

Introduction to Non-Linear Optics and Its Applications
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Lecture - 20
Manley - Rowe Relation, Energy Conservation in SHG

Welcome back student, to the next class of Introduction to Non-Linear Optics and its Application. So, today, we will going to have three very important going to learn very important concept watch.

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Topics

Nonlinear Optics

- ✓ Manley-Rowe Relation
- ✓ Energy conservation in SHG
- ✓ Different kinds of phase-matching

$\Delta k = 0$
Phase matching
 $\Delta k \neq 0$
 ΔX

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First one is the Manley-Rowe relation which is very important, then the energy conservation under second harmonic generation, and finally different kind of phase matching. In the last class we have learned about the phase matching and find out that there is a bandwidth Δk equal to 0 is a condition of our phase matching. So, Δk equal to 0, is the condition of our phase matching, but we find that even Δk is not equal to 0 we get some kind of phase matching.

And, we also able to figure out what is the bandwidth that if any external parameter X through which Δk is depending on, then what is a tolerance of this external parameter in terms of ΔX that for which we are getting the phase matching. And this phase matching Δk tolerance level we fixed at this 4 wave half maxima point. So, that we have already discussed in the previous class.

So, today we will going to discuss more important things like Manley-Rowe relation. So, let us go directly.

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The slide is titled "Evolution equations of E_1 and E_2 ". It contains the following content:

- Two coupled differential equations:

$$\frac{\partial E_2}{\partial z} = i \frac{\omega d}{c n_2} E_1^2 e^{-i \Delta k z}$$

$$\frac{\partial E_1}{\partial z} = i \frac{\omega d}{c n_1} E_1^* E_2 e^{i \Delta k z}$$
- Two intensity formulas:

$$I_2^{(2\omega)} = \frac{1}{2} \epsilon_0 c n_2 |E_2|^2$$

$$I_1^{(\omega)} = \frac{1}{2} \epsilon_0 c n_1 |E_1|^2$$
- Hand-drawn diagrams:
 - A box containing a blue wavy line labeled $E^{(\omega)}$ and a red wavy line labeled $E^{(2\omega)}$.
 - A red handwritten formula: $I = \frac{1}{2} \epsilon_0 c n |E|^2$.

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So, what is the meaning of Manley-Rowe relation? Before studying the Manley-Rowe relation; we will again go back to our original to mother equation which is evolution equation of E_2 and E_1 as shown here that inside the material if we have two waves E_1 and E_2 then E_2 and E_1 will going to evolve under this master equation which is the coupled equation.

Now, if I want to find out what is the corresponding intensity, then again we will going to use a very old expression that we have been using for last few classes. So, intensity is half epsilon 0 c n mod of E square this is the relationship between the intensity and the mod of E square term. Now, what happened what happened that we try to understand because of this energy exchange is there any quantity which are remained conserved. So, obvious question is, is the energies conserved here because E_2 is gaining some kind of energy. Initially we find that E_2 was not 0, but gradually it is getting some energy from E_1 .

So, now, in the previous class the way we have calculated E_2 was not physically correct because we consider E_1 as a constant. So, that means, E_1 is constantly feeding the energy to E_2 , but E_1 is not changing by itself. So, that means, the energy that is coming is not the total energy. So, the total if I calculate the total energy in under such condition

we find the total energy is gradually increasing because E 2 is gaining energy, but E 1 is not losing any kind of energy. So, that is why that treatment was not totally correct because that approximation is correct up to certain limit, but after that we need to put more rigorous calculation to find out exactly how E 1 and E 2 will going to evolve that we will do in the future classes.

But, today we going to learn that if I 2 and I 1 is the intensity related to E 2 and E 1, then what is the relationship between I 2 and I 1 this relationship is basically called the Manley-Rowe relation.

