Introduction to Non-Linear Optics and its Applications
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Lecture - 22
Type II Phase Matching, Symmetry in Nonlinear Susceptibility

So, welcome student to the next class of Introduction to Non-linear Optics and its Application course, this is now lecture number 22.

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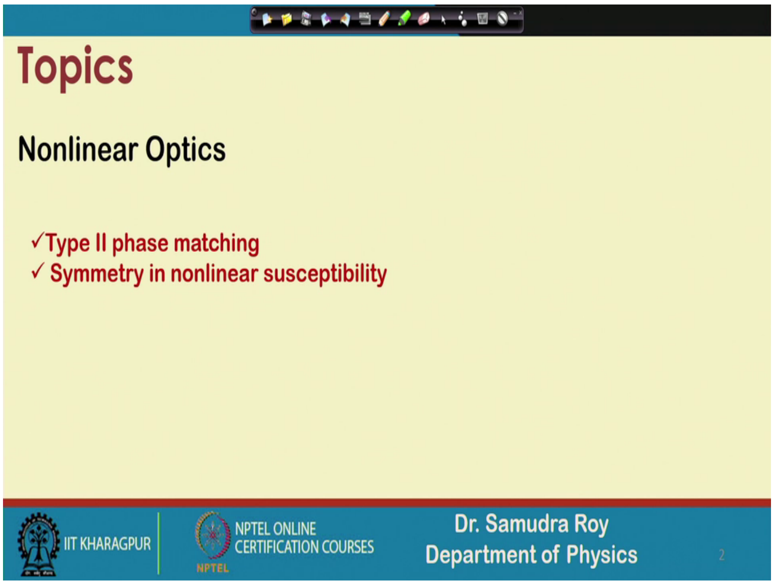

So, today we in this lecture we have the topic type of type two kind of phase matching and then symmetry in the non-linear susceptibility, so two very important topic.

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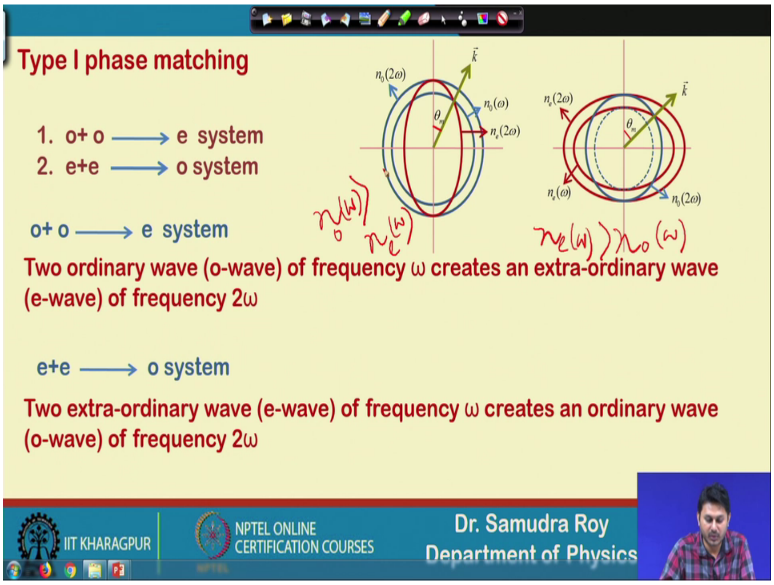

So, in the previous class what we have done is in type 1 phase matching. So, let me again remind. So, type one phase matching can have two kind of system. So, in type one phase matching two ordinary wave can merge and generate an extraordinary wave or two extraordinary wave can merge and generate an ordinary wave. So, two ordinary wave of frequency omega can create an extraordinary wave or two extraordinary wave of frequency omega can generate a ordinary wave of frequency 2 omega.

The index structure for two cases are distinct, if I now see here that in first case the condition has to be, n o of ordinary wave at particular frequency is greater than n of extraordinary wave of at that particular frequency. So, this kind of crystal is called the negative crystal because n o is greater than n e. On the other hand for other scheme that is e e is equal to o, this system; that means, two extraordinary wave when generate one ordinary wave for this system the reflective index of extraordinary wave at particular frequency is greater than the reflective index of the frequency omega.

