Introduction to Non-Linear Optics and its Applications
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Lecture – 26
SHG in KDP Crystal, Calculation of deff

So, welcome student, to the new class of Non-Linear Optics and its Application. So, in the previous class, we have studied how one can generate second harmonic in certain crystal and as an example we took KDP crystal whose d matrix is given and this is the negative crystal. So, we find that o plus o tends to e that should be the system to generate the second harmonics.

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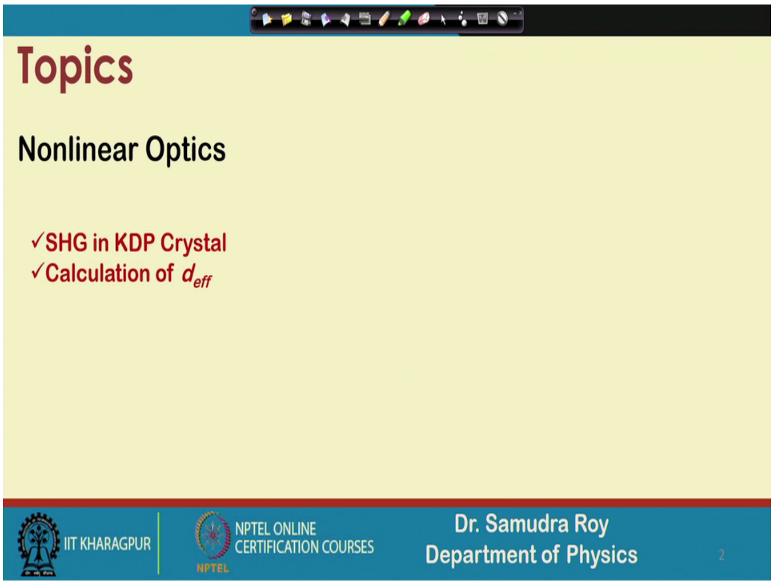

So, let us find what we have today. So, today we will continue this process because we find that even the phase matching is there for k vector, but still there is a possibility that there will be no non-linear polarization. So, phase matching may be there, but still there is a possibility that second harmonic will not be there. So, phase matching is not only the major condition. So, we need to follow or we need to achieve some additional condition also to get the efficient second harmonics.

So, in this particular course we will mainly confine to different kind of crystals where one can generate second harmonic. So, KDP crystal is one crystal and then after that we will going to take another crystal and whose d matrix is different which is the lithium niobate.

So, let us go back to the slide what we have in today's lecture is second harmonic generation in KDP crystal that is the continuation of the previous lecture. We will going to learn more about the orientation of k vector and find out what is the d effect, that is the very important, that even though we have a d matrix in our hand and we know that among this d matrix there are few coefficient that is involved in second harmonic generation process, but we find that effective value of these d may not be the same that we have in the d matrix. So, some additional quantity should be there some multiplicative term should be there and why it is there that we should understand very clearly. So, in this class we will going to understand how one can calculate the d effective for a given system.

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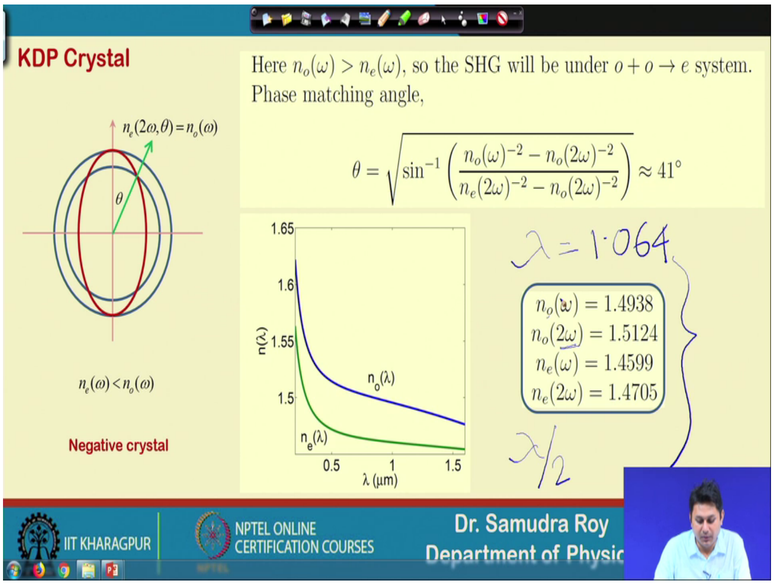

So, let us go back to our old slide, this is not a very new slide. Last day also we show that how the KDP crystal this is in this particular slide we show the nature of the KDP crystal. And in KDP crystal we know this is a negative crystal that is that is why the refractive index of ordinary wave at particular wave length is greater than the refractive index of extraordinary wave on that wavelength. And also we able to calculate the n values at different wavelengths are given. For example, here we can see these are the different n values. These n values are given at the operating wavelength lambda is equal to say 1.064 this is the operating wavelength at which we have this refractive index, but, there are 2 omega so, that means, we also have the value of the refractive index at lambda by 2.