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Manley-Rowe Relation

$$I_2^{(2\omega)} = \frac{1}{2} \epsilon_0 c n_2 |E_2|^2$$

$$I_1^{(\omega)} = \frac{1}{2} \epsilon_0 c n_1 |E_1|^2$$

$$\frac{dI_1^{(\omega)}}{dz} = \frac{1}{2} \epsilon_0 c n_1 \frac{d|E_1|^2}{dz}$$

$$\frac{dI_1^{(\omega)}}{dz} = \frac{1}{2} \epsilon_0 c n_1 \left(E_1 \frac{dE_1^*}{dz} + E_1^* \frac{dE_1}{dz} \right) = \frac{1}{2} \epsilon_0 c n_1 \left(i \frac{\omega d}{c n_1} \right) (-E_1^2 E_2^* e^{-i\Delta k z} + E_1^{*2} E_2 e^{i\Delta k z})$$

$$\frac{\partial E_2}{\partial z} = i \frac{\omega d}{c n_2} E_1^2 e^{-i\Delta k z}$$

$$\frac{\partial E_1}{\partial z} = i \frac{\omega d}{c n_1} E_1^* E_2 e^{i\Delta k z}$$

E1E2

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Well, this is my mother equation. These two equations are always there I 2 and I 1 already I defined in the previous slide. Now, next what we are doing, we just make a derivative with respect to z over I 2. So, if I make a derivative with respect to I 2 then eventually we are making derivative of this quantity because half epsilon 0 c n 1 all are constant; so if I make a derivative in the left hand side so, derivative will only be over this mod of E 1 square because E 1 is a function of z.

Here, please consider, that I am considering please note that we are considering E 1 as a function of z previously when we calculate when we solve this differential equation we considered that E 1 is a constant which is not critically true, but here we are putting more physical we put more physical emphasis and try to find out what happened if really E 1 is changing with respect to z.

So now, obviously, if E_1 is not function of z then we find dI_1/dz is basically 0, because there is no change if of I_1 , but here we are considering that change also. So, now, what is the value of this derivative? So, E_1 is nothing but $E_1 e^{i\Delta kz}$ when it is $E_1 e^{i\Delta kz}$ when I make a derivative there are two functions. So, first function derivative of the second function, second function derivative of the first function. Straightforward derivative rules we are using and then what we will do we just put the value of E_1 from value of this derivative from this expression.

So, when we use the value of this derivative from this expression in both the cases we find that $\omega d/cn_1$ and I_1 will come and if I take it common then it should be E_1 and multiplied by this quantity, but please note that this is dE_1^*/dz .

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Manley-Rowe Relation

$$I_2^{(2\omega)} = \frac{1}{2} \epsilon_0 c n_2 |E_2|^2$$

$$I_1^{(\omega)} = \frac{1}{2} \epsilon_0 c n_1 |E_1|^2$$

$$\frac{dI_1^{(\omega)}}{dz} = \frac{1}{2} \epsilon_0 c n_1 \frac{d|E_1|^2}{dz}$$

$$\frac{dI_1^{(\omega)}}{dz} = \frac{1}{2} \epsilon_0 c n_1 \left(E_1 \frac{dE_1^*}{dz} + E_1^* \frac{dE_1}{dz} \right) = \frac{1}{2} \epsilon_0 c n_1 \left(\frac{\omega d}{c n_1} \right) \left(-E_1^2 E_2^* e^{-i\Delta kz} + E_1^{*2} E_2 e^{i\Delta kz} \right)$$

Handwritten notes in red on the slide:

- $\frac{\partial E_2}{\partial z} = i \frac{\omega d}{c n_2} E_1^2 e^{-i\Delta kz}$ with $x+iy$ and $x-iy$ written next to it.
- $\frac{\partial E_1}{\partial z} = i \frac{\omega d}{c n_1} E_1^* E_2 e^{i\Delta kz}$ with $x+iy$ and $x-iy$ written next to it.
- $\frac{\partial E_1^*}{\partial z} = -i \frac{\omega d}{c n_1} E_1 E_2^* e^{-i\Delta kz}$ with $-iy$ and $-i\Delta kz$ written next to it.

So, if I make the complex conjugate of this quantity I will have a minus i . So, let me do that dI_1^*/dz so, it is $\omega d/cn_1$. This E_1^* become E_1 and E_2 become E_2^* and then e to the power i of minus Δkz , because we are taking the complex conjugate of this differential equation when I take the complex conjugate of this differential equation we have $E_1 E_2^*$. So, here you can note that we have $E_1 E_2^*$ and one E_1 was already there. So, this that is why, we have E_1^2 .

In the similar way, the next term we just multiply with E_1 because this term we do not need to make a complex conjugate because already this term is here. So, we will eventually have $E_1^* E_2$ and e to the power $i \Delta kz$. So, these two terms if

you note very carefully is a complex conjugate of each other. So, this term and these term are complex conjugate to each other with a negative sign, with a negative sign. So, now, if I say this is x plus $i y$ the second term is x minus $i y$ and if I subtract so, we have $2iy$ term. So, this i will come here and another i sitting outside. So, if I multiply these two and one negative sign here minus $2i$. So, I will get a plus real quantity here this has to be the case because in the left hand side we have a real term. So, entire term has to be real, anyway.

