Introduction to Non-Linear Optics and it is Applications

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Lecture – 03
Basic Linear Optics (Contd.)

So, welcome student to this Introduction to Non-Linear Optics Course. So, this is our third class. So, in the previous 2 classes we have learn about the Maxwell's equation and the corresponding wave equation and it is solution. And we find that it is solution is a special kind of solution, we called the plane wave solution. Here as a specific form why it is called the plane wave solution that is also discussed in the previous classes? Then we find out that there are different kind of media are possible.

If the media is isotropic; that means, the physical property of the media is same and irrespective of is the direction, then the problem if relatively simpler because electric field vector E and the displacement vectors D are parallel to each other. Also, the polarization which is the dipole moment per unit volume that is also parallel to the applied electric field; that means, if I apply the electric field in a particular direction, then the polarization will be in the same direction.

However this is not true in an isotropic medium. So, we introduce the an isotropic medium in our last class. So, we continue the concept and learn more about the anisotropic medium in this particular class.

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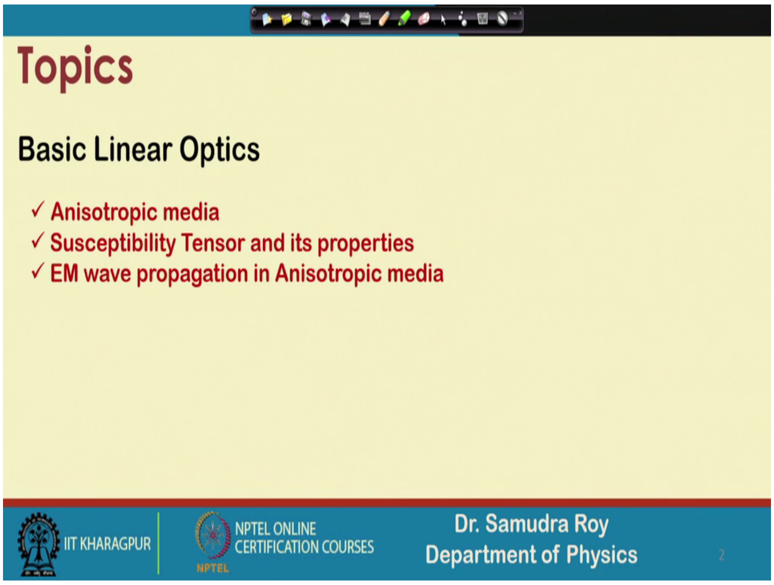

So, let us see what we have in this class? So, we have on the basic linear optics still we are in the basic linear optics domain. Today, we will going to learn the anisotropic media in more, susceptibility tensor and it is properties, and finally, how the electromagnetic waves will propagate through an anisotropic medium.

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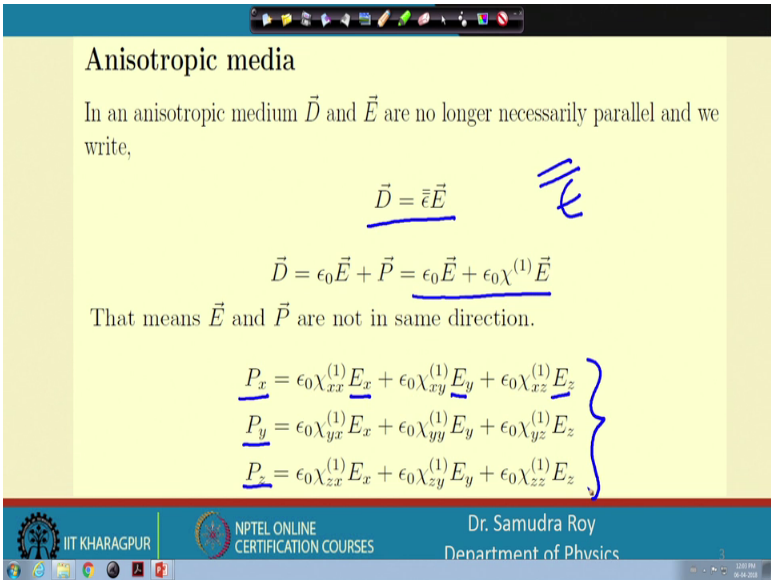

So, let us go to the next slide, this is the previous slide we have already introduced this slide in the last class, but let us do that once again this is some sort of recap. So, it the slide the expression suggest that in the slide, that in an isotropic medium D and E are not parallel, that is the main issue here; D and E are not parallel and it is not parallel because of the fact of these susceptibility tensor this is am susceptibility tensor, due to which D and E are not parallel.

Now, D is expanded in terms of polarization also and in this equation we can see that D and E are not parallel because P and E are not parallel; that means, the dipole movement that is generated due to the application of the electric field has different direction, that of the electric field. So, if I write the relationship between the polarization components P x, P y, P z this P x, P y, P z is no longer proportional to the E x component rather the E y and E z component are still there in the system.

