Introduction to Non-Linear Optics and its Applications
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Lecture – 28
Quasi phase matching (QPM)

So, welcome student, to the new class of Introduction of Non-Linear Optics and its Application. So, today is the lecture number – 28 and in the previous lecture, we have learnt a lot about the birefringent phase matching, which is mainly generated in crystal. We learn about the crystal symmetry and applying this crystal symmetry, how it is possible to generate the susceptibility matrix or d matrix, how the different element of the d matrix can be figure out by using symmetry operation, that we have also learnt. And after having all this information how we can use this information to find out the physics of second harmonic generation it is also been taught to you in the previous classes.

So, today, we will going to learn a new kind of phase matching because in our course we plan to cover two different kind of phase matching. The first one was birefringent kind of phase matching that we have already covered. Today we will going to start a new kind of phase matching called quasi phase matching.

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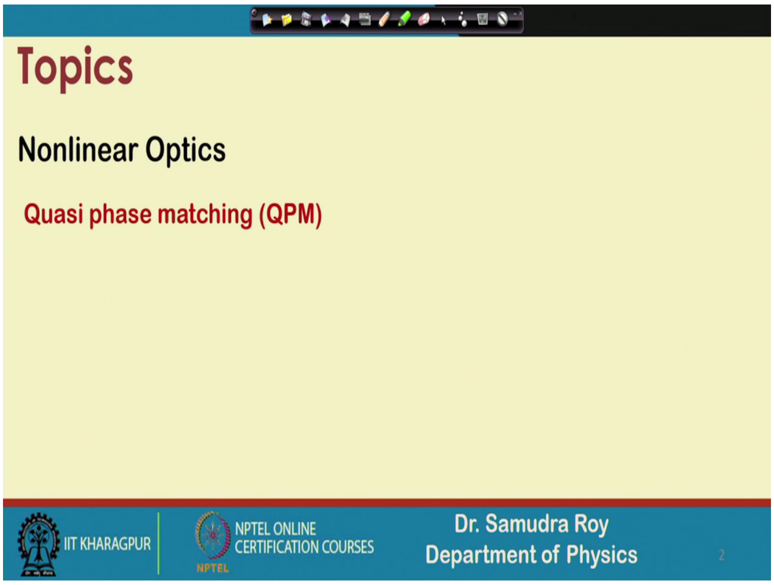

So, let us see what we have in quasi phase matching. So, this is the quasi phase matching today's topic.

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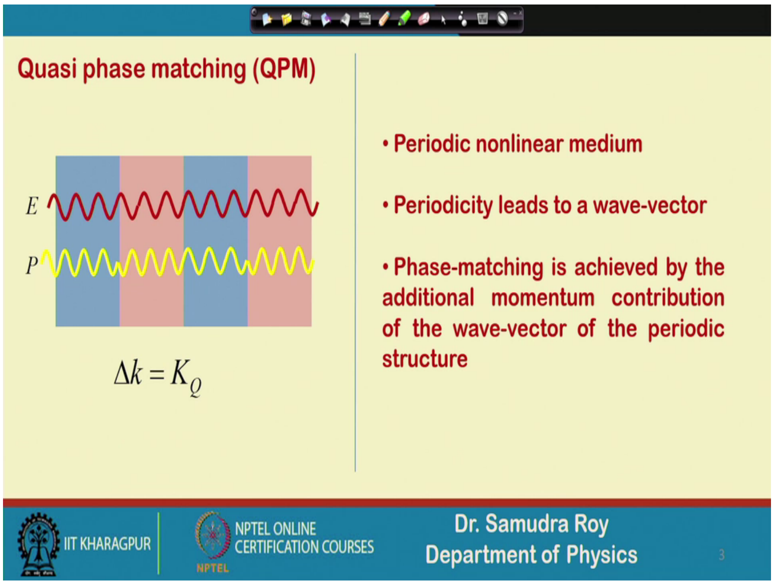

So, we have very briefly described these things to our earlier classes that in quasi phase matching what we essentially do that we will put some kind of periodicity in nonlinearity. So, that means, in a crystal we have several parts and this parts is continuing the non-linear coefficient, but this non-linear coefficient changes its sign in certain distance. So, there is a periodicity and over a period there will be a sign change. So, this periodicity of the structure basically leads to some kind of wave vector we call this wave vector k. So, using this wave vector we will going to we will going to compensate the phase matching delta phase matching term delta k, that is the aim here.

