Introduction to Non-Linear Optics and its Applications
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Lecture – 27
SHG in LiNbO3

So, welcome student, to the next class of Introduction to Non-Linear Optics and its Application. So, in the previous class, we studied that how one can generate second harmonic in the crystal like KDP crystal. So, one important thing there we find is that the launching angle of k. So, k is in such a direction that it matches that the phase matching is there, but also we need to find out a suitable azimuthal angle.

So, this suitable azimuthal angle eventually increase the d values and that is why instead of writing d we find d effect and also we find out that these d values in a in a d matrix there are several d values are there, but finally, when second harmonics are generated only one or two d values are there which are basically responsible for that. So, not all d value are used only one or two d values may be used to generate the second harmonic.

So, in the previous case we find that for generating second harmonic the orientation of k is important and also the direction of P z is important. So, P z is in z direction, but that will not going to give the second harm desired second harmonic because our condition is two ordinary wave will generate extraordinary wave. So, z direction was not the direction of extraordinary wave. So, we need to divide this component in such a way the pz component such a way one direction of the P z component can generate the extraordinary wave which is at 2 omega frequency then only we can generate the second harmonic properly.

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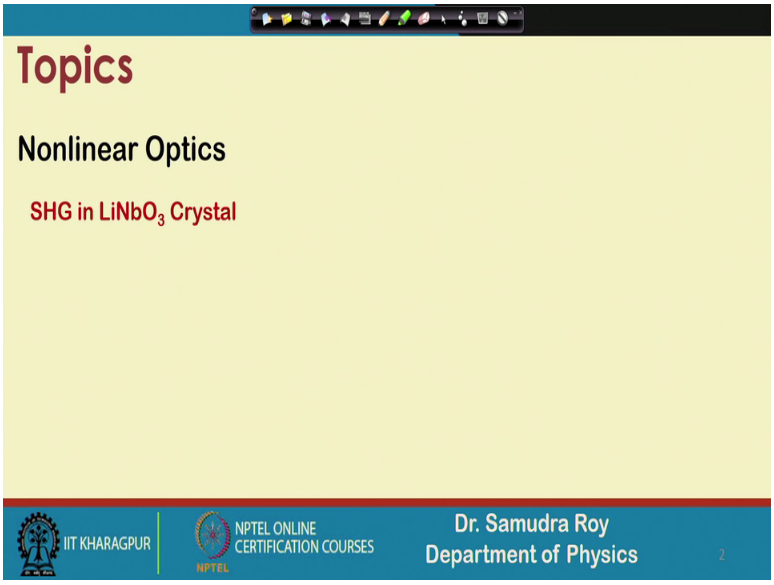

So, we will extend this concept. So, let us see what we have in today's lecture. So, today we will going to learn the second harmonic, but not in KDP crystal in a different crystal it is called the lithium niobate crystal. So, this crystal has different properties different d matrix. So, obviously, the condition of phase matching will going to change and also the launching angle or other issues will be there. So, how do will we take care of that issues. So, we will going to discuss in this particular lecture, ok.

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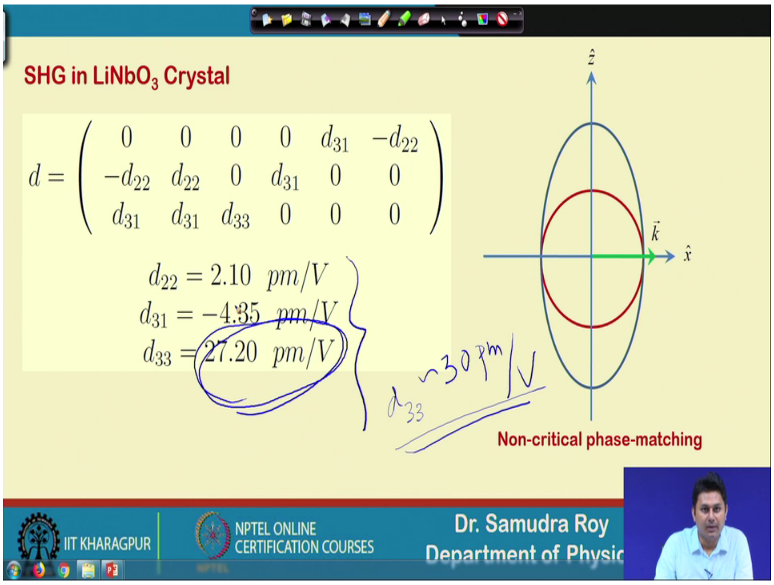

