Introduction to Non-Linear Optics and its Applications
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Lecture – 29
Quasi phase matching (QPM) (Contd.), Periodic d function

So, welcome student, to the next class of Introduction to Non-Linear Optics and its Application. So, in the previous class, we have started a very important concept regarding phase matching. This is called quasi phase matching and the basic principle of the quasi phase matching was to find out a suitable phase through d which is a non-linear coefficient. Now, d will be varying sinusoidally, so that it can compensate the additional phase that is happening due to the phase mismatch delta k.

So, delta k is compensated by another quantity we call K Q which is coming because of the fact that my d matrix or the d elements is now periodic in nature. So, that basically give rise to another quantity which is some sort of wave vector we call it K Q that will now going to compensate the delta k and that is why we will get a phase matching, a different kind of phase matching.

The interesting fact of this kind of phase matching is that we do not required any kind of birefringent crystal for that. Any kind of material we can use and today we will going to find out how we can eventually make this d value varies periodically which is not very easy job to do. But we can make a system where d can vary periodically, but not with a sinusoidal function rather a step function. So, if a step function is there then how to handle that thing and how the our calculation is modifying that we will going to do in this course.

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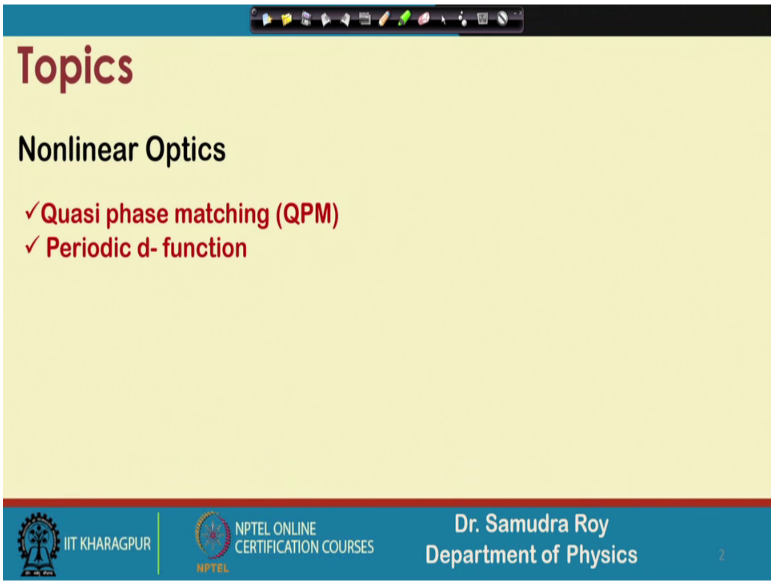

So, let us see what we have in this course. So, quasi phase matching which is thing that we have already started and then periodic d-function as we already mentioned that if the d-function is periodic what kind of thing, what is the consequence we have, that we are going to learn in today's class.

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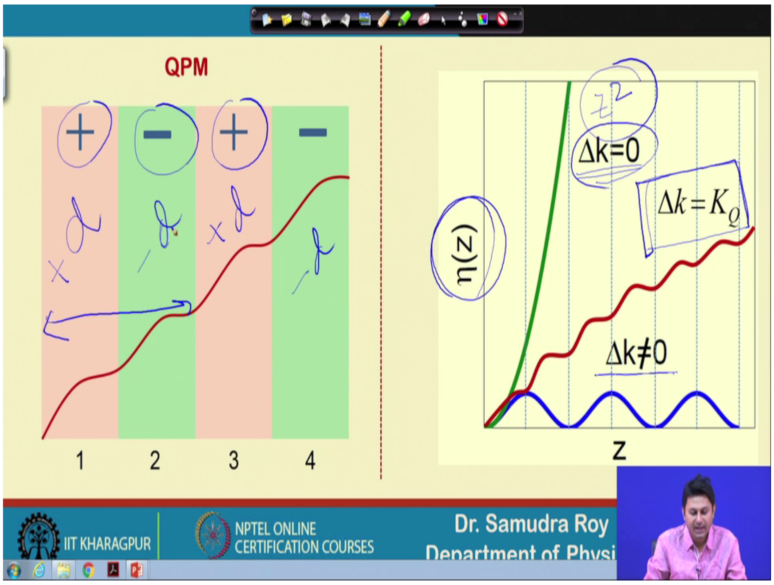

