Introduction to Non-Linear Optics and its Applications
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture – 30
1st, 2nd, 3rd order QPM, SHG under depleted pump

So, welcome student to the next class of introduction to Non-Linear Optics and its Application, this is lecture number 30. So, in the previous class so, we have learned something about the quasi phase matching mainly the calculation part, in quasi phase matching the important thing is that we have an arrangement where the value of d is periodically changing.

Here the periodical periodically changing this is the value the sign of d value is changing periodically from plus 1 to minus 1. So, now, we will going to calculate and try to find out that once we have this kind of function in our hand, how this can affect the evolution of the second harmonic. And the phase that is created due to the phase mismatch which is the delta k term k 2 minus 2 k 1, how it is compensating that term that we will going to calculate today.

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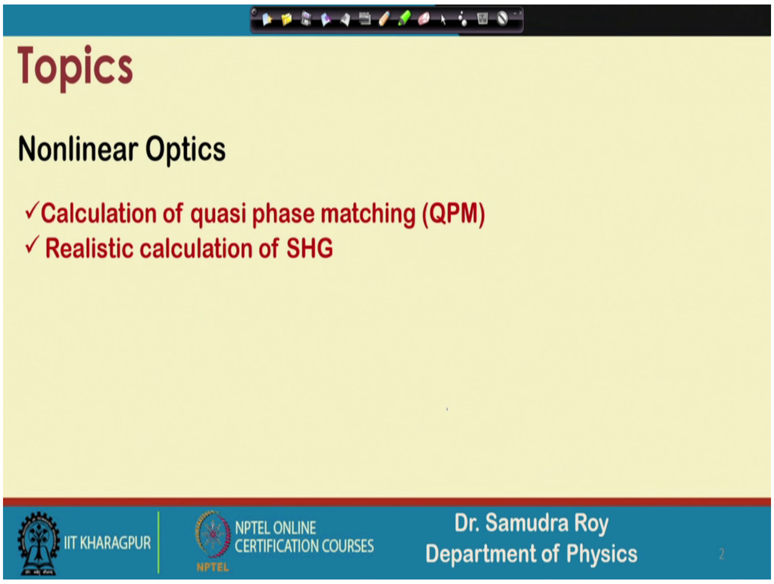

So, today we have two topics to discuss one is calculation of quasi phase matching and second is realistic calculation of second harmonic generation. So, realistic calculation of the second harmonic generation is a different part that we will start today, but let us first find out what is the quasi phase matching calculation.

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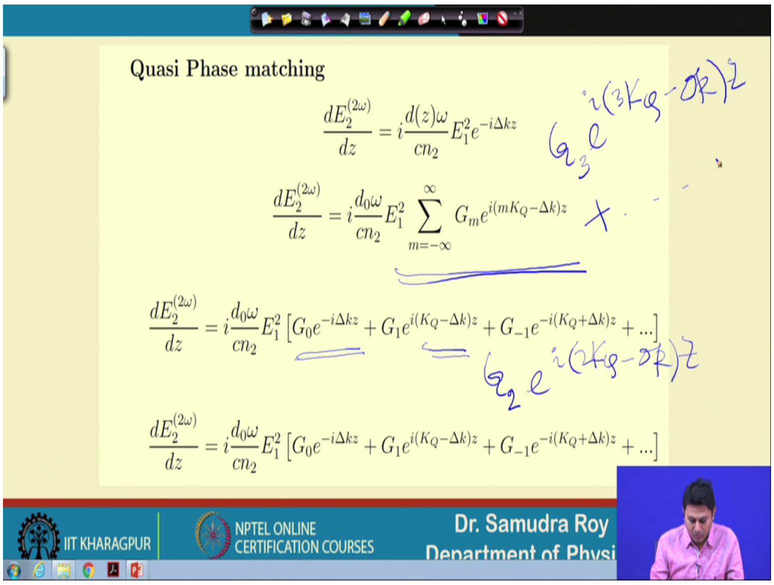

So, if you if we see that the quasi phase matching for quasi phase matching the d is now function of z that is the important part. The equation of second harmonic whatever the equation is written here in this slide is exactly the same that we have already derived previously only important inclusion here is now, d is function of z; that means, there is a variation over G d if d is a periodic function.

Then we know that d can be written as this form, we use this in the previous class dz can be represented in this particular form Gm e to the power i m KQ z. Where m goes to minus infinity to infinity once we write this dz then readily we have an expression or the relation over d which suggest that over a period lambda d has a same value. Since, we have this condition I can write d in this particular form we have already proved in the last class.

