Introduction to Non-Linear Optics and its Applications
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Lecture - 31
Realistic Calculation of SHG, 3 Wave interaction

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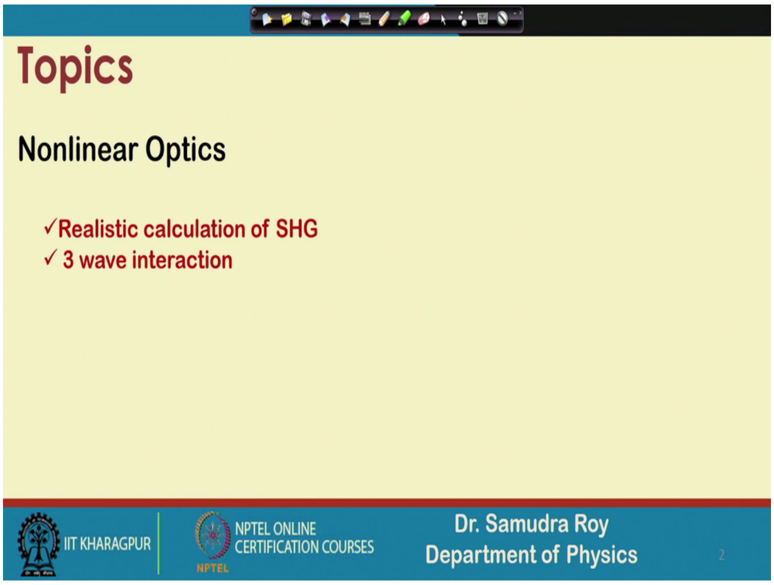

So, welcome student to the new class of Introduction to Non-linear Optics and its Application. So, today we will have lecture number 31. And in today's class we will going to learn two important things; one is realistic calculation of second harmonic generation and then, we will start the three wave interaction under second order effects.

So, in the previous, class let me remind you; in the previous class, we have started the second calculation of the second harmonic generation. And one of the major approximation that we have been taken that, during the second harmonic generation the fundamental wave which is a frequency omega is not changing throughout the distance; that means, the fundamental wave remain constant. But this is not the realistic case; we know that when the second harmonic is generating some of the energy from the fundamental wave is coming from that part to second harmonic generation part.

So, the second harmonic generation are a process where we have the energy exchange from the fundamental wave to the second harmonic wave. Since the energy is exchanged from fundamental wave to second harmonic wave; so what happened that physically the energy of the fundamental wave will going to reduce.

So, we will going to take account this issues in our next calculation and this calculation is realistic is nature, because this is the case one can expect in terms of energy conservation. So, that is why it is called the realistic calculation of second harmonic generation.

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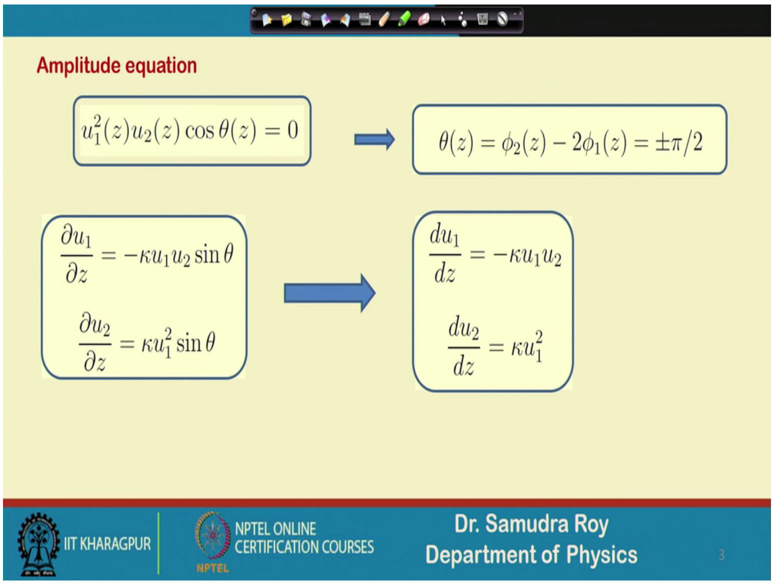