So, when this twos are, two conditions are there we have two different kind of system, but both the cases generate the second harmonic waves and able to generate also the angle at which this things can be possible and it is fall under type 1 phase matching. So, in type one phase matching we are using either negative crystal or positive crystal and the system is where we, we have two ordinary wave to generate one extraordinary wave or two extraordinary wave to generate one ordinary wave.

Apart from that there is a possibility that we can combine these things; that means, that means two, instead of two ordinary wave. If you see carefully here we have one extraordinary wave and one ordinary wave that can generate one extraordinary wave and one wave, one extraordinary wave and one ordinary wave can generate an extraordinary wave. So, here somehow it is become theta.

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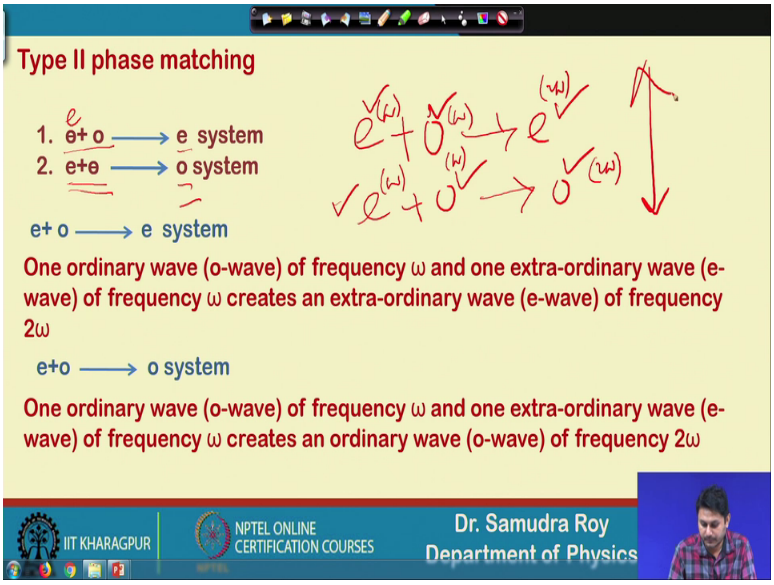

So, it should be e, so let me write e plus o can generate e this is one system and the type 2 phase matching e plus o can also generate an ordinary wave, this is the mixture of this things. So, this is called type 2 phase matching; so, let us understand what is going on.

So, one ordinary wave in this case, one ordinary wave, one extraordinary wave is creating one extraordinary wave this is one system. One extraordinary wave, one ordinary wave is creating one ordinary wave. Both the cases this is omega frequency, this is at omega frequency, this is at 2 omega frequency, this is at omega frequency this is at omega frequency and this is also has 2 omega frequency. So, this is these two conditions are there.

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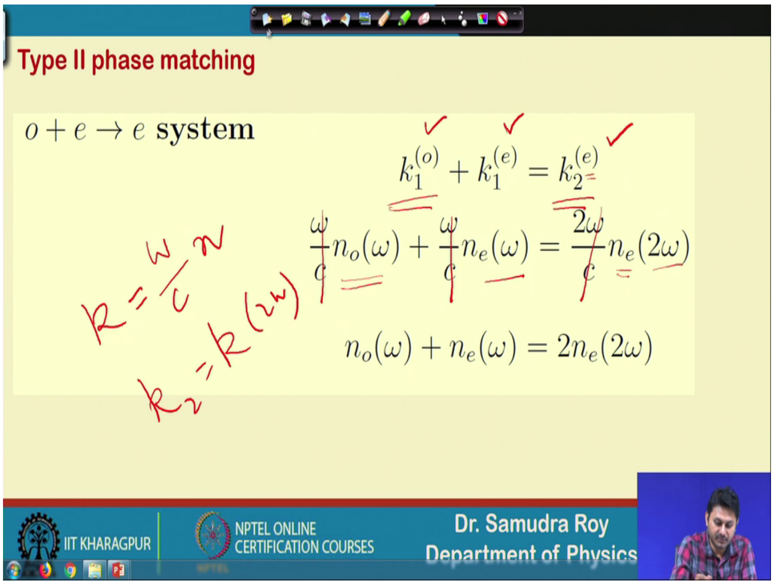

So, now after having the knowledge of this type 2 kind of phase matching, we can need to find out what is the reflective index representation for this type 2 and type ah, this type 2 phase matching with this system. So, one ordinary and one extraordinary leads to one extraordinary, in this system if I write in terms of k. So, k 1 is the frequency, k 1 is a propagation constant frequency omega 1 or omega. So, k 1 and ordinary, k 1 and extraordinary is generating k 2 at extraordinary.