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The slide contains the following equations and text:

$$\frac{dI_2^{(2\omega)}}{dz} = \frac{1}{2} \epsilon_0 c n_2 \frac{d|E_2|^2}{dz}$$

$$\frac{dI_2^{(2\omega)}}{dz} = \frac{1}{2} \epsilon_0 c n_2 \left(E_2 \frac{dE_2^*}{dz} + E_2^* \frac{dE_2}{dz} \right) = \frac{1}{2} \epsilon_0 c n_2 \left(i \frac{\omega d}{c n_2} \right) (-E_1^{*2} E_2 e^{i\Delta k z} + E_1^2 E_2^* e^{-i\Delta k z}) \quad 1$$

Handwritten note: $\frac{dE_2}{dz} = i \frac{d\omega}{c n_2} E_2 e^{-i\Delta k z}$

$$\frac{dI_1^{(\omega)}}{dz} = \frac{1}{2} \epsilon_0 c n_1 \left(E_1 \frac{dE_1^*}{dz} + E_1^* \frac{dE_1}{dz} \right) = \frac{1}{2} \epsilon_0 c n_1 \left(i \frac{\omega d}{c n_1} \right) (-E_1^2 E_2^* e^{-i\Delta k z} + E_1^{*2} E_2 e^{i\Delta k z}) \quad 2$$

$$\frac{dI_1^{(\omega)}}{dz} = - \frac{dI_2^{(2\omega)}}{dz} \quad \text{(Manley-Rowe Relation)}$$

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What should be the value of dI_2 / dz ? We calculate dI_1 / dz now we calculate dI_2 / dz . So, this is basically the procedure is exactly same. So, if I do the calculation then I will get exactly the similar kind of expression note that here, we will going to use this equation which is $i d\omega / c n_2$, then E_1 square e to the power $i \Delta k z$ that was the value of E_2 .

Now, what we will do we will just ok, there was a negative sign for E_2 . So, now, we make a complex conjugate of the entire quantity and then multiplied by E_2 when we do that complex conjugate gives this minus. So, that is why there is a minus and $d\omega / c n_2$ we can take common for both the cases with i and then this term become positive and E_1 become E_1^* square and then E_2 is multiplied. The next term is simply E_1 square E_2 e to the power $i \Delta k z$.

So, now if I put these two terms side by side, this is equation – 1 and this is equation – 2. We find that these two terms are related to each other with a negative sign. So, dI_2/dz is $\epsilon_0 c n_2$ and this n_2 , n_2 will cancel out here this n_1 , n_1 is cancel out. So, if I compare then find inside this bracket we have $E_1^* E_2$ with negative sign, but here we have $E_1^* E_2$ is a positive sign. Here also we have $E_1^2 E_2$ term, but here we have again $E_1^2 E_2^*$ term, but with a negative sign. So, over all we will have one very important expression and this expression is shown here which is a relationship between I_1 and I_2 in their differential form that dI_1/dz is equal to $-dI_2/dz$ and this is basically called the Manley-Rowe relation.

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$$\frac{dI_2^{(2\omega)}}{dz} = \frac{1}{2\epsilon_0 c n_2} \frac{d|E_2|^2}{dz}$$

$$\frac{dI_2^{(2\omega)}}{dz} = \frac{1}{2}\epsilon_0 c n_2 \left(E_2 \frac{dE_2^*}{dz} + E_2^* \frac{dE_2}{dz} \right) = \frac{1}{2}\epsilon_0 c n_2 \left(i \frac{\omega d}{c n_2} \right) \left(-E_1^{*2} E_2 e^{i\Delta k z} + E_1^2 E_2^* e^{-i\Delta k z} \right) \quad 1$$

$$\frac{dI_1^{(\omega)}}{dz} = \frac{1}{2}\epsilon_0 c n_1 \left(E_1 \frac{dE_1^*}{dz} + E_1^* \frac{dE_1}{dz} \right) = \frac{1}{2}\epsilon_0 c n_1 \left(i \frac{\omega d}{c n_1} \right) \left(-E_1^2 E_2^* e^{-i\Delta k z} + E_1^{*2} E_2 e^{i\Delta k z} \right) \quad 2$$