So, go back to our slides. So, this is the d matrix structure of lithium niobate. So, we can see that only three distinct elements are there in the d matrix. So, in previous lecture from the previous lecture we already find that this d matrix is start with 27 independent components and then by applying the symmetry it can reduce as to 10 components. And then further if I introduce some symmetry like Kleinman symmetry or using this Neumann's principle, where we can put some kind of operation over this d matrix and as a result we will find that few of the d matrix components are 0 and there are few nonzero elements and this non-zero elements are important.

Here we find there are few d matrix few elements of the d matrix, three elements are there three different elements are there, but most importantly this elements we should look d 33 this is the element which has the maximum value. So, this is really. So, what is the value of d 33. So, d 33 is of the order of 30 picometer per volt which is really a large value compared to the other values. This is almost ten times bigger than the d 22 and more than ten times rather and 1.5 times or so, bigger than d 31. So, obviously, this two things five times bigger than this rather. So, this 2 if I compare this components we can find that d 33 really having a large amount of d values.

So, we know that the efficiency is directly proportional to this d d components or this d values which are involved in the second harmonic. So, from the beginning our aim should be how to make our system. So, that d 33 should involve in generating second harmonic. So, that is our aim that how that second harmonic will be involving through d 33 that we need to think of.

Well, also one thing I should mention here, that this particular structure in noncritical phase matching system the k vector is almost in the direction of x vector why I am writing or showing this figure is important because for lithium niobate system the refractive index is temperature dependent. So, ordinary and extraordinary refractive index is depending on temperature.

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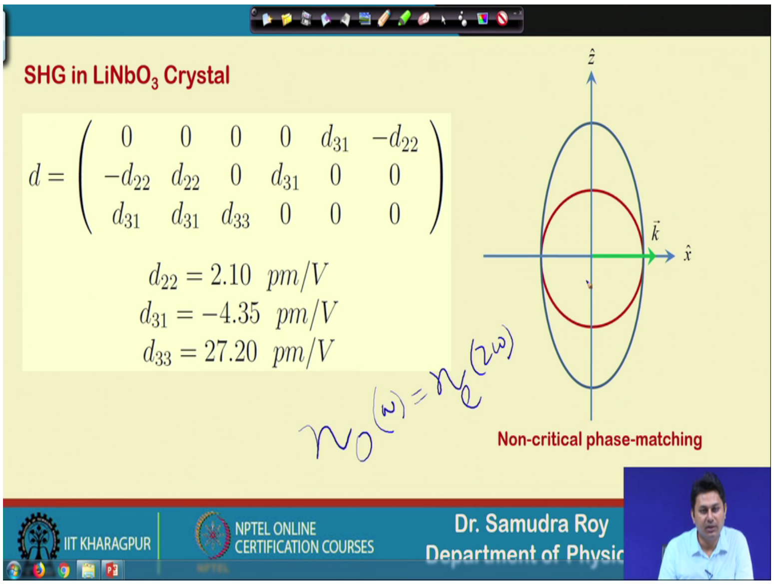

So, by varying temperature there is a possibility that we can make the condition we can make really make the condition n o of omega is equal to n e of 2 omega, but not only theta involvement. So, that means, this angle should be pi by 2 and that one can achieve in lithium niobate system, ok.

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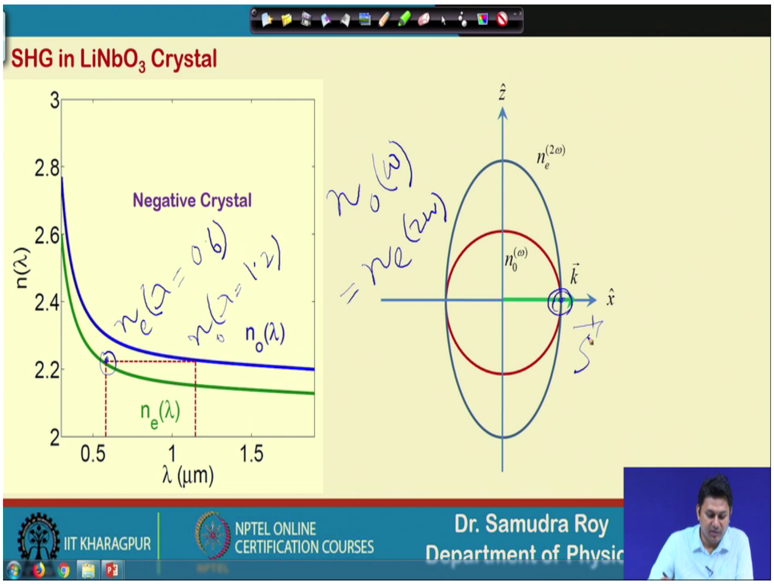