So, once we know this value n 0 omega, n 0 2 omega and n e omega it is not 0, rather it is o. So, n o omega n o 2 omega, n e omega, n e 2 omega if these values are given then there is a I mean using this we can find out what is the value of theta the phase matching angle and we find this is around 41 degree. So, this is the Sellmeier curve for the refractive index variation for KDP crystal. So, from Sellmeier equation we can easily find out what is the values of these coefficient or these refractive index at particular wavelength, ok. So, this information we already had in our previous class.

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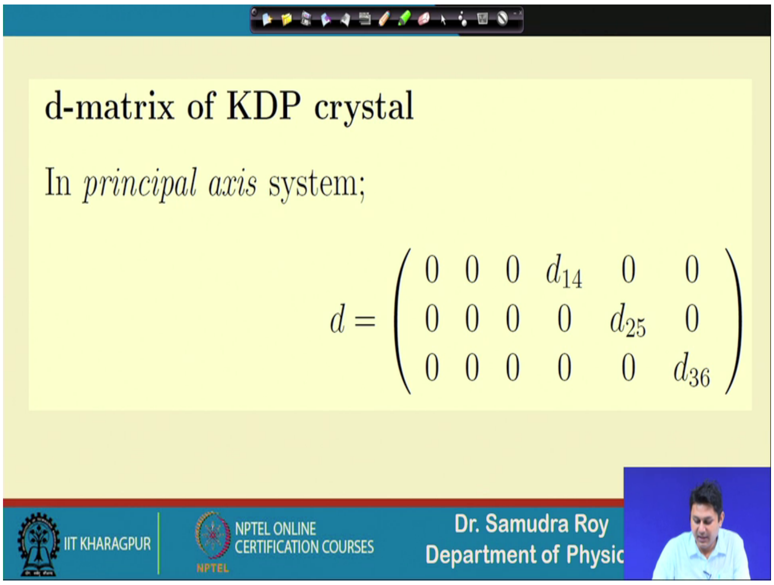

So, this is the d matrix structure. Again, I am showing this because in this d matrix structure you can see that only three different d matrices the coefficients are there. All the other coefficients are 0. Why it is 0 and how we can calculate that that we have already learned in our previous classes by applying this Neumann's principle and symmetrical operation we can figure out which coefficient is 0 and which coefficient is nonzero.

Here we find that there are three non-zero coefficients and if this three non-zero coefficients are there, obviously, they will going to involve in second harmonic process. But the question is which coefficient among this three coefficient which coefficient are dominant or which coefficient really generate the second harmonic, that is something we need to figure out.

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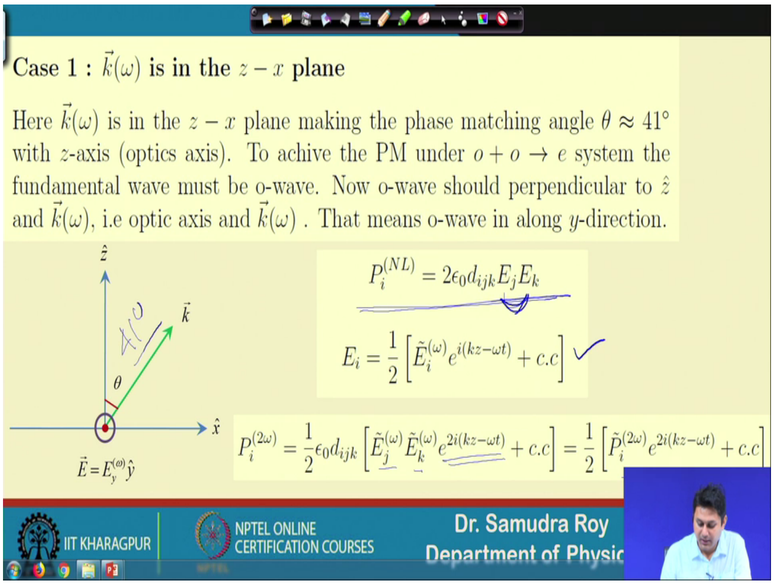

Well, this is again the previous problem, this is some sort of recap that if I launch the k vector in x-z plane then we find that there will be no second harmonic generation because I am launching the k vector in exit plane and this theta is 41 degree; that means, interesting thing is that this theta the value of this theta is 41 degree. So, that means, the phase matching is still there, but the orientation of k rather the azimuthal angle of the k is very important and in the last class, we find that for even though the phase matching is there is no non-linear polarization.

