different direction.

So, the phase matching may not be collinear in nature. So, there is a non collinear kind of phase matching. So, in that case how to find out the phase matching and other issues we will going to take care. But before we will before a going to the non collinear phase match, we will learn about the 3 wave interaction.

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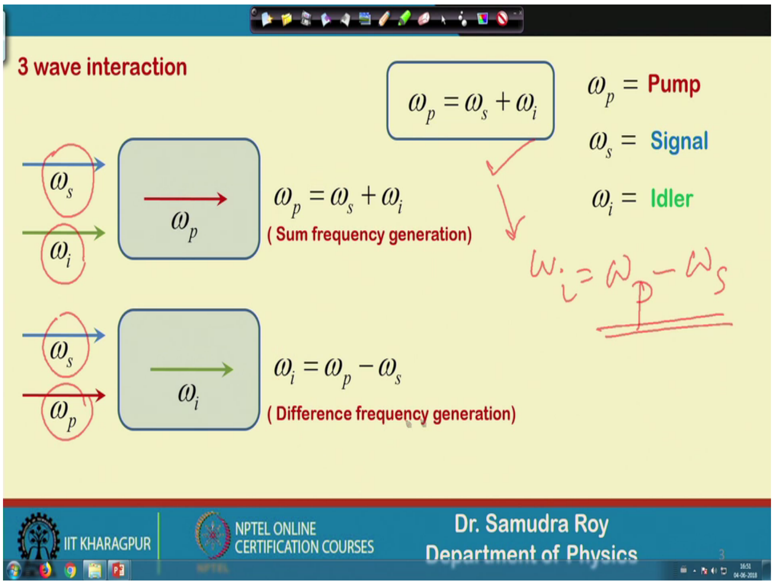

So, let us go back to the slides. So, 3 wave interaction as I mention this is associated with 3 wave as the name suggest. The nomenclature is I will going to launch one signal and this signal along with this signal if I launch another wave if I call idler, then this signal and idler they will going to form another wave we called it pump.

So, pump is nothing but the summation of signal and idler. So, this is if I look carefully this is nothing but the some frequency generation; that means, in the previous cases we use omega 1 plus omega 2 which was omega 3 this is some sort of sum frequency. So, here we are doing the same thing, but we change the name instead of using 1 2 one 3 now we change the name as pump, signal and idler that is first case or the first consideration. Second thing that one should remember here that in this treatment always the signal will be here. So, every time what we will going to launch is signal that is for sure, along with that we will going to launch idler or pump depending on which kind of wave we going to generate or we wish to generate.

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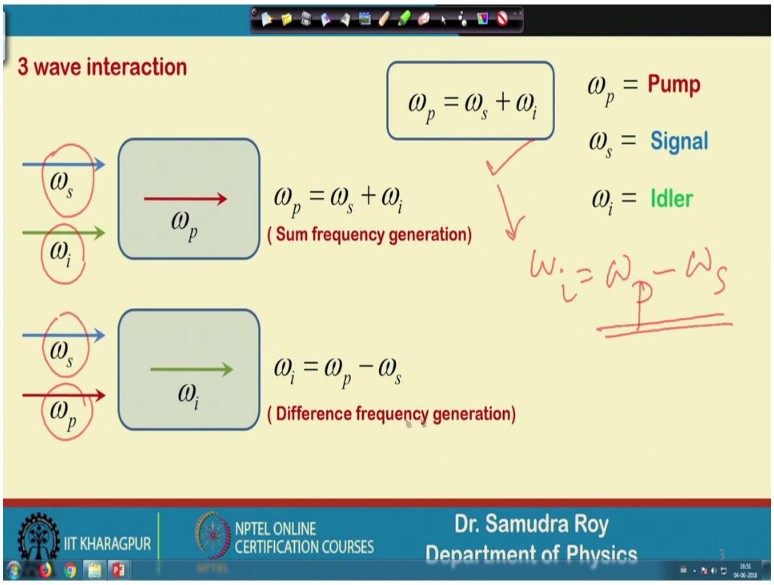

So, in some frequency generation it will be omega i because omega s plus omega i is equal to p this equation is a fundamental equation for this nomenclatural treatment, that we will always follows. And secondly, when we try to find out omega i which is nothing but the difference between these 2 frequencies omega p and omega s, from this equation we can readily find omega i is equal to from this omega i is equal to omega p minus omega s.