So, let us see this structure. So, in quasi phase matching the important thing is that this phase K Q is now related to another quantity delta k or delta k is related to another quantity K Q, this is the old figure. Because of this phase match we have the improvement in this eta value eta is a some sort of quantity which can give you what is the percentage of energy that is transformed from fundamental to second harmonic. So, this is something called efficiency. So, the how the efficiency is going to improve by introduction of this quasi phase matching thing is shown in this figure. When delta k not equal to 0, that is the most general case, we find that efficiency is there, but also it is going down if delta k equal to 0, that is the most ideal case, we find efficiency is going as a function of z square that will also learnt in the previous class

But, delta k equal to K Q; K Q is some sort of intermediate process, where we can improve the efficiency, but not the way we have here delta k equal to 0. However, delta k equal to 0, is the most on realistic condition because we find that delta k equal to 0 in this condition the efficiency which vary as a function of z square. When we consider a very important approximation that my pump; that means, the fundamental field is constant, but in real case this is not true. So, we will not going to get this kind of efficiency or the efficiency may not be a function of z square. So, we will find that in the later class how the realistic calculation gives the efficiency.

But, today we will going to learn about quasi phase matching. So, this kind of phase matching or this kind of efficiency we will get when we have a phase matching condition under quasi phase matching case. So, this is the figure in the left hand side; where we can see that this plus minus sign basically designate the nonlinearity of the system. So, here the nonlinearity d value is plus, here it is minus, here again it is plus here it is minus and so on and we have a period here. So, this is the period in one period we have the very we have the variable d which plus to minus.

So, once we have this kind of structure this minus d basically give rise to additional phase and this basically compensate delta k, that is the idea. So, today we will going to go more detailed calculation.

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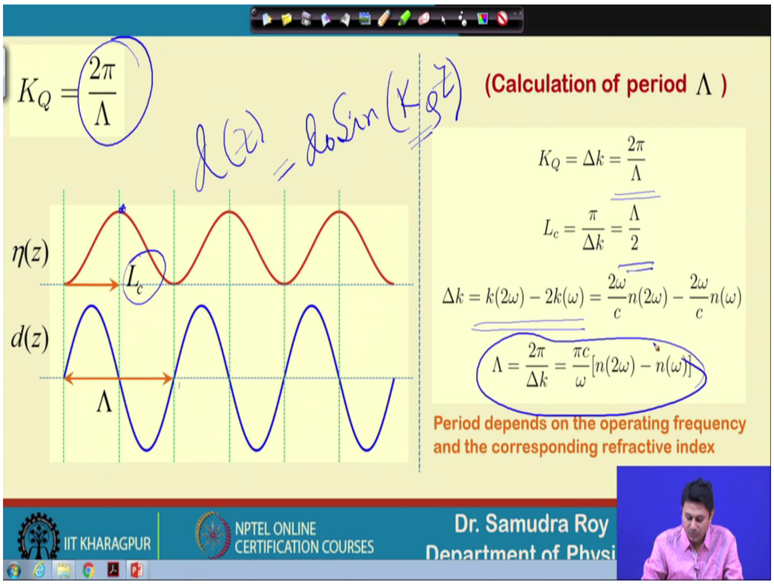

So, in order to find out this periodicity of d because d if you remember d this is the function of dz and it was d 0 sin of K Q Z this was the form of d. This K Q basically corresponds to the phase and this K Q corresponds to the period and this K Q is equal to 2 pi by lambda, this lambda is basically the periodicity of the system.

So, here we can see that in one period the d value will go maxima and then go minima and then again go to 0 and that is the standard sin curve. On the other hand if I compare this periodicity with the periodicity that we have in finding the efficiency in delta k not equal to 0 case, that we have shown in the previous slide that it will vary like this. So, there is also a periodicity and we know that L c is a length which is called the coherence length at which we have maxima here.

So, here we find that when we have a maxima and then after that this is going down. What happened in this case we will compensate this phase with this additional phase incurred by this d value. So, this is the concept and using this concept you can find out what is this lambda.