So, non-linear polarization can be represented in E j form like this and where E i is this. So, if I want to find out the second harmonic 2 omega term then we can find that what is the combination for which I am getting 2 omega term. So, E j, E k if I multiply both are having frequency omega then I will have a 2 omega term and then we can write this as a plane wave form and if I write this is a plane wave form it should be something like E i till 2 omega and e to the power i kz minus omega 2. So, P i 2 omega is amplitude P i 2 omega is a amplitude of a of the polarization. So, I need to find out what is the value of this amplitude and one can find very easily how this amplitudes are there or what should be the value of this amplitude by using this matrix form.

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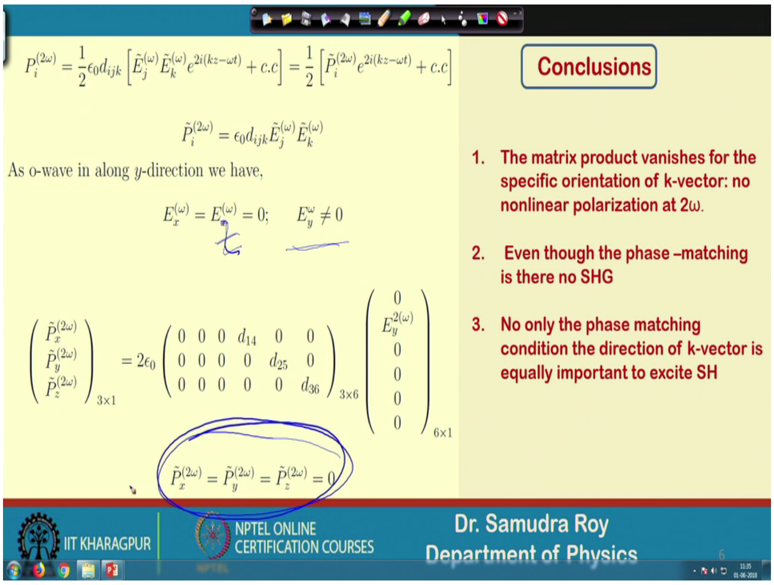

So, this is the matrix form that we also discussed in our previous class. So, before writing this matrix form we need to know that where is my electric field. So, electric field vector is important. So, if I now write here there is a small mistake it should be E z because x and z both components should be 0, because E is only in y direction. Since E is only in y direction all the other components will be 0 and now, what we will do, we will just put the e vectors here the way we put in the matrix form to figure out what is my P x, P y, P z or rather amplitude of the polarization at x, y and z direction. So, then we find that if my E z, E y is my launching with the input field if along y direction then all the components are 0 here all the components are 0.

So, since all the components at 0, phase matching is there because I am launching the vector with an angle theta equal to 41 degree, but my electric field is such that we are not able to get any kind of polarization out of that. So, there is no polarization at 2 omega frequencies that means, there is no second harmonic generation. So, very important example to show that the phase matching is not the only criteria to generate phase matching there are other thing that you should mind and also we can tune the second harmonic or increase the second efficiency of the second harmonic. So, how we will do, let us see in the next slide, ok.

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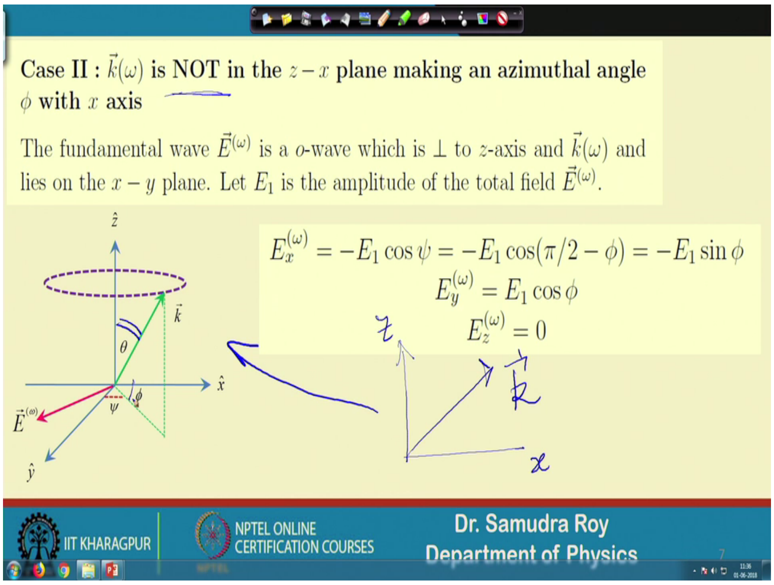

That was the case I, now in case II, what we will going to do; we will going to launch k not in z-x plane. So, please note that here we are not launching that. So, previously if you look this is the previous picture. In the previous picture that was in the z direction of z, this was the direction of x and k was in this direction and also in this particular plane, but now it you can see that the launching angle of k is same that is theta this angle is theta or 45 degree, but there is a azimuthal angle there.