So, we have a matrix like form and this relation is very important in non-linear optics. And, we will know in the future classes how this things will give leads to some other issues. So, let us go back to the next go the next slide.

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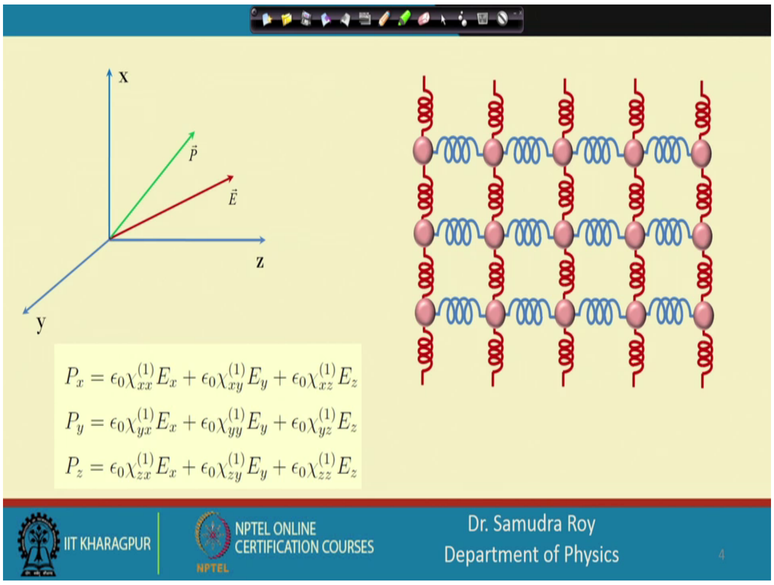

So, this is also a figure that we have shown the left hand side figure that applied electric field and polarization is not in the same direction they are not collinear. And the reason is shown in this in this figure. Here, we can see that, this matrix we have some kind of points this are some kind of points, this points are associated with the corresponding charge.

So, what happened that only electric field is applied the charge in the entire material which is bounded by the positive charge of the nucleus, they get shifted from their original position. And, if the electric field is vibrating with respect to time what happened? That it also starts vibrating; that means, the separation of the charge that also start vibrating.

So, now if I consider this vibration as a spring mass system, then this will be the entire matrix. Now in anisotropic system what happened? That this spring whatever is shown here with the blue and red, they are not in the similar kind of spring. So, the spring constant is different for these 2 cases. So, what happened that if I launch an electric field and due to which if something is displaced. So, this displacement may not be in the same direction of electric field rather this displacement is complex.

So, this complex displacement cannot be explained by this collinear thing so; that means, the displacement of this charge will be in a different direction and the electric field applied electric field in may be in different direction. So, the if I now try to understand the x y z component of this, then you can see that x y z component with respect to E field is already there. So, that means, the x component of polarization now depends on the x y and z component of the electric field that we have already discussed.

So, let us go to the next slide and see what we have here.

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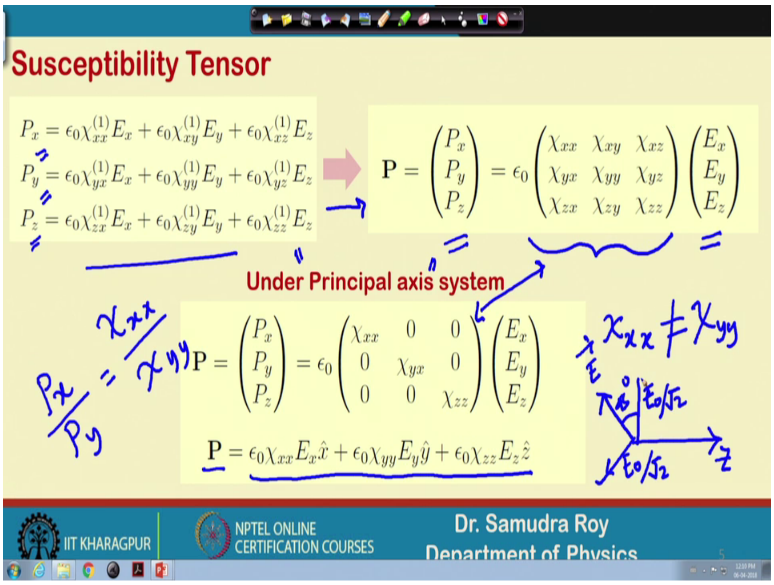

As, I mentioned ones I write the susceptibility tensor in this particular form, which is shown here the relationship between P and E is shown here. So, these leads to this particular arrangement, leads to a matrix form. And here we have a susceptibility tensor, which is in the matrix form with rank 2.