$\frac{d(I_1 + I_2)}{dz} = 0 \checkmark$

$$\frac{dI_1^{(\omega)}}{dz} = -\frac{dI_2^{(2\omega)}}{dz} \quad \text{(Manley-Rowe Relation)}$$

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This relation basically suggests that if I take these two terms on one side, this relation basically suggests that $I_1 + I_2$ is equal to 0. Now, $I_1 + I_2$ is a total intensity, so that means, the total intensity, the change rate of change of total intensity is 0. That means, the total intensity should be conserved under second harmonic process. So, this is the next thing, this is the next thing that we will go on to prove.

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Conservation of energy

$$U \propto I = I_1^{(\omega)} + I_2^{(2\omega)}$$

$$\frac{dU}{dz} \propto \frac{d}{dz}(I_1^{(\omega)} + I_2^{(2\omega)}) = 0$$

Handwritten notes:
 $\frac{dU}{dz} = 0$
 $U = \text{constant}$
 $\frac{dI_1}{dz} = -\frac{dI_2}{dz}$

Photon number

$N_{(\omega)} \rightarrow$ No of ω photon

$N_{(2\omega)} \rightarrow$ No of 2ω photon

$$U = N_{(\omega)}\hbar\omega + N_{(2\omega)}2\hbar\omega$$

$$\frac{dU}{dz} = 0 \Rightarrow \omega \frac{dN_{(\omega)}}{dz} = -2\omega \frac{dN_{(2\omega)}}{dz}$$

$$-\frac{1}{2} \frac{dN_{(\omega)}}{dz} = \frac{dN_{(2\omega)}}{dz}$$

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After doing the Manley-Rowe equation we can say that the total energy is conserved. So, Manley-Rowe equation is something which basically tells us the total energy is conserved. So, I_1 plus I_2 is here, I_1 plus I_2 is here. So, if I now make the total energy is proportional to I , so, rate of change of total energy is nothing but I_1 plus I_2 and then the derivative. Now, if I apply the Manley-Rowe relation that dI_1/dz is equal to minus of dI_2/dz . So, this basically gives me that this quantity will be 0, and when this quantity is 0 we have these expression which suggest that energy is conserved.

So, U which is a total energy is constant. So, very important physical outcome that during the second harmonic generation process whatever the expression we find out what is the amplitude the evolution of the amplitude of the second harmonic and the fundamental, but the equation is such that the total energy remain conserved and that should be the case because there is no external energy or absorption is there so; that means, there is no addition or subtraction of the energy, only there is a exchange of energy between the wave E_1 and E_2 and if that is the case then always we should have the total energy conserve.

Now, the next thing and the next thing is what is what about the photon numbers? The photon numbers is if I say N_{ω} is a number of photon of frequency ω and $N_{2\omega}$ is a number of photon of 2ω , then the total energy is N_{ω} total number of photon multiplied by h cross ω and $N_{2\omega}$ multiplied by $2h$ cross ω

because here the frequency is 2 omega. So, that is the total energy because for one photon we have the energy h cross omega. If N number of photon or photons are there we should have N multiplied by h cross omega to get the total energy for the photon having frequency omega.

So, this U is total energy. So, now, we try to use the relationship that we just derived by using Manley-Rowe relation that the total energy is conserved. If I do then it should be dU/dz which is equal to 0. Basically, this leads to very interesting expression that omega dN/dz is equal to 2 omega dN(2 omega)/dz. Mind it, we have a quantity like dN/dz. Now, dN/dz is basically the rate of change of photon numbers.

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Conservation of energy

$$U \propto I = I_1^{(\omega)} + I_2^{(2\omega)}$$

$$\frac{dU}{dz} \propto \frac{d}{dz}(I_1^{(\omega)} + I_2^{(2\omega)}) = 0$$

$$\frac{dU}{dz} = 0$$

Photon number

$N_{(\omega)} \rightarrow$ No of ω photon

$N_{(2\omega)} \rightarrow$ No of 2ω photon

$$U = N_{(\omega)}\hbar\omega + N_{(2\omega)}2\hbar\omega$$

$$\frac{dU}{dz} = 0 \Rightarrow \omega \frac{dN_{(\omega)}}{dz} = -2\omega \frac{dN_{(2\omega)}}{dz}$$

$$\frac{1}{2} \frac{dN_{(\omega)}}{dz} = -\frac{dN_{(2\omega)}}{dz}$$

Handwritten red annotations: E_1 and E_2 with arrows pointing to the right, and a red box around the final equation.