So, let us start with these. So, previous in the previous class we have done few things. So, one important thing that we have derived that one quantity is conserved over the distance and that quantity is so, let me go back to the slides yeah. So, one quantity is conserved and that quantity is this one; u 1 square u 2 multiplied by cos theta equal to 0. This is the term that was constant. So, this constant also we evaluated that at u equal to z z equal to 0 point, we know that u 2 0 was 0.

So, these things we used and then we find that this quantity u 1 square u 2 cos theta. This constant is nothing, but 0. From that we can find out what is the value of theta as the equation suggest. So, the theta has to be the value of plus minus pi by 2 because u 1 and u 2 are changing throughout the distance. So, both of them are function of z; cos theta is also changing. So, theta is a function of z, but theta is equal to phi 2 minus 2 phi 1 where phi 2 is the phase of second harmonic and phi 1 is a phase of the fundamental wave. So, they are related with phi 2 minus 2 phi 1 a parameter called theta a single parameter.

Now, theta is having a constant value pi by 2 throughout the distance, that it is that is interesting. And now if I put this value theta as pi by 2, then these 2 equation that is shown down stair is delta u 1 delta z and delta u 2 delta z which is the evolution of the amplitude of u 1 and u 2. These 2 can be written in terms of theta and kappa u 1 u 2 sin theta and kappa u 1 square sin theta these 2 are the set of equations.

Now if I put this value of theta equal to pi by 2, then these equation can be simplified and can be represented at d u 1 d z is equal to minus kappa u 1 u 2 and also d u 2 is equal to d z kappa u 1 square.

So, sin theta can be replaced by just 1 because we know that the theta is pi by 2; this is one condition. Also in the future classes, we see that we can take this pi by 2 as minus pi by 2, then the same equation will be there. Only one negative sign in the first equation kappa u 1 u 2 this negative sign become a positive 1 and kappa u 1 square become negative. So, that change will be there and because of this change, we will find that in one case we will get amplification in other case. We will get attenuation that thing we will come later.

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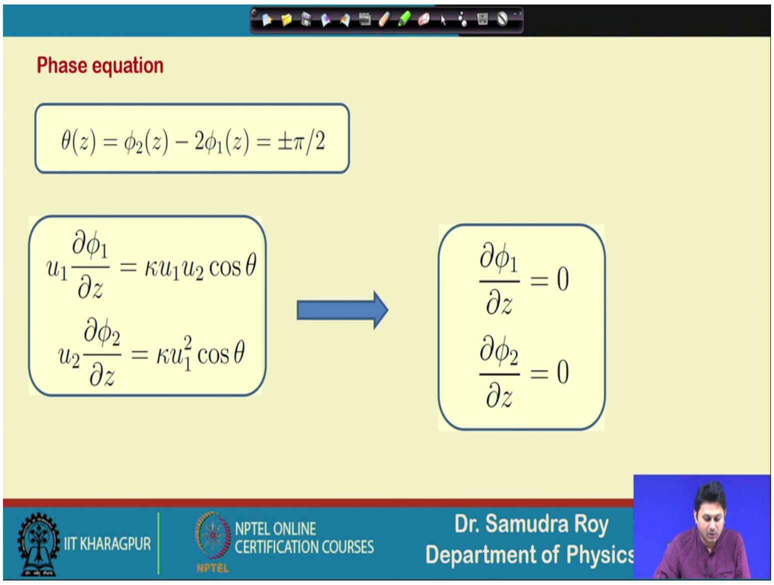

So, now what we will do that we will try to find out, what is the phase equation and the phase equation can also be one can derive. And this phase equation in the last class we have already derived and in the phase equation, if I put the condition theta equal to pi by 2, then we find that both the cases the phase del phi 1 del phi 2 remain conserved. So, the phases are not going to change throughout the distance. In this treatment this is the thing we fine.

