Introduction to Non-Linear Optics and its Applications
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Lecture – 21
Birefringence Phase - Matching (BPM),
Type I and Type II Phase Matching

So, welcome student to the next class of Introduction to Non-Linear Optics and its Application. So, in the previous class, we have started a very important concept which is called the phase matching. Two different kind of phase matching we discuss; one is the phase matching due to this called birefringence phase matching and another phase matching was the quasi phase matching.

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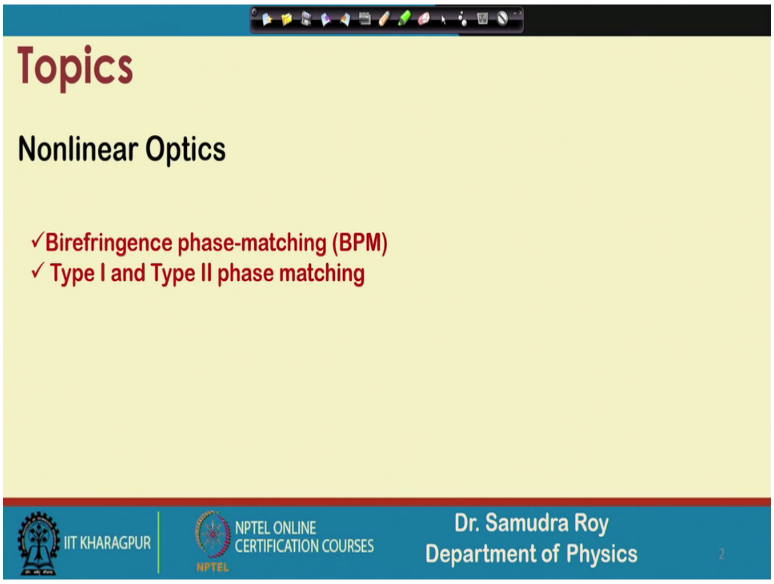

So, today we will extend ours study. So, bi birefringence phase matching condition we will learn in detail and in birefringence phase matching there are two types of phase matching are there type I and type II. We also going to learn what is this type I and type II phase matching.

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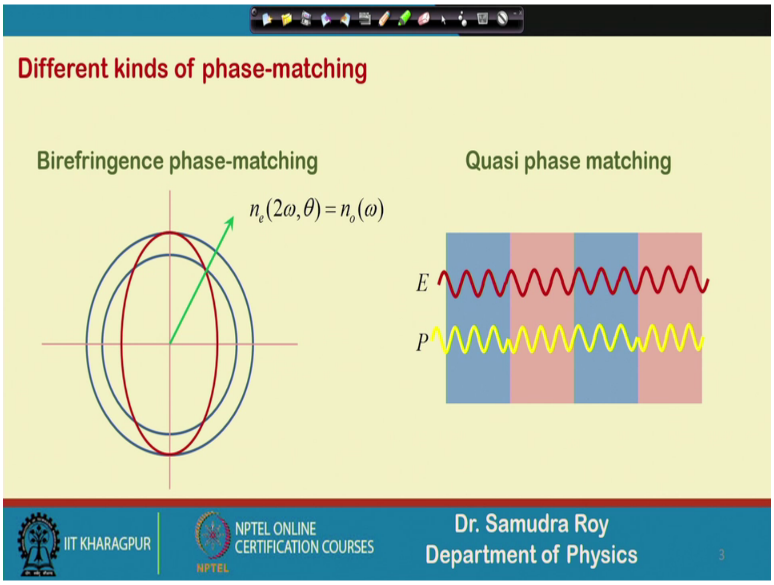

So this is the structure of birefringence phase matching and quasi phase matching. This is the old picture, whereas very important one. One should understand, what is the birefringence phase matching. In birefringence phase matching what happened that the crystal should have two different kind of wave; one is ordinary wave and another is extraordinary wave for inaxial crystal we have this kind of properties and this n o and n e basically representing the reflective index of ordinary and extraordinary wave.

So, n e is there is there reflective index and this reflective index is a function of theta. So, now, if I plot the index diagram then we can see that there is a specific point as shown here, this point at which we have at which we have the phase matching; that means, n e of 2 omega and that particular theta is matching of n o of omega that is eventually our phase matching condition. So, that means, if I launch a light in that particular direction the phase matching condition can be achieved, well.

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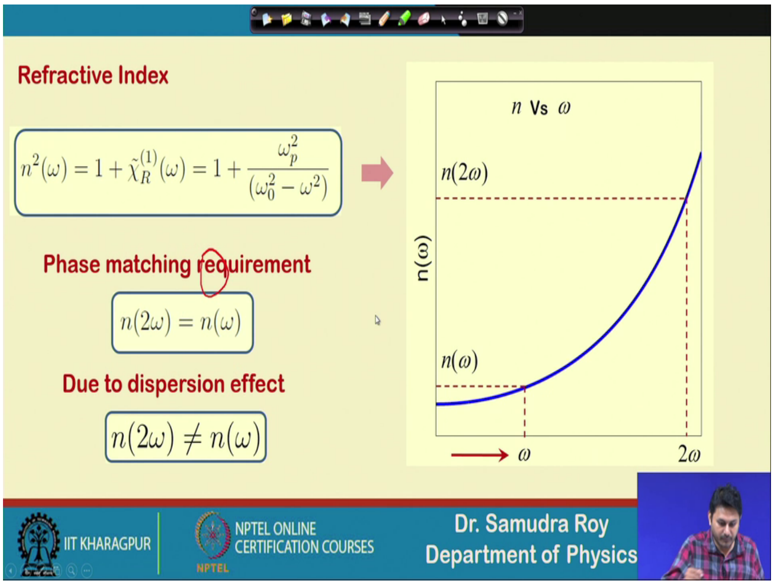