So, P and E are now in vector form this is a inspector form they are related to some kind of matrix we called is susceptibility tensor, these has some component. And this component basically related to the displacement of electric the displacement due to the launching of electric field.

And, now this components are here in this particular matrix 9 components are there, but we know that any matrix can be suitable modified under proper coordinates system. In this case, if my coordinates system is a principal we called a special coordinates system we called a principal axis system.

So, under principal axis system we can diagonalize this particular matrix, which may be helpful. Because, when we diagonalise this system; here we can see that this 9 component now, compress to 3 different components and only the diagonal components and there. And this is a very straight forward technique if a matrix is given to you and from this matrix you can find out what are the Eigen values of this given matrix.

So, the Eigen values are the diagonal elements. So, that is one way to understand that I can diagonalize a matrix and when I diagonalize matrix the quantity that is sitting in the diagonal and nothing, but the Eigen values of these things. And, if somebody asks how to diagonalize a matrix that is also well known that a matrix has some eigenvectors. So, if I use this eigenvector and form another matrix and make a similarity transformation then, because of the similarity transformation what happened? That I can diagonalize the given matrix, under that coordinates system and this coordinates made by the eigenvectors.

So, we can do that I mean this is not a very big deal we can do that. So, let us go back to the slides. So, here we can see that after diagonalizing the fact in that things. So, we can write P and E in this particular form. So, P vector can be now represented in relatively simpler form, because all the diagonal elements are gone, but still chi x x this component is may not be equal to this components.

So; that means, if I launch an electric field having same E x and E y forget about E z component, I launch an electric field and this electric field is distributed x and y. For example, this is the coordinates system this is my z direction and I launch in electric field here E and if this angle is 45 degree and if it is a vector E. So, I have a component here with E 0 root over of 2 and here E 0 root over off 2 the amplitude of the electric field are same. So, E x and E y are same.

So; that means, my electric field is well defined this is along 45 degree. If, that is the case what happened? If I put this into the equation I can also figure out what is my P x and what is my P y. And, if I make P x and P y if I want to find out the ratio you will find readily find that, if I make P x divided by P y this ration it comes out to be chi x x divided by chi y y. Here if I divide the E x E y component for example, let me erase this part yeah.

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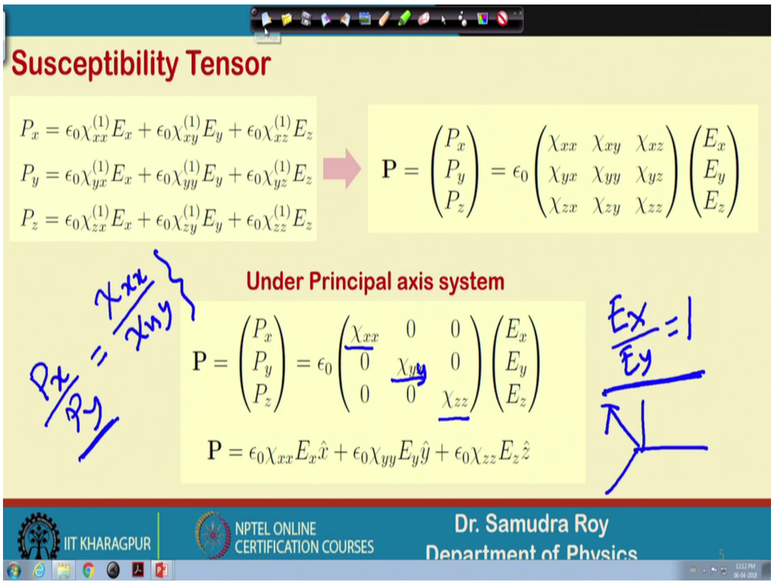

So, E x and E y component was 1 here as I mentioned if I do P x and P y they are ratio is chi x x chi y y. So; that means, the direction of E and the direction of P is different, because the ratio suggest that they are angle between x and y is 45 degree; that means, it is oriented along 45 degree in this plane. P x and P y is also; that means, the P is also in the same plane, but it is orientation is not 45 degrees because we have this ratio it is only true when chi x x and chi y y are same.

If chi x x is equal to chi y y then you can say that P x and P y in same direction; that means, P x and P y ratio is one; that means, E and P in same direction, but this ratio is normally not same in an isotropic system this ratio these values chi x x chi y x it should be x y and z z are not same they are different since they are different the angle between these 2 are different.