If we use this value if we use this value here the value of d, then we can write in total it is Gm e to the power iKQ minus delta k because of this minus ik term and z over some. Now once we have this I can expand this term as a function of m by changing the value of m. So, d 2 d d 2 d E 2 dz i d 0 omega cn 2 E 1 squared this is the constant term and then i expand this term so first term if I put 0 this term will be 0 this term will be 0. So, z

0 e to the power i delta kz that is some sort of term that we already have this is if there is no KQ then we will get some term like this.

Now, this from the second term the contribution of KQ is there, so if I put m equal to 1 we have KQ plus delta kz if I have km equal to minus 1 I should have z of minus 1 e to the power i KQ plus delta kz and so on. If I write what should be the term for m equal to 2 it should be G 2 e to the power i 2 of KQ minus delta kz what should be the term of m equal to 3 it is G 3 e to the power of i 3 KQ minus delta k into z and so on.

For the negative case I just put a negative sign here when I put a negative sign I need to put a negative sign. So, this negative sign and this negative sign I can take common then it should be minus of i 2 KQ plus delta kz when m equal to minus 2 and so on, so this term will go on ok.

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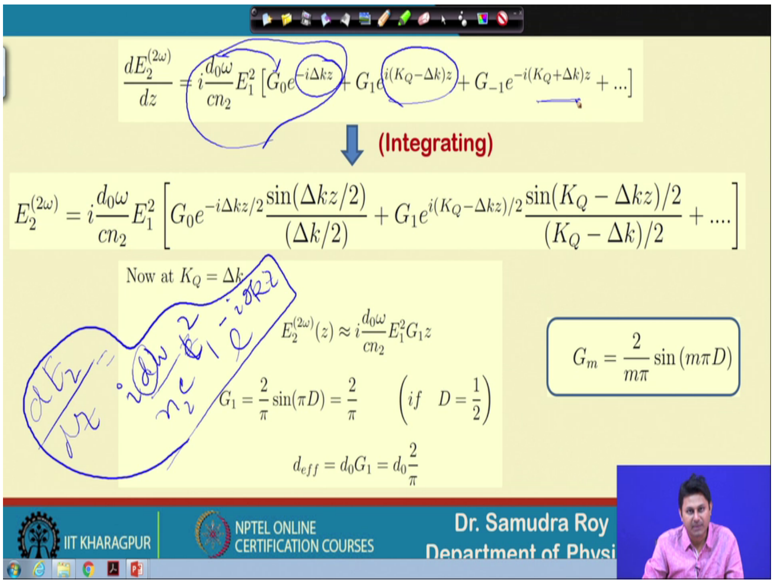

So, the next slide if I go this is the expansion of d E to dz, now the next step is to find out E 2 that is our goal, but before doing that I need to I need to make some comment here or try to draw your attention that here in G 1 term. We already have here in G 1 term we already have a phase which is KQ minus delta k, this is the phase that already there if I forget about the quasi phase matching term.

If I write only the second harmonic generation term E 2 this differential equation term it was initially i of d of omega divided by n 2 c e to the power E 1 square and the phase term was e to the power of minus of i delta kz, this was the term that we have without any kind of periodicity, d was only a constant term.

So, the first contribution if I see the first contribution of this term is something like this, but here d 0 and z 0 are the 2 terms, which is basically is d term slightly modified kind of d. But from the second term onwards what happened that we have a term in phase which is related to kq; that means, there is there will be first case what was the phase matching delta k equal to 0 is the phase matching condition.

But for the second case the phase matching condition appear when KQ is equal to delta k; that means, phase matching condition is now with KQ term. So, KQ is included and then here also we have some sort of phase term, but KQ equal to minus delta k is our phase matching, in the similar way if I go on with higher values of m.

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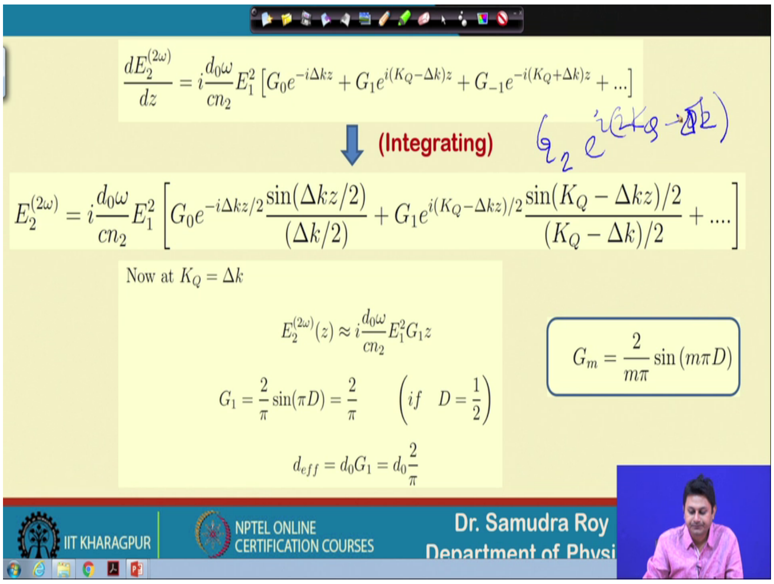

Then the next higher order term is G 2 we which we have already shown that here the condition of phase matching is this. So, 2 KQ minus delta k equal to 0 is our phase matching this is delta our phase matching well. So, once we have this expansion the next thing is to integrate it. So, if I integrate we know what should be the value for the first case it is some sort of sin function.