So, also in the previous class we find that the reflective index can be function of omega and as shown here in the picture that it is changing with respect to omega and when it is changing with respect to omega, there is no way that we can have a phase matching. So, due to the dispersion we never have n 2 omega is equal to n omega.

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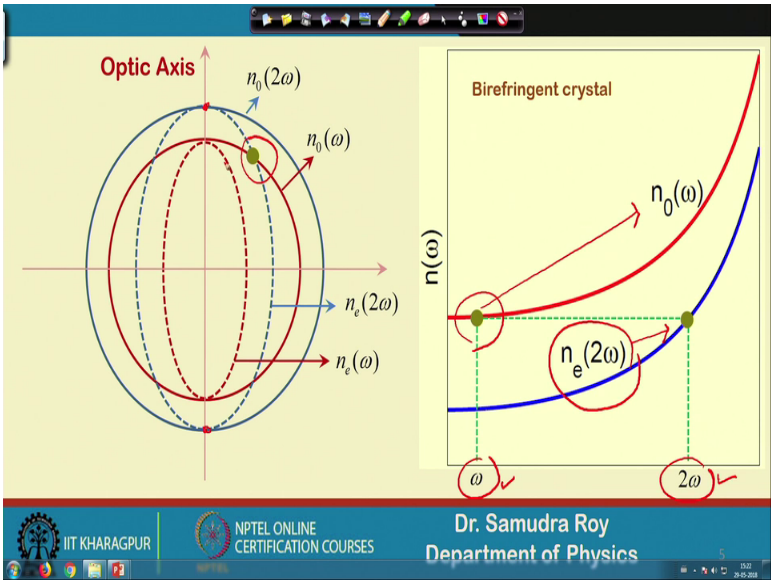

So, what is the then what should we do. So, this is very important picture of birefringence crystal, where we plot n omega and n 2 omega for ordinary and extraordinary wave. So, this is n omega of ordinary wave and this one the blue one is n e that means, a reflective index of extraordinary wave and this is launched for 2 2 omega.

So, when this is both are the functions of omegas, but at this point the value is this, at this point the value is this. So, what happened that you can if I make a horizontal line you can see that at 2 omega point whatever the value we have at omega point I will have the same value for ordinary reflective index; So, ordinary reflective index at omega frequency having the same value, extraordinary effective index at 2 omega frequency. So, this can be more clearly understood in this index figure when I plot this index figure when I draw this index figure this is optic axis, we know that in optic axis the direction of the optic axis the reflective index of ordinary and extraordinary wave are same, that is why this things are touching here and here.

Ordinary reflective index will never change and it will not be a function of theta. So, it will give us a circle, but extraordinary wave can change with theta. So, we have elliptical structure. So, we have ellipse in two dimension this things we have discussed in our very previous classes were we studied the basic linear optics, but here you can see a point where they are crossing. This is the crossing point between the ordinary and extraordinary reflective index and this crossing point is basically the point where two values are same. So, there is a possibility that we will have the phase matching at that particular point. So, that is the idea of birefringence phase matching.

We use some crystal to generate second harmonic, where the phase matching condition can be satisfied through this n e and n o this reflective index using this two reflective index.

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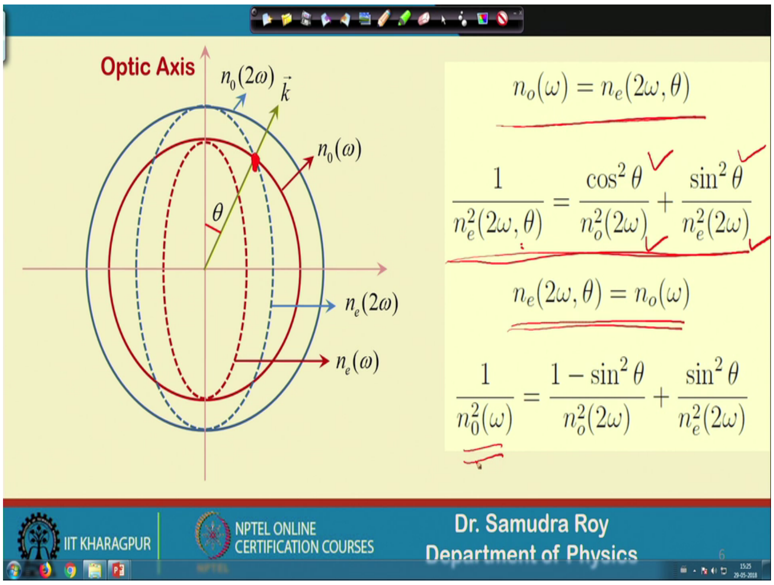

Well, now the next thing is to calculate what is the value of this angle? We are talking about that there is a phase there is a matching of reflective index, but this matching will be in particular angle. So, if k is the launched wave, k is the direction of the wave then it is making an angle theta for which we have the phase matching or for which we have the matching of the reflective index here, over this.