So, now our aim is here is to find out, how the amplitude is going to change which is rather important; that when the second harmonic is generating; that means, there is 0 amplitude of second harmonic at z equal to 0 point and gradually the amplitude will going to increase. So, we will going to find out how this amplitude is going to change over the distance.

When I consider the realistic situation; that means, the fundamental wave is also changing the amplitude of the fundamental wave is also changing. So, essentially what we will do, we will deal with these two equations here and these two equation which is d u d u 1 d z minus kappa u 1 u 2 and d u 2 d z is equal to kappa u 1 square. These two equations we will going to solve. And this is a coupled equation to we will solve this couple equation directly and try to find out how u 1 and u 2 will going to evolve over distance check, where u 1 is amplitude of the fundamental wave and u 2 is amplitude of the second harmonic wave that will going to generate because of the second under this second harmonic generation process. So, let me go back to our slides .

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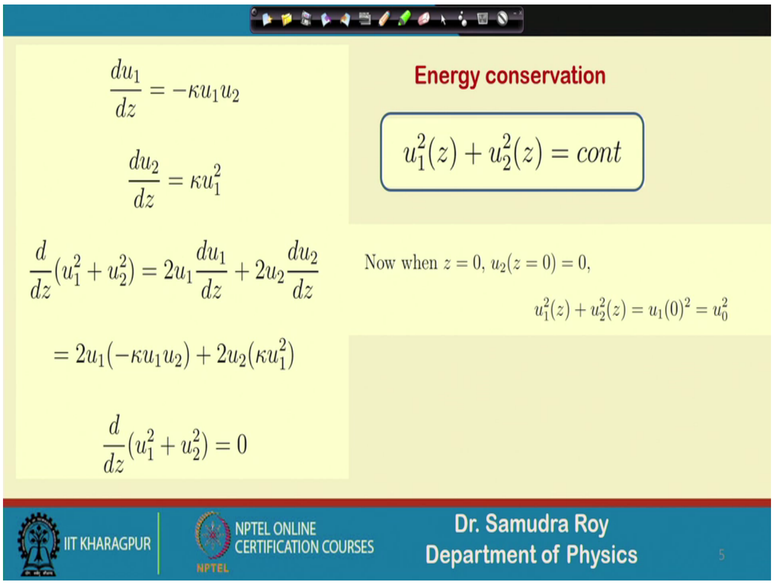

So, first let us try to find out, how the energy conservation is here applied. So, first d u 1 in the left hand side if I see the calculation, d u 1 d z is equal to minus kappa u 1 u 2 and d u 2 d z is equal to kappa u 1 square; these two equation is our mother equation. So, we will going to start from these two equation. And now if I try to find out; what is the value of our total energy, then the total energy is proportional to u 1 square and u 2 square.

So, now if I say that the total energy which is proportional to u 1 square and u 2 square, how this total energy is going to change; that means, we need to take a derivative of u 1 square are in u 2 square with respect to z. Once we take the derivative of u 1 square and u 2 square, we will going to find these equation 2 u 1 d u 1 d z plus 2 u 2 d u 2 d z.

So, just making a derivative of u 1 square and u 2 square and when you make this derivative, you we will simply get this form of equation. Now what we will do the coupled equation 2 coupled equation, the coupled equation is here, why? u 1 and u 2 is couple to each other which is that 2 coupled equation for fundamental wave and second harmonic.

So, what we will do that we will just replace d u 1 d z and d u 2 d z to whatever the value we have here in our mother equation. Once we replace these 2 equation; these 2 value d u 1 d z and d u 2 d z, then readily we find 1 equation 2 u 1 minus of kappa u 1 u 2, then plus 2 u 2 and then, if I replace the u 2 d z. So, d u 2 d z is replaced by kappa u 1 square.