So, sin divided by sin delta kz divided by 2 delta k by 2 this is a old term that we have already calculated, second term we will have exactly the similar form only delta k will be replaced by KQ minus delta k. So, we will have again have sin of these things and then here we have a mistake it should be KQ this z will be outside. So, here also this will be outside KQ delta k multiplied by z, so we will have a sin term also here and so on.

After integration the next thing is what happened we will put our phase matching KQ is equal to delta k, that is a phase matching that is a quasi phase matching we call the first order quasi phase matching. So, when we put KQ equal to delta k then you can see readily you can find that only the major contribution will come from this term here if I put KQ equal to delta k.

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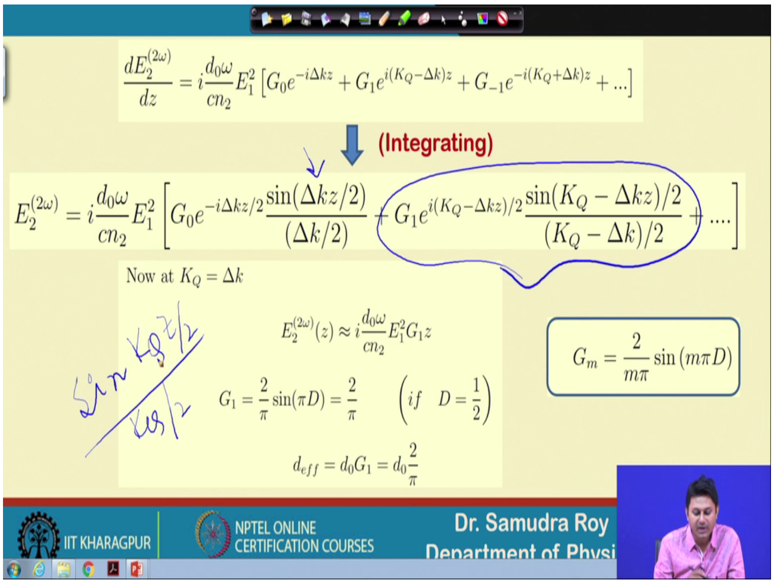

This term will be sin of KQ z by 2 divided by KQ by 2 it will not going to get any high value because it is some sort of sin function or if I write in terms of KQ it is a sing function. So, we know that the sin function got a maximum when these and these are 0 tends to 0, but here we find that it is not the case; What about the other terms, so higher other terms the higher other terms.

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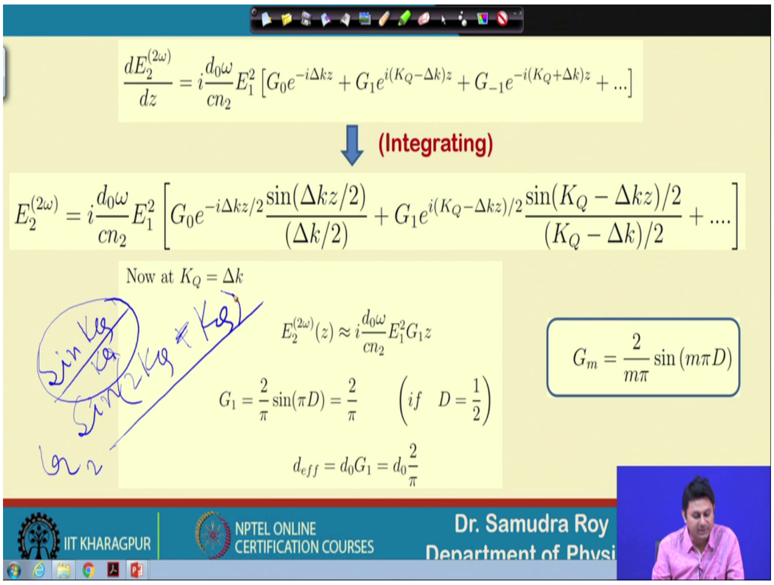

For example if I write G minus 1 and put KQ is equal to delta k we eventually have a term e to the power of minus i 2 of in in in terms of sin after integration we will have a sin term. So, we will have G of 2 sin this term will be 2 KQ and minus k, so minus k will be represented by plus minus KQ minus delta k is represented by minus KQ. So, we will eventually have sin of KQ divided by some term which is again KQ so again we will find that this term will not going to vanish because KQ is equal to delta k it is not 0.