So, this is the scheme of ordinary plus extraordinary is equal to extraordinary. So, now, k 1 can be represented at frequency. So, we know that k several time we are using omega by c and n. So, k 1 means frequency omega. So, omega divided by c and n, i need to write this is ordinary. So, ordinary omega k 1 at extraordinary omega by c in e omega, k 2 is what? K 2 is a frequency of at omega 2 omega . So, k 2 is a frequency at 2 omega. So, I write 2 omega divided by c, n of e because of e and 2 omega.

So, after having all this information, so we in the next step what we will do, I will just cut this omega divided by c term all together.

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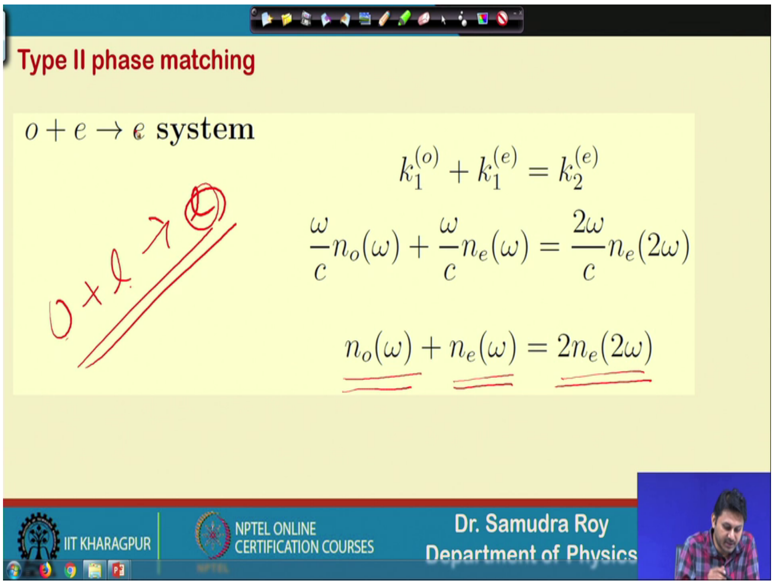

If I cut then we have a new kind of expression which is n of omega n e of omega is equal to 2 of n e of 2 omega, this is the type 2 phase matching condition, but this condition is for e plus o plus e tends to e system. In the similar way if you do o plus e tends to e system; that means, one ordinary wave one extraordinary wave is leads to one sorry, this is o because already we have done this things. So, let me write once again very clearly.

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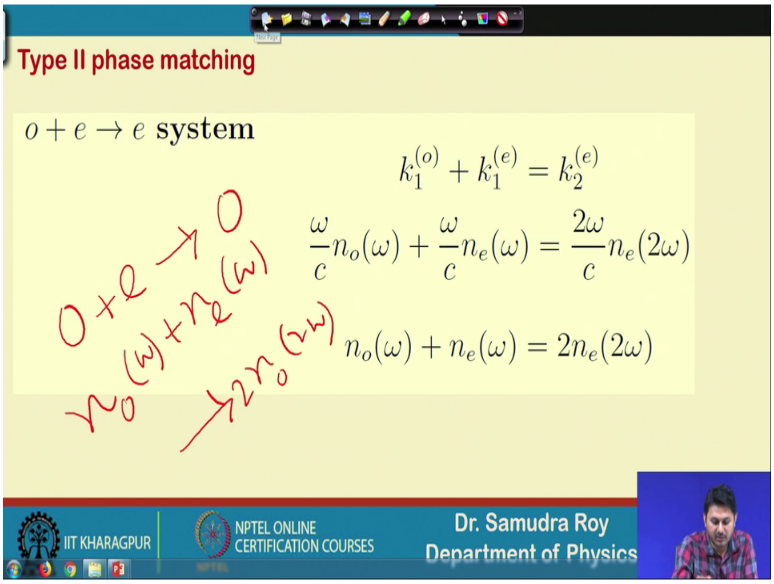

Ordinary wave, one extraordinary wave leads to one ordinary wave. So, what should be the expression the left hand side will remain same, but in right hand side it is different because we have ordinary wave of 2 omega. So, I will have two of ordinary wave of 2 omega, exactly the same calculation we have done in the next system ya. This is the system where we have one of the ordinary wave and one extraordinary wave that leads to one ordinary wave.