So, what we essentially doing in the previous case we find that propagation constant of two fields fundamental field and second harmonic field k 1 and k 2 if the quantity k 2 minus 2 k 1 which is equal to delta k is not equal to 0, we will have a non-phase matching condition. Say in order to make delta k equal to 0, we put lot of effort and in birefringent crystal we find that there is a possibility to do that if I launch my electric field in a specific direction the fundamental relative well in a specific direction, but here what we will do? We will use some sort of non-linear medium and this medium will be arranged in such a way that over a period it should have some kind of periodic structure and this periodic structure basically leads to some kind of wave vector and now this additional wave vector basically compensate this delta k.

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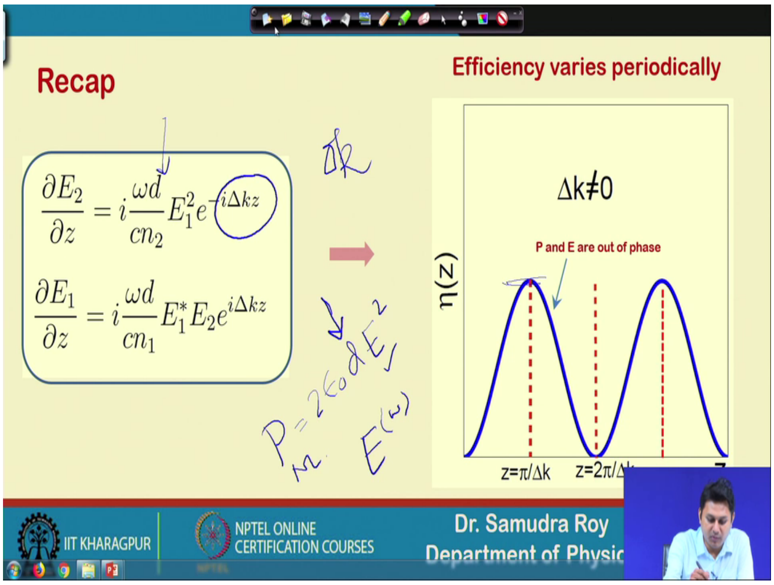

So, this is the concept and let us recap the thing. So, what was there in our previous lecture was the derivation of the second harmonic field and there was a coupled equation and this we considered our main equation or mother equation the phase mismatch term is sitting here, if you look very carefully is sitting here. Delta k is our phase mismatch term. Now, because of this delta k what happen that we need plot the efficiency up to this point we have some kind of efficiency and after that this efficiency go down to 0, and the reason behind that there is a phase mismatch between the polarization P and the electric field E.

The contribution of the polarization term is enter into the system by d, because polarization if you remember the polarization was the non-linear polarization was 2 epsilon 0 dE square if I write in this scalar way, this is a non-linear polarization. When I put this non-linear polarization into the non-linear Maxwell's equation then this non-linear polarization enter into the system through this value d. The electric field is already there, but there is a phase mismatch between electric field and this d and as a result the original electric field E and d this is the fundamental electric field that we have launched. And because of this, this phase mismatch between P and E what happened, we have loss of energy is here or the efficiency is going down.

So, our aim here is to introduce some process, so that these delta k can be compensated, how we can do that; so, one possible way that we can vary this d in such a way, that this phase can be nullified.

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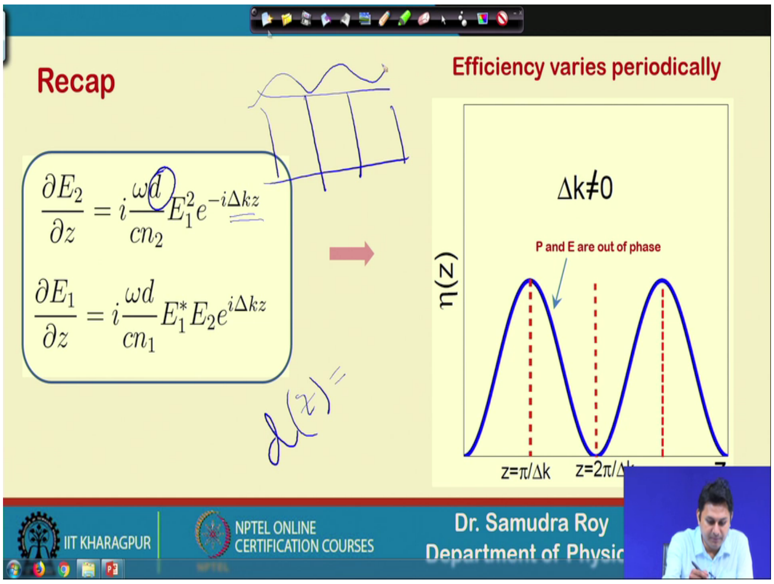

So, d for the so far we consider as a constant; constant means this is not a functional of the z, but now we consider d as function of z and it is periodic function because in the previous structure we find that the that was a structure that nonlinearity is changing its sign. So, that means, there is a periodicity involved in the structure.