Only contribution we will have this and this term tends to 0 this term tends to 0 and this term tends to 0; that means, this term tends to 1 when we have KQ is equal to delta k. Under that condition I can write E 2 is approximately equal to i d 0 omega c into E 1 square which we have already have multiplied by the coefficient G 1 and the limit at KQ minus delta k tends to 0 this limit this will gives you z so we will have a z here.

Now, G 1 is this because Gm is equal to 2 divided by m pi sin m pi d, so G 1 can be represented in terms of this quantity. So, G 1 is 2 pi if I put m equal to 1 it should be 2 divided by pi sin of pi d because m is 1. And now if I say my duty cycle d is equal to half then it should be 2 divided by pi because d half means sin of pi by 2 it is 1. So, we have the d half so if I now write this G 1 and d 0 together we will have an effective d this effective d is nothing, but d 0 divided by 2 pi.

So, we can see that the first contribution which is containing G 1 term in this case this d effective is less than d 0 by a factor, because d effective is d 0 multiplied by 2 by pi and pi is 3 point 1 4 so; that means, we will have a quantity which is less than 1 that is multiplied to d 0. So, d 0 is reduced a bit, but we have still have a contribution and this contribution basically give rise to second harmonic. All the other contribution will not going to give any kind of phase matching, so this term will die out very soon. So, only term that will be there is this so if I now try to find out what are the other possibilities of phase matching.

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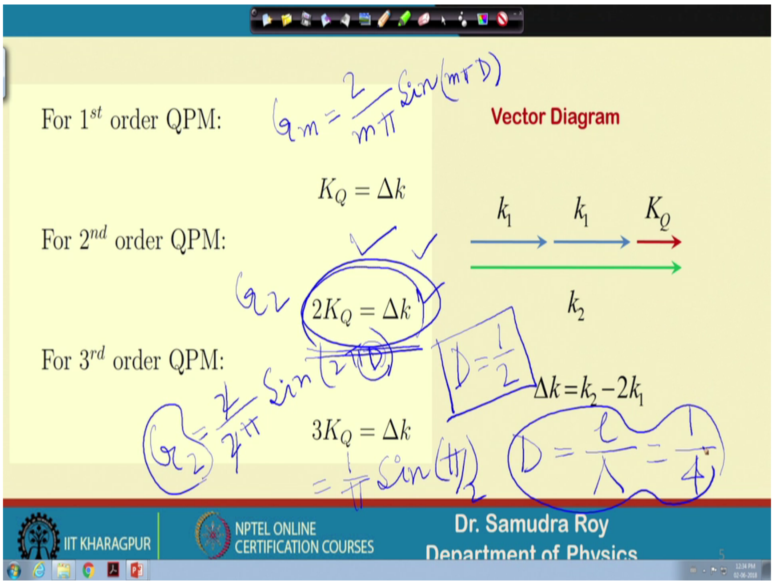

Then KQ equal to delta k is not only not only the ff 1 phase matching condition that one can have in this quasi phase matching, but 2 KQ is equal to delta k that is another one 3 KQ is equal to delta k is another one. So, on I will find the higher and higher order phase matching, but you have to be remember that if I for example, we can have a system where 2 KQ equal to delta k is our phase matching condition then which of the term that is responsible is our G 2.

Now, if I try to find out G 2 with the duty cycle d is equal to half then G 2 is what was our Gm Gm was 2 divided by m pi and then sin of m pi multiplied by the duty cycle d that was the form of Gm. Now suppose we have second order phase matching for second order phase matching the value of m is 2. Now I try to find out what is my G 2 because G 2 will be effective on gen on generating of second harmonic.

So, G 2 will be 2 divided by 2 pi because m is 2 and then sin of 2 pi multiplied by d this first term 2 2 will cancel out. So, we will have 1 by pi sin of pi because d is half, but sin of pi we know that this is 0 sin of pi is 0; that means, even though we have a second order phase matching condition we can have G 2, but the contribution of the G 2 is 0.

So, second order phase matching the contribution of G 2 is 0; that means, we will not going to get any kind of second harmonic generation even the phase matching is there be careful about that because the coefficient G 2 will going to vanish. Why it is vanishing that is interesting and it is vanishing because of the d value we are using this duty cycle equal to half, but the duty cycle d is represented by l divided by big lambda. Now, if I say l divided by big lambda this ratio is 1 by 4 because this is in our hand, then you can readily find that d is 1 by 4 and this value is pi by 2 and then again you start getting G 2 because here the G 2 is not equal to 0, but 1 by pi.