So, that is mean. So, that is we will going to launch these 2 and the difference between these 2 basically gives omega i. So, this is nothing but the difference frequency generation, but the nomenclature is different one should remember the nomenclature omega p is pump omega s is signal omega i is idler, omega p is normally greater than signal and idler.

So, omega p is omega s plus omega i that one should remember, and also in all cases one should consider that omega s is launched always. That means, signal is always in the left hand side it will going to launch we will going to launch signal every time. So, in some cases pump will going to generate and in some cases the difference frequency will going to generates or ideal idler wave will going to generate.

Next once we have the nomenclature in our hand, we need to express the fields for 3 cases. So, these are the field expression for 3 cases. It is exactly the same thing that we have been using when we use the nomenclature as 1 2 1 3 here we will going to change slightly that for omega pump we write omega p and at the same point we will write amplitude as E p.

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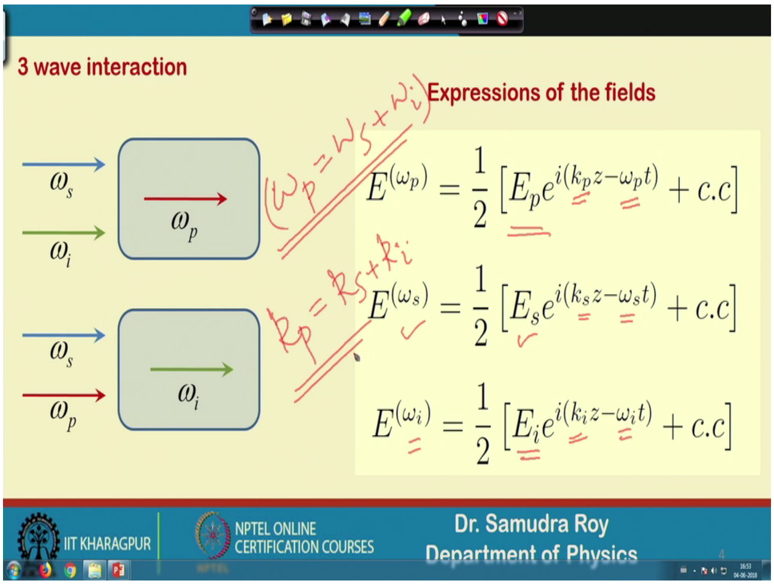

E p is amplitude of the pump wave k p is the corresponding wave vector and omega p is a frequency. For signal it is same omega E s is the amplitude of the signal k s is the wave vector, and omega s is the corresponding frequency. For idler also I can write E i is the amplitude k i is the corresponding wave vector and omega i is the corresponding frequency. All the cases the equation omega p is equal to omega s plus omega i is valid. So, this is our fundamental equation, in terms of k it is k p is equal to k s plus k i. So, momentum and this is the energy conservation equation and this is some sort of momentum conservation equation that is here.

So, after having the nomenclature, now it is time to find out one important thing and that is the corresponding non-linear polarization term; when these 3 waves are there. So, now, these 3 waves already I mention this is the 3 waves that we have in the system. Once we have 3 waves the total field E is now represented by the pump field, the signal field, and the idler field this is the total field.

What should be the value of polarization where the total field is represented by these 3, again I have written here in the right hand side what about the polarization?

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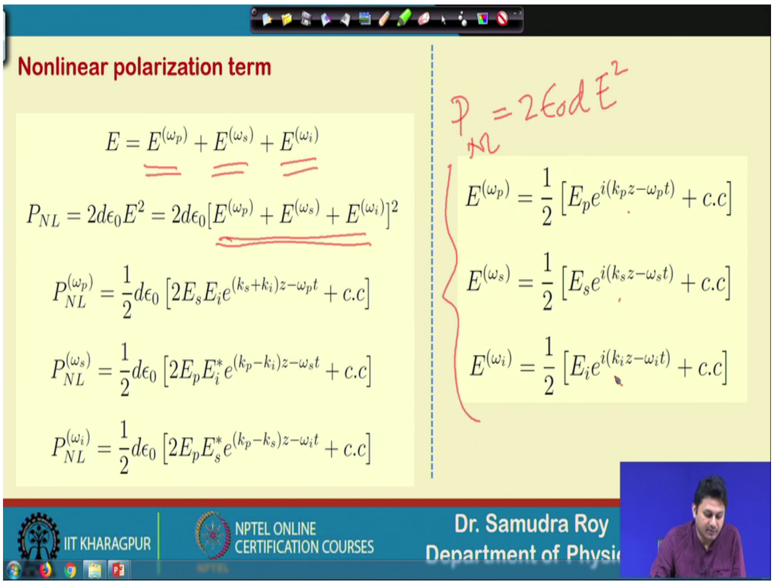