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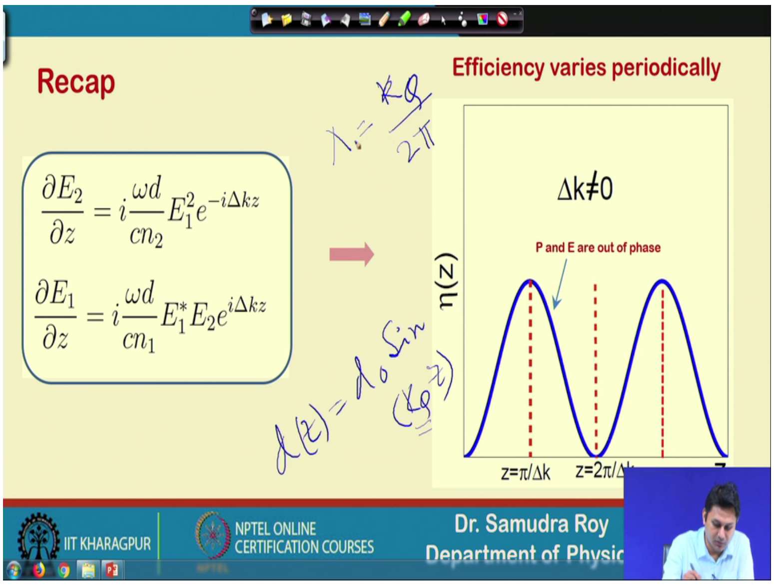

So, if I consider this periodicity, then d can be approximately written as a sinusoidal function and since it has some periodicity I write this Kz, like where Kz is the corresponding wave vector if the periodicity is lambda then obviously, k and lambda will be related like this. So, this K Q is by 2 pi, now that is 2 pi so, sorry.

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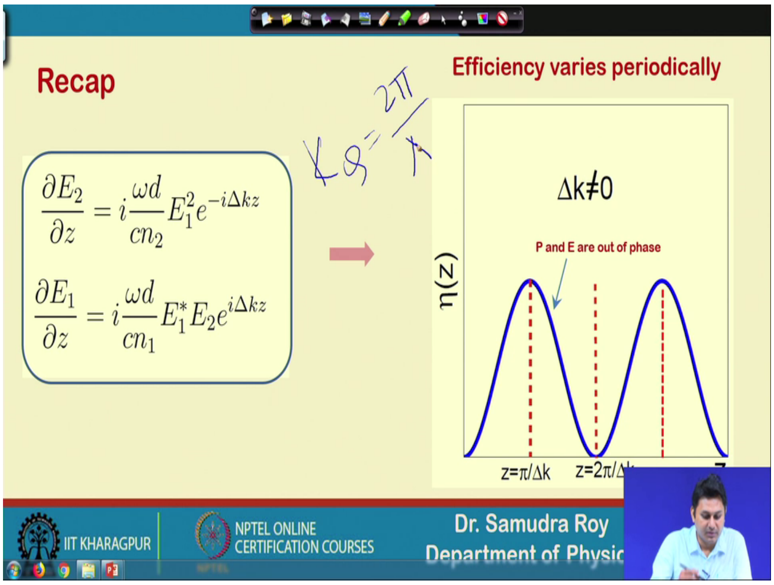

So, K Q should be 2 pi divided by lambda, this is the relationship between K Q and the periodicity lambda at which we have the d.

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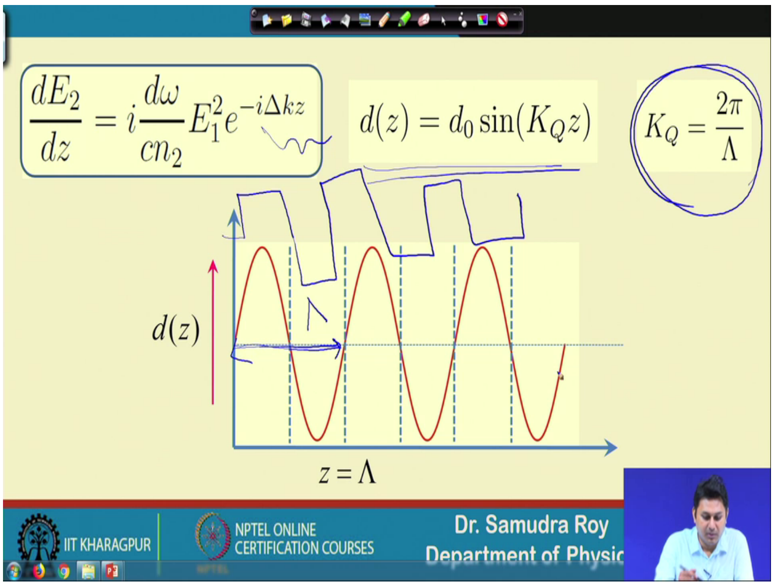

