Introduction to Non-Linear Optics and its Applications
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Lecture – 19
Phase Matching of SHG, Gain Band Width Calculation

So, welcome student, to the class of Introduction to Non-Linear Optics and its Applications. So, this is lecture number – 19.

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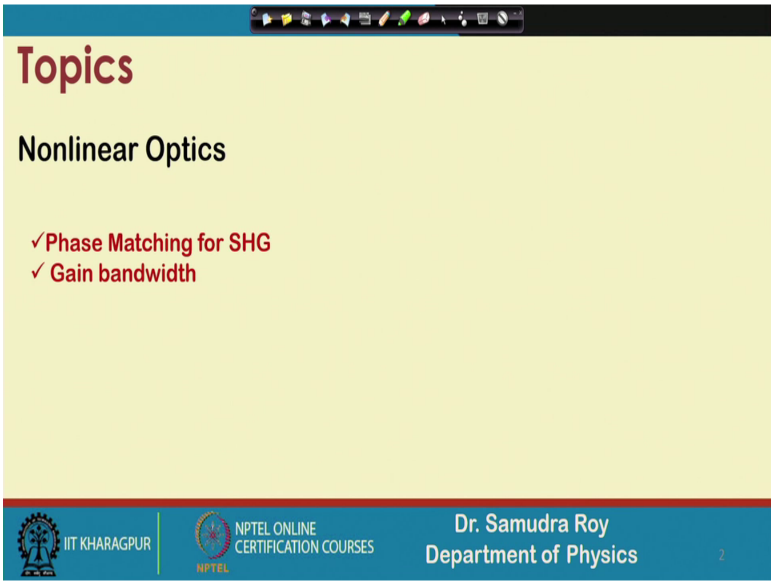

In the previous lecture, so, let us see what we have in this lecture. In the previous lecture, we learn about the second harmonic generation. The mathematics of the second harmonic generation we derived and now, today we will going to learn about the phase matching and then the gain band width of the efficiency curve. So, efficiency is something which also we discussed in the lass class.

So, let us find, what is the meaning of phase matching in second harmonic.

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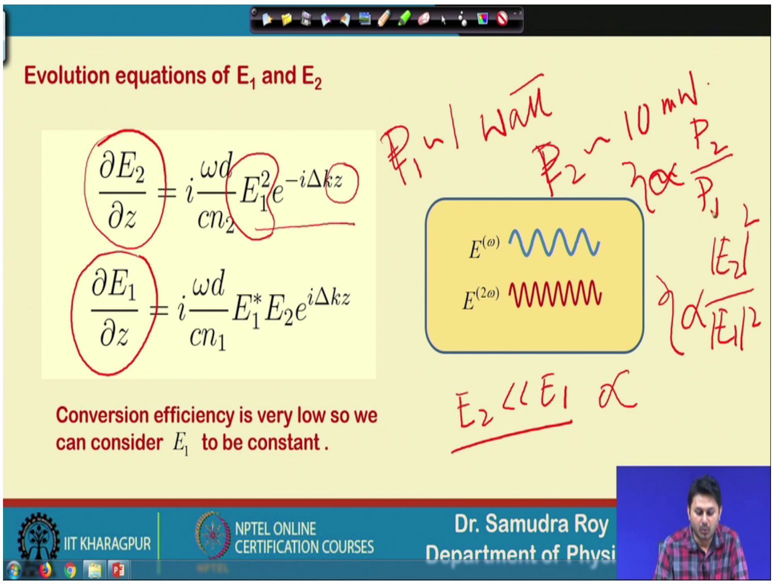

Well, this is our expression that we have derived in the last class. So, once again let me remind you what is the meaning of this. So, this is the evolution of this term is the evolution of the second harmonic and then this term is evolution of the fundamental wave. The evolution of the second harmonic wave which is defined by E 2 is function of z, obviously, that is why the derivative with respect to z in the left hand side gives us something. Here, the function z is appearing we can see clearly, but also E 1 suppose to have function of z.