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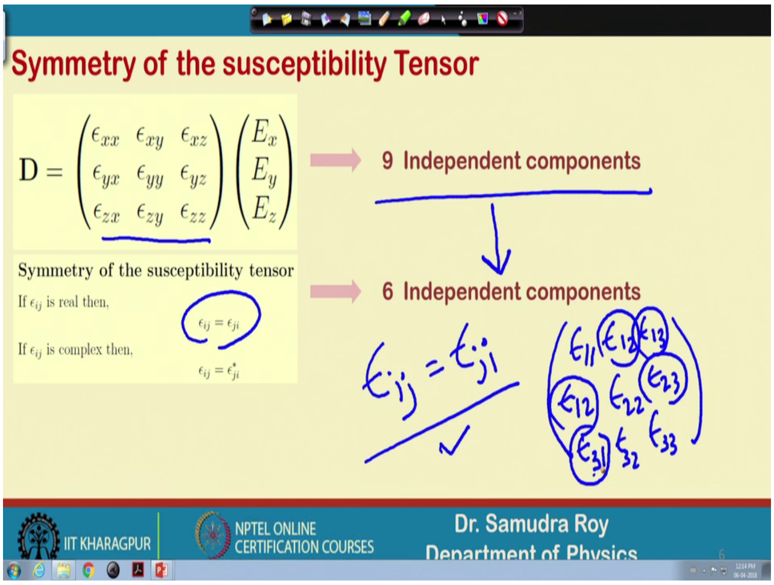

So, the same thing D matrix can be D vector can be represented in terms of the susceptibility tensor and here the susceptibility tensor is written in this form it has 9 different components as is written, but this 9 different components may not be may not be different, because susceptibility tensor has some kind of some kind of symmetry here the symmetry is if they are real quantity then epsilon i j is equal to epsilon j i.

Considering this is a real quantity in case of complex the i j is equal to the complex quantity so; that means, they are forming some kind of hermitian relationship. So, if is susceptibility tensor has this property. So, readily we can understand that this 9 component now shrink to 6 different components. So; that means, whatever the value I have in the diagonal case say 1 1 2 2 3 3 it is different.

Now, 1 2 1 3 2 3 these 3 components 1 2 1 3 and 2 3 is same as here 2 1 and then here E 3 1 E 3 2. So, these 3 components this 3 components are same, because if I interchange this things they are same 1 2 is equal to 2 1 that is why I write 2 1 1 3 will be equal to 3 1. So, I can also write it is 1 3. So, this 3 are same. So, that is why I have 6 independent components ok.

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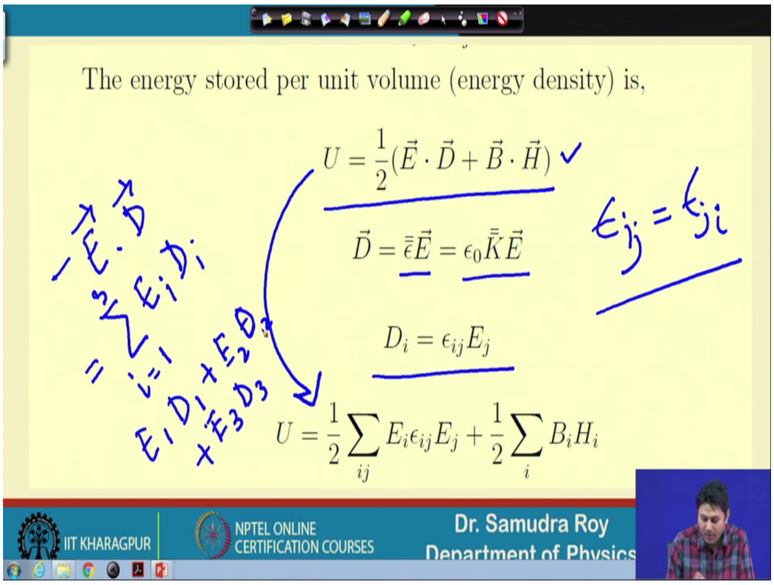

Now, let us understand why this, these components, which is epsilon i j is equal to epsilon j i try to understand this in detail. So, one way to one way is to use the energy expression. So, in many places you find that energy density can be represented in this way if this is the case. So, this is some sort of proof that epsilon i j is equal to epsilon j i. So, they are symmetry in nature I mean this coefficient are symmetric.

So, U is now energy density. So, energy stored per unit volume this is the energy density definition of energy density and I can write this as this way. If I write this then D is epsilon I epsilon bar double bar I or epsilon double bar K epsilon 0 double bar K E this is the definition of these relationship between D and E in anisotropic system. So, in component form I can write D i is equal to epsilon i j and then E j once I have this then I can put it here in this equation.

So, I put this relationship here. So, if I make the dot product between E and D it comes out to be the summation of if I make the dot product E and D if I make the dot product of that. So, the result will be E x D x E y D y E z D z. So, if I write in component form it will be E i D i, I running from say x y z so, 1 2 3. So, E 1 D 1 plus E 2 D 2 plus E 3 D 3 something like that, where E 1 D 1 are the x component E 2 D 2 is a y component and so on.