So, let us go to the next slide may be it will be clear to you. So, again here is to find out some wave so that this phase can be compensated. So, that is why I introduced a d in terms as a function of d, where d as a function of z and this is a sinusoidal kind of function. So, once we use the sinusoidal function we have a periodicity in our hand and this periodicity is involved with the propagation vector or K Q to P by lambda.

So, this is the plot of d this is a sign kind of function. So, if I plot these things we have the structure of the d is this; that means, I am changing the value of d. So, if you look very carefully here the value of d is positive, here the value of d is negative, here again it is positive negative positive negative and so on. So, that means, over a period from here to here this is a period, we call this period as big lambda again we have another period and so on.

So, structure wise if I put this structure so, nonlinearity is changing like this; this is negative, this is positive, this is negative, this is positive and so on we will show this structure once again this is just roughly showing how this things are related. But, here we should note that here this function is not a step like of function which is s a sinusoidal function, we will take care of this issue in the later classes.

But, for the time being let us consider d is the function of z and it is varying sinusoidally and this is it is functional form if we introduce d in such a way it could introduce some sort of phase associated with that. So, this is the amplitude thing and this is the phase term. So, this phase will be introduced through d which can compensate this delta k term, so that I can have the entire exponential term 0. So, how do we how one can do that. So, let us see.

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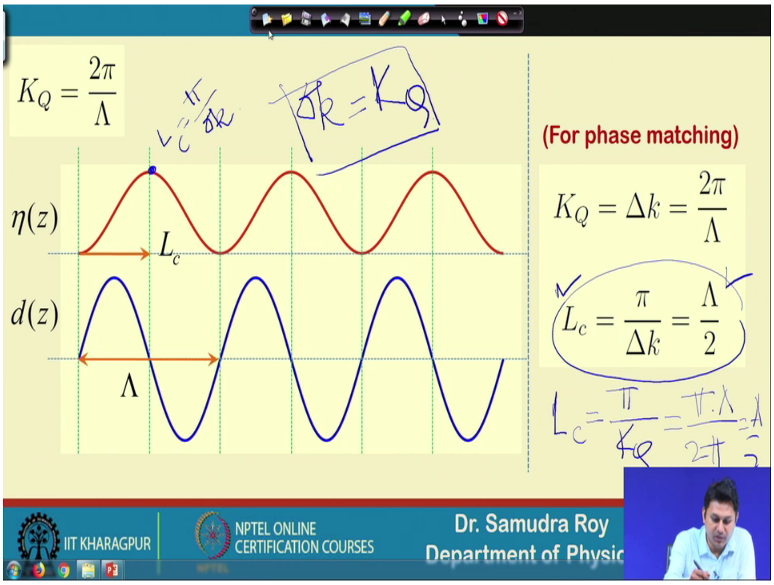

Well, for phase matching what we try to do is let us. So, what was a aim to do the phase matching and for phase matching this delta k which was equal to 0. So, the phase matching condition, but now we will compensate it with K Q. So, this is our phase matching. If this is our phase matching then we need to with very careful about choosing the periodicity of d. The efficiency is changing like this and we know that this is the length at which we have my efficiency maximum this length is called the coherence length. So, L c this coherence length, where pi by delta k.

Now, our phase matching condition suggest that delta k has to be equal to K Q. If delta k is K Q then we can write that L c is equal to pi by K Q. Again, K Q is 2 pi by lambda because K Q is related to the periodicity of the d. So, if I write it is pi by 2 pi into lambda which is lambda by 2, as shown here. This is the relationship between the periodicity of the efficiency and the periodicity of the d that we are proposing. If I compare we will find a very interesting correlation between these two that whenever we have a change of phase the d will produce a additional phase, so that this will be compensated and this deviation can be removed.

Here, we can find that efficiency is increasing. So, also the phase of the d is increasing, here it is decreasing. So, additional contribution of the d basically give rise to a condition where it can compensate this phase. Also, it is increasing so, d is increasing when it is decreasing d is also decreasing so, we have a compensation he here and so on. So, this compensation how this compensation will physically appreciated, let us consider let us find out through our next slide.