So, when I replace this thing then we can see that these 2 things are cancelling each out; cancelling out each other. So, 2 u 1 square u 2 with the negative sign multiplied by kappa and then that should be equal to 2 u 2 kappa and 1 additive term is there 2 u 2 kappa u 1 square. So, when we add this two term because of this negative sign, what we find that these two terms will be equal with opposite signs. There will going to cancel each other.

Once they cancel each other, then readily we find out the d d z of u 1 square and u 2 square. This quantity is 0; so, u 1 square u 2 square, the derivative of these quantities 0; that means, that u 1 square and u 2 square are constant. So, they are not changing throughout the distance so; that means, the energy conservation is there. So, the energy is conserved.

So now, another things we should note that when I say u 1 2 1 u 1 square and u 2 square are conserved so; that means, u 1 square u 2 square that plus u 2 square that is a constant quantity. Now if I put the boundary condition that what happened of these quantities are z is equal to 0 point, then we find that u 2 at z equal to 0 is 0. But this summation is conserved so; that means, u 1 square u 2 square which is at particular z point is equal to the u 1 0 whole square or if u 0 square; that means, the total energy that was there at z equal to 0 point is equal to the energy distributed by u 1 and u 2 and that is why u 1 square z plus u 2 square z is u 0 squares u 0 square. So that means, the total energy is nothing, but the energy that we have at the input which is nothing, but the total energy of the fundamental wave.

The input we have the total energy in terms of fundamental wave and when the second harmonics is going to generate, it will take some energy from the fundamental wave. So, that the summation of these 2 energy remain conserved and this constant value is nothing, but the total energy that was incident at the input; that means, at z equal to 0 point.

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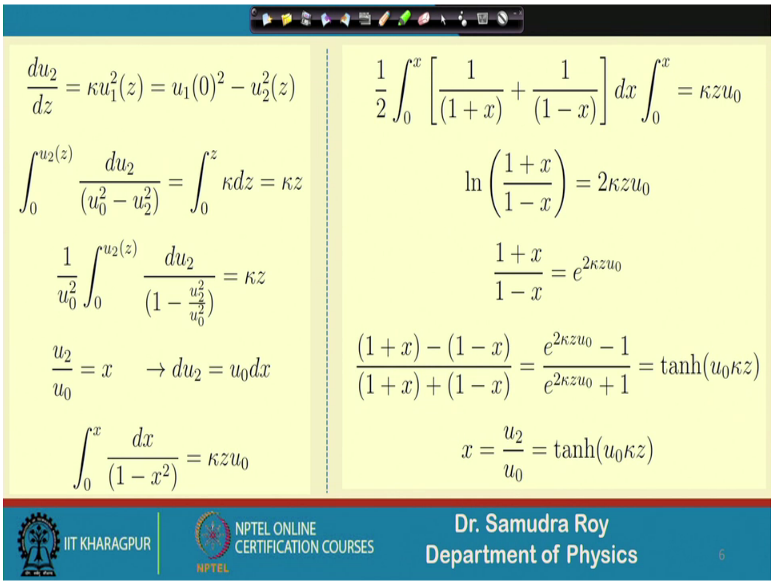

Well if I go to the next slide; we will now in this slide is important slide and we will going to find out what should be the value of u 1 and u 2. So, function u 1 and u 2, we will going to find as a function of z. So, now if I begin with d u 2 d z because I my aim here is to find out u 2. So, I need to solve this equation and there is trick you way to solve this equation this coupled equation rather.

So, kappa u 1 square d u 2 d z is equal to kappa u 1 square that we have in our previous slides. So, this is the equation we are going to use this d u 2 d z is kappa u 1 square. So, that we know. So, now, what we will do we will use this equation here and then u 1 square 0 square minus u 2 z, I replace these things because; obviously, 1 kappa should be multiplied here.