So, that means, k is not in exit plane this is hanging in the in the wave. So, so k is now hanging in this space and x and it is not in x and z plane. So, what happen, if I change the launching angle of the k the immediate consequence of these things will be the change of the electric field, the direction of the change of the electric field. So, how the directive electric field will going to change its direction, because we know that ordinary wave always perpendicular to the z axis and k let it me.

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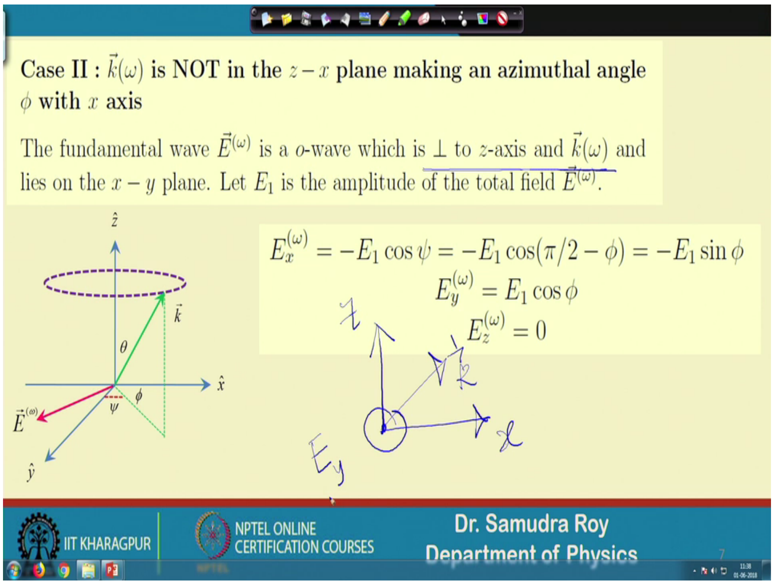

If I go to the my previous picture where it is z, this is x and this is k then all are in same direction and if I say that what should be the direction for which we will get both k and z perpendicular. So, the direction that is perpendicular to both k and z will be the direction perpendicular to this plane as simple as that. So, E was perpendicular to the plane previously which is nothing, but the direction of y.

Case I, we have studied this problem, but here the situation is slightly different because if I want to find out what is the direction which is perpendicular both to z and k; that means, we need to find out the direction which is perpendicular to the entire plane containing z and k. So, in this figure we can see that k which E which was in initially in y direction it is now going to shift to an angle and this angle is nothing, by nothing, but the angle phi because I am rotating this k to phi angle, this is azimuthal angle and as a result k will change E will change.

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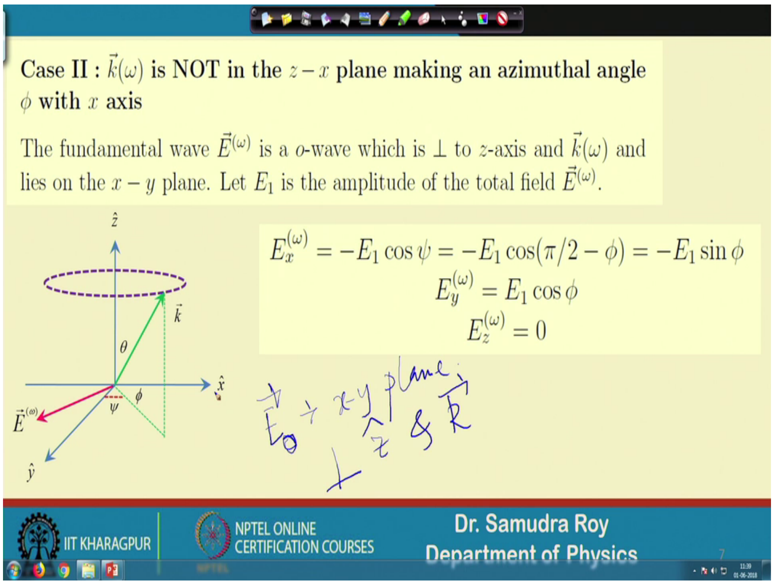

Obviously, E will be in x-y plane and it should be perpendicular to z axis and k vector. So, we know that this electric field is related to ordinary wave. So, this ordinary wave is now in this direction given by this red line which is perpendicular to z and k both and lying on x-y plane. So, now, the next thing we need to find out what are the components of this electric field because when we put the matrix form them the components are required if you remember E x, E y, E z these components are important otherwise you cannot able to write the matrix form completely. So, we can divide this electric field to its components and if I do then we find that E x which is a x component of that should be E 1 cos phi this is.