We know that the polarization non-linear polarization rather is equal to 2 of epsilon 0 d in the scalar form simply E square. So, this E is now E omega p e omega s plus e omega i. So, once I put this E in terms of omega p omega is omega i should be e omega p plus e omega s plus omega i and whole square of this quantity. When I make a square of that now the interesting thing will come, because I am making the square of this plus this plus this.

So, because of this square term the frequency mixing will be there. But we need to find out what is the value of different what is the value or what is the different frequency that I am looking for that is important here. So, let me let me see it here. So, P N L of omega P; that means, this is a P N L term this P N L term contain many terms because when I make E 1. So, this is a plus b plus c whole square something like that, but also the complex conjugate terms of their. So, you are making square of that. So, we will have a square b square c square plus 2 a b, 2 b c, 2 c a this kind of combine term will be there. Whenever I have a b b c c a then we will have the addition of all these terms.

So, omega p plus omega omega s omega s plus omega i or omega p plus omega i these kind of term will appear not only that because of this complex conjugate we will have omega p minus omega s or omega p minus omega i this kind of terms is also there. So, I left this problem to the student to find out how many different frequency component; you can have for this case when E p E omega s and E omega i is shown here. But by just inspection you can find many thing that is the important thing you need to look very carefully and when you look very carefully by inspection you can find many things here. For example, there are many terms in P N L very frequency terms, but if I want to find out what should be the frequency term, what should be the P N L containing the frequency omega p, then we need to look this term very carefully.

So, how the omega p can be generated first term is omega p, but we have a squared term; that means, it should be omega p squares will not going to get omega p. So, omega p can only be generated with 2 term because in our hand.

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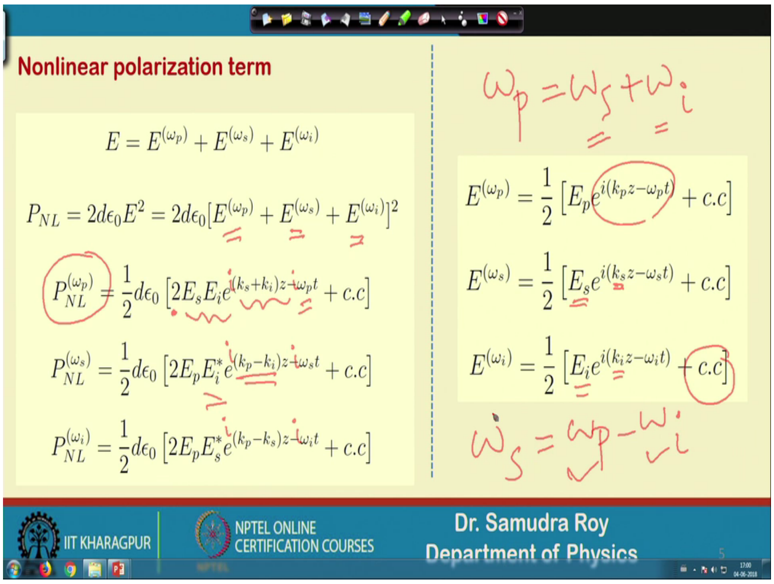

We have omega p is equal to omega s plus omega i this is always true so; that means, omega s and omega i when these 2 term will multiply each other, then I can have omega p frequency component term.

So, in this a plus b plus c whole square we will have one term 2 a b c, 2 a b, 2 b c to c a. So, here this term and this term when these 2 term multiplied then we will have a frequency component omega s plus omega i. Omega s plus omega i is nothing but omega p. So, now, if I take these 2 term from here omega is an omega I can multiply these 2, we will have omega s omega i which is here a 2 term will be there, because it is 2 a b 2 b c 2 c a. So, this 2 will be there because of this degeneracy factor and in the exponential term we have the since we are multiplying these 2 terms, we will have k s k i with a addition.

So, we will have k s plus k i and also we have omega s plus omega i with a negative sign, which is nothing but minus of omega p. So, that omega p is nothing but the omega p component that we are looking for. Also the complex conjugate term one complex conjugate term will be there, because all the complex conjugate term is sitting here, when you make a square of that we will get all the complex conjugate.