So, now we have this in my hand n o of omega is equal to n e of 2 omega at theta point. Now, we have the equation in our hand that one this equation is a very old equation. So, one of n e square 2 omega at theta I want to find out the theta dependency. So, if I want find the theta dependency in the right hand side cos theta and sin theta storm is there. So, it is cos quare theta sin square theta and n e 2 omega and n e 2 omega n o 2 omega square.

So, now, if I put this things here the condition here so, what happen that n o in the in the left hand side I put n e of 2 omega just to remove this omega and I put just n o of omega square and in the right hand side I just replace this cos to sin terms, so that everything become in sin. Our goal here is to find out the theta angle for which I can get this angle. So, when I replace this cos to sin then we have 1 minus sin square theta this sin is already there, this quantity we know, this quantity we know this angle I want to find out, but my phase matching condition suggest that this is equal to n o, I just replace n.

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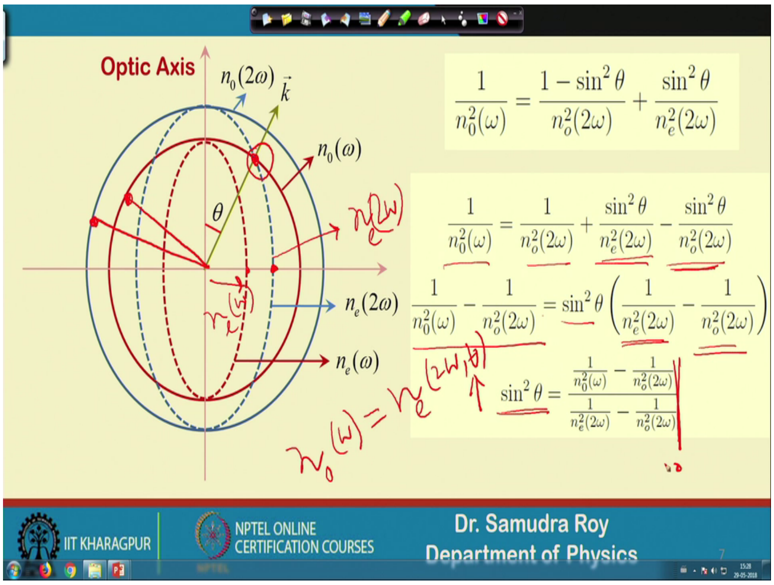

Once I have this form then the next thing is easy we just replace this so, the next step is to simplify. So, 1 minus n o square 2 omega I will put it here, I will put it here and then I will just write minus of sin square theta n o square 2 omega here. So these two terms are function of sin theta and we have another 2 terms I put this terms in the left hand side and I am getting 1 divided by n o square omega minus 1 divided by n n o square 2 omega. So, this is the ordinary reflective index. So, this is the ordinary reflective index of 2 omega and ordinary reflective index of omega.

And, in the right hand side we have sin theta if I take sin theta common then we have 1 divided by n e 2 omega and one divided by n o 2 omega. So, n e 2 omega and n o 2 omega again these are the values in this particular diagram if I want to find out where which value I am talking about you can readily understand that which value is are this. So, if I say n n 0 or n o 2 omega n o omega; that means, I am talking about the radius of this quantity.

So, radius is n o of 2 omega, n o of n o of omega n o of 2 omega is the radius of this blue line. So, this is this radius is basically the value of n o omega, this radius amount of this radius is basically the value of this quantity, n e of 2 omega is which value at this point and at this point a values are different. So, I am basically talking about this amount this is basically n e of 2 omega and this amount from here to here is basically n e of omega.

So, all these values one can find out with this index structure; mind it, is function of lambda. So, omega is a fixed value here for different omega we will have different structure, but the important thing is that if these values are provided then we can verily find out the angle at which we have these condition value. That means, these condition wherein omega n o of omega is equal to n e of 2 omega with theta angle. So, with this angle is very important. So, we find the direction at which the phase matching condition is valid.

So, in order to find this direction these are my expression. So, once we have this expression in my hand then verily I can calculate the angle at which the phase matching condition is there, that means, maximum efficiency of generating second harmonic will be achieved at that particular direction. So, that means, inside the crystal if I launch a fundamental wave then the launching angle is very important criteria. I cannot launch in any arbitrary direction, but I need to launch a very specific direction for which compared to optic axis I launched to a certain angle, so that my phase matching condition is valid and once the phase matching condition is valid to we will have more efficiency second harmonic generation. So, this is the process to find out this angle.

So, if I know all the values of ordinary and extraordinary reflective index then we can readily find out what should be my angle.