So, again in terms of k we have the similar thing, only thing that is changing here is o. So, at end of the day we are generating the ordinary wave having 2 omega frequency.

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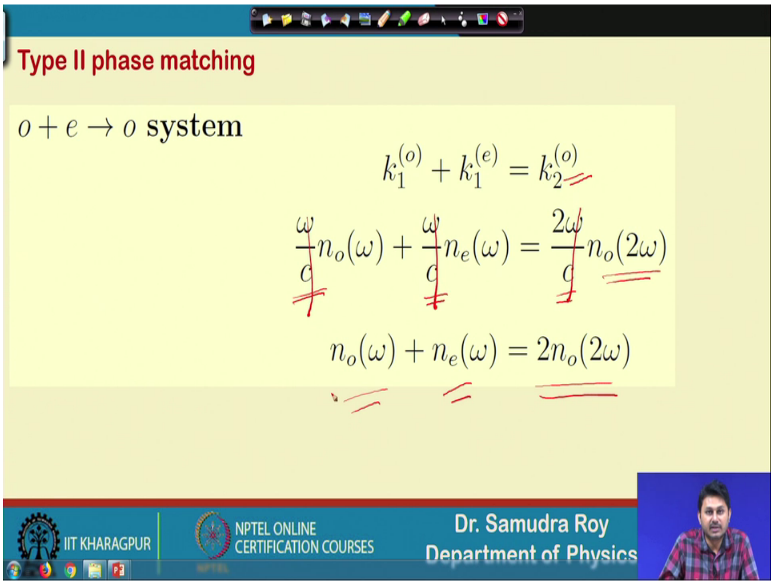

So, we have omega c, omega c, 2 omega c, n of 2 omega and in all the cases if I now cut omega by c omega by c, then we will have n of omega, n e of 2 omega is equal to 2 of n o of 2 omega. So, this two are type 2 kind of phase matching, but this type 2 kind of phase matching in order to achieve is it not easy we need to maintain many things that, many conditions are there for which we can have these things. But we can always generate a the angle theta for which this can be possible. So, let us check what we have in the next slide.

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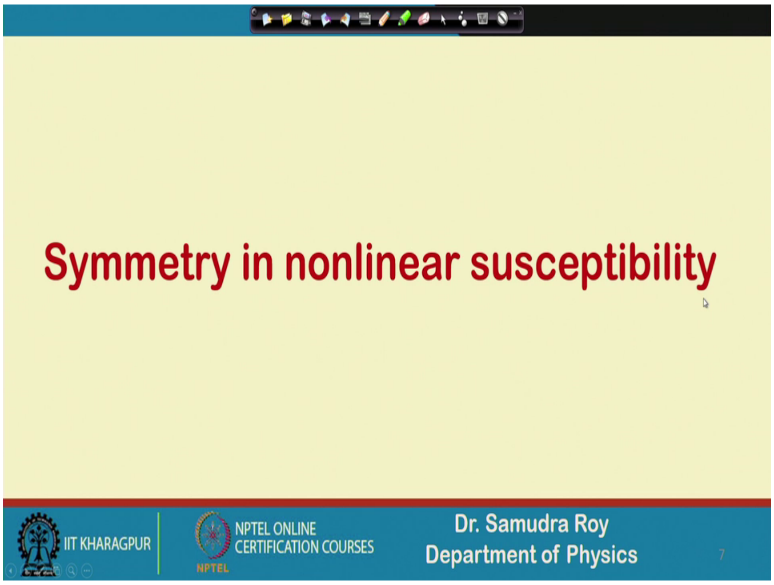

Next thing we will going to learn is Symmetry in non-linear susceptibility. So, here let me go back to this type 2 phase matching, in type 2 phase matching we have two different condition this and this, but for this two different condition again it is possible to find out the direction, but it is the complicated one. But one can have, one can find out the angle theta by using this expression, it is very complicated kind of expression, but it is possible to find out some angle at which these things are valid.

So, next symmetry in non-linear susceptibility, this is very important thing that we are going to study or eh in this particular course that the symmetry and all this.