So, now I can write this in this particular form this component form and now I also use the relationship between D and E. So, I have this. So, E i is there in place of D i i play I write epsilon i j E j in the similar way B can be represented that this, but I am not writing any other way B and H, because I am not bothering about this term this is a magnetic term due to the magnetic effect, but as I mentioned in my last class that we are we are using this kind of materials where the magnetic effect is not there, only I am I am interested in the and if the system has some electric property of the dielectric constant is important here ok.

So, I write u in terms of this component form once I write this E in this component form then this is the way 1 can write. So, then what I try to find out is the power flow in unit volume once is say power flow the unit volume.

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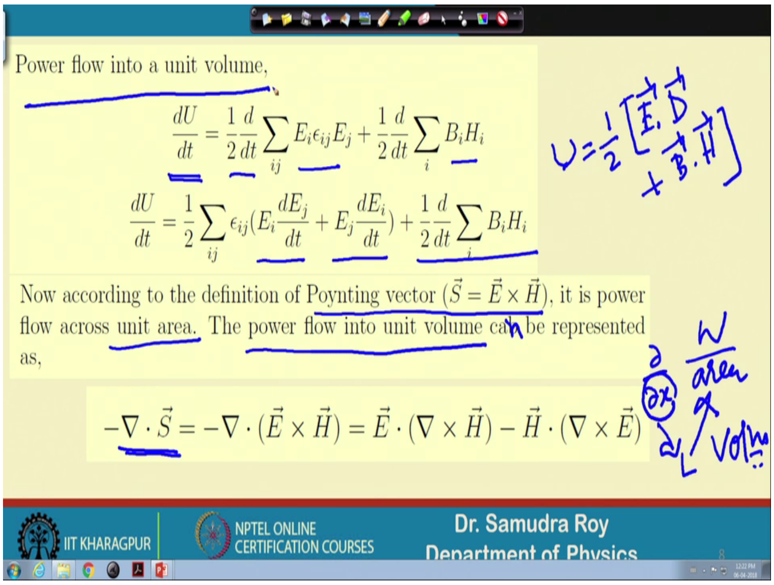

Then, there is a rate associated with that. So, the rate when the rate is associated that; that means, I try to find out with respect to time. So, that is why I make a derivative of this U with respect to time this is the amount of power flow per unit volume, because U is a energy density. So, amount of power flow per unit volume.

So, when I make this things make a derivative. So, the derivative will be over this quantity and this quantity, because U was half of this E i j stuff. Now, in the second step what we do that we make a derivative with respect to t and this quantity has E i and E j, since this quantity has E i and E j we write this derivative into 2 parts E i E j d t and E j d i d t this is the 2 part.

We know that if the function is E i and E j, then make a derivative if this to function are multiplied to each other, than one function multiplied the derivative of the other function plus the other function multiplied by the first one derivative of the first one. So, this is the rule we know. So, using that rule I have these 2 terms. And the rest of the part is same and not as I mention the magnetic term I should not bother ok. So, this is the amount of power flow we have when I use the density, when I use the energy density of the form half E dot D plus B dot H when I use this form.

But, there is another way to find out the amount of energy flow per unit volume and that is to the pointing vector. So, pointing vector also we define in our last class it is the amount of energy flow per unit time per unit area or another what, the amount of power flow across unit area. So, the power flow per unit volume if I want to find out this quantity which is the same thing that we have derived in terms of U. So, now, we are doing the same thing we try to find out the power flow per unit volume. So, what we will do what we will do here there is mistake. So, I should be written I can. So, what we will do we can make a divergence of E.

So, that is the power flow into the system the negative sign suggest, that it is the power flow and this power flow is inside the system. So, into the volume if the power flow I calculate I need to do these things. So, unit wise if you find is something, where the amount of power flow per unit area. So, the what divided by unit area means the area will put there. So, it is what divided by area? So, now, when we make a divergence of this quantity; that means, you are making some kind of operation like del x x is sitting here, which is a space variable and have should have a dimension it should have a dimension like distance.

So; that means, length. So, length dimension is here. So, length into area this will be a dimension of volume. So, eventually what will have this quantity give the power flow per unit volume, which is the same thing that we have derived here by using this U. So, now, what happened E can be represented in terms of S can be represented in terms of E and H and when I write E and H, then I can write this identity vector identity in this simple way grad dot E cross H can be represented in this way, this is the standard way to represent the grad we know that then what we have.