Now, one can also find out the lambda. So, if I consider this is a phase matching. So, K Q is equal to delta k and 2 pi by lambda L c is equal to lambda by 2, very easily we can see that the L c should be the half of the period whatever we have in periodic d and delta k which is k 2 omega minus 2 k or the k 2 minus 2 k 1. So, now if I use this expression together then we can find that delta is these value. So, if I know the refractive index at 2 omega if I know the refractive index at omega, then readily we can find out what should be my periodicity. So, this we have already done in the previous classes, this is some sort of review of the previous class.

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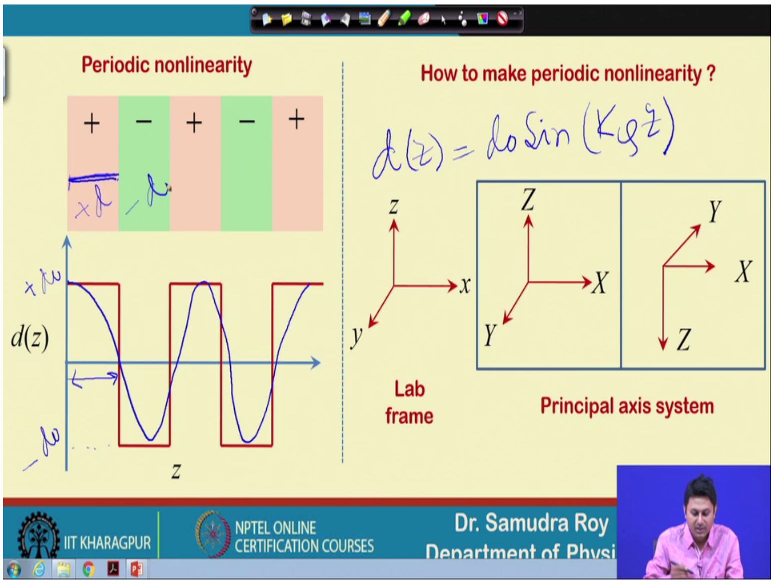

So, today we will going to learn more important thing that we mention in the previous case that d is varying sinusoidally which is the most ideal case. But, in fact, it is not possible when we try to do in realistic case. So, in realistic case what happen? So, we have a periodic nonlinearity, so that means, the value of d here so, the value of d from here to here not going to vary. So, it will be a constant value.

For sinusoidal case if I plot here, for example, for sinusoidal case it will be something like the variation will be something like this is roughly the sinusoidal variation, but the sinusoidal variation can be approximated by this step like functions because we know that from here to here which is from this point to this point what happened, the d value will remain constant. So, there will be no change in d value as shown here in sinusoidal case the d value is changing, but if I put some crystal here the d value remain constant from here to here.

Now, what happened, after this point we will put the crystal in such a way that the value of d is negative, so, suddenly we have a drop. So, d value will go to negative sign. So, if this point is plus d 0 this point will be minus d 0. So, it will vary like this, it will not going to vary sinusoidally.

So, we need to take care of this issue because when we develop the theory in the previous class, we consider d as a function of z as d 0 sin of K Q of z this was our assumption that d is varying sinusoidally, but in real case we find it is not varying sinusoidally it is varying, but it is varying in a step like function from positive to negative.

So, we will handle this issue today, but before that we need to know how we make crystal positive d to negative d, that is important, because I am using the same crystal and I put this two part of the crystal in such a way that in one portion of the crystal I have a positive d value and in another portion of the crystal we have negative d value. So, how one can make this we need to find out. So, the process is not that difficult to be very honest. So, we can just one treatment is if you just rotate the crystal or change some put some kind of operation of the crystal. So, that the d component become minus d then the thing can be done very easily.