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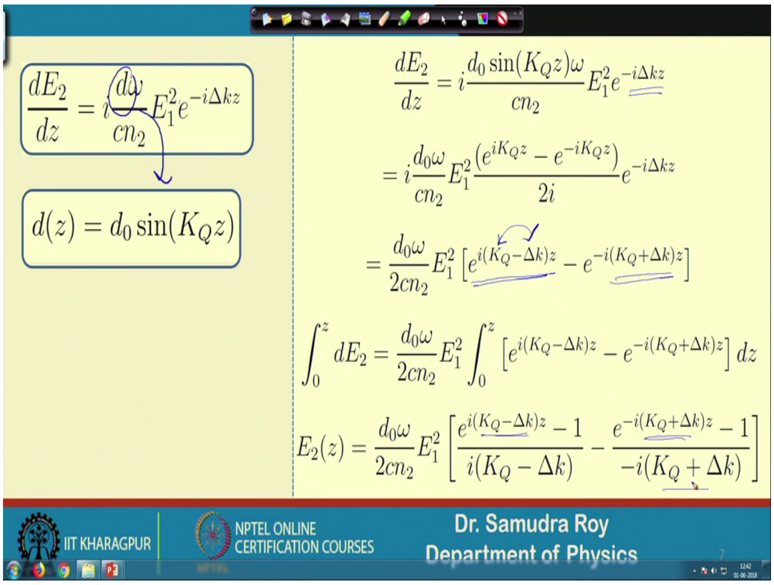

So, the mathematics is straight forward. So, the d is now a function of z and represented as d 2 sin kz. So, our other equation was i dz cn 2 E 1. So, this is a second harmonic equation of second harmonic generation. So, what we will do with thus we have just replaced this d with this one which we have done here. Once we do that then in order to find out the phase we can write this sin term into e to the power i Kz minus e to the power minus of i Kz divided by 2i this is just I write the sin term in terms of E so that I can combine these things with this. So, after doing that we have e to the power K Q minus delta k and e to the power K Q plus delta k.

So, we can see that readily this K Q and k are come together so that we have a possibility to compensate these two this delta k with K Q which is basically the coming through the period of this d function. So, next we going to integrate that to find out what is my E 2. When I integrate so, this term will be simply e to the power i K Q minus delta kz minus 1 and then K Q minus delta k.

So, this is my total term, I integrate it, this is a constant term it will be like here and when I integrate we will find these two terms which were exactly the same process that we have done in our previous classes. So, we have this equation and then we consider this is a constant if I integrate this two then it will be e to the power i kz divided by kz by z.

So, now we have an additional phase associated with the d. So, that is why in state of e to the power iK I have K Q plus delta k and here we have these two terms.

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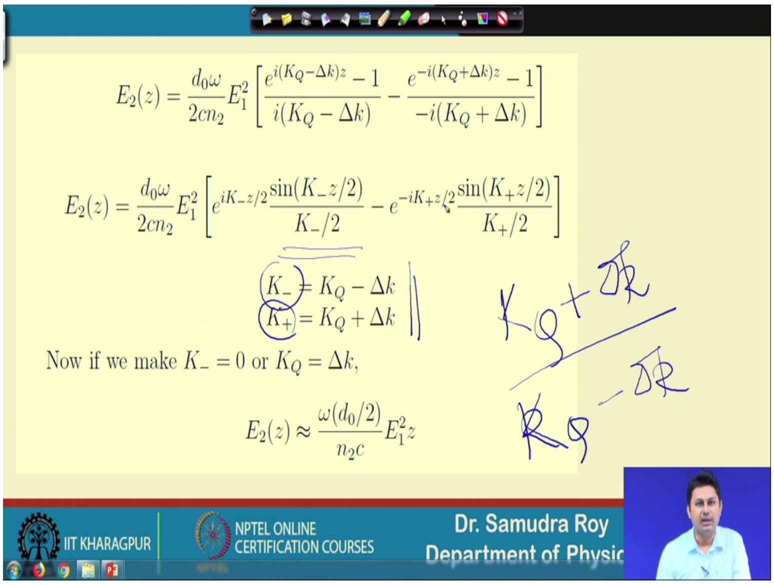

So, after that what we get? So, this was my E 2 after integration. So, E 2 can be again represented in times of in terms of this sinc function. So, previously also we come across this sinc sinc function, here we are also doing the same thing. So, we will going to have a sinc function and this sinc function one term is K K minus 1 term will be continuing this K Q plus delta k and another term will contain K Q minus delta k. So, K Q plus delta k, I write K minus K plus and K Q minus delta k I write K minus. So, these two are associated with these two term. So, I just replace here to make it a more compact notation.