So, complex conjugate of this term one can also find. So, here one i is missing. So, we have to be careful enough. So, one i is missing here, i is missing here. So, this there will be i is actually during the typing some amount is not there. So, what about the omega s term? Omega s term is something which also we can find from this equation what about the mother equation or this main equation I am saying. So, omega S here I can write omega S is equal to omega P minus omega i. So, omega P and omega i these are the 2 components that will be important here to find out omega S.

So, omega p; that means, this term and omega i with a negative sign means the complex conjugate of this term. So, when I make a complex conjugate term multiplied by this term then I will really get this omega s term. So, complex conjugate if I multiply I will have k p here. So, I will have k p minus k i. So, that is why here we have k p minus k i and also if I have E p and complex conjugate; that means, I have E i star here. So, I have E i star here and a frequency components simply omega s, because I am getting omega p minus omega i.

So, minus omega i is coming from this complex conjugate term and I am getting this. Also P N L omega i P N L omega i how do I get that, P N L omega i is again I can find out from this equation.

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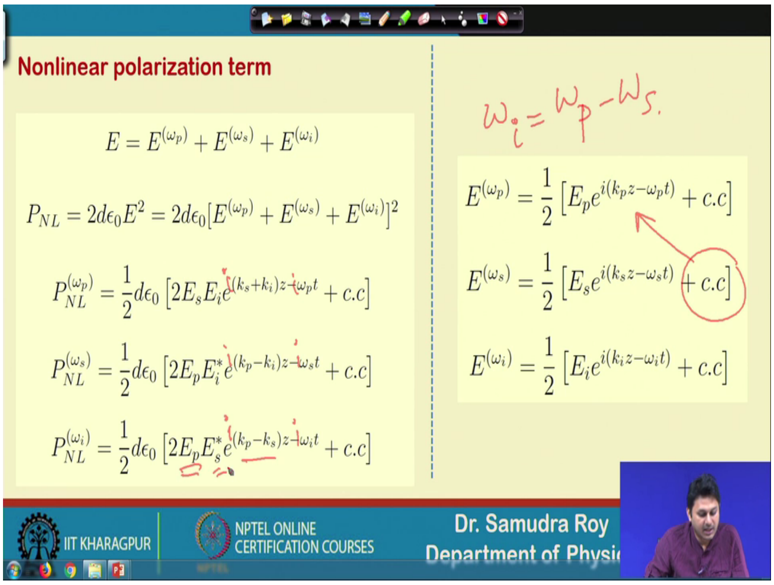

So, omega i mind it here we should have one I i sitting here i i it is said typing mistake that is why this i is missing, this one of from here we can see that when I multiply this and this i will remain.

So, what about the last term? So, omega i will be simply omega p minus omega s. So, omega s is negative so; that means, I am using this complex conjugate multiplication of this complex conjugate with this term. So, when this 2 term will multiply to each other in this a plus b plus c whole square term under this term, then we will have a k p e p which is this k p minus k s as usual as the previous case and this complex conjugate. So, we have E s star. So, E s star.

So, once we have this 3 equation then we have the equation of p non-linear for omega p omega s omega i and also the total fields. So, now, it is time to derive once we have these things. Now it is time to derive the corresponding equation for that the corresponding equation for that.

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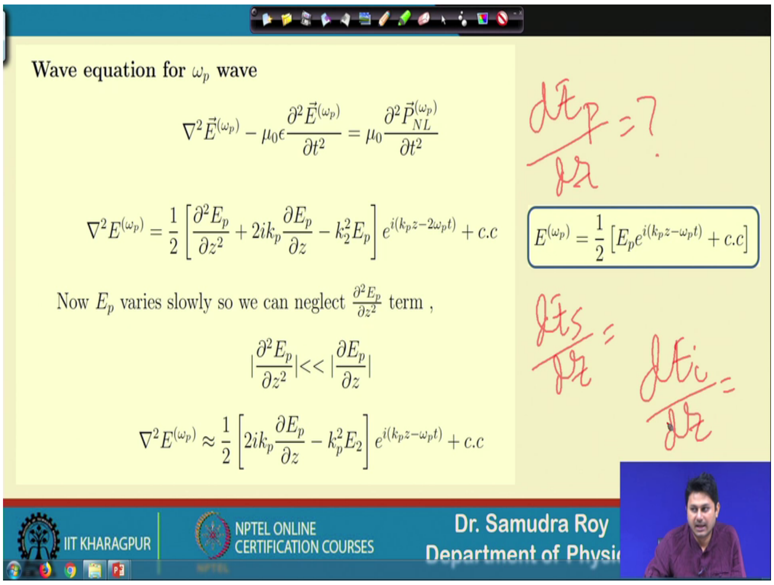

So, equation means I try to find out the evolution. So, my goal here is to find out the evolution of evolution of this E p d z how this term will be there? D E s d Z how this term will be there and d E i d Z.