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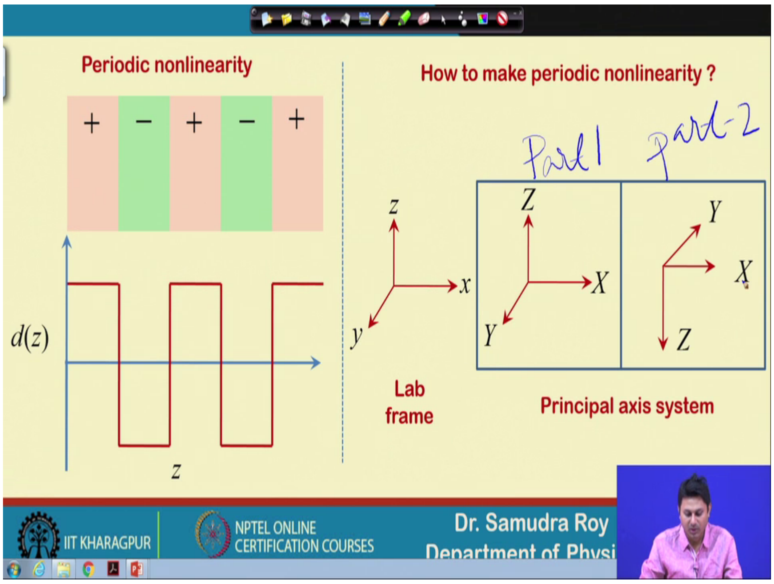

For example here suppose we have a crystal this crystal has two part; this is part – 1 and this is part – 2, this is a laboratory frame and we know that for any crystal we can write our d matrix for a given coordinate system. So, normally the d matrix is written in terms of principle axis. So, big X, big Y, big big Z, capital X, capital Y, capital Z is a coordinate system which basically gives us the principle axis. So, this is under principle is big X, big Y, big big Z or capital X, capital Y, capital Z.

In part – 1, what happen, this coordinate system is suppose coincide with the lab frame. So, we have a d matrix here for this particular system or this particular orientation. Now, what we will do in part – 2, we will make a rotation of say theta equal to pi; that means, this z will go down and since this is a right handed coordinate system this is a left handed coordinate system. So, X, Z goes to here, Y goes to this point and X remain same because I am making a rotation around X point.

So, now my principle axis is changed, compared to my lab frame axis. So, whatever the d value we have under this rotated coordinate system should be different from the previous one. So, let us take one example, then this things may be clear.

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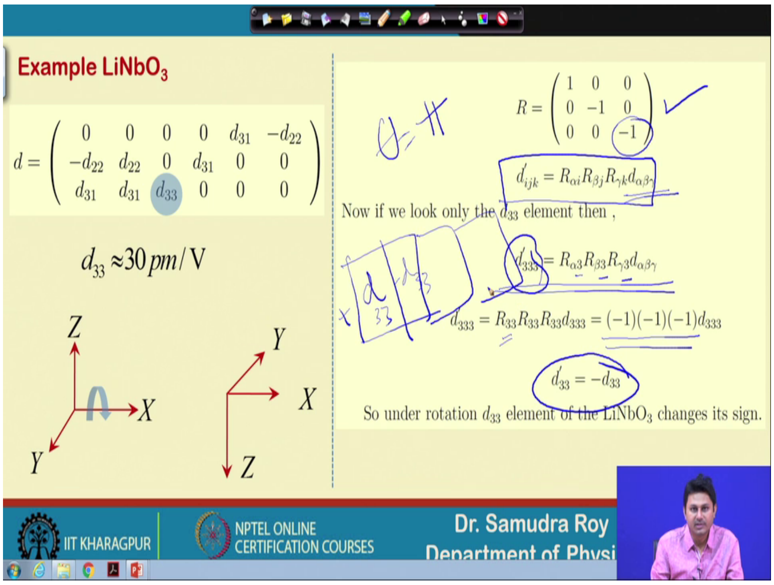

So, we will going to do the samething for lithium niobate crystal. We know that for lithium niobate crystal the quantity d 33 is maximum, it is of the order of 30 pecometer per volt. In the last class I think we discussed this issue. Now, what happened that we will going to make a pi by pi rotation around X axis. So, this is our principle axis for this principle axis the d matrix is written here and now, for example, to have the quasi phase matching the d 33 is a quantity that we are going to use. So, there are many d components here, but d 33 which has a maximum value we will going to use this 33 to achieve quasi phase matching.

So, that means, our aim should be if I rotate this system to theta equal to pi around x axis then what should be the value of d under this rotated system. So, we know the what is the form when the matrix or a tenser is rotated to certain angle or make some kind of transformation. So, if I put some kind of transformation over that, then what happened, that the coefficient is going to change and we know the coefficient can be written in this general form.