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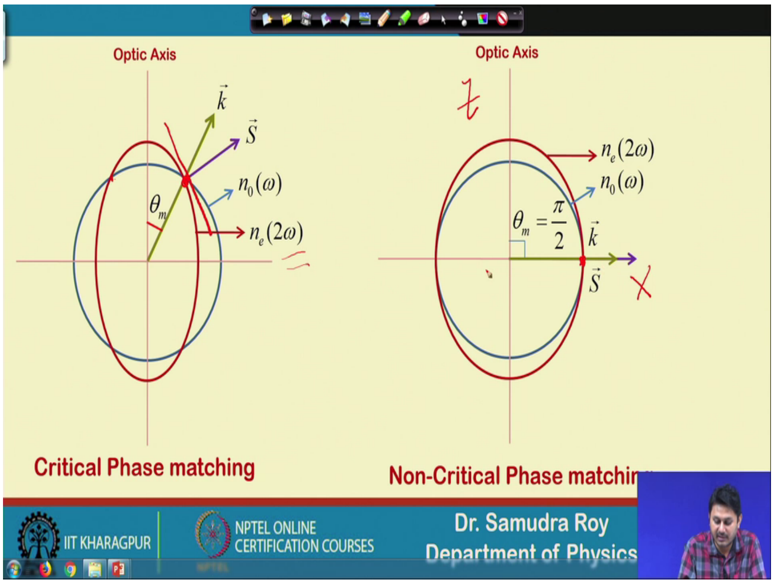

So, this there is a cutting point as I mentioned at angle. So, this is called basically the critical phase matching once we have this angle at this point they are matching. So, k is an this direction and when the k is in this direction. So, the S has to be in different direction because S should be perpendicular to, if I draw a perpendicular line here in the surface then S has to be perpendicular to that. So, there is a there is a angle difference between k and S. So, we know that this is called that walk of angle. So, k and S will not be in the same direction for critical phase matching condition and in critical phase matching what happen, we have a matching of n omega and n 2 omega n e of 2 omega, but with finite angle.

Now, there is a possibility that we this angle can increase up to pi by 2 and then the figure will be something, where this cutting point merges to one single point. There are four different direction by the way, this, this two are other direction also, where the cutting is possible. But, here we can see that if I increase this n 2 omega then there is a possibility that their cutting exactly over x axis because this is the x axis and this is my z axis this or optic axis.

So, along these direction what happened, k and S will be parallel to each other. So, non-critical phase matching is a very important thing where we have the additional property that k and S both are parallel to each other and if I deviate slightly then there is still there is no problem because they are parallel to each other. So, that is why it is called non critical phase matching.

So, criticality is somehow less in this particular case where the phase matching angle theta m for here as we mentioned is pi by 2. So, when theta m is pi by 2 we have certain angle means certain direction, where the phase matching is possible and this direction is nothing, but along x direction. So, this is a very important ah kind of phase matching. And, normally by changing the temperature there are few crystals if you change the temperature of the crystal what happened the reflective index, extraordinary reflective index may change.

And, when there extraordinary reflective index change there is a possibility that it will reach to this that value by it not changing the ordinary or by adjusting the temperature it is possible to make this angle pi by 2. So, from critical phase matching so, we can go directly to the non critical phase matching domain and non critical phase matching is much superior than the critical phase matching.

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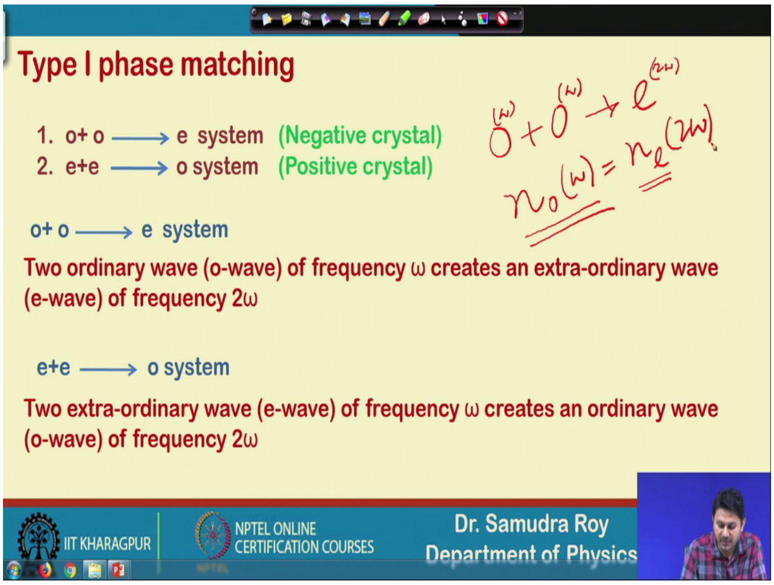

Ok. Now, the next thing is type I and type II phase matching. So far, we are dealing with the phase matching for which we are generating. So, what was the phase matching? So, let us write this things clearly. So, in type was type I phase matching two systems can be possible as shown. For negative crystal what happen that one ordinary ray one ordinary ray and one ordinary ray can generate one extraordinary ray. Phase matching condition should be n o of omega is equal to n e of 2 omega.

So, n o of omega means ordinary waves at fundamental frequencies this ordinary wave and fundamental frequencies ordinary wave and fundamental frequencies they are generating extraordinary wave. So, extraordinary waves are generating because of the combination of two ordinary waves because this phase matching condition in this is the process.