So, this term we will going to find out, the evolution of pump signal and idler. So, now, we will going to use our old Maxwell's equation with non-linear polarization terms. So, this is our Maxwell's equation in non-linear polarization term E is given. So, when I make a grad square term because first term when I evaluate, I will get the second derivative of the first term second derivative of this function which gives this thing and 2 of the first derivative of first term and second term. This is the standard procedure that we have already used in our previous calculations then we will have this things with exponential term sitting here.

Now, this in the derivative, the second order term is much much smaller compared to the first order term. So, what we will do we will going to neglect these things. So, will going to neglect this thing; once this things is neglected the equation is relatively is simpler and we will get 2 i of k p del E p del z is equal to minus of k p square e 2 and then multiplication of these exponential term and complex conjugate will always be there. So, one thing you should remember that here we are try to find out the evolution of the pump, you can do the same thing for signal and idler only thing is that you need to just put proper P N L and proper electric field function so that you can solve this equation in terms of their amplitudes.

Well, once we have a the value of these things the value of the grad square, then we will have the second equation part that mean time derivative. So, time derivative this is the total expressions. So, I am making a time derivative. So, when I am making a time derivative with respect to that, then only this exponential term containing times.

So, we will have minus of omega p.

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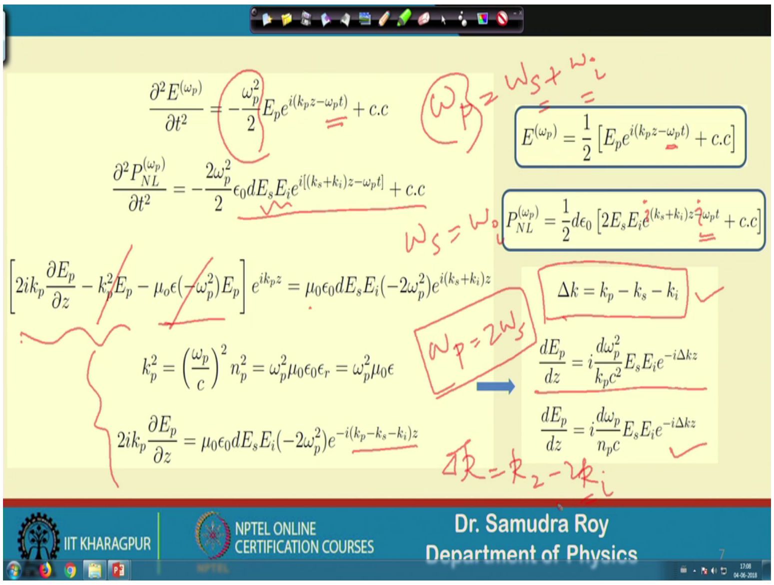

So, once we have minus of omega p we will get this term here with a half because half is already there what about the polarization term again here i is missing. So, we have to be careful in enough that one i should be there in all cases exponential i somehow missing. So, when I making a derivative with respect to t for p non-linear the time is sitting here only. So, we will again have 2 term omega p twice i omega p and i omega p should be there a negative sign will be there, and omega p square term will be there 2 is already there. So, 2 and 2 will cancel out. So, we will have term like this.

So, now all the 3 terms are in our hand derivative terms space derivative time derivative for both the cases is there. So, I just put these 3 time together in the non-linear Maxwell's equation, then the first contribution is this the time contribution here I can write in -this way because omega p square is multiplied by mu 0 epsilon in the equation and then omega p is omega p t is same for all the cases. So, I will just cancel it out. So, when I cancel it out I will have only the term containing k p.

So, in the right hand side we will have this term and this term is multiplied by mu 0. So, mu 0 multiplied by epsilon 0 as usual and d E s E i which is the amplitude term here, and 2 omega p square with a negative sign is sitting here. So, now, we will see that this term and here term will cancel out why this is cancel out? You can see from here this is the equation this is the expression which suggest that how this 2 term will going to cancel out.