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Now, inside the system what happened, the second harmonic waves are generated from the first fundamental wave. So, if this is E 1 E 2 is generating. E 2 is generating means the number of photon is there which is not conserved. So, E 2 is generating means initially the number of photon may be less, but gradually it is getting some kind of energy through the photon number. So, the number of photon may not be same. So, that is why there is a evolution of number of photon also. So, finally, we have one expression which suggest minus half dN/dz is equal to dN(2 omega)/dz. That means, we are getting number of photons, the number of photon change the number of photon change for omega frequency and half of that will be equal to the number of photon gain of 2 omega frequency.

So, this is the number of photon change for omega frequency and if I make a half of that; that means, the rate of change of photon that is changing photon number that is changing for omega photon. The negatives signs is means it is reducing the rate of gain of 2 omega photon is half of that and that is true because we know that 2 omega is generated because of the fact that there are 2 omega frequencies are there.

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Photon picture of SHG

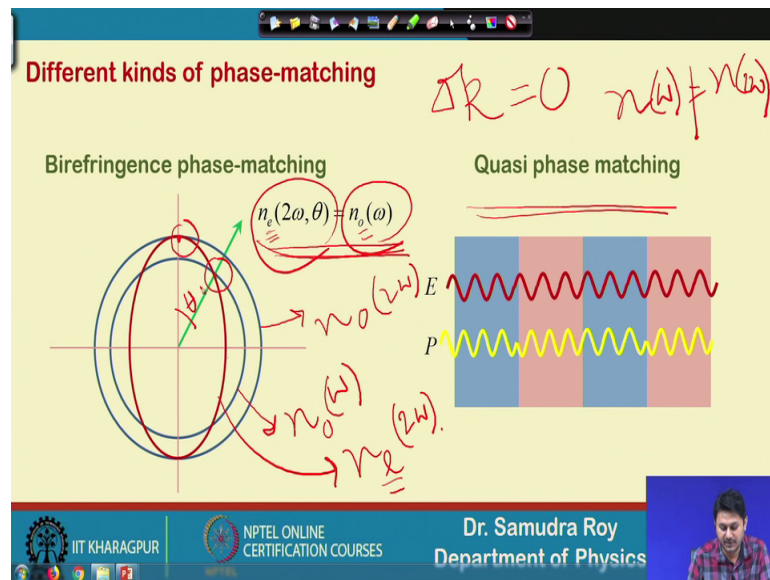
Two ω photon collapse and form a 2ω photon.

$$\frac{1}{2} \frac{dN_{(\omega)}}{dz} = \frac{dN_{(2\omega)}}{dz}$$

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If I now try to find out this picture we can say that 2 omegas are merged to generate two one 2 omega photons. So, there are two photons with omega frequency, they are collapse and they form 2 omega. When they collapse; that means, there is a change of photon number. And since there are two photon that is changing if I make a half of that then we can say that the number of photon that is generated because of the collapse of two photon is $dN_{2\omega}/dz$. So, this is the photon number in the in terms of photon number we can have the Manley-Rowe relation. So, this is a Manley-Rowe relation, but in terms of photon numbers, ok.

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After having this expression now, we will go to a very important concept or very important thing in specially in second harmonic generation and that is phase matching. So far, we mentioned that Δk equal to 0, is our phase matching condition, Δk equal to 0 is our phase matching condition. So, let me write once again Δk equal to 0 is our phase matching condition, but we never mentioned how to make Δk equal to 0. We assume that if Δk equal to 0, we have a phase matching, but now we need to understand that what happened when I mean Δk equal to 0 we know, but now we need to understand that; what is the procedure to make this Δk equal to 0.

So, one way is that this is called the birefringence phase matching. There are two phase matching system we will going to cover in this course one is birefringence phase matching and another is quasi phase matching. So, let very briefly describe what is a meaning of birefringence phase matching and what is a quasi phase matching. So, in birefringence phase matching as the figure suggest that for this phase matching if you remember our condition was $n(\omega)$ has to be equal to $n(2\omega)$, but due to the presence of dispersion these condition never valid. But, if $n(\omega)$ is not equal to $n(2\omega)$ then one thing we can do that we can equate n_e of 2ω and we can equate to n_o of ω .