So, in order to excite the second order quasi phase matching condition the important thing that you need to take care is the duty cycle. If you change the duty cycle there is a possibility that you will get G 2 not equal to if you retain with a previous duty cycle that is half. So, with this periodicity what happened we will not going to get any kind of second order quasi phase matching even though the phase matching is there, but the effective contribution will be 0.

So, next here we will going to conclude our topic ok, so before that we have something to say regarding the vector diagram. In vector diagram this is a collinear kind of phase matching if I write this vector diagrams you can readily see that k 2 delta k is equal to k 2 minus 2 k 1, and if I write the first order it is KQ minus delta k is equal to 0.

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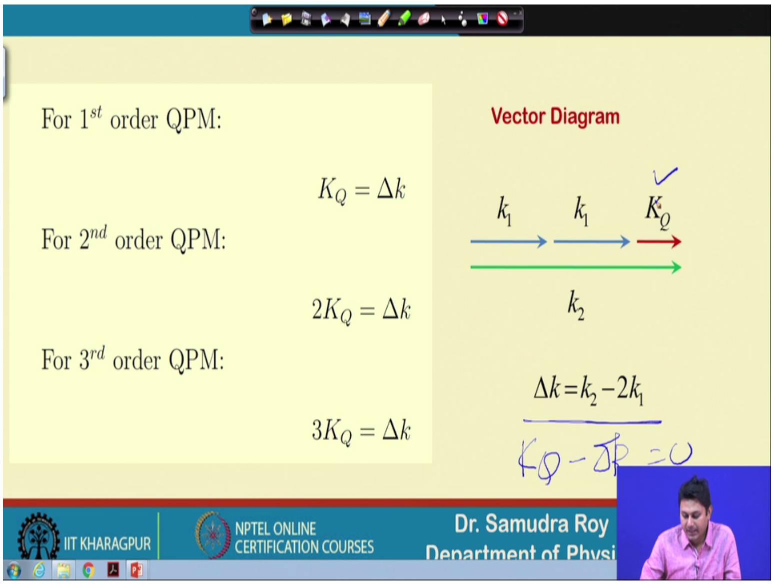

So, delta k can be in the vector diagram can be represented with this and this KQ is basically the vector which is associated because of the periodicity that is there because of the periodic d function. So, after that this is the end of the study of quasi phase matching. So, now, we will going to start a new topic which is the realistic calculation of second harmonic generation.

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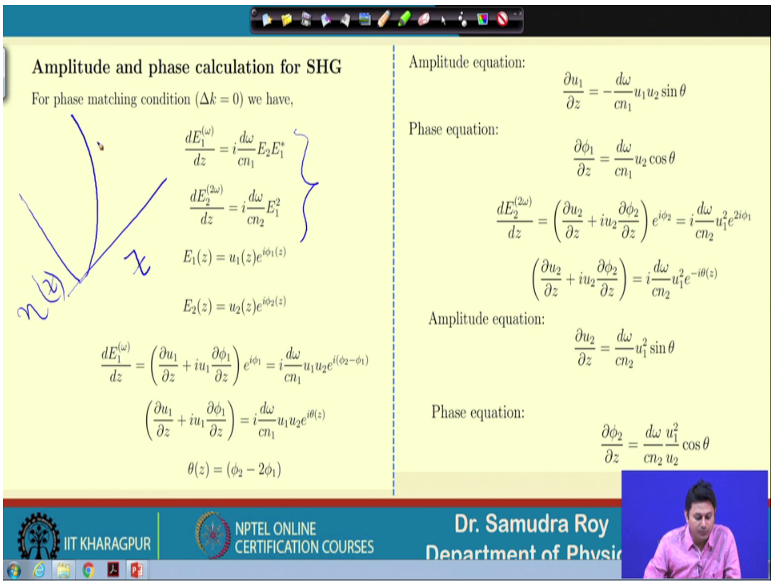

So, this realistic calculation was important, because in the realistic calculation we will not going to consider the electric field having the frequency omega or the fundamental electric field is constant because it is also changing we have 2 coupled equation. So, if I see these equations so let us go back to the equations. So, under phase matching condition delta k equal to 0 we have 2 equation in our hand to remember.

Both the equations are coupled to each other and in order to find out k 2 initially what we have done we take this E 1 as a constant and then integrate when we integrate we find that E 2 is proportional to z or the power is proportional to z square. So, when we put the efficiency curve it was like this, but we also mention that this is not realistic. So, over z eta of z was something like this, but we also mention that this is not the correct representation of the correct representation of this efficiency. So, we need to find out the realistic calculations.