But, if we say that the efficiency is very low; that means, if I launch a fundamental wave of the order of say 1 watt and we are able to generate. So, this is E 1 of the order of 1 watts, say. Now, if we generate the amount of second harmonic if we generate this of the order of say 10 milli watt, then E 2 is very very less than E 1 and efficiency is efficiency eta is proportional to mod of E 2 square divided by E 1 square which is essentially the ratio of the power.

Now, this quantity should be very small. So, here E 1, I should write not E 1 because it is a unit of watt. So, better to write it is P 1 and P 2. So, P 1 and P 2 is of the order of 1 watt and 10 milli watt then the efficiency which is proportional to this quantity or efficiency which is of the order of which is proportional to say P 2 divided by P 1 will be very very small. When the efficiency is very small then we can consider this E 1 as a constant because E 1 not going to change much.

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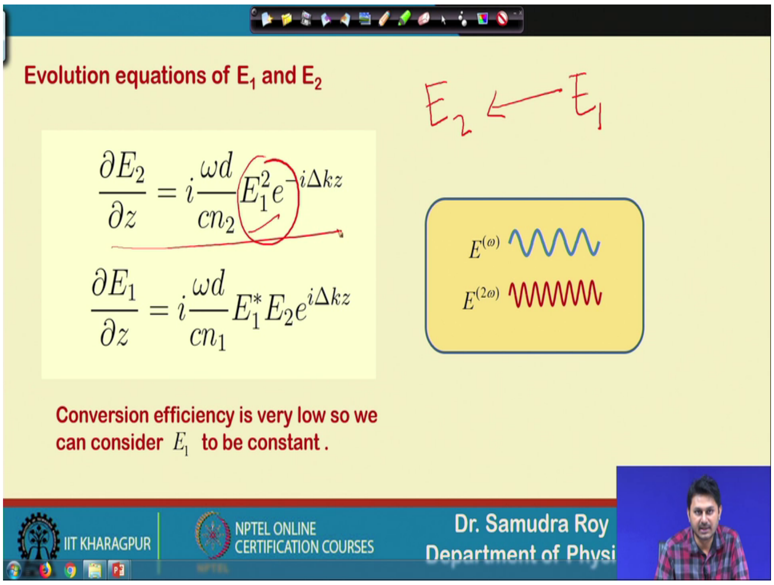

Because, the power is coming from E 2 is coming from E one. So, E 1 basically generate gives the power to E 2 and that is why in the source term E 1 is sitting. So, very little amount of power can be transformed from E 1 to E 2. So, we can consider that E 1 is constant. Under such condition we can able to find out what is the E 2 value; that means, directly integrating this equation. When we directly integrate this equation we get something. So, let us find out what we are getting.

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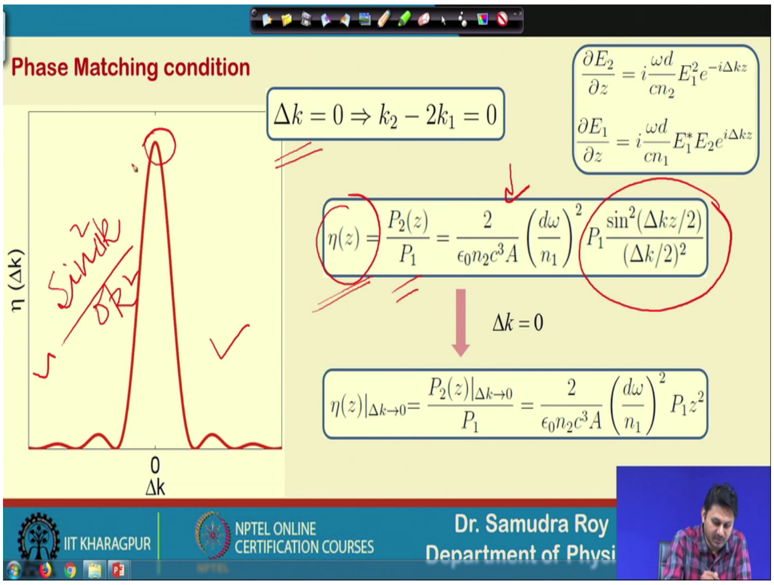