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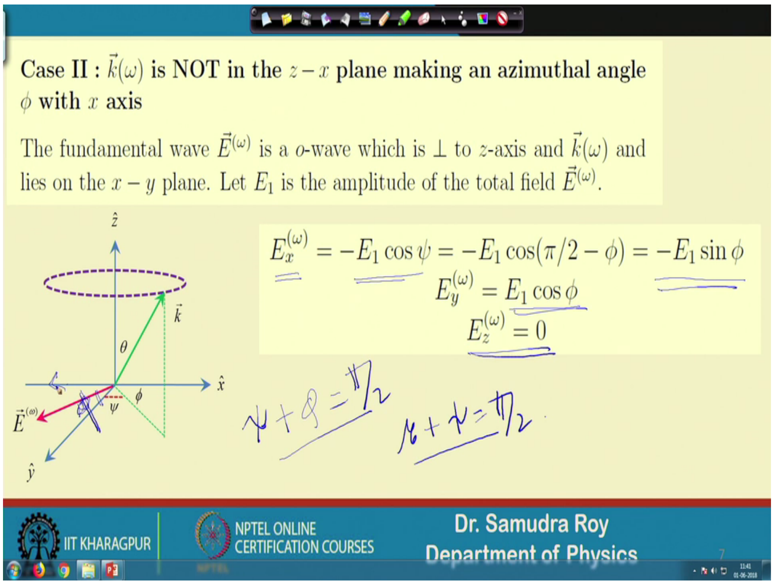

So, if I write in term terms of psi. So, psi plus phi, this is pi by 2 this angle is pi by 2 because this angle is angle between the x direction and y direction which is perpendicular and also E is perpendicular to this plane, so that means, E is perpendicular to this dotted line, E is perpendicular to this dotted line.

So, if this angle if I say this is this angle say mu so, mu plus psi also pi by 2. So, from these two equation we can say that this angle is phi. So, if this angle is phi then we can readily derive the different components. So, what will be the y components? It will be E 1 cos phi as simple as that. E z, there will be no E z component because E the vector E is lying on x-y plane. So, there should be should not be any kind of E z component and this component along x direction there will be one component and this component will be simply minus of E 1 sin phi, the direction is negative. So, that is why we write this component as minus of this.

So, these tthree components if able to evaluate, then the next step is to find out what is the matrix form what is the total matrix form. So, now, I am launching the electric field and launching the electric field in the direction of k which is making an angle phi with x axis which is the azimuthal angle. Initially the phi previous case the phi was 0, but here we have some amount of phi value; that means, now k is not in x-z plane it is hanging into the space and now we need to find out under this condition if I change the launching angle in terms of it is azimuthal distribution then what will be the fate of second harmonic generation? Second harmonic is really going to generate or not, that we need to find out. So, next in the next slide we will try to find it out.

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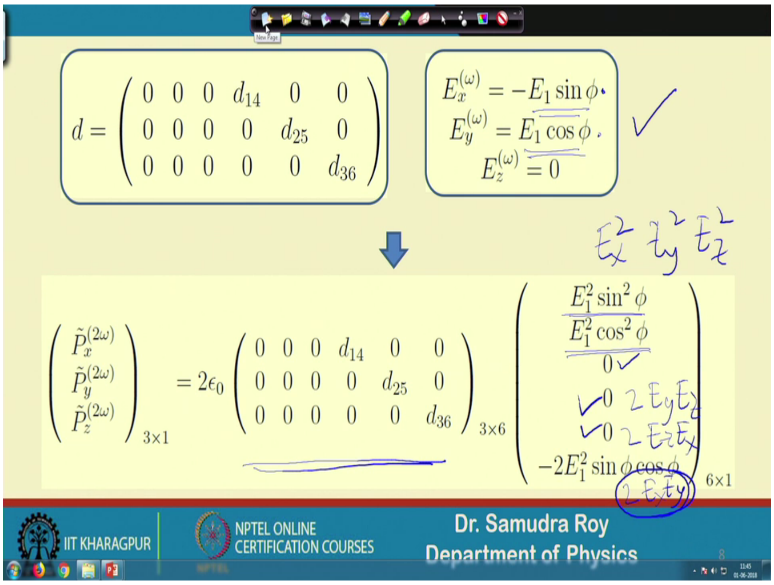

So, now we have d matrix in our hand we have the E components in our hand. When these two are in our hand, we can readily write our d matrix and the E components in vector form, so that I can figure out this P x, P y, P z this non-linear polarization component at 2 omega frequency.

So, here we have d matrix which is written here and we have the column matrix with the components of E field and this term will be in terms of phi. So, now, E x is E 1 sin theta. So, I will have E 1 sin square theta because this is E x square then E y should be E 1 cos theta. So, we should have E 1 cos square theta this will be E z. So, E z square is 0, this should be 2 of E x E z E z again 0. So, we have 0 and the fifth term will be 2 of E z, E x as written here this is also going to vanish and finally, we have 2 of E x E y and this 2 of E x, E y term will going to survive because E x is not equal to 0 and E y is not equal to 0. So, we will have 2 of E 1 square sin phi cos phi or in other way one can write as E 1 square sin of 2 phi.