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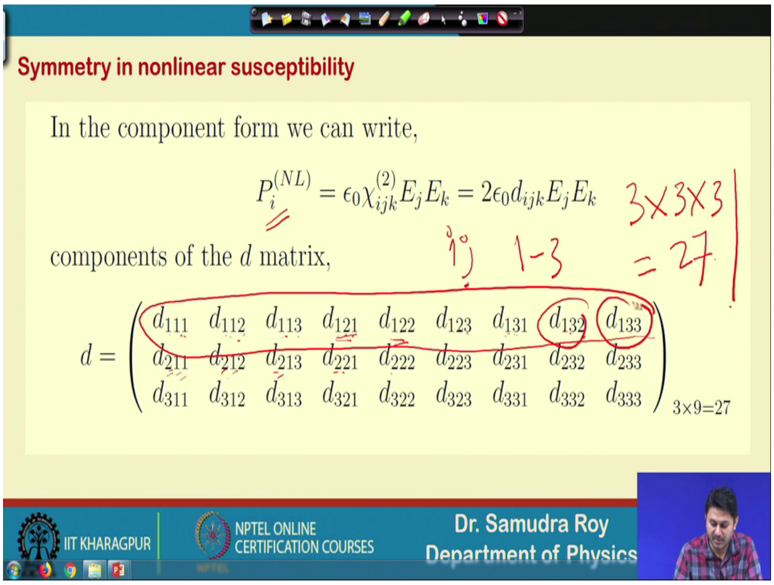

So, let us go back to our original equation to non-linear polarization and electric field, well. This is the component form of non-linear polarization and electric field. So, ith component of non-linear polarization can be reached in terms of j and k and this is the susceptibility components, the tensor components of the susceptibility ijk.

So, now jk is is a indices which is, which is repetitive here. So, we know that when this indices are repetitive eventually we have this summation sign here, where ijk is something in this case not ij it should be jk, jk ok. So, it is something like 1 summation sign is here and this sum is over j and k, but we never use this summation sign because of this sign notation we just removed that. It basically reduces lot of energy and this combustion equation become more compact.

Now, the question is if we have ijk here in terms of d matrix this is 2 of epsilon 0 dijk. So, how many de components are there. Now, if I calculate ijk there are 3 components each one component say i and j fixed, if i and j fixed then k components can vary 1 2 3 . So, I have 3 components, for each three components I will have another 3 components for j and for each j components I will have another 3 component of i. So, we will eventually have 27 different component of this d when d ijk all are vary from 1 2 3.

In this matrix form i try to write all this 27 elements and if you see, what is my first term 1 1 1, then 1 1 2; that means, I am not changing i and j, i and j remain same I just changing, I am just changing k value. So, this is 1 1 3 then for d I have 1 2 1, 1 2 2; that means, this is for this case I am just fixing i and j at 1 and 2 and then changing the last one 1 2 3 then i 3, 1 1 3 and then 2, I am not changing 1 and 3, 1 and 3 and 3.

In the similar way now I am changing the i value 2 1, 2 1, 2 1 1 2 3, 2 2, 2 2, 1 2 3 and so on. So, if you write all this things in some sort of matrix form you will find that there should be 27 elements, this 27 distinct elements one can have once we have the entire picture. So that means, for ith term, only for ith term if i is fixed we should have that much of element, how many elements are there here 1, 2 3, 4, 5, 6, 7, 8, 9. So, only for ith element in the right hand side I have a 9 different elements in my hand. So, that is the very combustion thing that in order to have one polarization component in right hand side I have 9 different num different d d elements or d matrix ok.

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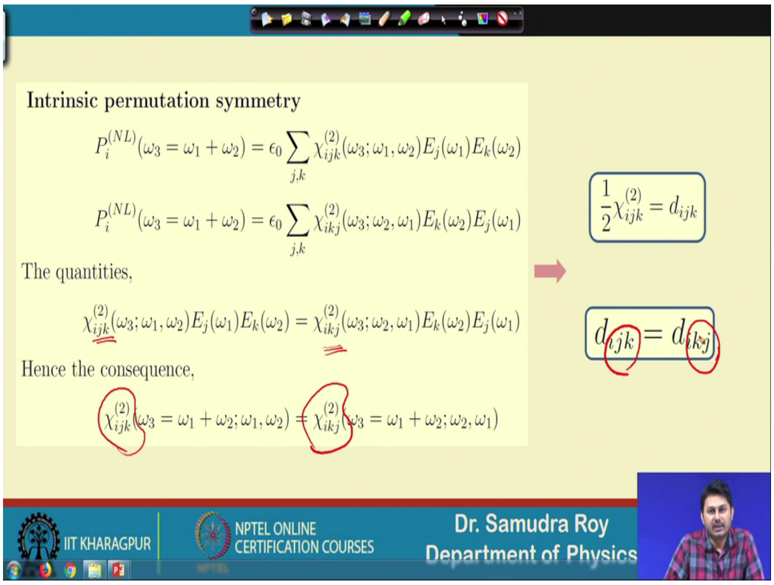