So, let us see what we have in the next slide. So, in the next slide basically we plot refractive index for this lithium niobate structure and this refractive index plot suggest that at particular wavelength say if I look very carefully here we have some wavelength say around 1.2 or something and here we have this is the value of n lambda. So, this is n of omega which or n let me write once again. So, this say n of lambda equal to say around 1.2 and if I go back here in this point so, this value is the same value of whatever the value we have here. This is n, this is ordinary so, I should write o. So, this is n e because this is over extraordinary line, n of e of lambda equal to say around 0.6 which is half of that.

So, that means, if I write in terms of frequency n o at omega is equal to n e at 2 omega which is which is a condition that one can achieve when the phase matching is critical in nature; that means, k vector is moving in x direction, where this two points are coinciding. So, this two points are coinciding so, there is no theta dependency here; so, that that is the direction where we should launch our light.

So, if I do that then this noncritical phase match has some facilities. Since is a noncritical k vector and S vector both will follow the same. So, S vector is also going to the same direction. So, these are additional advantages that we have. So, if we have this kind of phase matching issues then we need to find out few other things**.**

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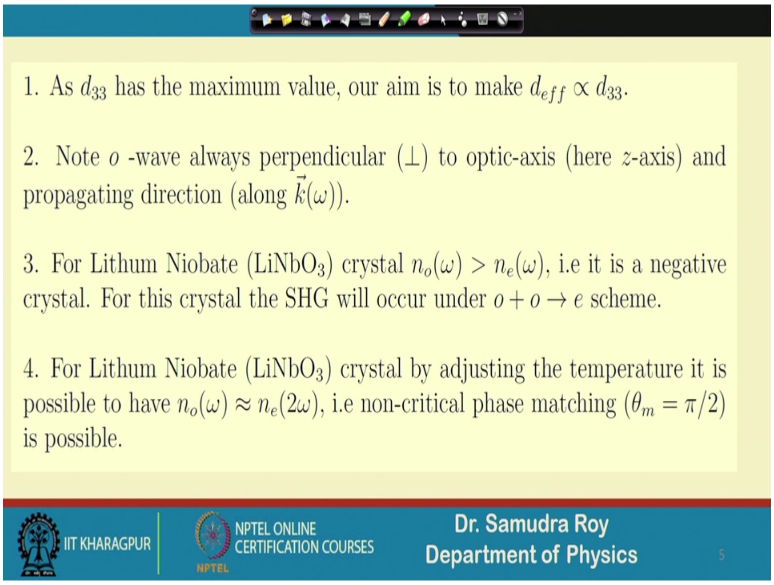

So, here we whatever we have discussed it is written in a compact form in a note form. So, what we have let me read it once again that d 33 has a maximum value. So, our aim should be d effective proportional to d 33 as I mentioned that the second harmonic should be in such a way that it should be proportional to d 33.

Now, o-wave always perpendicular to the optic axis and the propagation direction vector k that you should always know that whenever we calculate the second harmonic and if it is o o plus e system then we should be careful that the fundamental wave should be ordinary wave and the generated wave that is 2 omega wave, the generated wave which is having the frequency 2 omega should be extraordinary. So, that that thing we should remember and we should do these things very carefully that how this waves are generated.

For lithium niobate crystal n o is greater than n e as I mentioned and it is a negative kind of crystal and since it is a negative kind of crystal the phase matching condition that one should have is o o plus o o tends to e system. So, this is a scheme through which the second harmonics will generate. And, as I mentioned for lithium niobate crystal by adjusting the temperature there is a possibility that we can have n o omega n 2 omega that we have already shown and that means, my phase matching angle is around pi by 2; that means, k is making pi by 2 or k is perpendicular to the optic axis and that is the direction along which we have phase matching.