So, all when we have all this term in our hand so, now, it is time to find out what is my P x, P y, P z. So, widely we can see that P x will be 0, because in order to get P x the fourth term this term will be involved and here we should have some term here, but here we have 0, so, P x will be 0. What about P y? P y we have d 2, so, this term is responsible for generating some term corresponds to P y, but in this matrix we have something here, but we find the quantity here is 0. So, that means, also we have P y 0.

So, one we so, we have P x 0, P y 0 exactly the previous case we got P x 0, P y0, but also P z was 0 there, but here we find that for P z we have d 36 one term sitting here that should be multiplied with the last term which is non vanishing right now because this term is non vanishing so, we will get some kind of contribution out of that.

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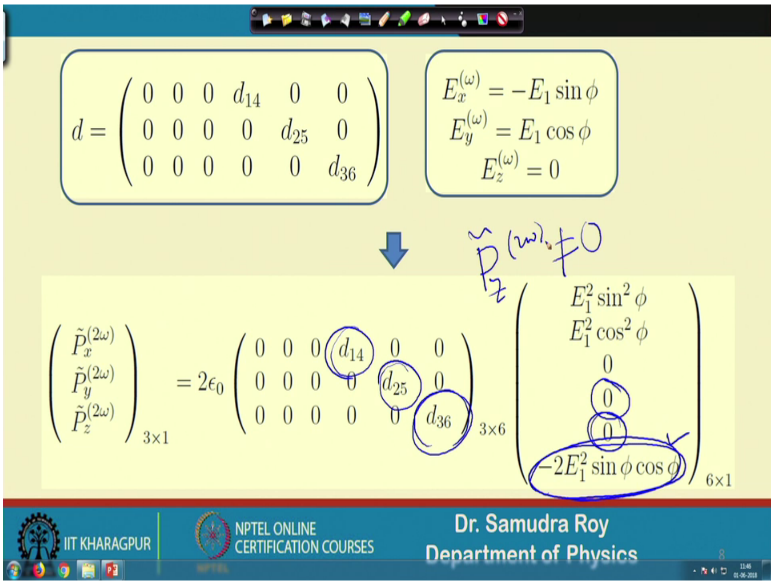

So, P z the most important thing that we figure out that P z 2 omega this quantity is now not equal to 0; that means, this will going to contribute and generate some field having frequency 2 omega. So, that means, second harmonic generation might possible under this structure. So, let us see what we have in the next slide.

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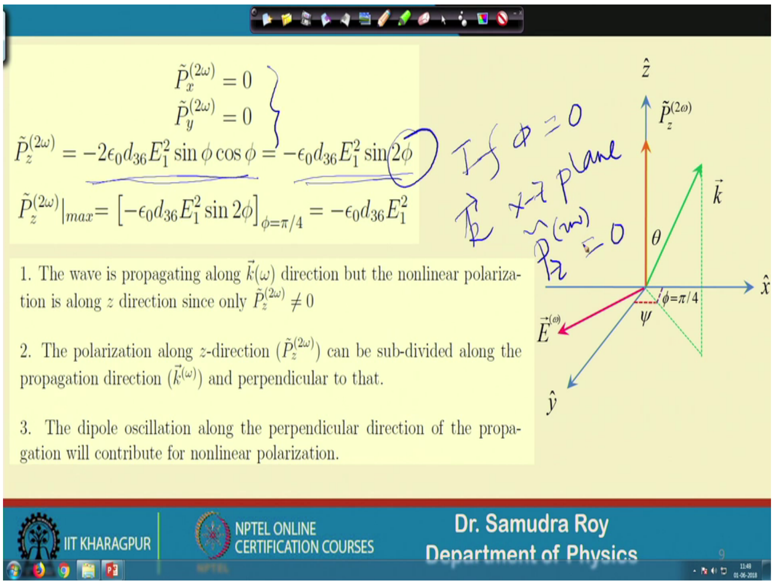

So, next slide we will have the same thing that we were discussing that P x, the value of P x is 0, the value of P y is 0 and the value of P z is the multiplication of that two term which we discuss. So, if I go back to the previous slide multiplication of multiplication of this term and multiplication of this term will be P z.