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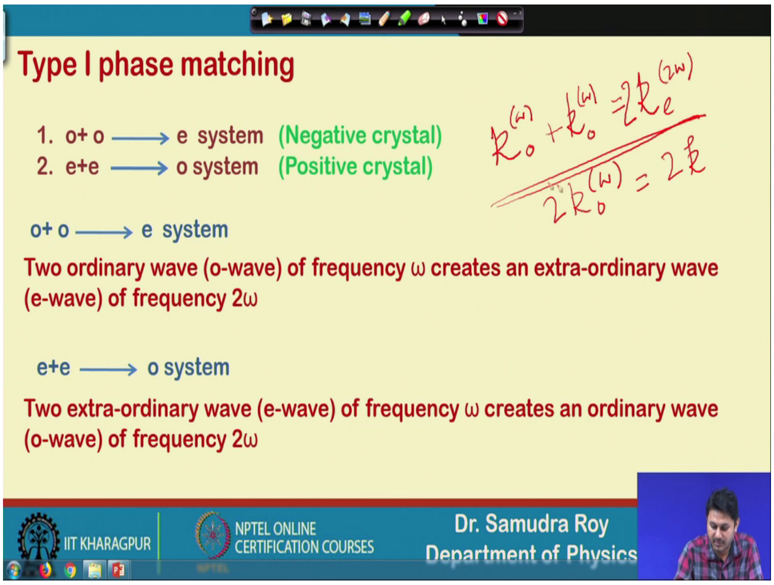

So, let me let me write in terms of k, I think that will be easier to understand. So, k of ordinary wave with frequency omega plus k of ordinary wave with frequency omega can generate k of extraordinary waves with frequency 2 omega 2 of k of ordinary wave frequency 2 omega. Now, if this is thus this is the structure in terms of k, then what happened I can write again say here as it written that two ordinary wave of omega of frequency two ordinary wave or o wave of frequency omega creates an extraordinary wave of frequency 2 omegas are in k domain or for k this is the expression.

So, now if I write it is 2k ordinary omega is equal to 2 of n o it is just k 0. So, let me write it once again. So, here I will do that later. So, two ordinary wave of omega merge to generate one extraordinary wave, this is one structure. Another structure is still possible where we have in positive crystal we have a structure where two extraordinary wave two extraordinary wave of omega can create an ordinary of 2 omega.

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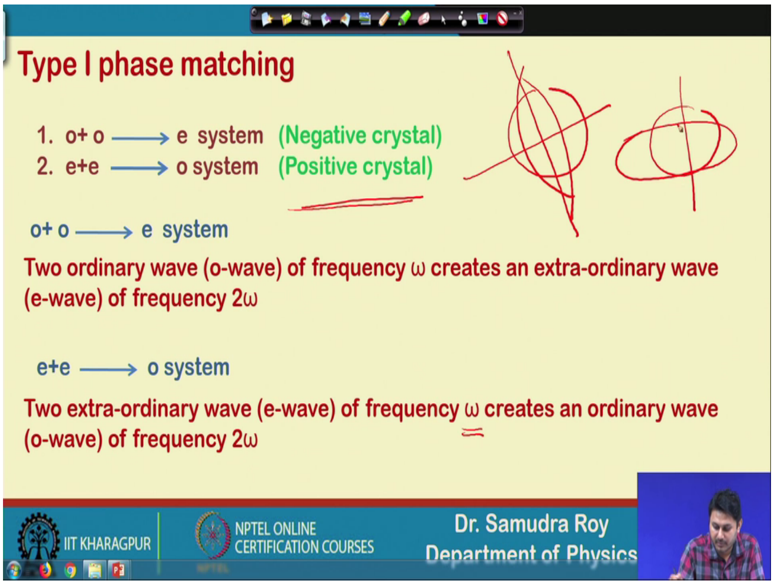

In the previous case we have a structure like this, but another structure is still possible we will show, where we can have this kind of thing. So, the drawing is not proper. So, maybe we can have this figure in the next class slide, yeah.

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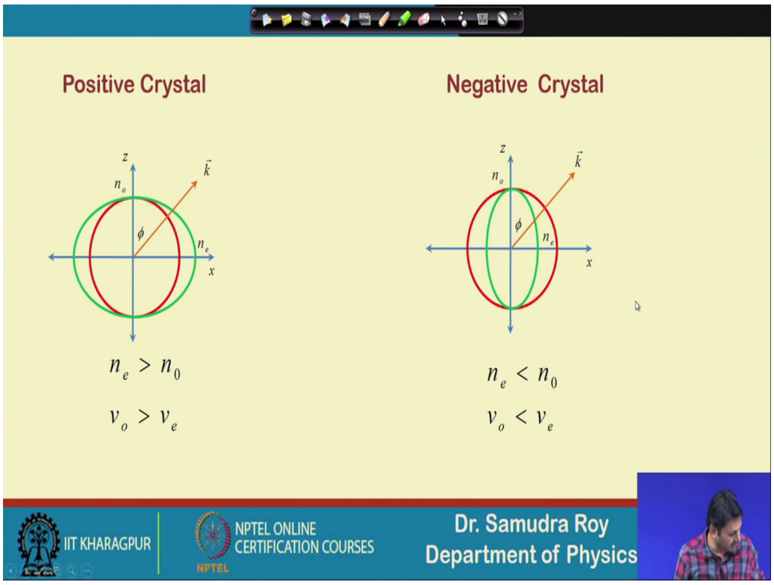