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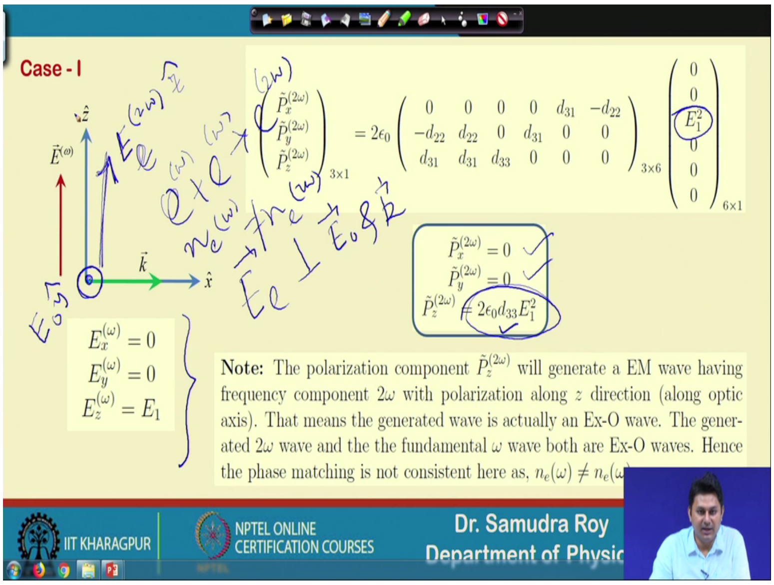

So, next we will try to understand the second harmonic generation in two different cases like we have done for KDP crystal. Here the two different cases are; in first case the electric field is now z direction. If the electric field around z direction it is perpendicular to the k, because k is along x direction. So, the this one possibility because k has to be in x direction, the phase matching is in this direction only. So, electric field that we can launch this is a trivial case that I launch in z direction.

Now, if the electric field is launch along z direction. So, readily you can understand that this fundamental electric field is not a ordinary wave this is an extraordinary wave, but anyway if this is the case let me figure out what should be the value of P x, P y, P z but from the beginning we know that this wave is extraordinary wave, but still we need to figure out what is going on here to understand the system.

So, E x is 0, E y is 0 and E z is 1, this is our this is our field the field components x, y, z components of the electric field. Now, if this electric field components are this and d matrix which is already given in the previous slide so, P x, P y, P z for second harmonic we can write this in the matrix form. And if I write this in a matrix form only the y component will going to survive, all the other component will be vanished because E x and E z are there sorry only z component will be survived the other two will going to vanish.

Now, here unlike the KDP crystal we can see there are many nonzero d elements are there. In that case only three here only three terms were there which was nonzero. Most of the cases with 0 coefficients were there. So, that is why most of the terms was 0, but here there is a possibility that we have some kind of nonzero term and indeed we find that P x is equal to 0, P y is equal to 0 and P z that is generating which is not 0.

And also proportional to d 33 that was our initial aim that my polarization at second harmonic or polarization at 2 omega should be involved with d 33 coefficient. So, d 33 coefficient should be involved in generating the second harmonic and here that is the case. So, that means, my polarization would generate a second harmonic along this direction, propagation will generate the second harmonic along this direction because P z is along this direction. So, it will going to generate a 2 omega along this direction.

Now, the important question that we have already mentioned before starting this discussion that what is the what kind of wave E 2 omega is? E 2 omega is perpendicular to k and it is along the z direction, it is along the z direction. If it is perpendicular to the k and if it is along z direction so, z and k are in the same plane. So, what should be the direction of ordinary wave? Ordinary wave is along y direction that we first to need to figure out, this is the direction of ordinary wave. Ordinary wave is along y direction, that is for sure.

If ordinary wave is along y direction, k is along z direction, what should be the direction of extraordinary wave? We know that extraordinary wave is perpendicular to ordinary wave and k that is the rule. So, in this particular case extraordinary wave will be along the z direction. So, that means, E 2 omega which is along z direction which is our second harmonic field that is generated by P z 2 omega, the z component of the polarization is eventually an extraordinary wave. So, I should put the E here. Now, we can easily find that the fundamental wave which is generating this extraordinary and the second harmonic that is generating is also an extraordinary wave.