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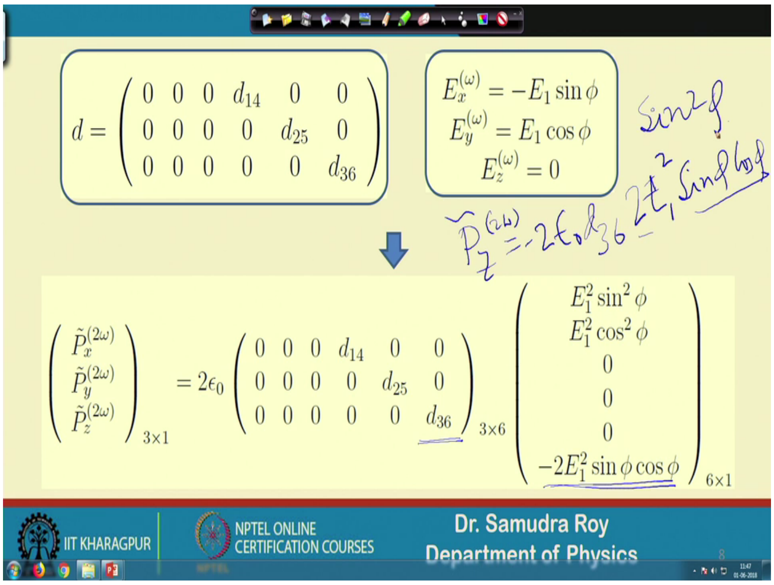

So, P z will be let me write it here, so that you can understand P z tilde 2 omega this term will be simply 2 of epsilon 0, this we always have then d of 36 and then with a negative sign another 2 term is sitting here, E 1 square will be there and sin phi cos phi. So, in compact if I write these two sin phi cos phi as sin of 2 pi I can compact this term and this term will be 2 of epsilon 0 d 34 E 1 square sin of 2 phi.

So, in the next slide actually we have written this thing here if you look carefully P x is 0, P y is 0, no point no doubt about that. So, P z is this quantity which is the multiplication of d 36 and the last term which is 2 epsilon 0 E 1 square sin phi cos phi which is this. Now, one thing one should note that this P z 2 omega this term is depending on phi this phi. So, you can see that if this phi is 0, that means, phi is which term here in the figure you can see that this is our phi term. If phi is 0, that means, k is in which plane, exit plane then P z 2 omega is 0, which is nothing, but the previous case which is which is something that we have already find in our previous calculation.

So, here we will see that this phi is important term because if it is 0, then P z 0, that means, if I change this orientation of k, then there is a possibility that I can maximize this P z because it is depending on phi. So, one can easily find out what should be the maximum value of phi the maximum value of the phi when this quantity is 1, because sin theta has a maximum value of 1.

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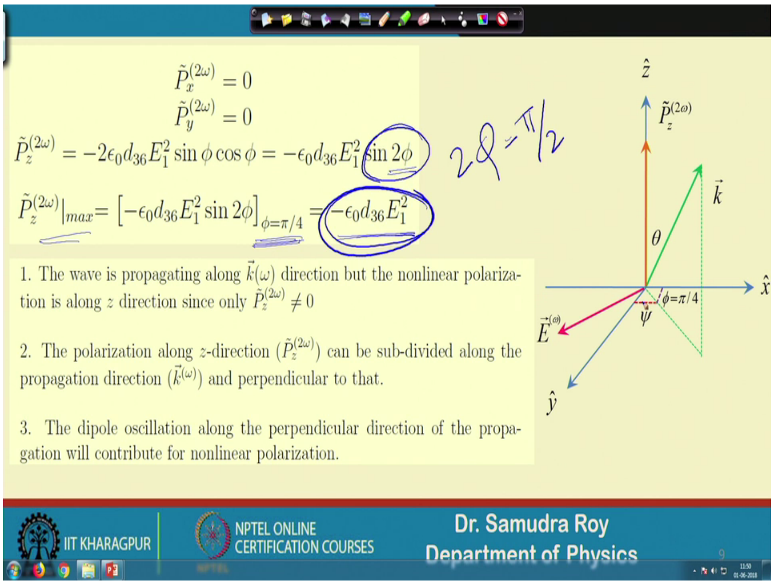

So, when this value will be 1 when the angle 2 phi is equal to pi by 2. So, sin at sin pi by 2 we know that we have maxima. So, here the angle is 2 phi. So, phi has to be pi by 4 for where for p will be the angle pi by 4, for which we will get the P z maxima. So, P z maximum I have written here if this quantity at phi equal to pi by 4 which is simply epsilon 0, d 36 E 1 square. So, we figure out that my P z can be maximized by launching the k suitably and we need to find out what should be the angle of this azimuthal what should be the this azimuthal angle for which we maximize the quantity and once we do that we figure out what is my P z.