So, this positive crystal, what happen? In the positive crystal we have the velocity of ordinary wave is greater than velocity of extraordinary wave, that means, the reflective index of extraordinary wave is greater than reflective index of ordinary wave and in negative crystal opposite happen. So, this is the direction of k both the cases and this is negative and positive crystal. This is the old slide I am using because we have already learned it to recapture what is going on we show that.

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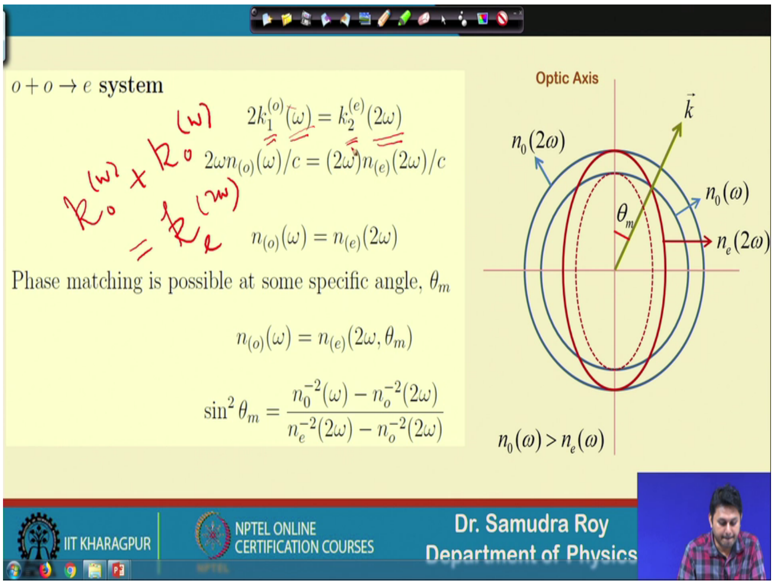

So, now we like to understand what is happening the o wave system, that is exactly the thing that I was trying to do just few minutes ago. So, let me do that once again that two ordinary waves so, here again let me write. So, o, o system what happen, that two ordinary wave are generating one extraordinary waves; that means, 2 of k 1 0 here k 1 means ordinary and k 2 is extraordinary written as o and omega o and e and here the frequency is 2 omega and omega for k 1. So, that is why I write it as k 1 and k 2 here.

So, if I use this expression and then if I convert e 2 in terms of reflective index. So, k 1 is omega divided by c n of omega n of ordinary wave with omega frequency.

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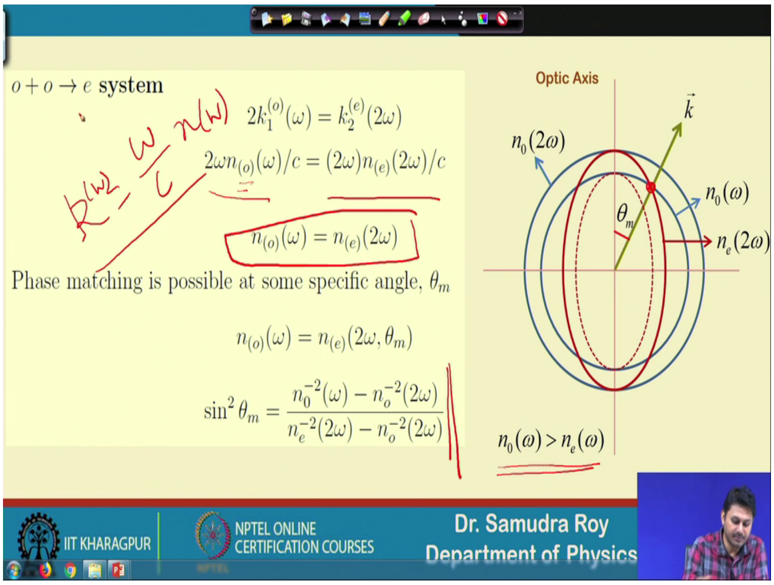

Here, also k so, just we use this k is equal to omega divided by c k of omega is omega divided c n of omega, we just use this expression both the cases. One case we use extraordinary reflective index n e because my k vector is extraordinary. k vector for ordinary we just replace n of o and when I do these two things together we have the expression where we can see that ordinary reflective index of frequency omega is matching with extraordinary reflective index of frequency 2 omega.

So, exactly this is the figure that we are looking for. So, for this kind of phase matching theta m, one can easily figure out and that is the value of theta m also we have derived this value and this is happening for this kind of crystal, where n o is greater than n e.