So, next let us see what we have. So, now, fortunately we have some kind of permutation symmetry in this particular system, what is the permutation symmetry? So, let us write here. So, permutation symmetry suggest that if I want to find out a polarization at particular frequency say omega 3, this omega 3 is generated due to 2 waves of omega 1 and omega 2. Then I can have the susceptibility with frequency omega 3, it is generated omega and omega 2 and epsilon this eh electric field of jth component and electric field of k th component have this frequency omega 1 and omega 1. Now, if I interchange the i and j components then we will have a similar expression. So, here what we are doing we are changing the ith and jth component, see if I change the ith and jth component then this term here and this term here and this ij will also shift their location, but in the left hand side we will have the same quantity again.

So, that means, if I, if I try to understand; that means, susceptibility i j k and susceptibility jki kj both the cases will have the same, if this is equal to this then we can widely say that susceptibility at ijk, susceptibility of ikj are same. So, that means, I can interchange these 2 things. So, this two things, two things are same, now this ijk and ikj they are same due to this intrinsic permutation symmetric then the d matrix significantly there are 27 elements. So, this 27 elements can significant reduces to 18 element. So, how this elements reduction are possible, let us see.

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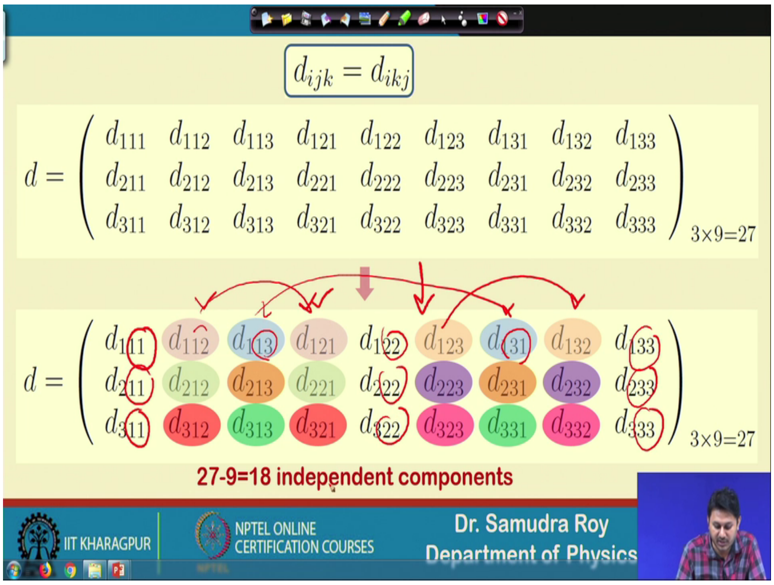

Well here we can see the d matrix has 27 different elements, in the first one, but we have this condition in our hand. So, this is the condition due to the permutation symmetry I have this in my hand that dij, d i j k is d ikj. Once we have dij ijk is dkj, then if I look very carefully then here in this from here to here we find that this con components and this components have same. All the components which are marked with the different colors are same, if you, if you find this pairs are same value, d 1 2 and d 2 1 these are the same because if I interchange 2 to 1, 1 to 2 this will be the same quantity.

Again here we have 1 3, some where I should have 3 1 and 3 1 is sitting here. So, these are the same color. So, these two things are same. So, this these same this two things are same, here in the first row also d 1 2 and d 3 2, d 1 2 3 and d 1 3 2 again this two things are same. We have another components also d 2 1 2 and d 2 1, 2 2 1 again this two components will have the same value, this two things will have the same value. But, if you note that these components 1, 1, 1, 1, 1, 1 of will not going to change because even if you change it, it will remain in the same quantity. Here also 3, 3, 3, 3, 3, 3 will not going to change, 2, 2, 2, 2, 2, 2 not going to change. So, how many independent components we have after this permutation, using this permutation symmetry the answer is 18.