If two waves are extraordinary the phase matching condition. So, we will never going to have two extraordinary wave to merge an extraordinary wave. This is a frequency omega, this is a frequency omega, this is a frequency of 2 omega, then my phase matching condition would be n of extraordinary of omega will be n of extraordinary of 2 omega which never possible. So, that means, the phase matching angle we make in this way, but there is a possibility that I can generate e along z direction I can launch e on z direction also P z is not equal to 0; that means, P z is still generating. P z is generating means it will going to generate a second harmonic wave along z direction.

So, second harmonic wave can also generate along z direction there is a possibility, but that will going to contradict the fact that my phase matching things where e n of e omega should be equal to n of E 2 omega which basically not possible.

So, even though all the terms are nonzero we will not going to find any kind of second harmonic. So, that is the very important example even though it looks like trivial from the beginning, but we should understand the fact that if the term is nonzero that does not mean every time it will going to generate second harmonic. So, there are few condition that should follow and this condition when followed strictly then only the second harmonic generation is possible under this birefringence phase matching. So, that means, second harmonic generation is a very critical phenomena and you have to be very careful in generating the waves and second harmonic and you should know the physics behind that and every time you should keep that in mind.

So, in the note we mention that the polarization component P z will generate electromagnetic wave having the frequency component 2 omega with polarization along y direction along optic axis; that means, the generated wave is actually an electromagnetic extraordinary wave as we mentioned. The generated 2 omega wave and the fundamental wave omega wave both are extraordinary and that will that that is the reason why the phase matching should not be. So, there should be 2 omega here, it is again a typing mistake. So, n of omega will be never equal to n of 2 omega. So, that means, this orientation of e is not a valid orientation. So, we need to do something new.

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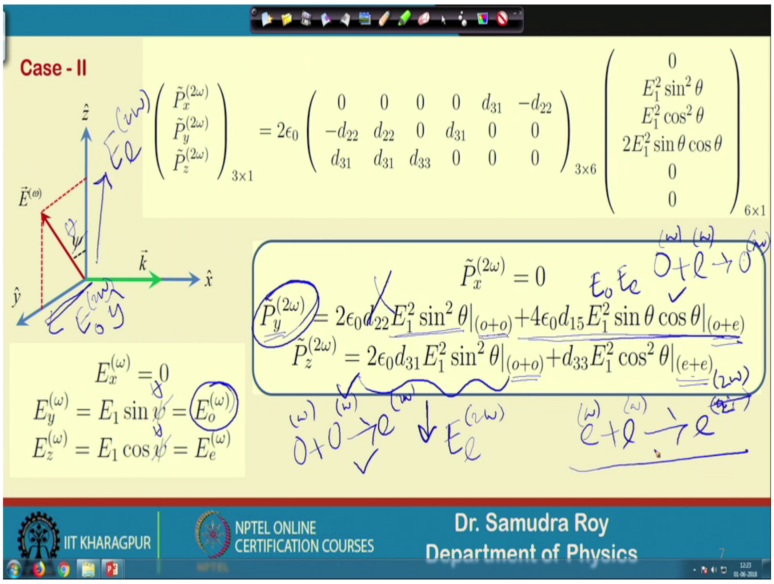

So, now what we were doing that we are launching the electric field making an angle phi with the z axis. So, previously E was in z direction now E is in arbitrary direction with an angle psi. Thus figure is shown here E is k is along this direction this is the direction along which the phase matching is there, but the orientation of E basically is important. Even though the phase matching is there P z is not equal to 0, still we find that there is no electric field second harmonic generation. So, here we need to find out what is going on in terms of this E. So, E is launched with an angle psi.

So, that means, we can figure out what are the components. So, first thing we need to find out what are the components. So, E x should be 0, because E is eventually in y-z field or y-z plane, E y is the component along the y direction. So, it should be E 1 sin phi sin psi and z is E 1 cos psi. Now, you need to be careful enough to write these component in terms of ordinary and extraordinary because along y direction we mention it should be a ordinary wave. So, I write this E 1 sin phi is a component that is generating ordinary wave and the other component which is along z direction should be the extraordinary wave that issue we have already deal with in the we have already discussed in our previous slides, that if the electric field along the z direction the electric field is an extraordinary wave.