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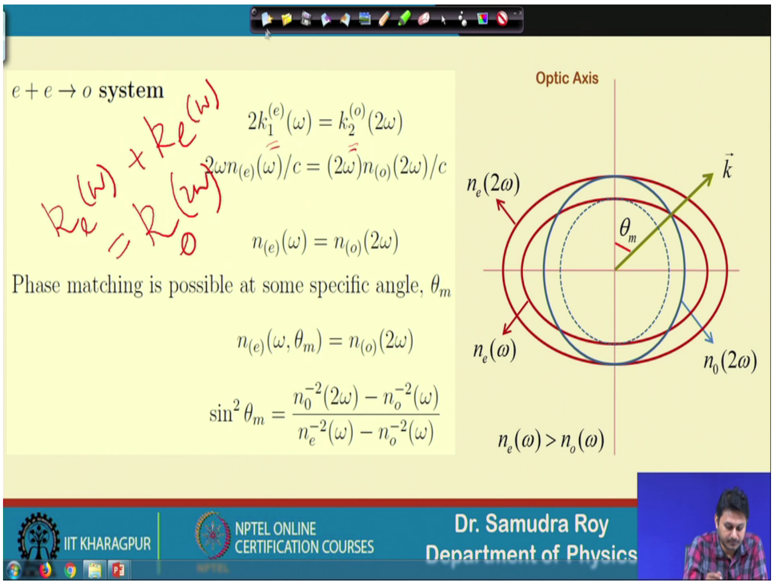

So, what happened for o, o for e system we understood the next thing is that what happen for other scheme; that means, what happened for e, e to o system; e e to o system what happen, two k 1 vector with extraordinary wave. So, k e at omega plus k e of omega, these two extraordinary wave vector will be equal to k o of 2 omega; k of omega is represented by k 1, k of k of e at omega represented by k 1 and k of 2 omega is represented by k 2, this ordinary and extraordinary are symboled like this way.

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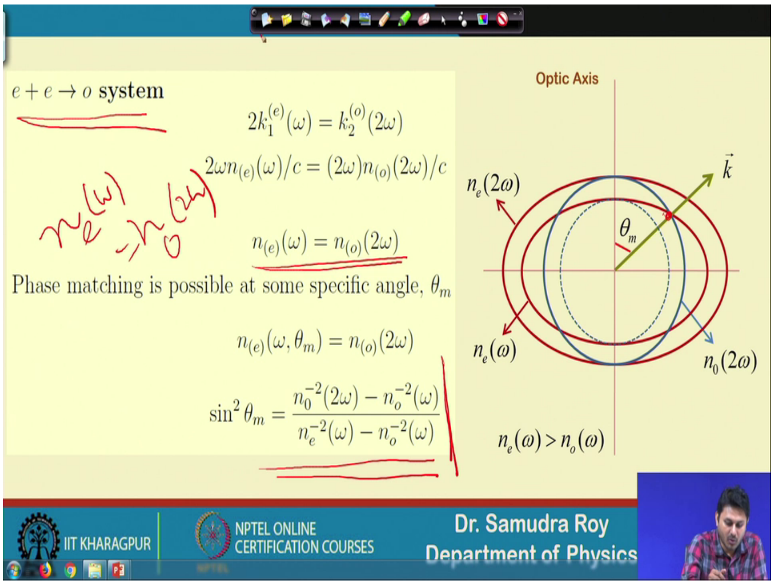

Now, again we are doing the same thing, but here we are find one important condition that now, instead of n e equal to n e of omega is now equal to n o of 2 omega. So, if I now try to find out the picture then obviously, n e of omega has to be greater than n o of 2 omega. So, this is a crystal where this is possible at this point as shown here. Now, one can again find out the theta m value. So, I can I can I can find out what is my theta, but there is a difference there is a small difference between this expression and the previous expressions, you have to be very careful. The previous expression we have already calculated.

In the similar way if you calculate you can have a result something like this. So, this is the result for which we are getting second harmonic, but the type of the second harmonic is it is a first type of, but the this is a system, where two extraordinary wave is generating one ordinary wave. In the previous case we find one two ordinary wave is generating one extraordinary wave, in this case we find two extraordinary wave is generating one ordinary wave. So, go back to this expression once again. So, here it is n o, n o to the power minus 2 omega n o to the power minus 2, 2 omega and then in the denominator we have n e and n o.

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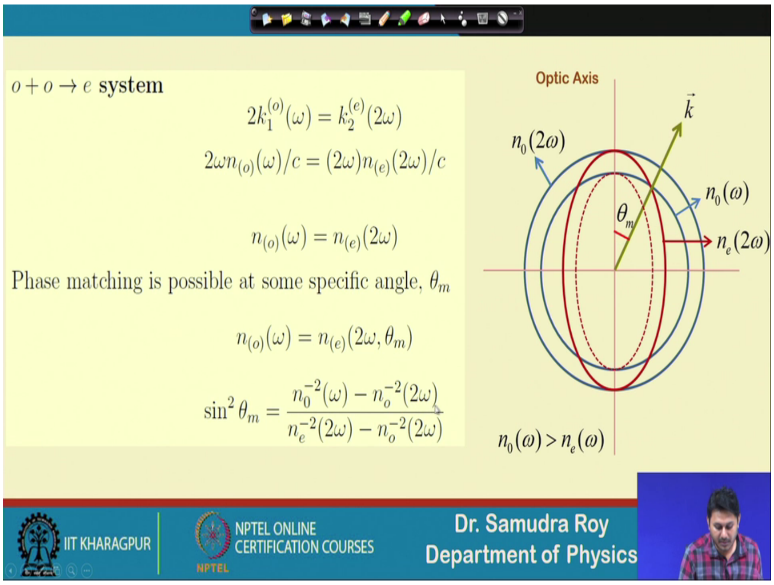