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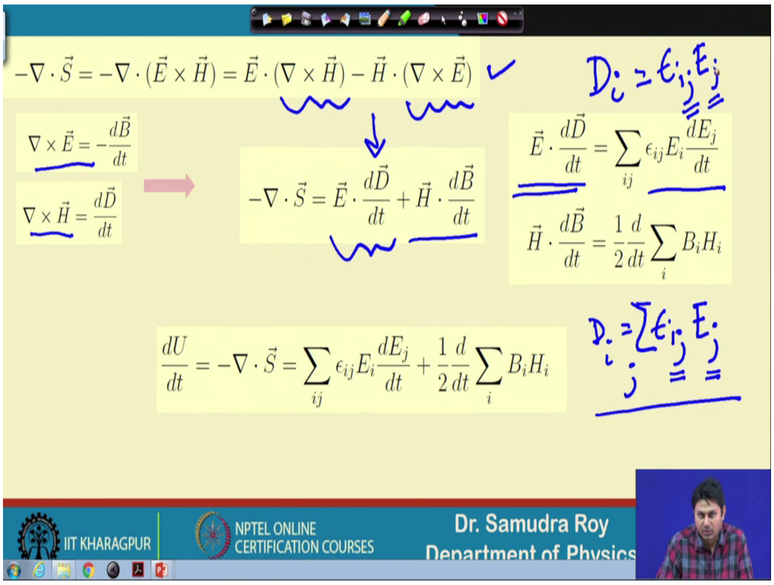

So, in the next slide we have the same expression that we have shown. Now, what we will do we just replace this and these with our known quantity; that means, this is the from Maxwell's equation I have curl cross E is minus del B del t and curl cross H is del D del t.

If I replace these 2 thing then this equation because of this replacement from here I can have expression which suggest the power flow per unit volume is E dot E dot d D divided by d t H dot d B d t, a dot product again can be represented in terms of summation. So, if I do this dot product in summation form it should be something like this. Again the relationship between D and E is used here because vector the ith component of the D is represented by i j and then E j.

By the way one thing I should I should mention here that this notation is called Einstein notation. When, we have a repetitive index here in this case the repetitive index is j so; that means, there is summation associated with that. So, sometimes we write this same expression as this way, summation over it j epsilon i j and then E j. This equations suggest that the submission is over j and when the j is repeated then the submission is over j.

So, now, Einstein notation suggests that if these repetitive indexes are there we should not use the summation sign. So, that is why I write the D i epsilon i j E j j is a repetitive index. So, that is why the summation is not required the summation sign is not required.

However, here I put the summation sign. So, make understand that over which the summation is because dot product itself as a submission over i and this d as a submission over j. So, that is why there are i j sitting here.

In the similar way h and b is there, but there is no j. So, only 1 summation is there and that is over i. Now, if I put these 2 things together then I have a relationship d u d t and this things S minus of divergence of S and have a form like this. So, d u d t if you remember d u d t this particular form we have already derive in our previous slides. So, now, if I compare which I will do in the next slide if I compare these 2 things then I will have the same expression, but 2 different forms.

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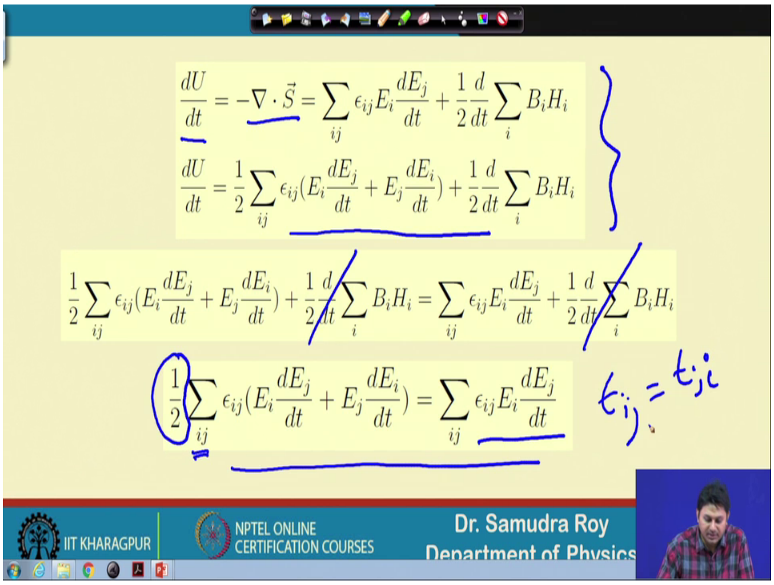

So, this is the comparison between 2 expressions. This is coming from this pointing vector and this term is coming by using the U. So, these 2 things are eventually the same thing, but we have 2 different equation. Now, we put this equation side the side when I put this equation side by side, I find that this term and this term will be cancel out because this is the same term we have both the side. Then we have interesting expression in our hand and this is interesting expression suggest that half of epsilon i j E i d i d j d t E j d i d t is equal to this quantity.