Yeah, so, after integrating we get the value of E 2 which is proportional to z square that we have shown in the last class and then we figure what is the value of the efficiency. So, this is basically efficiency defined by eta which is the ratio of P 2 by P 1. I just mentioned in the previous slide which have some value related to this is a quantity this is a constant quantity, which we should not bother about that, but this is the quantity which is very important because the efficiency is entirely depend on that.

Now, what happened if I want to improve this efficiency? We know that if the value of efficiency itself is very small, but still it is a possibility that we can improve this value. So, in order to improve the value of this efficiency eta it is better that we can we plot this as a function of delta k. So, delta k is what delta k is the phase mismatch. So, if I plot this entire quantity for a fixed z, if I plot this entire quantity efficiency as a function of delta k then we will get this kind of function which is nothing, but the sinc function. So, it is order of say sin square delta k divided by delta k square. So, it is something like this. Here, in state of delta k we are plotting delta k z by 2 and delta k by 2.

So, now if we plot this things we can see very clearly that we have a maxima here. So, this point the function reaches to a maxima and the other points you can see that there is the efficiency is very less value. Say in order to have the maxima, what is the condition if I see here delta k is 0 is the condition for which we are getting maxima. So, when we get the maxima at delta k equal to 0. So, this is the condition and this is the value of the efficiency and this efficiency we can see is now proportional to z square, that we have mentioned, ok. This portion we understood because this is the old thing that we are going to do.

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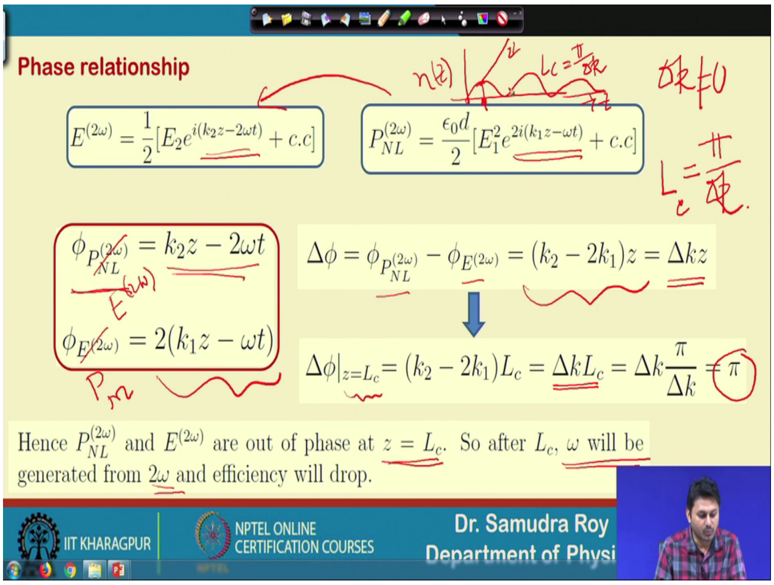

Now, the question is what is the phase relationship between so, in the previous curve we see that electric field is launched here and second harmonics is generating here. So, second harmonic basically is generating because P non-linear 2 omega is generating.