So, d ijk prime is the prime the coefficient of ijk of d matrix at prime frame and it is related to the coefficient of nonprime frame the alpha beta gamma and R alpha I, R beta j and R gamma k is the component of the matrix that basically leads to this kind of transformation. So, in this case it is rotation so, we know that for rotation matrix what is R and now if I put say theta equal to pi; that means, on rotating pi along X axis so, my R matrix will be having these particular form. So, I know what is my R 11, R 12, R 13 and so on.

So, now if I try to put this rotation try to find out what happened the d 33 component under this rotation then we going to use this and when we going to use this expression for d 33 I need to put d 33 prime because I need to know what is the new form or what is the value of d 33 in prime frame. So, once I put d 33 I need to put ijk is equal to 33 and 3 then I put alpha 3 beta 3 and gamma 3 and alpha beta gamma should be some over; So, then alpha beta gamma should take 1 2 3 value for all the cases.

But, we find that R matrix is such a way that only the diagonal elements are there all the other elements is 0. So, when I write R alpha 3 the alpha has to be 3 because only R 33 coefficient is nonzero that is this one all the other elements is 0. In the similar way, R beta 3 also can be represented as R 33 because other elements are 0 and R gamma 3 also represented by R 33 because other elements is 0. So, since the other elements are 0, then R 33 R 33 R 33 we put the value of R 33 which is minus 1 minus 1 minus 1.

If I put 3 minus 1, then we find that d 33 prime is related to d 33 with a negative sign; that means, in the rotated system or if I rotate the crystal and then if I try to find out what should be my d value then you find that the d value at least the component 33 component is now a negative value of the previous one. That means, before rotation whatever the d value we have after the rotation in rotated coordinate system this value is changed and this value is a negative of whatever the value we have.

So, that means, if I just change or rotate the crystal and coupled it with the previous crystal then we have a structure like that where in this part we have d plus say d 33 which is plus and this part we have d 33 minus and if I do the same thing with by putting all these things together then we will have a structure what we have a variable d. So, one portion it is d positive, another portion d negative, one portion positive negative and so on.

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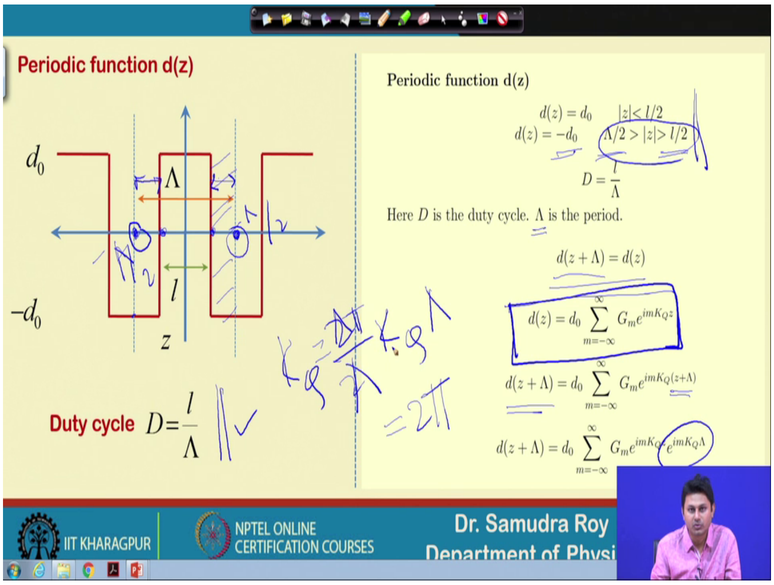

Ok. So, now, since we have as I mentioned the d-function is not sinusoidal, it is a periodic function, but it is a some sort of block kind of function. So, we need to take care of this issue and we need to find out the Fourier components of this, so that we can find out what is the corresponding Fourier component that we will going to use.

So, let us try to understand this d function first. So, here we plot the d as a function of z. So, this is the period from this point from this point to this point suppose we have the period. So, this period is lambda, you can see that this particular structure will be there for right hand side and left hand side. So, this is a repetitive structure. So, this structure that is why from here to here this point to this point we write it as my period lambda.

From here to here we say this length is l. So, the ratio lambda and l is called a duty cycle. So, that we will going to use in the later, but we should know. So, the function is something which is shown here d is equal to d 0 when delta of z is less than l by 2. So, this point is l by 2 this point is l by 2. So, when the delta z; that means, from here to here the value of d is positive. When delta z is mod of mod of z is less than delta 2 greater than delta 2, but less than big lambda by 2. So, we are basically considering from here to here this point and here to here this point.