So, this quantity is equal to kappa multiplied here. The kappa is by mistake is not there. So, d u 2 d z is equal to kappa u 1 square and u 1 square plus u 2 square z is equal to u 1 0 squares. So, I will just replace u 1, in terms of u 2 and I will get this with a kappa value. And after having that what we will do? We will going to integrate that.

So, if I integrate then d u 2 divided by u 0 square minus u 2 square 0 u 2 is a limit, then that will be equal to 0 z and k d z because in the right hand side, we have k kappa sorry kappa d z. So, kappa d z will be integrated over and we will have kappa z.

In the left hand side, I can manipulate this term I take u 0 square common. So, 1 by u 0 square will be here d u 2 and then 1 plus u 2 square divided by u 0 square that is the term. We have in the right hand side, kappa z as usual; then I take u 2 u 0 as x.

If I take u 2 u 0 as x, then d u's u 2 become u 0 d z. We replace this term here. When we replace this term here as d x what happened that 1 u 0 will be here and this 1 u 0 divided by 1 u 0 square will give you 1 divided by u 0. So, if I take 1 u 0 on the right hand side, it will multiplied with kappa z u 0. Once it is multiplied will kappa z u 0, in the left hand side simply becomes d z divided by 1 minus x square.

Now, this is a standard integration. Most of us can use the standard integration. But in sometimes we find that it is difficult to remember the standard integration, but one can derive it directly; because if I do this standard integration, it become a law kind of thing, but you can directly put the value or you can use the standard procedure.

If I use the standard procedure, then 1 divided by 1 by x square can be represented by 1 by 1 plus x plus 1 by 1 minus x. It is some sort of factorization. When I factorize the half term has to be there, because here in the numerator 1 2 term will appear if I do that. In the right hand side, we have kappa z u 0 as usual disintegration sign will not be there anyway.

So, in the here, we will have a from one equation because when I integrate this part when I integrate this part, we will have log these 2 I can put here. So, so that I have 2 kappa z u 0. So, we will have the integration of this part is log of this quantity and integration of this part is again log and once a it is going from 0 to x. So, that is why I can write it is log of 1 plus x divided by log of 1 minus x by putting the boundary conditions u 0 to x and then 1 plus x divided by 1 minus x in the right hand side, l n of that in the right hand side it is 2 kappa z u 0. So, it will become exponential 2 kappa z u 0.

So, 1 plus x divided by 1 minus x is this quantity and from that we can readily figure out what is my x. I just make a simple algebra calculation and when I make this algebra calculation, this term becomes simply tan hyperbolic of u 0 kappa z. So, here is my goal is to find out what is u 2, but u 2 is represented in terms of x. I figure out what is x; x is nothing, but u 2 divided by u 0. So, from here I can write that u 2 is nothing, but u 0 multiplied by tan hyperbolic u 0 kappa z.

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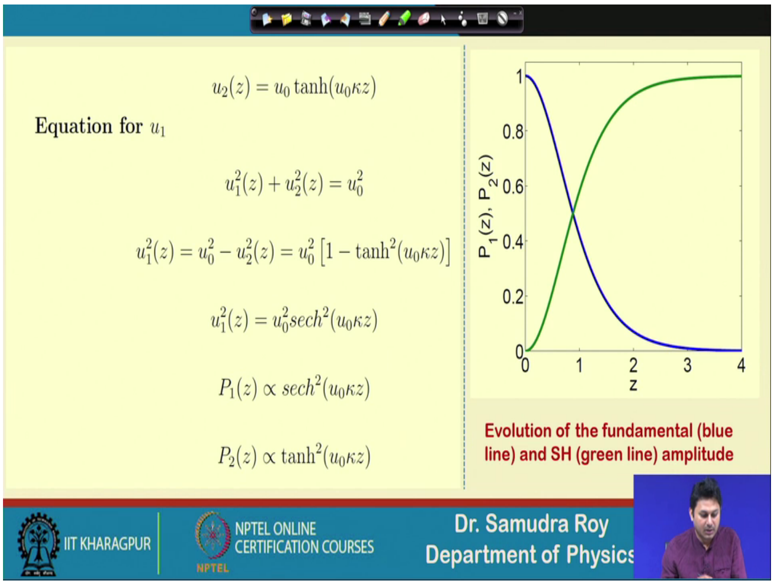