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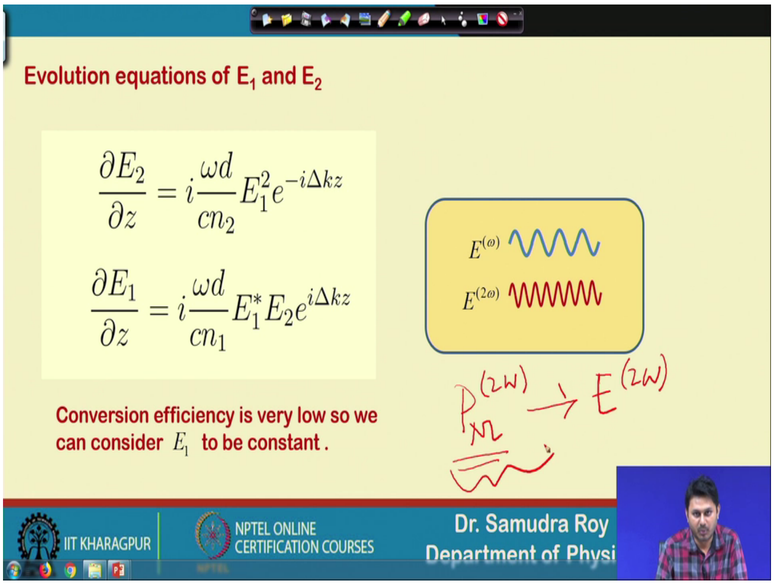

So, this is the reason why E 2 is generating. So, P non-linear 2 omega is a quantity which is important because this is the source term for second harmonic. So, now, the next thing is to understand what is the phase relationship between E 2, which is generated and the corresponding source term that is constantly feeding the energy to E 2; that means, P non-linear is a non-linear polarization term that is vibrating with a frequency 2 omega and because of that we are getting the frequency the field with frequency 2 omega, but one thing you should note that this is also a travelling wave because of this term.

So, both are travelling wave and we find in the next previous class that if the velocity of these two travelling wave was same, then we will get the efficiency maximum. So, the conversion of energy become maximum and the reason is that. So, though these two waves are propagating together and the velocity phase velocity is matching and if their phase velocity is matching that eventually leads to the phase matching condition tilde k equal to 0.

So, here we will try to find out what is the corresponding phase. So, the phase of this quantity is written as phase P non-linear 2 omega, this is a phase of non-linear polarization term which is nothing, but k 2 this is written wrongly. So, this phase should be this phase. So, this quantity should be E 2 omega. So, this is the phase of 2 omega frequency. So, 2 omega frequency basically give rise to this phase k 2 z 2 omega and the phase of this non-linear polarization the suffix is now reversed here somehow. So, please note that then the phase is 2 k i z minus omega t.

So, now if I want to find out what is the difference between this two phase we will get an interesting result. So, difference between these two phase is nothing, but this minus is this. So, if you do the difference then we will have k 2 minus 2 k 1 z which is delta k into z. So, delta k into z is the phase difference between this two.

So, now if we try to find out what is the phase difference at the coherence length; we know what is the coherence length, the coherence length is the length at which we have the efficiency maxima when we plot this things under delta k naught equal to 0 condition and then L c if I put it here in place of z we will have delta k multiplied by L c.

Now, we know from the previous lecture that L c was pi divided by delta k. So, at this point at this point we will have the efficiency maximum when delta k not equal to 0, this curve you should remember. So, it was something like this and then one efficiency is going. So, it is efficiency as a function of z. So, this was proportional to z square, but this is a sinc function and we are plotting as a function of z here. So, this point here we have L c which was pi divided by delta k, that was the figure we had in the previous class.

So, now, if I want to find out exactly this point what is the phase difference between this two we find that the phase difference is exactly pi. So, pi phase difference is there at this point where the efficiency is maximum delta k equal to 0. Now, after that what happened, if I go beyond that then we find that the efficiency is now dropping down. So, that means, P NL and E are out of phase exactly at the point z equal to L c. So, after L c what happened omega will not going to generate 2 omega rather omega will be generated from 2 omega. So, that is why the efficiency will go down.