So, now, we will going to do that, in order to do that what we do is we divide the total electric field E 1 into 2 part 1 is amplitude part u 1 z and in phase part u E to the power i phi i phi z phi 1 z. So, both are now varying with z for E 2 also we will divide into amplitude and phase in this way. Once we have the amplitude and phase separately where u 1 and u 2 are the real quantity also phi 1 and phi 2 has a real quantity this complex amplitude we can also divide into E to the power i r E to the power i theta forms.

So, here it is something like this so now, we will have a derivative over E 1. So, when we make a derivative over E 1. So, since E 1 is represented by E 1 E to the power i phi 1 so that derivative of E 1 and phi 1 can give this form du 1 dz plus i u 1 d phi 1 dz through our i phi 1. And this quantity is in from the right hand side i write it is i d omega cn 1 and E 2 E 1 can also represent in term terms of E 1 and E 2 u 1 and u 2 multiplied E to the power i phi 2 minus phi 1 because there is a start. So, that is why minus phi 1 is here, so this phase is associated with that.

So, in the left hand side we have du 1 dz plus iu 1 d phi 1 dz i d omega cn 1 u 1 into E to the power i theta z. Where theta is phi 2 minus 2 phi 1 this is the phase difference between these two, but not the total phase difference this is some quantity phi to minus 2 phi 1 the phase difference should be phi to minus phi 1, but 2 term is here. Now, the amplitude equation we had an equation here you should look carefully this is our equation. So, this equation is a complex equation in the left hand side the real and complex.

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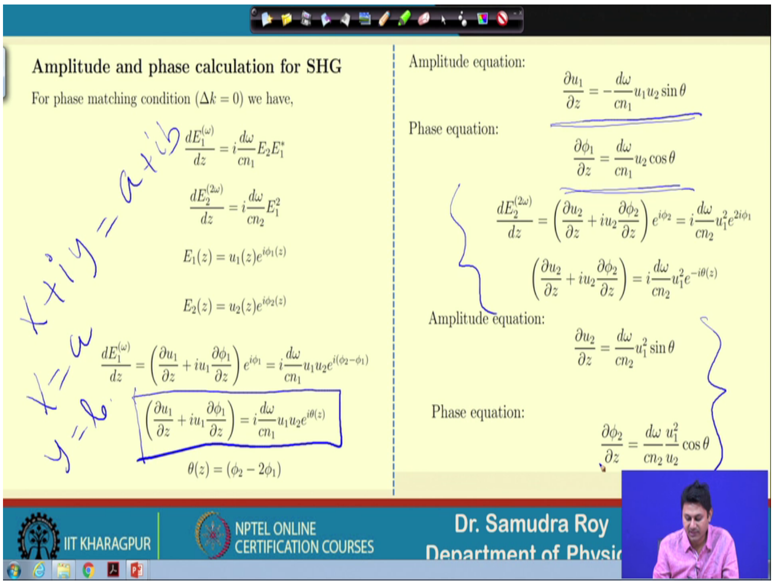

So, we know that if 2 x plus iy is equal to a plus ib then we can write x is equal to a and y is equal to b. We will going to use the same thing the real part of the left hand side will equal to real part, and if we do we will find this equation that du 1 dz is equal to u 1 u 2 sin theta multiplied by d omega cn 1 and the imaginary part which is again equal and the eq phase equation i will get this quantity.

In the similar way we can find the same thing for E 2 and I will not going to explain the entire process because it is already there. So, I ask the student to do this things by yourself to find out these 2 equation for E 2 and you will find that we will get this 2 all the calculations already there, but I am still ask you to do that by yourself this is a very straight forward calculation So, real an imaginary part you can extract out from this 2 so after doing all these calculation when we extract the amplitude and phase equation together.

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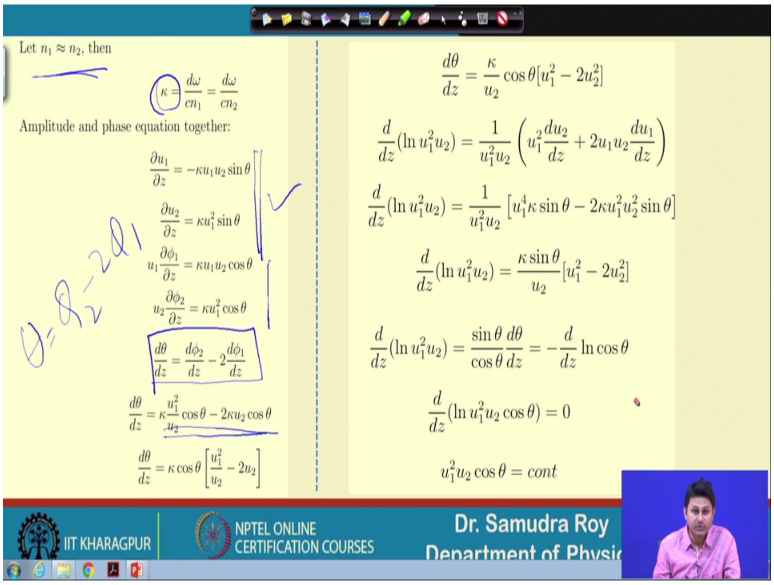