So, that means, n_e means there is a possibility that we can have the refractive index there is a possibility that we have a refractive index which is equal to the refractive index

at ω , but this refractive index for extraordinary wave and this is refractive index for ordinary wave. So, these curves suggest that these values are for, this is a circle, so, that is a refractive index of ordinary wave at frequency ω . This is a refractive index of ordinary wave at frequency 2ω and if this quantity is there this is a refractive index of 2ω . But, this any, will change as a function of δ this function of θ . So, if this θ is there so, for different θ we have the refractive index different this is optics axis. So, that optics axis these two quantities are same.

But, what happened that at that particular point there is a possibility that we will have two things are same. So, this is called the birefringence phase matching. So, we will discuss in detail what is a meaning of birefringence phase matching and how we will we will going to achieve that. So, this is a one part, second part is quasi phase matching. Quasi phase matching is also very interesting. In quasi phase matching what we will do we just change the phase by rotating the crystal structure.

So, if the direction of the crystal is this if I rotate the crystal in opposite direction, so that we know that if I change the crystal compare with their axis the what happened that the their coefficient is also going to change. So, if I rotate the crystal there is a possibility the coefficient of d will going to change and that means, d now become a function of z .

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The slide is titled "Different kinds of phase-matching". It is divided into two main sections:

- Birefringence phase-matching:** This section shows a diagram of two concentric ellipses representing the refractive index surfaces for ordinary and extraordinary waves. A green arrow points to the intersection of the two ellipses, with the equation $n_e(2\omega, \theta) = n_o(\omega)$ written next to it.
- Quasi phase matching:** This section shows a diagram of a crystal structure with alternating blue and red layers. Two waves, labeled 'E' (red) and 'P' (yellow), are shown propagating through the structure. Red arrows indicate the periodic modulation of the crystal's properties. A red arrow labeled 'z' indicates the direction of propagation.

At the bottom of the slide, there is a footer with the IIT Kharagpur logo, the text "NPTEL ONLINE CERTIFICATION COURSES", the name "Dr. Samudra Roy", and the title "Department of Physics". A small video inset of Dr. Samudra Roy is visible in the bottom right corner.

So far, we are dealing with the other quantity, but considering d as a constant, but here we can see that d can also change periodically. If we change d periodically then there is a

possibility that we can if there is a phase mismatch between E and P polarization electric field we can by rotating the crystal we can fix it and when you fix it they should be equal. So, this is the concept of quasi phase matching. So, again we will learn this quasi phase matching in detail.

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Birefringence Phase-Matching (BPM)

Phase matching requirement

$$n(2\omega) = n(\omega)$$
$$\Delta k = 0 \Rightarrow k_2 - 2k_1 = 0$$
$$k_2 = 2k_1 \Rightarrow \frac{2\omega n_2}{c} = 2 \frac{\omega n_1}{c}$$
$$n_2 = n_1 \Rightarrow n(2\omega) = n(\omega)$$

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So, now, what is a birefringence phase match as I mentioned? This birefringence phase match $n(\omega)$ has to be $n(2\omega)$. And this is the phase matching condition from this phase matching condition one can easily find that my condition of phase matching has to be this, which we have already discussed in the previous class.

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Theory of dispersion

$$\tilde{\chi}^{(1)}(\omega) = \frac{Nq^2/m\epsilon_0}{(\omega_0^2 - \omega^2 - i\gamma\omega)}$$

$$\omega_p^2 = Nq^2/m\epsilon_0 \quad \checkmark$$

$$\Delta = (\omega_0^2 - \omega^2) \quad \checkmark$$

$$\tilde{\chi}^{(1)}(\omega) = \frac{\omega_p^2}{(\Delta - i\gamma\omega)} = \frac{\omega_p^2(\Delta + i\gamma\omega)}{(\Delta^2 + \gamma^2\omega^2)} = \tilde{\chi}_R^{(1)}(\omega) + i\tilde{\chi}_I^{(1)}(\omega)$$

$$\tilde{\chi}_R^{(1)}(\omega) = \frac{\omega_p^2\Delta}{\Delta^2 + \gamma^2\omega^2}$$

$$\tilde{\chi}_I^{(1)}(\omega) = \frac{\omega_p^2\gamma\omega}{\Delta^2 + \gamma^2\omega^2}$$

$$n^2(\omega) = \tilde{\epsilon}(\omega) = 1 + \tilde{\chi}^{(1)}(\omega) = 1 + \frac{\tilde{\chi}_R^{(1)}(\omega)}{1} + i\tilde{\chi}_I^{(1)}(\omega)$$