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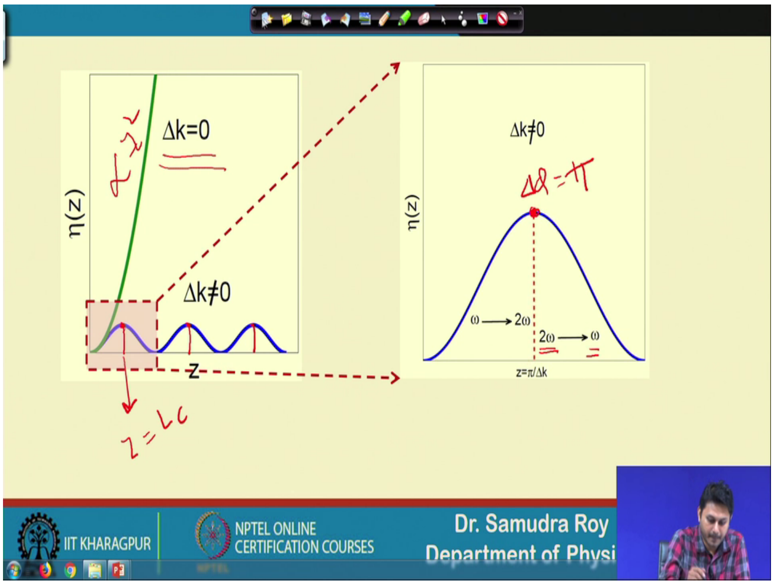

So, let us go to the next slide to understand the fact this is the figure and this figure is basically gives you the entire scenario what is going on. So, here the efficiency versus z curve and this is basically the condition this curve is proportional to z square and it is happening when delta k is equal to 0 that we have already figured out. But, this is the more realistic case when delta k not equal to 0, the efficiency will be not that much, but we still have some value where efficiency reaches to a maxima; for example, these are the points.

Now, these points here when efficiency is maxima it is called the coherence length. So, at this particular coherence length what happened, the efficiency reaches to a maxima. So, from this figure if I now expand this portion only then we find that x this goes to a maxima at this particular point and then what happened it is go down.

So, here the exactly phase mismatch delta phi was pi. So, the phase of the propagating non-linear polarization term and the phase of the electric field having frequency 2 omega component is now entirely out of phase and as a result what happened? The efficiency go down because now, 2 omega is generating from omega is generating from 2 omega; exactly the opposite phenomena is happening because their phase is now reverse, that is why the efficiency is going down, ok.

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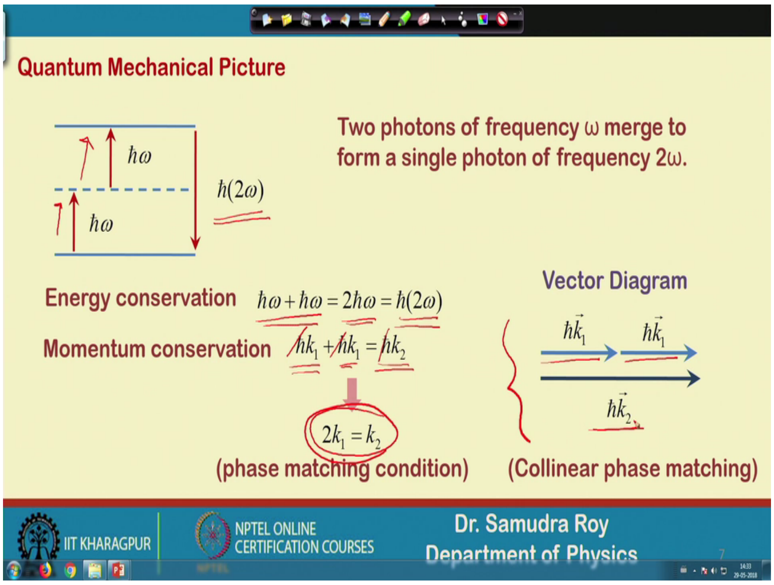

After that what we have, the quantum mechanical picture of the entire system which is important. So, we know that in second harmonic generation omega frequency from omega frequency we are getting 2 omega frequency. So, that means, from energy conservation we can have that two photon, this is a photon picture, two photon can merge to generate one photon of frequency omega. So, 2 photon of frequency omega and omega where the energy is h cross omega and h cross omega, if I add this two we will have 2h cross omega. So, that means, two photons are merging together to generate a photon of 2 omega frequency.