So, u u 2 z is now I find, if I considered the realistic calculation; I find that u 2 z is changing as a function of tan hyperbolic. So, it is interesting because if you remember in the previous case and the previous case we consider that there is no pump depletion; that means u 1 is constant. So, in that condition, whatever the amplitude we figure out for second harmonic was proportional to z; that means, it is increasing linearly and it is independent.

Since it is increasing linearly, so if I increase z, it will increase along z in the linear fashion. So, there is no saturation on or something like that. So that means, I am getting huge amount of energy, second harmonic energy when I just increase my z value and;

obviously, that is not the correct case. We will show in the next slide by putting these 2 figures side by side; obviously, that was not the correct calculation because in that calculation, we approximate a every and this approximation was that the fundamental wave is constant.

So, it will not going to change, but fundamental wave is constant and still I am getting some kind of energy from the fundamental wave. This is the situation that can happen when the efficiency or the transfer of energy from the fundamental wave to the second harmonic is small.

But if the amount of generation of second harmonic is high or the efficiency of conversion of energy is very high, then we should not take this kind of approximation. And this is right now whatever the calculation, we are doing this is the correct approach to find out what is really the most a realistic form.

So, now, the tan hyperbolic function suggest, if I know provide this tan hyperbolic function in terms of power, then this power is equal to tan hyperbolic square. So, the power of second harmonic will going to change as tan hyperbolic square and here the green curve basically shows, how the tan hyperbolic square curve we will going to change at z equal to 0. It is 0; that means, there is no second harmonic and z equal to 0 point and if I go forward, if I go with increasing z; then what happened that gradually the second harmonic will going to change and it will give some kind of energy. Amplitude is going to increase.

This is in normalized form and which is to a maximum and then saturates; that means, when the second harmonic is gaining the maximum energy out of the fundamental wave, so there is no way that it can change is amplitude. Because the total energy is conserved. In the total energy is conserved, it should be it should be a saturation. There should be a saturation because the energy is conserved and most of the energy, I can take for out of from are all the energy. In this case if there is no loss, I can assume that all the energy from the fundamental wave is now converted to the second harmonic wave. So, that I will have a saturation at 1 point; that means, this is in normalize unit. So, all the energy is no transfer from here to here.

Now, this is the energy thing for tan hyperbolic. When is what is the evolution of e 2, but u 1 the evolution of e 1 is equally important; that means, how the fundamental we will going to change and here we can see that u 1 square plus u 2 square u 0 square that is our that is equation for energy conservation.

Once we have the energy conservation equation, I can write u 1 is u u 0 square minus u 2 z square u 2 z square is u 0 tan hyperbolic. So, I can replace this things. Taking u 0 common, we have 1 minus tan hyperbolic square u 0 kappa z; 1 minus tan hyperbolic square theta is nothing, but sech hyperbolic square, theta as I can just to replace this as sech sech hyperbolic square u 0 kappa z with u 0 square as an amplitude. Then the power is proportional to this quantity the power of 1; that means, the power of fundamental wave and the power of the second harmonic wave is proportional to the sin hyperbolic square that we have already shown.

In this plot we shows; I mean in this plot we have shown how this P 1 and P 2 will going to change as a function of z. And in one case, when it is second harmonic wave, we find that it will change as a tan hyperbolic square kind of function and when it is the fundamental wave, it will reduce very rapidly and it will reduce in the form of sech hyperbolic square.