Now, after cancelling out these 2 term we will have 2 i k p this things e to the power this and this I can write this e to the power k term here. So, it will be e to the power minus of i k p minus k s k i and in the left hand side we have this. So, just solving in one or 2 step, I can write this is my expression where delta k is this quantity. And finally, we will have the expression of the pump field and the evolution of the pump field and evolution of the pump field of the amplitude of the pump field rather is d E p d z is equal to i d omega p n p C p s E i e to the power the phase term.

This is exactly the same term that we have derived in our second harmonic generation process. In second harmonic generation process the process was d generate because from omega I can generate omega 1 from omega I can generate 2 omega, but here what we are doing here we are generating this. So, please note this change here my formula is omega p is equal to omega s plus omega i

Now, in this case from omega p is generated from omega s and omega i. So, omega s and omega i are generating omega p, if omega s and omega i are not same to each other then it is non-degenerate k. But if it is omega s is equal to omega i then we can have omega p is nothing but 2 of omega s which is nothing but the phase matching the second harmonic for this and the phase matching here is also like same.

So, in that case if you remember delta k was k 2 minus 2 k 1; now k p if I considered 2 then k s and k i if I say this is same because omega s and omega i consider same. So, here also I can write k s and k i are there same. So, k s and k i can be represented 2 of ks which is nothing but here i so, the fundamental right or the signal. So, we are getting the similar kind of expression, but since we are dealing with 3 with. So, we will supposed to get 3 equations.

So, one equation I am getting. So, now, it is your job to find other 2 equation if we do if you find the other 2 equation, then the other 2 equation will come in the similar way and you will get these 3 equation in your hand. This is the 3 evolution equation of the wave please note carefully.

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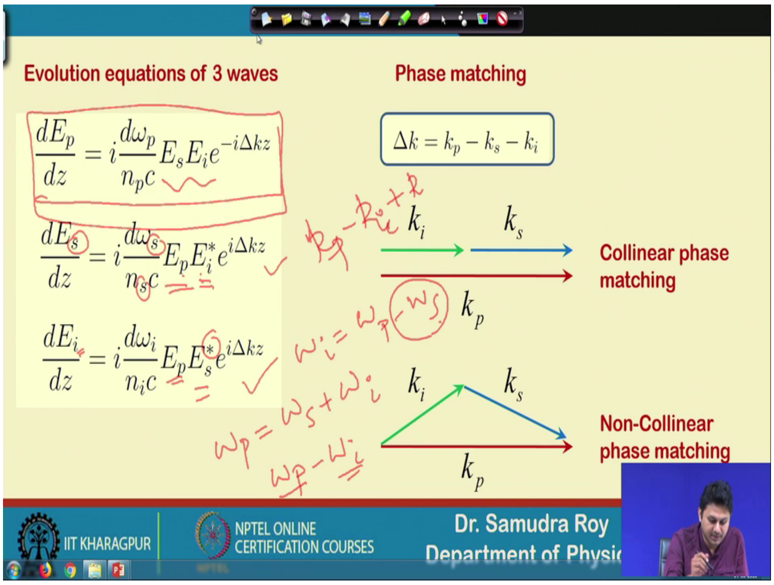

That this is the equation we have derived here in this class. So, these 2 equation I asked the student to derive it by himself or herself and one can readily find out the relationship between these two.

So, for signal what is changing? Here omega is will come n s will come and these 2 term you can see that omega p if you remember then you need to do not need to if you remember this simple thing, you do not need to remember all this calculation or all this equation you can it can come very nicely to your mind that if omega p omega equal to omega s plus omega i. So, electric field containing the frequency omega s can be represented in terms of omega p and omega s omega i is like this.

So, omega p and omega i omega is negative. So, the field here has to be p and for negative omega i it should be E i star. This 2 combination can only give you omega s in the similar way you can understand for E i; that means, idler; idler is how much omega p minus omega s so; that means, we will have omega E p here and for this minus omega s we have E s with a negative sign. For phase matching also we will have the same thing, here we are dealing with omega s omega p; omega p is minus of k s k p k s k i according to our notation.

So, once we have minus of k s k i these things, when I have k s. So, what essentially will do that k s will come forward k i will come and then k p will go so; that means, one negative sign additional negative sign will be there because here we can see that E p multiplied by E i star. So, E p contain k p and multiplied by k i star will be minus k i, but also I am having a term which is class of k s. So, let me write it clearly. So, the phase term here, it is k p minus k s minus k i.