And, this point in this region this basically this point is lambda by 2 this coordinate is lambda by 2 this coordinate is minus lambda by 2 minus lambda by 2 this point. So, minus lambda by 2, 2 minus l by 2 the value of d will be minus d, l by 2 2 lambda by 2 the the value is also minus d. So, this minus d obviously, in this range, this is the range where the d value is minus d.

So, we defined the d value or we defined the d-function and based on this function we can try to find out what should be the functional form. So, now, one thing one should note that d has a periodicity lambda. So, d-function should be written in such a way that dz by lambda should be equal to dz this is the form of a periodic function we know that. So, suppose we write d as this is basically a Fourier form where these are the Fourier coefficients. So, I am making a sum over this quantity to represent a periodic form. We can always represent a periodic function in this way. So, if the function is periodic we represent this where G m is a Fourier components of that.

Now, if I write dz plus lambda then it should be G m e to the power iK in place of z I write z plus lambda. So, in the next step it should be G m e to the power imK z into e to the power iKm K z, this quantity K e to the power im K Q lambda; K Q lambda we know it is how much? It is 2 pi because K Q was lambda divided by 2 pi divided by lambda. So, K Q into lambda is equal to 2 pi.

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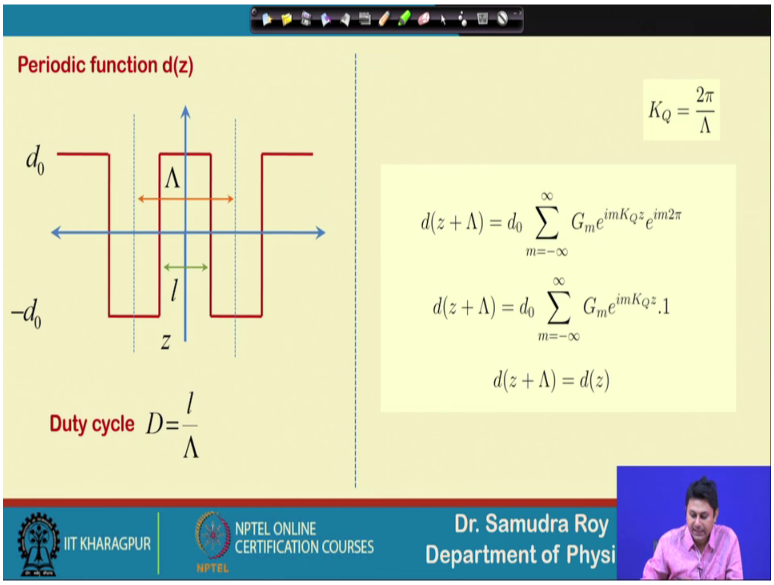

So, once we have this thing then this thing we can replaced by e to the power of i 2 pi into m, where m is a integral. So, this quantity is eventually 1. So, if I go back to the next slide then we can see that since K is 2 pi by lambda dz plus lambda is equal to this quantity and when I put this is something which is valid. So, that is why the point is d can be represented in that summation form.

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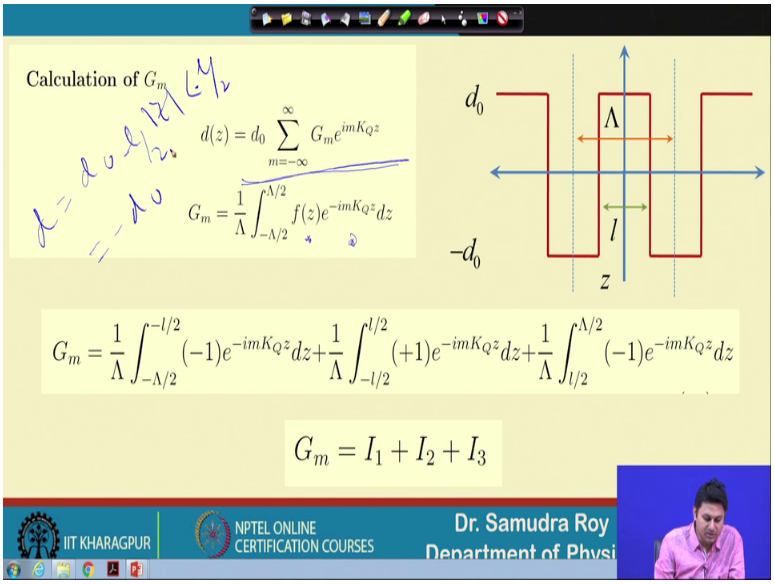