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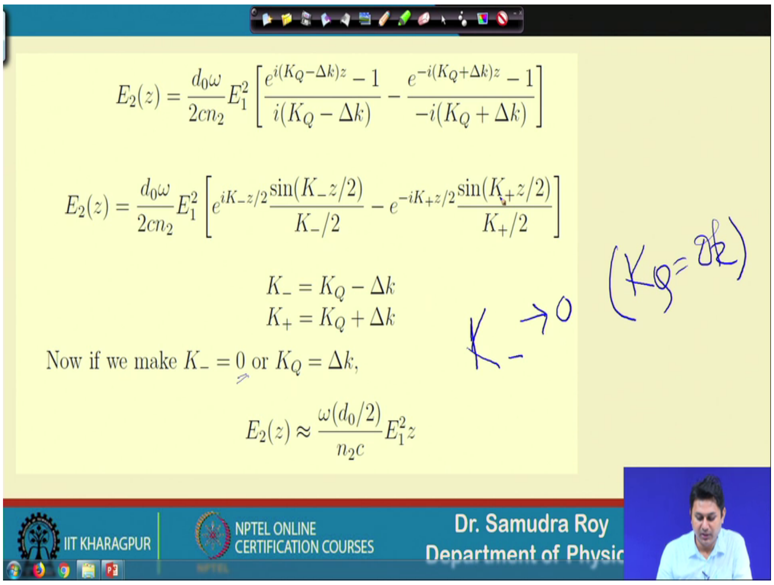

So, now we have two term in our hand. So, from this two term, if I try to understand so, we will find that if K Q equal tends to 0, if k minus; that means, k minus tends to 0 which is essentially K Q is equal to delta k. So, this term has a huge contribution, this term will be build large compared to the other term because this is k K 1 plus and this is not 0.

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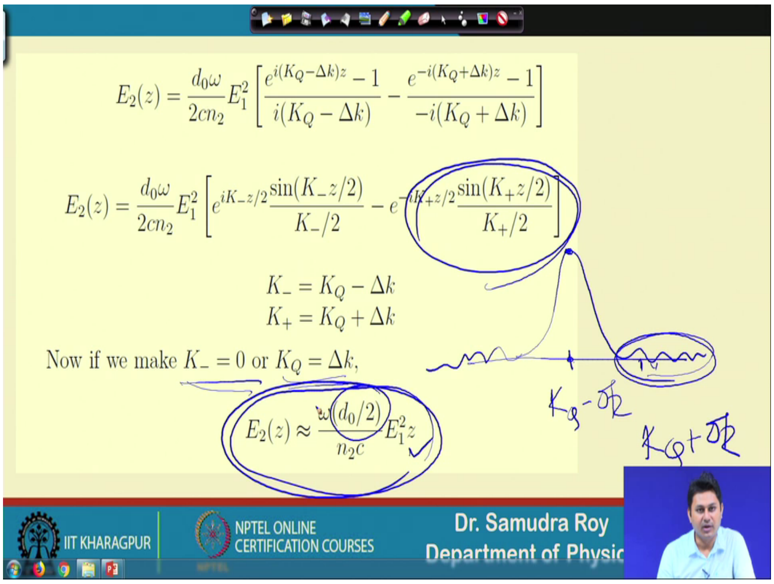

And, we know if I plot this things, we know how the sinc function will look like. So, if I now put this is K Q minus delta k which is now 0. So, at this point what happened we have a maxima here. So, what about K Q plus delta q delta k this term is somewhere here or here. So, we find that the contribution of this portion is very much small compared to the portion where we have K Q minus delta k term. So, that is why this term which have a very low contribution and the prime contribution will come through this term when this K Q equal to 0 condition is satisfied.

So, k K minus is equal to 0 condition is satisfied or K 2 equal to delta condition is satisfied. When K Q is equal to delta delta k then we have this term in our hand this if you look very carefully this is exactly the similar kind of term that we have already figure out in our previous calculation, where this depends on z, here also we are getting z. But interestingly you should note that here my d term is not d, but d 0 divided by 2, where d 0 is the amplitude. So, amplitude divided by 2. So, some weightage factor is coming here and this weight weightage factor is coming because of this quasi kind of phase matching issues, ok.

So, we calculate my E 2 under the phase matching and this phase matching is quasi phase matching and in quasi phase matching the condition is K minus equal to 0, here K minus is equal to 0, means K Q this is equal to delta k.