So, now, if I plot together we find that this blue line which is swing how the fundamental wave we will going to decrease as a function of z. And as a result, what happened the energy is coming from these to these and second harmonic wave is evolving and it is evolving to a maxima and then saturates.

So, this is the real picture. And in all the cases; all the times if I see that at z equal to 0 point, all the energies are of fundamental wave. When I go to some z point, then I find that the total energy is now totally converted to second harmonic waves. So, all the energies and now going to second harmonic wave and the fundamental wave energy of the fundamental wave is almost 0 or vanish. At each point if I some this to 2 point, we will find that the sum will give you a constant value. For example, here 1 and 0 is there. So, if I some this point it is 1 plus 0; so 1. At this point both the cases, it is 0.5 and 0.5. If I add, it will be again give 1.

So, this point and this point basically give me the addition of these 2 point are basically constant. That is the case basically we figure out for the total energy conservation and this is in fact, total energy conservation condition when if I add these 2 curve, then we will get a constant value and in this case is constant value is 1.

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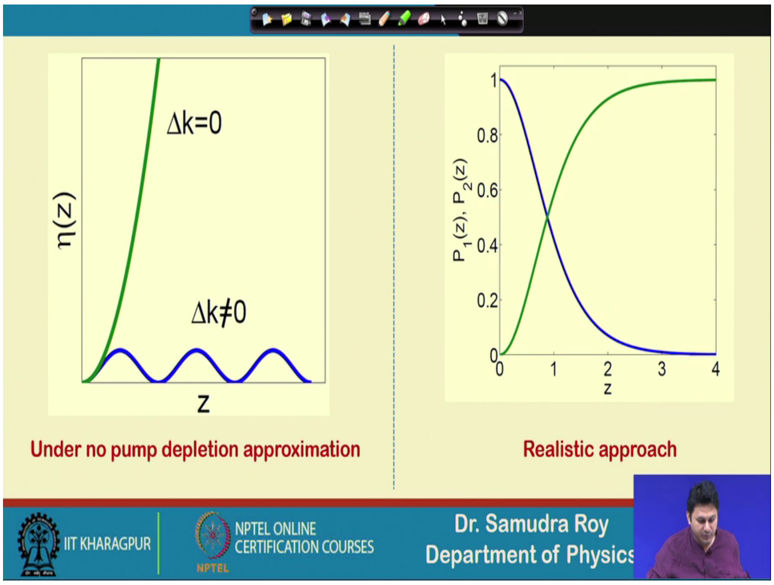

Well after that, we will side by side try to understand what is going on. In this is the old figure and this is efficiency figure old efficiency figure and this old efficiency figure we show that how delta k equal to 0 case. This is changing very rapidly and this change this is a function of z square and z is here in the x axis and you can see that when k is not equal to 0, the efficiency is changing, but there is a sinusoidal change here and efficiency reaches to a maximum its then goes down.

In with this realistic approach, we can find that whatever the value you are getting at z k equal to 0 point, it it is initially the structure is something like that. Here also it is increasing and we will also getting here some kind of increment, but after that it is not saturating. But for realistic case we find it is saturating to some point; which is rather more realistic.

So, this approach suggests that the total energy remain conserved, but here this condition was not considered. Because we considered the pulp; there is no pump depletion; that means, the energy of the fundamental wave is not changing, but here we consider that both are changing. So, that is why we get more realistic picture and that is the correct one.