So, these 2 pairs. So, how many components are 9 here and 9 here. So, all this 9 components are same, this 9 components and this 9 components are same. So, now, that means, we can reduce this 18 components to 9 components and then 9 components are already there. So, 9 plus 9 so we will have 18 components. So, we will reduce from those 27 to eighteen we reduce 9 components because of using of this permutation symmetry.

Once we reduce this permutation symmetry the next thing is to rewrite the d matrix in a more compact way.

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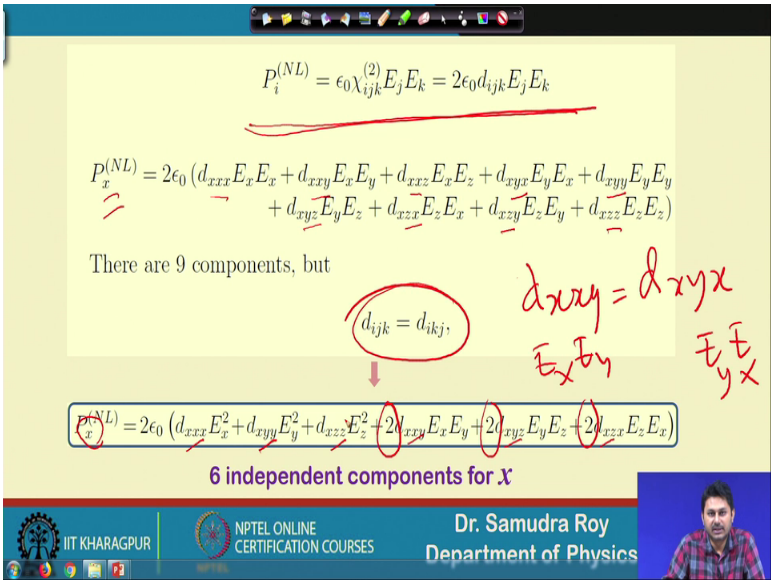

So, before going to write this de matrix in a more compact way a we need to find out why should be the value of P x. So, this is the same things we are writing. So, P x is epsilon 0 ijk and now if I want to find out the xth component I should write d xx, d xy, d xz also d x yx d yy then d x is same for all the cases. So, just I am changing the next 2 indices and this.

So, there it should be 9 there should be 9 components 1, 2, 3, 4, 5, 6, 7, 8, 9 components, but again I am using this permutation symmetry and once I use this permutation symmetry, I will reduce this to 6 components. So, for 1 px initially it was 9, similarly for P y we have 9 components for P y we had 9 components. So, total 27 components all this 27 components at different d, but for this permutation symmetry we can reduces to 9 components. So, this is one component x x, this is one y component, one component this 9 independent components are now reduces to 6 independent components with this permutation symmetry.

So, once we have this things you will find that d of x of xy is equal to d of x of yx, this terms are here both the cases because in both the cases this multiplication term ex ey is the same, here also the multiplication term will be ey, ex so we will have a 2 term commonly. So, we have 2 here, 2 here, 2 here for all this course term xx, yy, zz should have independent and we have just square term here. So, this is the way to write p in terms of e with d matrices. So, if I now wrote this things in contracted form then we have something different.

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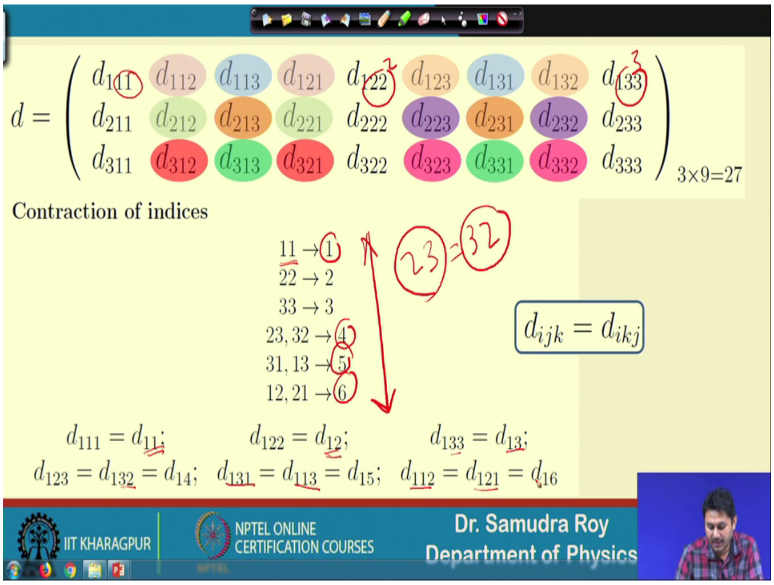