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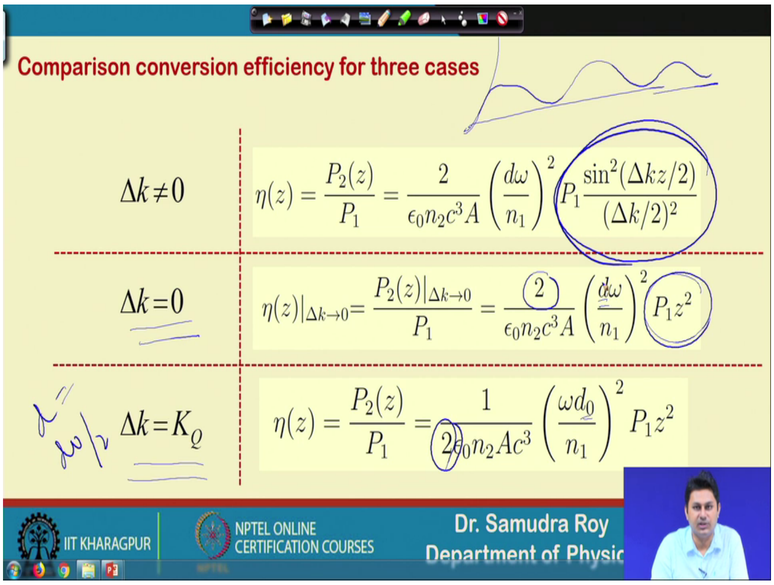

So, if I now, compare the efficiency in all the cases then we will make it tabular kind of form. So, when delta k not equal to 0, that is the most standard case we had our efficiency which is proportional to the sinc square. So, if you remember the figure when delta k is equal to not equal to 0, this is not a sinc this is sin square. So, it is depending on sin square set. So, if you plot this things if you remember the plot was something like this. So, it was a sin square variation.

If my phase matching condition is achieved that is delta k equal to 0, the condition the proportionality constant is now changing and it is changing very rapidly. So, previously it was a sin square delta kz divided by 2 proportionality and now it is proportional to z square term. So, if you plot it grows like this, we have already seen this figure.

Now, we are doing a quasi phase matching kind of stuff if I do the quasi phase matching kind of stuff we have almost did the similar expression that we have in delta k equal to 0, only thing here is that here we use d, but here we use d 0 which is the peak value of d. Here an additional two term is there; this two term is coming because here also we have a two term, but d is basically now replaced by d 0 by 2 in this particular case.

So, we have reduced the efficiency because my d value which was the peak value and now become some sort of average value. So, we are doing the similar kind of treatment here assuming that d is now average having an average value and this average value is d 0 divided by 2 and if you replace d as d 0 divided by 2 this and this expression will be identical to each other.

So, this is the relationship between this efficiency as a function of z with different k values what we have here so, different phase matching condition.

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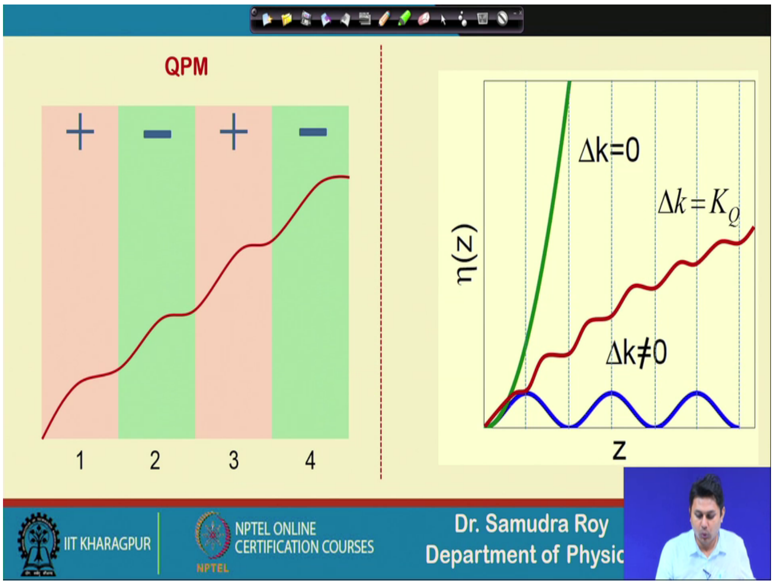

So, now if we go to the picture then this is the picture actually I try to figure out and in this figure in this figure it is shown how because of the quasi phase matching how efficiency is improving. So, these three condition which were shown here in the first case it was delta k not equal to 0 and in first case it was depending on sin square. So, that is why this is the term this is the structure we have this is the sin square figure and this is basically the condition for which delta k not equal to 0, this is the figure for that.

When delta k not equal to 0, we have a sin square dependence of the efficiency. But, when delta k equal to 0, we have a z square dependence and this z square dependence is basically shown here in this figure by this green line; that means, the efficiency increase very rapidly as a function of z it increase very rapidly with z. If it is increasing very rapidly with z, so, at some point the efficiency will be very large.