Now, this equation will be consistent only when epsilon i j will be equal to epsilon j i. So, if you try to understand this you need to make a summation over that when you make a summation over that then there are 2 summation here i j. So, epsilon i j will be epsilon i and j. So, I will take some value j and j will take some value i. So, if i same these are the same thing then we will have a term that is twice. So, that things will cancel out this half term and because of the cancel out of this half term eventually we will get a single equation like this.

So, this equation will be consistent only when epsilon i j is equal to epsilon j i. So, with this treatment we can prove that this is the case here ok.

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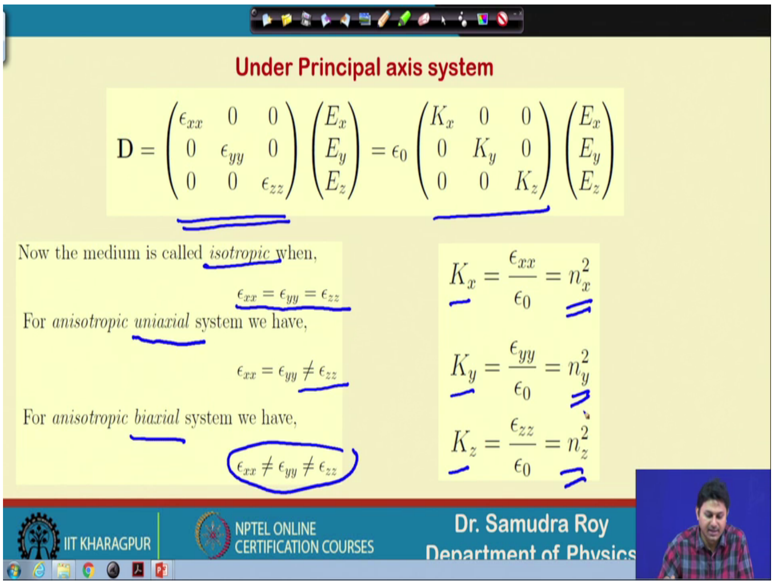

So, go back to our an isotropic system. So, under principal axis system this is our expression of D and E epsilon can be represented in terms of k like this also K x K y K z. And, if the system is isotropic we know that epsilon i i epsilon epsilon x x epsilon y y and epsilon z z are same. If epsilon x x and epsilon y y are same, but epsilon z z are not same this belongs to the anisotropic system and we call this anisotropic uniaxial system.

In the similar way if all the 4 3 terms are different it is called the anisotropic biaxial system. So, uniaxial biaxial anisotropic system in isotropic system all the terms are same, but here in other case it is different. So, now, what happened if I write K x K y K z I can find that K x is nothing, but the refractive index of x y and z, which are different. So; that means, the refractive index along x y and z direction is different for anisotropic system, it is very important outcome the refractive index of x y and z are different.

Now, if I put y y and z z same as x x then we find that in isotropic system the refractive index is same in all the 3 directions. So, an isotropic system the refractive index is different for 3 different directions ok.

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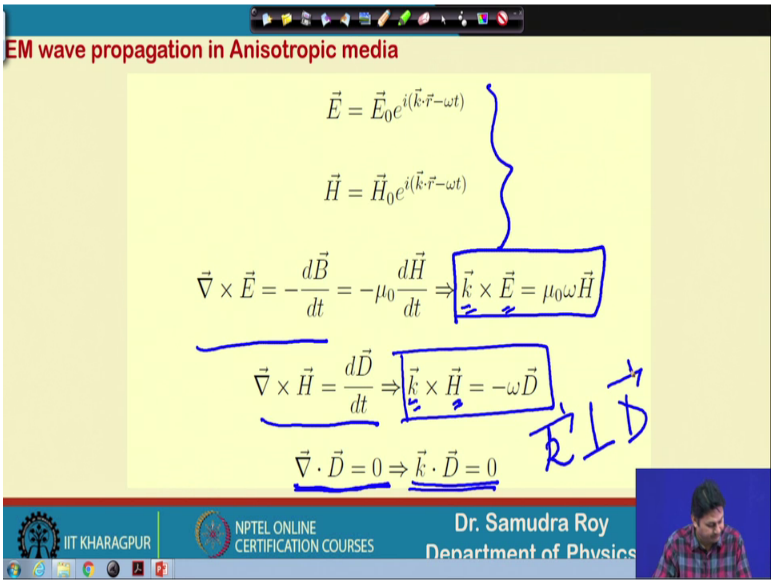

Now, finally, we like to understand how the wave is propagating in an isotropic medium that is a very important thing, because it is not trivial something say not trivial.