So, after having the knowledge that, for this permutation symmetry this d matrix now reduces to from 27 to 9, 27 to 18 and then for each we will have for each we will have 6 component, 1 for 1 piece we have 6 components. So, let us go back to this d matrix once again, for this d matrix this terms are similar terms, all the terms having the same color on a similar terms. So, once we have this similar terms then we will not doing to write all this things once again because this is called the contraction of indices.

Because if 2 3 is equal to 3 2. So, we will not going to write every time this 2, instead of this we will write some different and here this different thing is 4. So, 1 1, so 1 1 this components I write at 1, 1 2 2; that means, this one I write 2, 3 3 I write 3, 2 3 and 3 2 which are eventually same thing as I mentioned here, I write 4 3 1, 1 3 which are eventually same, I write 5 and 6. Since there are 6 independent components I write this 6 independent components in more short and notation or I sink the indices. So, d 1 1 I write d 1 1, d 1 2 2 I write d 1 2, d 1 3 3 I write d 1 3, d 1 2 3 I write d 1 4, d 1 2 3 and d 3 1 2 is same.

Similarly, d 1 3, d 1 1 3 is same d 1 1 2, d 2 1 is same and I write this as 1 6. So, once we have this contraction, so the d matrix become more compact and this is the final compact form of the d matrix initially we have 27 components, now we have 18 different components.

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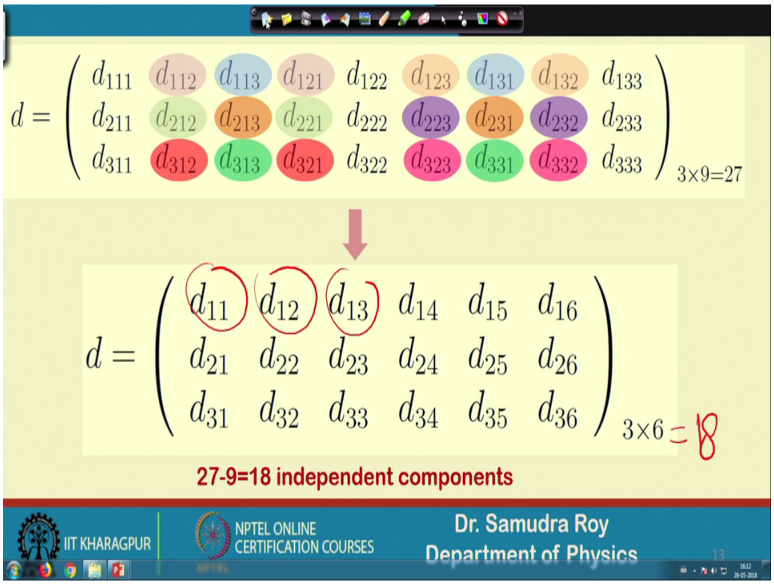

So, here we have this or this we have this things 1 2, 1 3, 1 4, so 2 1, 2 2, 2 3, 2 4. So, now, interesting thing is that previously we have 27 components, now after using permutation symmetry and then contracting the coefficient or the indices I now reduces the 18 different components.

So, this is the form the compact form of d matrix. So, in the in the next lecture we will find that again we can use some other symmetries. So, that I can reduce the elements of the d matrix further and then by using some kind of symmetry operation we are also reduce the d matrix, the elements some of the elements will be same, some of the element will be 0. So, initially even if we can see that there are 18 different elements can be possible to d matrix, but later we will going to find that most of this coefficient are going to vanish and we will deal with only few elements of the d matrix and in all the cases we will use some kind of symmetry operation.

So, with this note let me conclude this class. So, in the next class we will start from this d matrix and contractions of element etcetera and try to find out what the symmetry operation is and by applying the symmetry operation how do I find the exact value of this element so.

Thank you for your attention, see you in the next class.