So, here I need to find out what is my s. So, when the s is here. So, there will be negative sign because k is these 2 things have to be positive and then we will get the rest of the results. Well I believe the students that they can derive these 2 equation the way it is shown here. So, they can be like that. So, now, one thing we need to discuss here quickly, that is the collinear and non collinear type of phase match. For collinear phase matching we can see that they are in the same direction, this kind of phase matching this is the phase matching condition. So, delta k equal to 0 is a phase matching condition, and in vector diagram we can say that it is happening when these 2 are same direction.

But there is a possibility that they are not in same direction, if I consider this is a vector notation. So, this vector and this vector they can add to generate this vector and they can follow a triangle of rule to get this non collinear kind of a phase match. So, this is a different kind of phrase match appear when we are dealing with 3 different waves.

So, now how to find out the phrase matching, what should be the angle that is important for non collinear kind of phase matching. So, this is our delta k.

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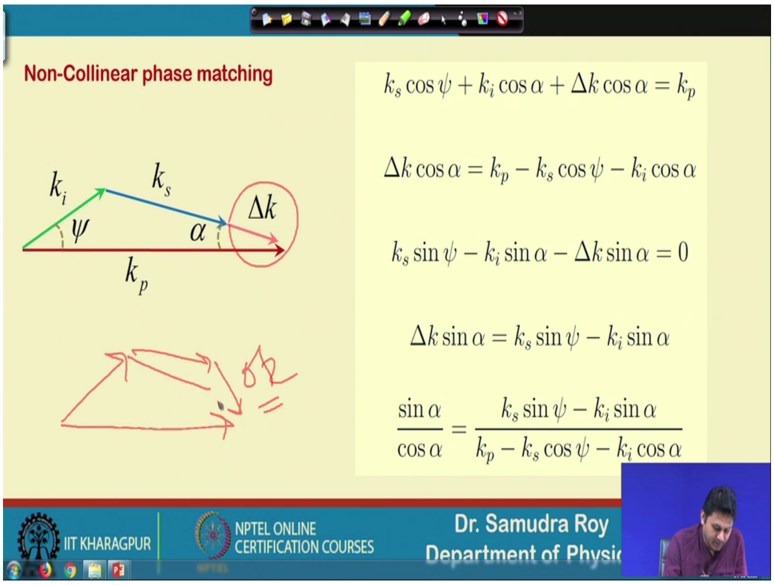

Delta k means this is the mismatch that we have this mismatch will be minimized one can see very easily this mismatch can be minimized, when this is in same line for example, if we have this, this and if I this and if this is my delta k

So, this direction this direction is not the preferred direction, because this delta k is not the minimized delta k. So, these delta k will you minimize when it is in the same line as shown here. So, how to find out this alpha and what should be the amplitude for which we will getting the phase matching is shown here. Once we know this is in the same line so that means they are following a triangle that is the condition. So, if this is psi and this is alpha, psi is angle between the i and k p where k p is the propagation constant on pump and I is the propagation constant of idler.

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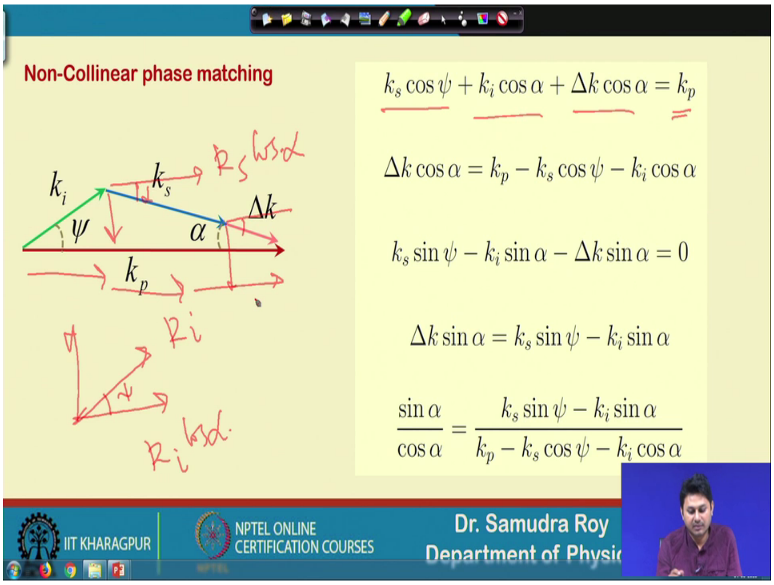

So, now if I try to find out the different components you can see that the components of k i k s and this is the components the x components, and if I add this all the x components I will get the total k p. Because k i can be represented in terms of this and this if this is k i if this is angle is psi. So, this component is k i cos of alpha. In the similar way if I write k s k s also I can divide into 2 part one is this and is a perpendicular component.