So, let us start with our electric and magnetic field vector, which is these 2 and then I use the Maxwell's equation and from this Maxwell's equation I can find one expression that k cross E is equal to H. This expression this is not a new expression this expression we have already figure out in the earlier classes. So, k cross E means the H vector is perpendicular to k and E, that is outcome that is a first outcome that H vector; that means, magnetic field vector is perpendicular to the vector k and the vector E.

Next expression what we have is this; this expression is coming directly from the Maxwell's equation that is forth Maxwell's equation. And these equation suggest that D vector perpendicular to the vector k and H, previously we find that H vector is perpendicular to k and E. Now, we find that the D vector is perpendicular to k and H. Another, equations this is the first Maxwell's equation we have. So, from this Maxwell's equation we have a very important outcome k and D.

So, k and D are related to k dot D equal to 0; that means, k and D are perpendicular. So, we have k perpendicular to D. So, vector is perpendicular to D.

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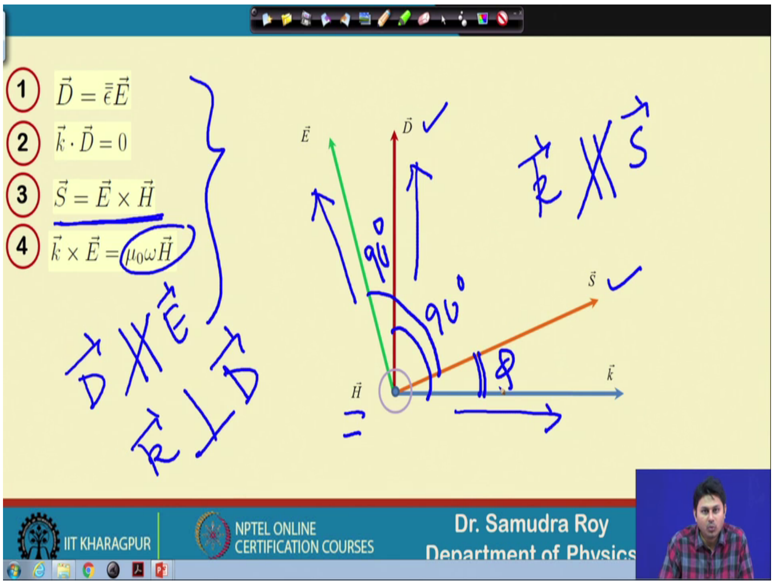

So, few informations we have so, now I find out what is the significance, what are the significances of this expression? So, there are 4 expression written here there are 4 expression written here, this 4 expression suggest first expression suggest D vector is not parallel to E this is the first conclusion; that means, if I have a D in this figure if I look if I have a vector D. So, E will be in different direction.

So, this is the direction of D and this is the direction of E that is the first conclusion we have. Second thing is that k dot D is 0; that means, k vector should be perpendicular to the D vector, if that is the case D vector is given. So, the k vector should be perpendicular. So, these is 90 degree this is 90 degree. Another expression is here which is S and E, these expressions suggest that S vector is perpendicular to E and H.

Before considering the s let us consider the fourth expression and this forth expression suggest that H vector is perpendicular to k and E both. So, now, k and E in this plane so, H vector has to be the perpendicular to this plane. So, here we find that H vector is perpendicular to this particular plane once H vector is perpendicular to the plane is also perpendicular to D it is also perpendicular to S, that is for sure.

Now, S vector is perpendicular to E and H both; that means, S vector and E vector are perpendicular. So, I can write here this is ninety degree. So, one thing very interesting we figure out here is that k vector and S vector are now not in the same direction. So, k vector is not parallel to S vector. And in the previous case we find that the S vector is something, which gives us the direction of the energy flow. And in anisotropic isotropic system you find that the energy flow in the same direction of the k, that is why in the previous class we put this k vector inside this intensity term.

So; that means, the energy is flowing in the same direction of the k, but that is only true when the system is isotropic. And in anisotropic system we find that there is a difference the direction is different with k and f. So, k is a flow of the field, but energy is flowing completely different direction and there is a difference phi this phi is an angle and we call this angle as a work of angle. So, we will learn more about work of angle in our future classes.

So, here we like to conclude today's class. So, today's class will learn few interesting concept that susceptibility what is the susceptibility tensor and how I diagonalize this susceptibility tensor and this diagonal elements are nothing, but the refractive index along x y and z direction. And then also we figure out that how the electric field vector and the displacement vector and S; that means, the pointing vector and the k vector are related to each other in anisotropic system. And we find a very important thing that the k vector and the energy flow S vector are not in same direction.

So, the wave is propagating along one direction and the corresponding energy can flow in a different direction. And there is a gap between these 2 angle between these 2 and these angle is called the work of angle. We will learn more about this we will do more rigorous calculation to figure out how this things is going on in our next class.

So, with this note so, let me conclude here thank you very much for your attention. So, see you in the next class.

Thank you.