Then the next thing we can do is to use this thing, now if we use that this d omega cn 1 and d omega cn 2 are very if n 1 and n 2 is very close to each other we can write d omega cn 1 and d omega cn 2 very close to each other and we write is general coefficient kappa. So, when I do that then we will find these things will reduce the calculation significantly

Now, 2 amplitude equation I can write du 1 dz is equal to minus kappa u 1 u 2 sin theta and du 2 dz is kappa u 1 square sin theta this was the 2 amplitude equation mind it, the term kappa is used in two different cases because d omega cn 1 we assume n 1 is very close to n 2. So, we assume that they might be same for amplitude case also we have amplitude equation and this amplitude equation is u 1 d phi 1 dz is equal to k u 1 u 2 cos theta and u 2 d phi 2 dz is equal to kappa u 1 square cos theta.

Now, we know theta is equal to psi 2 minus 2 psi 1 so when we make a derivative of theta, we will have an additional term d phi 2 dz minus 2 phi 1 dz phi 1 phi 2 we can have from here. When I use this phi 1 phi 2 we will have an expression of d theta which is u 1 square u 2 cos theta minus 2 kappa cos theta, why we were doing all this calculation we will find in the right hand side basically because in the right hand side eventually we get 1 quantity which is conserved over the distance; that means, it is not going to change.

To figure out we need to use this methodology this calculation and in the Boyed book this calculation is there if somebody is interested or see you can go to this book, but all the calculation is here in this slide. If you do this calculation by your own then you will find that it is not a very critical calculation rather it is a lengthy, but it is not a very critical it is a straight forward calculation.

Next we will find one very important quantity and that is log u 1 square u 2 when we try to find out log u 1 square u 2, if you if I make a derivative of z 1 u 1 u 2 is be there because I am making derivative. So, u 1 square then the derivative of this quantity which is u 1 square this plus the derivative of 2 u 1 u 2. So, we will have this again we will going to use this du 1 dz and du 1 dz from this equation these two equation and we will find one expression like this.

If I simplify this equation we will eventually have one expression d dz is equal to log of u 1 square u 2 kappa sin theta divided by u 2 and u 1 square minus 2 E 2 square. Now in the right hand side this quantity, we already get here in terms of theta if I slightly manipulate this thing this cos theta and all this things I can write d cos theta by z divided by if I put this cos theta here.

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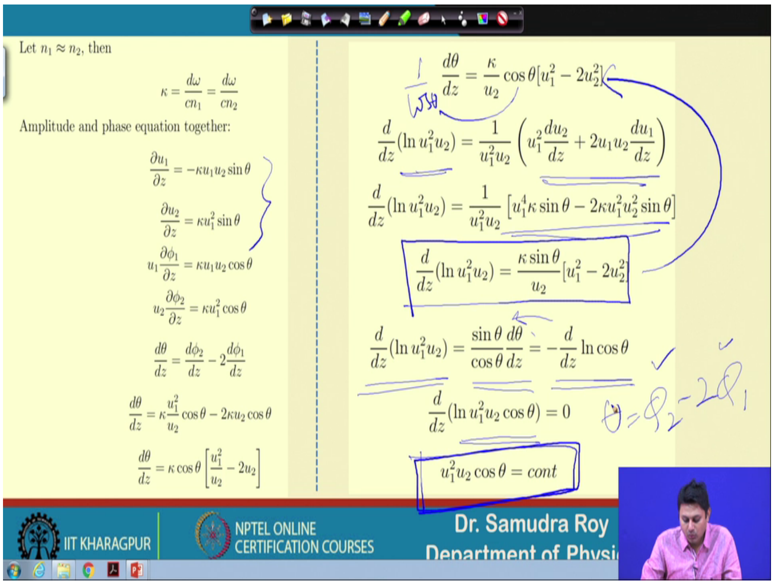

So, 1 by cos theta will be this term and now sin theta it is multiplied sin theta by cos theta d dz is equal to log of d dz of ln of u 1 square u 2 this 2 quantity this quantity can again be represented as log. So, minus of dz ln cos theta should be equal to this quantity you look carefully you can really understand what is going on. Now if I combine these two the important very important expression that will come is d dz of ln u 1 square u 2 cos theta is equal to 0; that means, u 1 square u 2 cos theta is equal to constant this is a very, very important expression that we end up that u 1 u 2 and cos theta both are varying independently.