So, here that is the z component in extraordinary wave. So, once we have the E x, E y, E z, the next thing is to find out my polarization. Now, you can see that there are many nonzero terms sitting here. So, E x was 0 E y was not 0. So, I put this E z was not 0, 2 of E x, E y 2 of E y is z is not 0, but other two terms is 0 because both the cases it is involved with E x term.

So, now if I calculate term by term so, P x turns out to be 0, there is no P x; P y there is a combination of term if I calculate P y how to calculate these multiplied by this 0, d 22 this which is nonzero, this 0 multiplied by this is a 0, d 31 multiplied by this is again nonzero. So, this term and this term, two terms are there and also we find out what is my P z. P z is 3 11 and another is d 33 E 1 cos square theta.

So, there are many terms are there are many terms that is in this in this matrix. So, as a result we have different term in P x, P y and P z. So, P z is containing two terms, P y is containing two terms and P x is containing no terms it is 0. So, now, you should be careful enough to find out that what is the combination. So, in both the cases if I find it is E 1 square sin square theta, so, that means, E 1 sin theta, ok, here is a notational problem actually here if I write it psi. So, it has to be psi, but I have written is theta. So, let us consider this angle is theta. So, this angle should be theta, this angle should be theta.

So, if I consider this thing here that sin squared theta E sin E 1 sin square E 1 square sin square theta is basically the components of ordinary wave. So, two ordinary waves are merging to generate the polarization along y direction. Also, here we have E 1 square sin theta cos theta; that means, here we eventually have one ordinary component of field one extraordinary component of field. So, ordinary plus extraordinary component can generate polarization along y direction and in P z also two ordinary waves are generating and as a result I am getting some kind of wave a some kind of P z. Here also in the similar way two extraordinary wave are combined to generate the P z term.

So, here one by one if I consider so, you can understand that these two ordinary. So, what should be the electric field associated with P y. P y will going to generate an electric field of 2 omega along this direction. This is electric field along 2 omega and the direction is y. So, that means, this is this has to be an ordinary wave. So, two ordinary wave will going to generate something ordinary which is again violating the phase matching. So, this term the contribution of the these term will not be there. But, here we have a situation where one ordinary and one extraordinary wave can generate an ordinary wave of 2 omega, this is omega of ordinary wave, this is omega of ordinary wave. It is going to generate a ordinary wave up to omega this is a type II kind of phase matching.

So, type II kind of phase matching is here. What about P z? Two ordinary wave P z will generate an electric field along this direction. This is the direction of extraordinary wave which is generating 2 omega. So, once we have electric field 2 omega along z direction; that means, this contribution this contribution will going to generate an electric field which is extraordinary in nature. So, what should be the scheme here? The scheme is ordinary plus ordinary which is generating extraordinary. So, this is the frequency component omega, this is the frequency component omega, this is the frequency component 2 omega. So, this is also valid. So, this term is also valid.

What about the last term? Last term, again two extraordinary wave will going to generate P z. P z will going to generate an extraordinary wave. So, extraordinary two wave will leads to one extraordinary wave of 2 omega. So, omega, omega it will going to generate 2 of omega. So, again it is violating. So, this is 2 of omega again this is violating because the condition is not valid.

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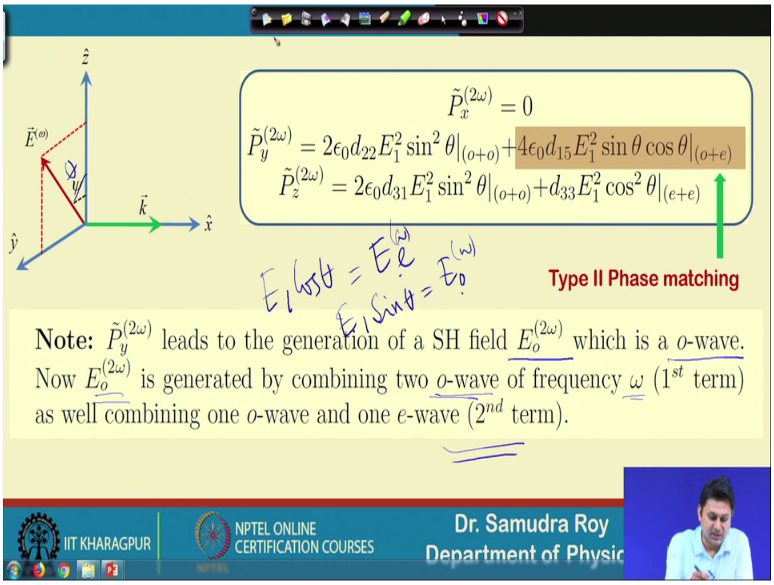