So, next thing that is important is how to find out this G m because, G m is something which is important. So, d can be dz can be represented for this periodic function, this is the realistic function of d. So, we can write d as summation of m tends to infinity m from minus infinity to infinity. So, Fourier as a Fourier series so, G m is a Fourier component. So, Fourier component can be represented if the function is given we know that if this is given then G m can be represented over the full period then the given function and e to the power i minus m K z dz.

So, if the functional form is given, and then if I integrate to over the period with 1 by period, then we will have the value of G m; So, now, the function is given, so, fz is given to us which is already shown in the previous slide that d is equal to d 0 and equal to minus d 0 with mod of z is less than l by 2 minus l by minus l by 2 l by 2 this and ok. So, let me go back to previous slide.

Then, the d-function definition of the d-function is there. So, it is less than l by 2 and greater than l by 2 when I put the mod sign I do not need to put the other limit. So, this is the definition of g definition of d function.

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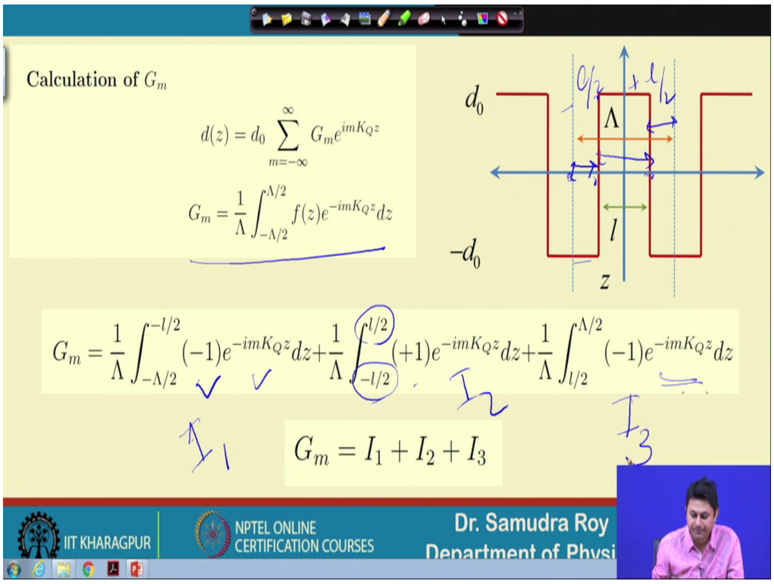

So, now what we will do here we will find out what is my G m. So, G m is defined by this so, we will integrate part by part. So, first we integrate from this point to this point which is minus lambda by 2 to l by 2. So, when minus lambda by 2 l by 2 we integrate the value here is minus 1, because I have already put this row outside. So, function here is basically minus 1. So, we put function minus 1, e to the power imKz, ok.

Then the next part the next part is from integrated from here to here first then I integrate in this region here to here. So, this point it is l by 2 minus and this point it is l by 2 plus. So, minus l by 2 to plus l by 2 I integrate and here the value of d is plus 1, so, this is plus 1, e to the power minus imKz, z will be there and finally, we have 1 by lambda from here to here the length is the limit is here plus l by 2 to plus lambda by 2; So, minus 1 e to the power imKzz.

So, we can divide the total integration into three part the first part I can say this is I 1 and second part I say it is I 2 and third part integration is I 3. So, now the next thing we need to evaluate the I 1, I 2, I 3 independently and then I find what is my G m.