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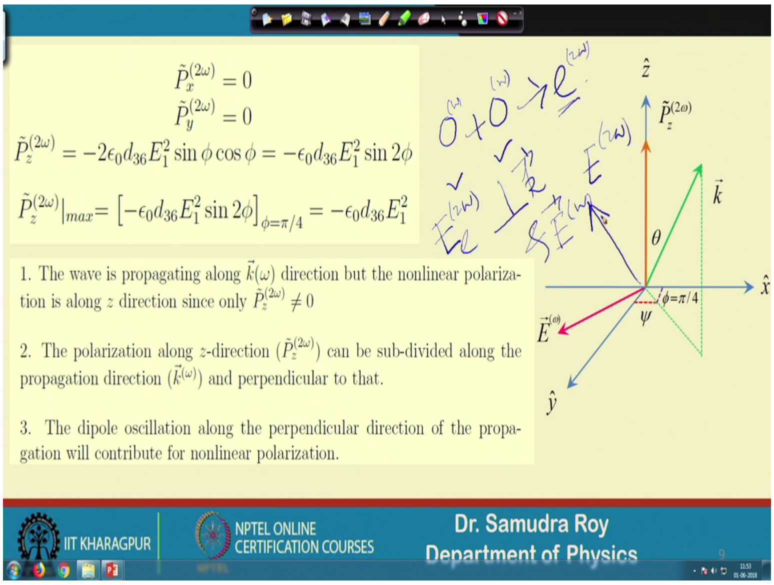

So, now, after maximizing the P z there are few things. So, P z means it will be in this direction. So, electric field that will going to generate the electric field that is going to generate is along this direction. So, E of 2 omega will be along this direction and what will be this field? This field is E, this is not E rather. So, this field this is a field let me write once again. So, the corresponding field will be E of 2 omega and this field will vibrate along z direction, but the question is in this particular case let us let us see what we have so far.

So, we have propagation along k direction, but the non-linear polarization along z direction since it is non-equal not equal to 0. So, non-linear polarization is along this direction. The propagation along z direction can be subdivided along the perpendicular direction and propagation direction and perpendicular to that. So, the next thing that we need to understand is that here my scheme has to be this, that means, two ordinary wave will give rise to an extraordinary wave of frequency 2 omega this is omega, this is omega. So, two ordinary wave can give rise to a frequency 2 omega which is extraordinary wave. So, now E 2 omega is not extraordinary wave. So, it what is the condition of extraordinary wave? So, extraordinary wave of 2 omega should be perpendicular to k should be perpendicular to k and also E should perpendicular to k and also E.

So, now if I want to find out what is the components, what is the components which can perpendicular to both this thing oh we can see that it can be divided into two part, along this direction and along the perpendicular direction so, somewhere here.

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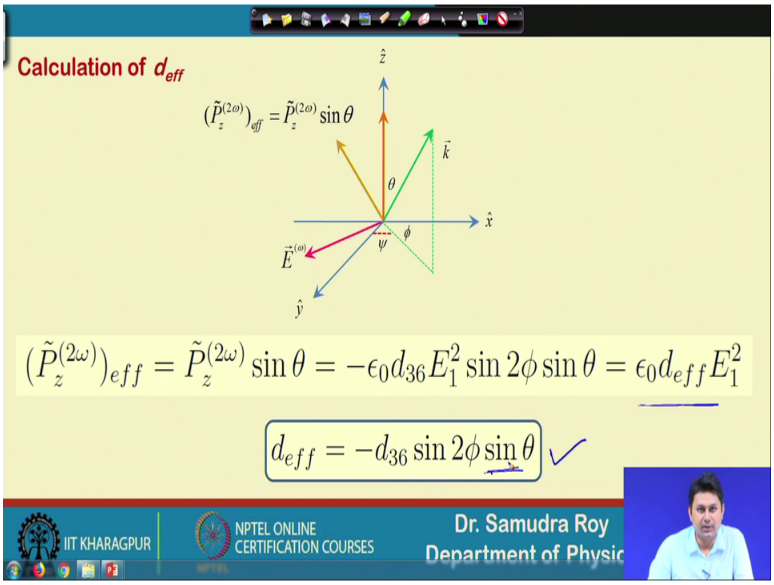

So, if I find what we have in the second slide then we find that this is the direction we can make is perpendicular. So, this direction is basically the direction at which we have our umm electric field that is really my extraordinary field. So, effective P so, the P will generate P 2 omega will generate here, but the effective thing we need to subdivide into two part. So, if this is theta so, this part will be the sin theta of that. So, sin theta will going to generate some effective d. So, sin theta will going to generate some effective d and this d effective is eventually this.

So, we find that even though the polarization is generating the polarization direction is not in such a way that we can generate the phase matching properly. So, in order to generate the phase matching properly we need to find out the direction along which really the second harmonic will going to generate. So, following that we find that it should be perpendicular to k vector. The direction should be perpendicular to the k vector. So, the electric field of 2 omega which is generating should be perpendicular to the k vector.

So, in order to do that we need to divide this field into two part; one is along this direction and along one is along perpendicular to that. So, if we divide the perpendicular portion then we will have a sin theta term that should add to get the effective polarization term. So, here we find the d effective is not only d 36 d 2 2 phi also a sin theta term has to be added with that. So, this sin theta term is important because this is the effective one. So, this effective things is important because we are find out we need to find out what is the second harmonic of this particular system where two ordinary rays merging to generate one extraordinary ray.

So, in the next class we will discuss more about these issues. So, with that note let me conclude here. So, see you in the next class, and thank you for your attention.