But we have already mentioned that this is not the case because we have put a very significant approximation here and the approximation was the generation of the second harmonic is very weak. So, that is why the efficiency is very weak. So, that means, up to initial few distance it is valid and after that if it is increasing like this it is a nonphysical kind of thing and we need to take care of the other equation which suggest that E 1 is also changing, but here we consider both the cases in all cases we consider E 1 is not changing.

So, delta k equal to 0, delta k not equal to 0, this condition we have already discussed, but here a new thing we have discussed that is delta k equal to K Q. This is basically the quasi phase matching condition. In quasi phase matching condition what is happening initial structure is same, so, it is up going up like this. So, when in delta k not equal to 0, when it is going down what happen the d term which is having some sort of sinusoidal effect basically compensate the phase which is responsible for this going down. So, it again going increase to up to this point. Again after that it is increasing so, d is also increasing and we have this after that when it is going down. So, again there is a compensation by d and so on.

So, that is why we will have k efficiency curve like this which is gradually increasing the increment is not like z square kind the increment is not like delta k equal to 0 like a sinusoidal kind of increment the increment is in between these two. So, using these quasi phase matching term we can improve our efficiency the generation of the second the generation of this second harmonic.

In the left hand figure, we show schematically the figure which is already shown here the improvement of the efficiency and you can see this stated region is basically the region where the value of the nonlinearity is shown plus and minus show that the value here is positive and the value here is negative. So, this is the negative positive negative positive kind of arrangement and for this kind of arrangement basically we introduce some kind of phase and this additional phase basically compensate, the phase that is already there into the second harmonic generation system which is delta k. So, delta k is basically compensated, but there is increment, the increment is not smooth.

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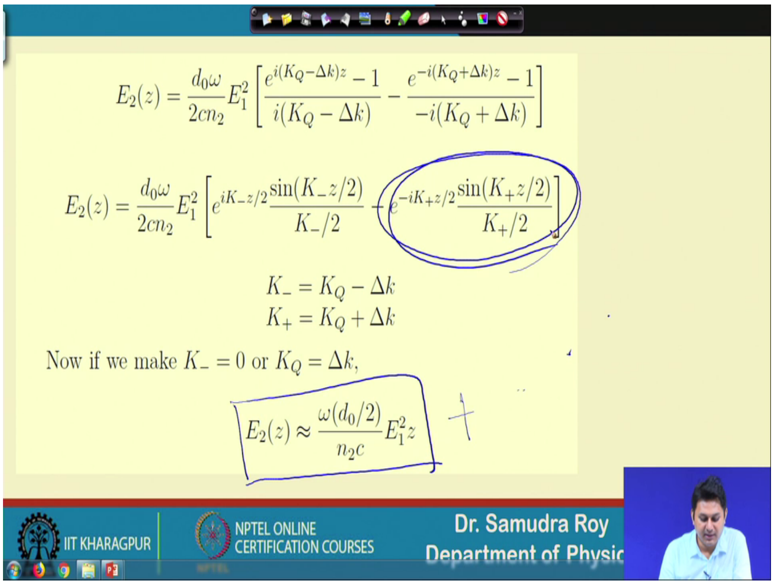

And, the reason behind that is this approximation if you look very carefully in this case, so, you will find that we had a term here E 2 during the calculation and we approximate this term. So, this approximation basically it is approximately z as a function of z, but there should be another term added with that and this term is coming to this contribution which we basically neglected. So, if we not neglect this term then that will be the actual case I mean actual case we find that the improvement of this efficiency or the efficiency curve will be something like this, it will be not exactly the z square increment it would be in between something.

So, with this note, so, let me conclude today's class. So, today we have started a very important concept regarding phase matching, this is a second kind of phase matching which we will call the quasi phase matching. So, in quasi phase matching what happened that we introduce the nonlinearity in such a way that it will going to change it is sign over distance. And as a result what happened that d will introduce an additional phase and this additional phase basically compensate the phase that is already there and the phase matching condition is now slightly different.

So, what is the advantage of this system, the advantage is that we do not require any kind of birefringent kind of phase matching, a medium which is non isotropic or isotropic in nature, but non-linear can be used also in this system to generate second harmonic.

So, when in the next class we will going to find more on this quasi phase matching system and we will learn how we can use this de function and this d function is not in real system sinusoidal. So, in real system it is a block kind of function. So, how from this block kind of function we can realize this d as a sin function and how the Fourier components can be take part into the phase matching condition, this we will going to learn in the next class.

So, with this note let me conclude here. So, thank you for your attention and see you in the next class.