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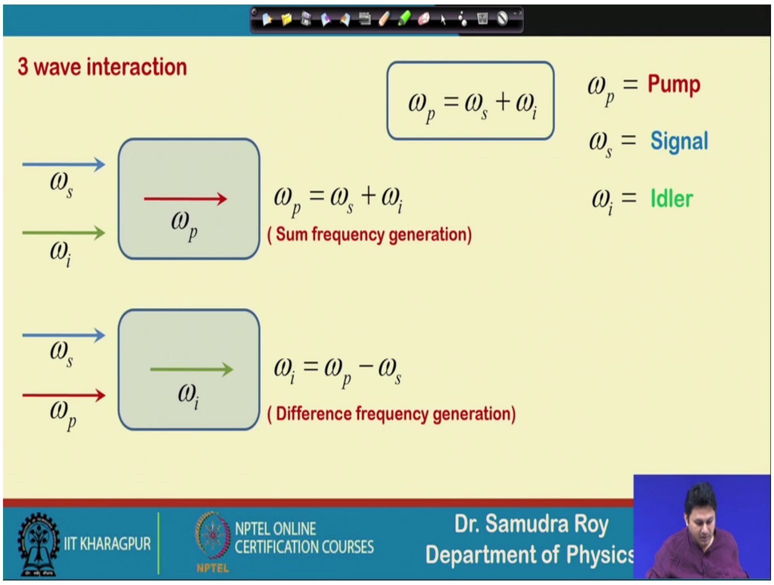

Well next we will going to see something about the 3 wave interaction. So, so far we are dealing with a second harmonic generation and in second harmonic generation, what happened? The 2 wave 1 fundamental wave is incident and because of this fundamental wave we are getting a wave whose frequency is twice than the fundamental wave so; that means, if I launch omega, then 2 omega will merge and there will going to generate omega plus omega. But in three wave case, what happened; that omega s and omega i, these 2 are there. When omega s and omega i are considered as signal and pump.

Now we put some kind of nomenclature that a instead of having 2 wave, we can say there is 3 wave because omega 1 and omega 2 can combine to each other and generate omega three it may be omega 1 plus omega 2 or omega 1 minus omega 2. In one case, it is a sum frequency analysis and in other, it is a different frequency, but the thing is we will have three wave in our hand.

So, now we will going to change the nomenclature and try to find out what happened for is three wave interaction. In the first case we can say that 1 wave we called pump which is nothing, but omega s plus omega i; that means, omega signal and omega idler these are the frequency of signal and idler. So, 3 waves are incident are considered here 1 is pump 1 is signal and 1 is idler.

So, in the input what we will do that we will incident the signal, all the cases. If with the signal we incident some kind of idler and if omega p is equal to omega s plus omega i are valid, then we can generate a pump wave omega p; that is some sort of some frequency generation. The same thing we are doing, but just change the name. So that we can understand the process more clearly. But when we do the difference frequency generation instead of launching idler, we can launch a pump. Signal will always be launched at the input. So, omega is will always be in the input. Only change we will change the idler or pump.

So, when we change the pump; idler to pump, then equation is same omega p equal to omega s plus omega i and omega i is generating. So, omega is nothing, but omega p minus omega s. So, this is nothing, but the difference frequency generation. So, sum and difference frequency generation are there, but now we will going to understand what happened for these 3 wave and this is called the 3 wave mixing. So, these 3 wave mixing, we will learn in our next classes. So, today I think we should conclude here.

So, in the conclusion I can say that we will in today's class, we have learn 2 important things; one is for realistic calculation, what happened when the total electric field is conserved and in total electric field under the conservation of total electric field, the second harmonic are generated and the second harmonic is generated at the same time, the fundamental wave is also losing energy.

So, that the total energy remain conserved and from realistic calculation, we see that it is really possible to find out how this things are increasing and decreasing. In one case, we find it is increasing as a function of tan hyperbolic square. An other case, it is decreasing with the sech hyperbolic square kind of function. The differential equation, I solve the differential equation when getting this. And in other case a and finally, we find that a the 3 wave mixing is something which we can also do in the next class that so, far we are dealing with sum and difference frequency generation. But today we will find that the sum and difference frequency generation, how we can do that using this new kind of nomenclature? namely pump, signal and idler. So, with this note I would like to conclude my class here. So, see you in the next class and in the next class we will start this a 3 wave mixing process in detail.

So thank you for your attention and see you in the next class.