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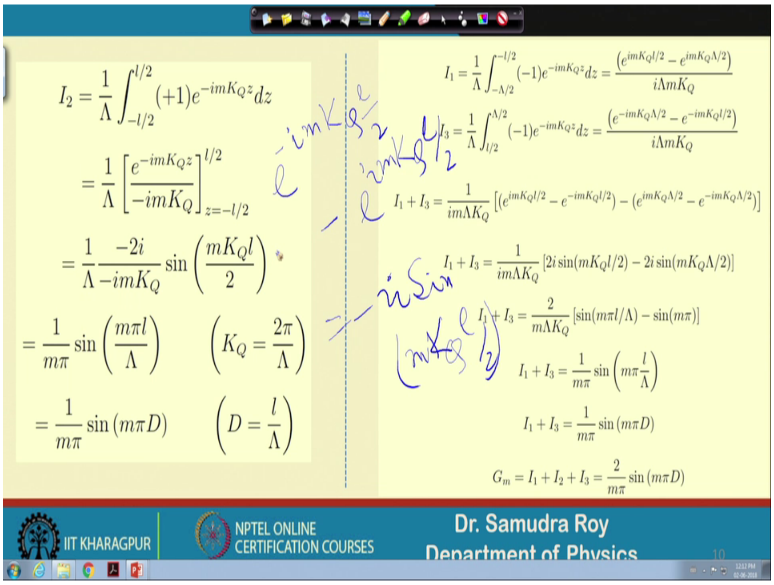

So, first I 2 part which is easier because I am making a limit minus l by 2 to plus l by 2. So, the function is plus 1 here e to the power i. So, I just readily integrate. So, when I integrate it will be e to the power imKzz divided by imKz with a limit z equal to minus l by 2 to plus l by 2.

Now, if I put this things here, then what we will find? e to the power here we can find e to the power minus of im Kz l by 2 minus e to the power of imK Q l by 2. So, first we will find e to the power imK Q l by 2 and then we will find e to the power plus imK Q l by 2 divided by imK Q. So, if I take this common and if I consider this it is nothing, but minus of 2 i sin of mK Q l by 2, which is shown here this is the quantity we will have.

So, from here we can easily find that this is sin and now, if I put the value of K Q then K Q is 2 pi by lambda. So, 2 pi by lambda if I put here then readily we can find that it value this value is nothing, but 1 by m pi sin of m pi l by lambda.

Now, I can also use this quantity which is a duty cycle, very important quantity. So, this become I 2 become 1 by 2 pi and if I put this ratio l by lambda to d. So, m pi by d, but what about I 1 and I 3? So, I 1 also exactly in the similar way if I do then we will find it is e to the power iKmK Q l and pi l by this quantity, I 3 also we can do the similar way and we will have a similar kind of quantity. When we add this two find mind it the limit is different here. Here the limit was minus l by 2 to plus l by 2 that is we can readily have the sin form, but here it is minus lambda by 2 to plus lambda by 2. So, that is why the limit is different and since the limit is different we will not going to get the sin come directly.

So, what we will get if I now add this two thing, then one term here and one term here one term here and another term here can be from these two, can be considered. And when this two term we consider we will find two term like sin and sin one is one for one the argument is mK Q l divided by 2 in other case the argument is mK Q lambda by 2.

So, these two when we add then we find that K Q lambda by 2 basically gives me sin of m pi which is 0, because sin of we know that sin of m pi by m is an integer this quantity is0. So, eventually we will come that I 1 plus I 2 is 1 by m pi sin of m pi l by lambda. Now, l by lambda is d. So, we will have 1 by m pi sin of m pi d.

If you look here we have already got the same thing for I 2. So, I 1 plus I 2 plus I 3 is eventually 2 divided by m pi sin of m pi into d. So, this is the important quantity because in the future classes we are going to use you need to find out what is the value of G? G m is a general term if I what to find what is G 0 then if I put 0 here you will find at the limit say G tends to 0 this limit goes to 1 or some value because I need to put a d multiply. So, it should be around these value should be 2 divided by d or something or if this value is around 2.

So, in the similar way we can find what is my G 1, G 2, G 3 and so on, that this kind of value we will going to use in the next class. So, important thing is you need to make sure that you know what is the value of G when the function of a function of d is periodically changing when the function is periodically changing the value of G the Fourier components can be calculated and it should be like this.

So, with this note let me conclude today's class. So, today we learnt mainly how the quasi phase matching terms are there if the periodicity of d is there, it is not sinusoidal. If it is a block kind of function, then how to handle this kind of things we need to find out the corresponding Fourier components. And we find the corresponding Fourier components in terms of G m, which we will going to use in the next class.

So, today we like to conclude here. So, let me finish the class here. So, see you in the next class and thank you for your attention.