Because, u 1 is a amplitude of the fundamental wave u 2 is amplitude of the second harmonic and theta is a difference between their angles different between their phases, but the phase directly it is not different. So, theta is phi 2 minus 2 phi 1, phi 1 and phi 2 are their phases. So, theta is difference of this 2 quantity, but they are also changing with respect to z; that means, theta should be a function of z.

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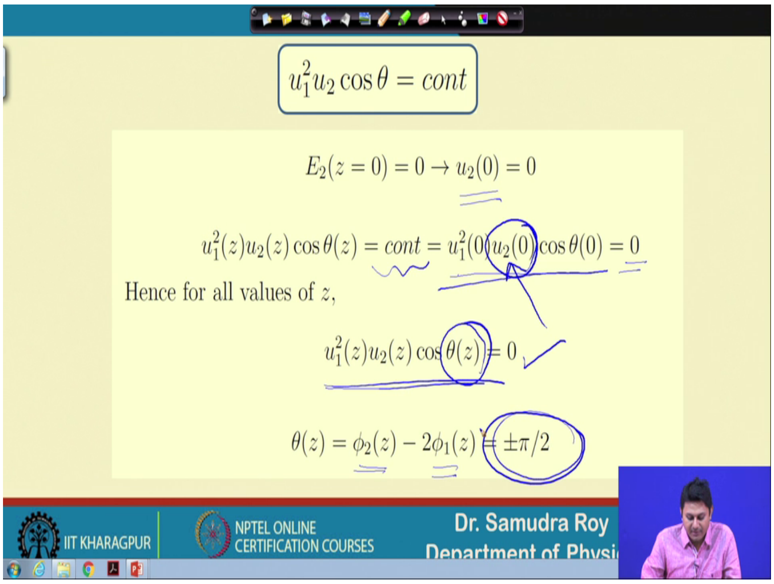

So, u 1 u 2 so if I now write u 1 u 2 cos theta is equal to constant. So, this means u 1 which is a function of z u 2 which is a function of z cos theta is a function of z is constant. Now, if I put some boundary condition if I put some boundary condition E 2 at z equal to 0 is 0 because at the beginning there was no second harmonic so; that means, u 2 at z equal to 0 has to be 0. When u 2 at z equal to 0 is 0; that means, u 1 z u 2 z cos theta equal to 0 constant which is same as all the term at 0 point.

So, u 1 0 u 2 0 cos theta 0 that should be equal to the this quantity, because it is a constant throughout the motion. But now we find that this quantity this quantity is 0; that means, whatever the constant we are try to say is 0; that means, u 2 square u 2 u 1 2 z u 2 z and cos theta z is 0. Now, again we find a very interesting outcome from this u 1 and u 2 u 2 is 0 at z equal to 0 that is true, but after that u 2 is not equal to 0 u 1 is also not equal to 0 this is multiplied by cos theta and this entire quantity is 0; it is only possible it is only possible when the theta has some relation. So, theta z which is phi to z 2 phi 1 z if this quantity is plus minus pi by 2 then only we can say that this is a valid equation.

So, we will find a valid equation here and this valid equation suggest that when they have a phase relationship so; that means, when the second harmonic is generating there is a phase relationship between the second harmonic and the fundamental wave. And this phase has relationship is phi 2 minus 2 phi 1 is equal to plus minus phi by 2.

So, we will in the next class we will going to use this phase relationship to find out the parametric generation and all this issues will come. But important thing is this calculation is lengthy, but important thing is 1 quantity that is conserved during the propagation is u 1 square u 2 multiplied by cos theta. And this cos theta is something where theta is represented by the phase of the fundamental and second harmonic wave; this phase relationship phi 2 minus 2 phi 1 and phi 2 minus 2 phi 1 these phase relationship is maintained throughout the distance. So, phi 2 minus 2 phi 1 is always plus the minus pi by 2 depending on the situation and then it is conserved throughout the propagation.

So, we find a very important outcome today that 1 equation that is not changing and using that we will find out a what should be the amplitude and phase revolution over the distance. And we will find in that case that is not a z square dependency, but we have a realistic calculation. So, with this note let me conclude today’s class. So, in the next class we will start from here and try to find out more about the second harmonic generation. And how the second harmonic field will going to evolve under realistic condition that we will going to find using today’s outcome. So, with that note let me conclude here see you in the next class.

Thank you for your attention.