Handwritten notes: $n(\omega)$, $= n(\omega)$, $n(\omega)$

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So, now the condition we need to find that $n(\omega)$ is equal to $n^2(\omega)$. Now, $n(\omega)$ is equal to $n^2(\omega)$ is it never possible and why it is never possible let us try to understand through our dispersion theory. So, we know that this susceptibility can be represented as a function of frequency in this way and then we can write this susceptibility, if this is a plasma frequency. And if I put this is Δ this is the thing we have already done so, I am not going to very detail about that. So, I believe all of you understand. This susceptibility, first order susceptibility will be a function and I can write it as a real plus imaginary and the real part is this and imaginary which basically gives you the absorption is this one.

So, now, refractive index is ϵ_0 and 1 plus this. So, if I expand these things then it should be real plus imaginary and we know that 1 plus this quantity basically gives us the refractive index, the real part is basically refractive index gives us the refractive index. This quantity this susceptibility is function of ω . So, refractive index has to be function of ω . So, once we have 1 plus this and now the important thing is that the explicit form is given here, how this quantity will vary as a function of ω is given here. So, we have this functional form and if I try to find out what is my refractive index, then we just we just plot it.

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Refractive Index

$$n^2(\omega) = 1 + \tilde{\chi}_R^{(1)}(\omega) = 1 + \frac{\omega_p^2}{(\omega_0^2 - \omega^2)}$$

Due to dispersion effect

$$n(2\omega) \neq n(\omega)$$

n(ω) Dispersion

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And, when we plot it we find quite interesting thing and that is this. So, as I mention my refractive index n square become function like this and this is the condition let me go back, so that you can understand what I am trying to say.

(Refer Slide Time: 24:05)

Theory of dispersion

$$\tilde{\chi}^{(1)}(\omega) = \frac{Nq^2/m\epsilon_0}{(\omega_0^2 - \omega^2 - i\gamma\omega)}$$

$$\omega_p^2 = Nq^2/m\epsilon_0$$

$$\Delta = (\omega_0^2 - \omega^2)$$

$$\tilde{\chi}^{(1)}(\omega) = \frac{\omega_p^2}{(\Delta - i\gamma\omega)} = \frac{\omega_p^2(\Delta + i\gamma\omega)}{(\Delta^2 + \gamma^2\omega^2)} = \tilde{\chi}_R^{(1)}(\omega) + i\tilde{\chi}_I^{(1)}(\omega)$$

$$\tilde{\chi}_R^{(1)}(\omega) = \frac{\omega_p^2\Delta}{(\Delta^2 + \gamma^2\omega^2)}$$

$$\tilde{\chi}_I^{(1)}(\omega) = \frac{\omega_p^2\gamma\omega}{(\Delta^2 + \gamma^2\omega^2)}$$

$$n^2(\omega) = \tilde{\epsilon}(\omega) = 1 + \tilde{\chi}^{(1)}(\omega) = 1 + \tilde{\chi}_R^{(1)}(\omega) + i\tilde{\chi}_I^{(1)}(\omega)$$

Handwritten notes:
 $\chi_R(\omega) = \frac{\omega_p^2}{\Delta^2 + \gamma^2\omega^2}$
 $\chi_I(\omega) = \frac{\omega_p^2\gamma\omega}{\Delta^2 + \gamma^2\omega^2}$
 $\omega_0^2 - \omega^2$

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So, this delta so, χ_R it is delta square plus gamma square omega square, delta is a omega 0 square minus omega square. If we consider my damping is very small, if we consider my damping is very small then this equation I can approximate as this. I can neglect this term if I neglect this term we will have omega p square divided by delta,

delta where this things. So, omega p square divided by omega 0 square minus omega square will be the functional form of xi R which is the real part of this.