So, once we know these components carefully then we can readily find out that what is going on. So, now, as I mentioned this term this shaded term is a type II kind of phase matching, where P y leads to generation of second harmonic along y direction we call it E 2 y. So, which is a which is a ordinary wave and E 2 omega is generated with a combination of one ordinary wave of frequency omega, that is the first term here and another combination of 2 omega, which is the second term here. So, first term was E 1, first term was again this notation was which it should be either it should be theta or it should be psi. So, there is a notational mismatch, but anyway.

So, this angle is theta; if this angle is theta so, E 1 cos theta is along z direction. So, this is an extraordinary component and E 1 of sin theta is ordinary wave because it is along y direction. So, both the cases it is omega component. So, these two combination basically one ordinary and one extraordinary wave generate this term and this term basically gives us a type II kind of phase matching.

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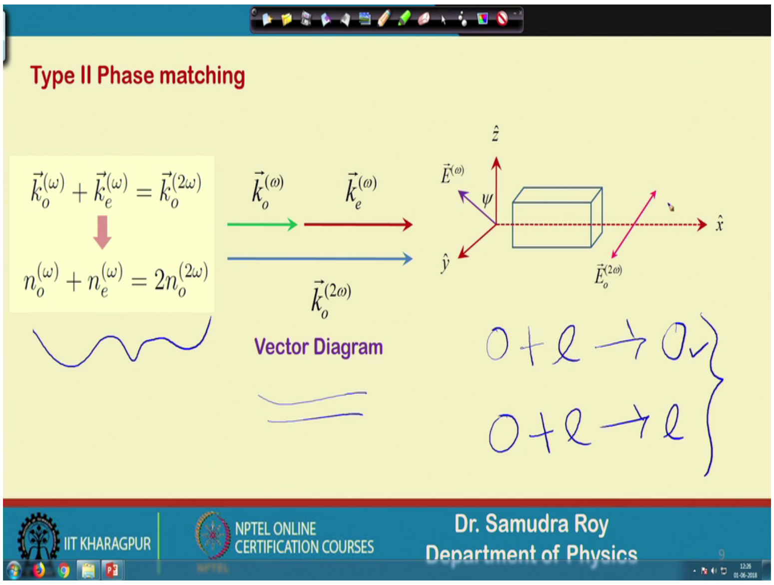

Well, we are almost at the end of this class. So, type II phase matching if I look carefully what is going on so, in this picture so, in type II kind of phase matching the wave vector k omega of ordinary wave though vector k omega of extraordinary wave they will going to merge and generate k omega of an ordinary wave. So, that is one combination, but you should remember that another combination is still possible. So, we have already learnt that.

So, there are two possibilities ordinary waves plus extraordinary wave can this two and ordinary wave or ordinary wave extraordinary wave can these two one extraordinary wave both the cases the phase matching can be possible and this kind of phase matching is called type II, kind of phase matching which we just learnt in the example that is the second harmonic generation one part of type II phase matching can be possible. And the refractive indexing term of refractive index we have the relation between the refractive index like this is a phase matching condition in terms of refractive index.

If we plot the vector diagram we can see that one ordinary one extraordinary wave, they can merge to generate one ordinary wave and in the figure also we can see that if I launch an electric field in this direction when it pass through the system, if it generate the electric field which is ordinary in nature then it will vibrate along this direction. So, we will start from this electric field lounge electric field, but the second harmonic field will change its polarization state and ordinary wave that will vibrate in y direction, it will be in y direction. So, this will be a schematic figure to show how the second harmonic will be generating in lithium niobate crystal where type I and type II phase matchings are there together.

So, with this note let me conclude here. So, we have learned a very important in the last two classes we have learned very important problem; one is second harmonic generation in KDP crystal and in second harmonic generation in lithium niobate crystal and in lithium niobate crystal you find the orientation of the k is fixed, but if I change the orientation of E, then there is a possibility to generate type II kind of phase matching.

So, with this note let me conclude here. So, see you in the next class and thank you for your attention.