What happened in this case it is n o, n o, n o, n e, n o, n o, n o, n e. But, if you look carefully here we have omega in the previous case here we have omega in the later case we have 2 omega. So, there are two system, please note that, this is o o e system and in this particular system I have reflective index of ordinary wave here, but the frequency at omega and here we have reflective index of ordinary wave, but frequency 2 omega. Other terms are look quite same, but in this case 2 omega, 2 omega, 2 omega, but here omega, omega, omega.

So, I suggest you to please calculate this things by your own way. This is this is the calculation that we already done in this calculation we have already done. So, that means, this calculation. So, let me go back to the previous slide. So, that you can understand yeah here you can see the calculation for the crystal where two ordinary waves are margin to generate one extraordinary wave. So, this calculation is already done.

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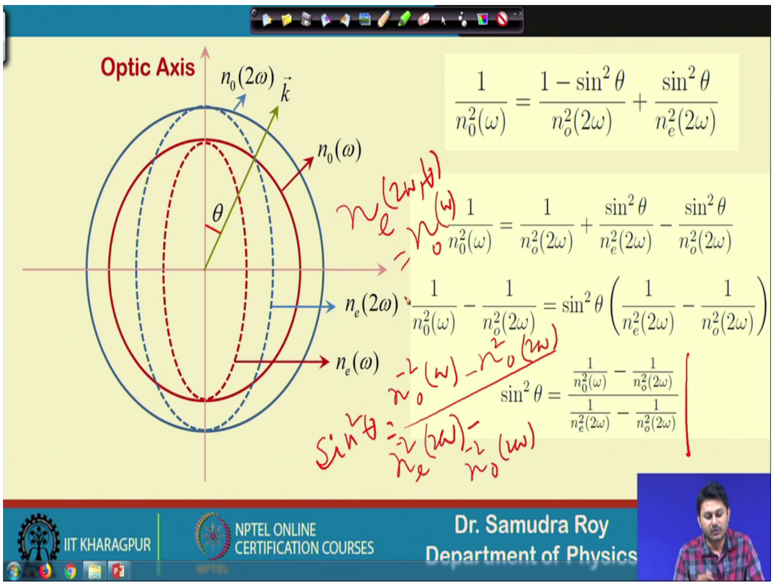

So, here we can see that it is sin square theta if I now write in terms of. So, n o of minus 2 omega. So, please note that one by n o square is just represented as n o of minus 2, to just reduce the complexity of the this expressions. So, here we have 2 omega, this is my outcome that we derived here both the cases we have a negative sign, negative sign. So, this is the way we have calculated and in order to calculate one important expression that we have used is n of e of 2 omega theta is equal to n of o of omega.

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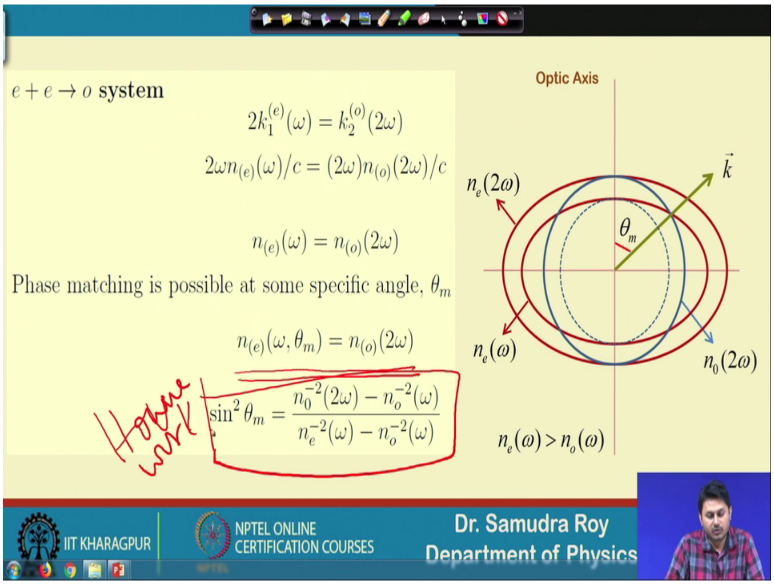

So, exactly the similar treatment you need to follow in the next case. In this case, but you need to careful about that you need to use this expression that now my condition is slightly change at any of omega theta m is equal to n o of 2 omega. So, I can put it has home work or the class work and I want you it will be very a very easy calculation. So, I want the student to please do this calculation and match whatever the expression is shown here is a same thing that you are getting or not.

So, with that note let me conclude here. In the next class, again we will start from this level and try to understand more about matching condition and the crystal symmetry is also be cover in the next class. So, with that note let me conclude. Thank you very much for your attention and see you in the next class.