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Theory of dispersion

$$\tilde{\chi}^{(1)}(\omega) = \frac{Nq^2/m\epsilon_0}{(\omega_0^2 - \omega^2 - i\gamma\omega)}$$

$$\omega_p^2 = Nq^2/m\epsilon_0$$

$$\Delta = (\omega_0^2 - \omega^2)$$

$$\tilde{\chi}^{(1)}(\omega) = \frac{\omega_p^2}{(\Delta - i\gamma\omega)} = \frac{\omega_p^2(\Delta + i\gamma\omega)}{(\Delta^2 + \gamma^2\omega^2)} = \tilde{\chi}_R^{(1)}(\omega) + i\tilde{\chi}_I^{(1)}(\omega)$$

$$\tilde{\chi}_R^{(1)}(\omega) = \frac{\omega_p^2\Delta}{(\Delta^2 + \gamma^2\omega^2)}$$

$$\tilde{\chi}_I^{(1)}(\omega) = \frac{\omega_p^2\gamma\omega}{(\Delta^2 + \gamma^2\omega^2)}$$

$$n^2(\omega) = \tilde{\epsilon}(\omega) = 1 + \tilde{\chi}^{(1)}(\omega) = 1 + \tilde{\chi}_R^{(1)}(\omega) + i\tilde{\chi}_I^{(1)}(\omega)$$

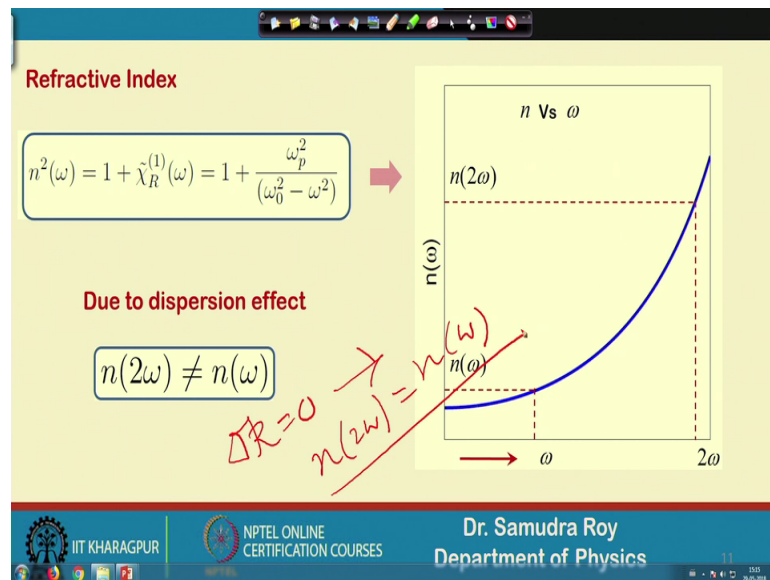
Handwritten note: $\chi_R = \frac{\omega_p^2}{(\omega_0^2 - \omega^2)}$

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So, that means, xi R will be omega p square divided by omega 0 square minus omega square. So, when I put this xi R in to my equation; equation of refractive index then what we find let us see. Well, as I mentioned I just replace this omega R to omega R to this quantity when I put this quantity omega p square divided by omega 0 square minus omega square, then we have a functional form. Now, if I put this functional form here. So, what happen that we will have a curve like this? This is called the dispersion curve, this is called the dispersion curve, n is a function of omega is called the dispersion.

So, this dispersion suggest that if I increase omega, then if omega is here then omega then n 2 omega will be somewhere here. So, n of 2 omega will be always greater than n of omega at least is in this normal domain, this is called the normal dispersion. In normal dispersion what happened n of 2 omega. That means, n at higher frequency always be greater than n of the lower frequency. So, now, the dispersion curve readily suggests that this condition is not valid. If this condition is not valid the next important issue that what to do in order to get the phase matching.

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Because, my phase matching suggest that my n has to be equal to n of 2ω . My phase matching condition basically Δk equal to 0, suggest that I should have n of 2ω will be equal to n of ω . Now, if a material has dispersion and if these dispersion curve look like this which suppose to be the case then we will never get the phase matching condition. So, if we get then if we never get the phase matching condition then the only situation we can have to use some kind of crystal where we have two different kind of refractive index; one is ordinary refractive index and another is extra ordinary refractive index, and maybe there is a possibility that ordinary refractive index and extra ordinary refractive index may match at certain angle where we can have our phase matching condition.

So, we will in the next class we will be going to learn what should be the crystal and how we will get this phase matching for this kind of crystal and detail calculation. So, with this note let me conclude my class here. So, see you in the next class.

Thank you.