So, this component which is here is simply k s is of cost of alpha because this angle is alpha. In a similar way for delta k also I can divide into 2 part here and here since this is alpha. So, we have this is alpha. So, once we add all this k x components it will be this plus this plus this. So, these 3 things will give you the total k p k p is a amplitude which is in this direction.

So, from here we can write delta k is this quantity, in the similar way I can divide these into perpendicular component. So, if I do. So, perpendicular component this direction is perpendicular direction this, this. So, if I divide if you look carefully it is in down stair, it is also in down stair not in up stair. So, if add this 3 things. So, has to be 0 because there is no perpendicular component of k p, it will not going to equal to k p any component of the k p so, that is why this is 0.

Once this is 0 since this perpendicular component are vanishing each other, then I can have delta k sin alpha into this quantity. From here to here if I divide then this is coming as sign of alpha plus cos of alpha is coming as this. After having this equation what we do we just simplify this is the equation that we are having. So, we are simplify this, and when you simplify you multiply these and these we will find that one term sin alpha cos alpha multiplied by k sin alpha cos alpha multiplied by k with the negative sign that will cancel out, and eventually we have tan alpha is this.

So; that means, I now have a direction along which the k should go if this psi is given to me if k i k p and k s amplitude is giving to me, there is a possibility. So, case k i k s this amplitude is given. So, I can find out what is the value of this alpha. Also the value of k p I can find because we know that if in the triangle this 2 angle these 2 is given the axis, this 2 sides are given and if one angle is given then this square; that means, k p square, k s square and the angle between these 2 this has to be k s actually this is has to be k i.

So, this thing can give me the value of delta k, because this things and k plus k s k i plus delta s should be the value of the other sides.

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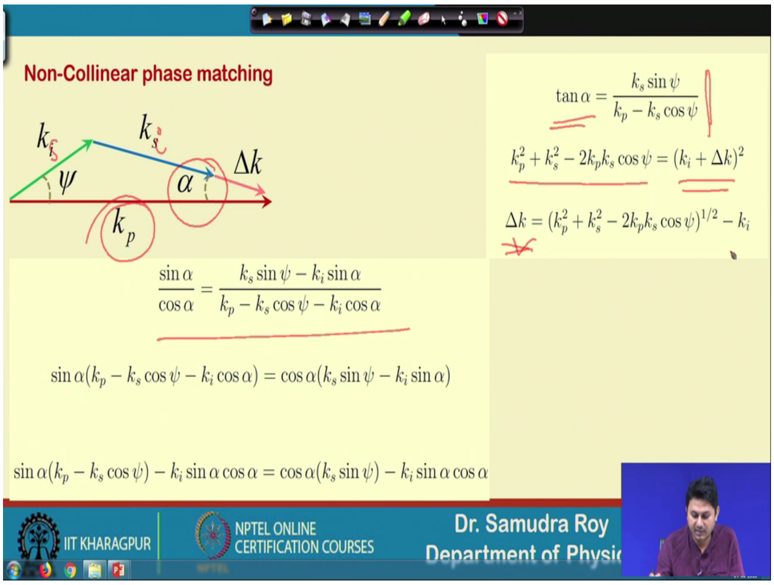

So, this square plus this square minus these things is a site, which will be equal to this. So, when we calculate these things we will also find out what is the value. So, k p k s and their angle is given and k i is also given, then I can optimize the value of delta k delta k is the phase matching. So, phase matching is 0 you can see very easily from this equation. If delta k is equal to 0 or if psi is equal to 0, then we will have the collinear form of equations. So, this is a general form and this general form we can find out what is the value of delta k and their angle for non collinear case.

So, with this note let me conclude here. So, today we have learnt the 3 wave mixing case in detail. So, in the next class we will go forward with other issues. So, with this we will like to conclude the class here.

Thank you for your attention. And, see you in the next class.