So, this is the quantum mechanical picture, where we can see that one photon and another photon is merged to generate another 2 omega photons. So, this is the photon with frequency 2 omega. In the similar way if I try to find out the momentum conservation then we can say that the momentum of two photon which is h cut h cross k 1 plus h cross k 1, where k 1 is the propagation conserve propagation constant of the photon having frequency omega and then the momentum conservation suggest that if this two are added then we are getting another momentum h cross k 2, which is the momentum of the wave that is having frequency component 2 omega.

Now, if I simplify this or if I just cut this h altogether h cross then whatever the expression we have is nothing, but the phase matching equation. So, the phase matching condition is nothing, but the conservation of the momentum. So, when the phase matching condition is there we can say the momentum is conserved and when the momentum is conserved or under that condition we have greater efficiency.

So, in vector diagram we can write this things in this way. So, one photon with h cross momentum and another photon which h cross k one momentum, they add up to generate another photon which having the propagation constant k 2. This is the collinear kind of phase matching because all this vectors are in similar direction. But, in the future class we will find that there are non collinear phase match is still possible where more than two waves are associated to generate a third wave. So, three wave mixing process are there. Then we will have a different condition of this phase matching in terms of this vector picture or this vector diagram well.

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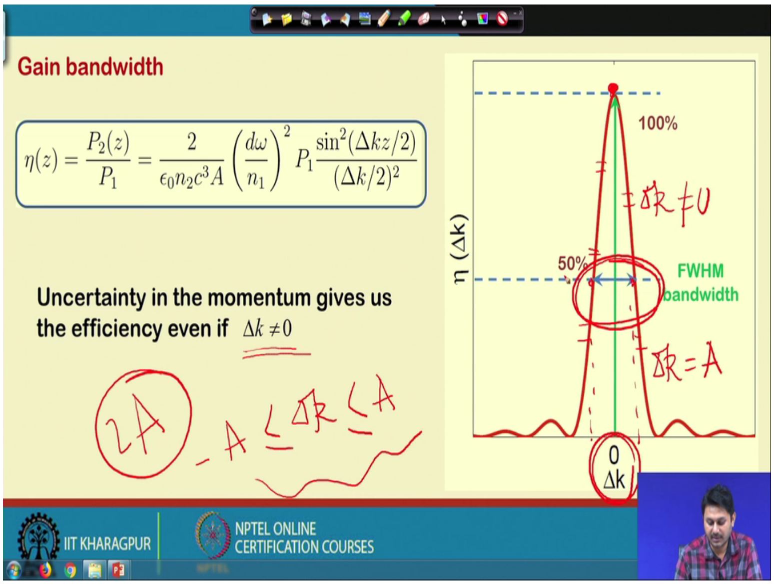

After having the knowledge of all this things, now it is time to understand what is the meaning of this gain bandwidth. So, what so far we find that efficiency reaches to a maxima when delta k equal to 0, that is the thing we have already find.

So, this is the point where efficiency reaches to a maxima under the condition that delta k equal to 0. So, delta k equal to 0 or in absolute phase matching condition we have a maxima. Now, the question is if I plot this things as a function of delta k we can see that even if delta k is not equal to 0, for example, this point, this point, this point, this point or this point, where delta k is not equal to 0 in all the cases delta k is not equal to 0, we still have some amount of efficiency.

So, delta k not equal to 0, still give some amount of efficiency. Obviously, the efficiency is going down very sharply both the sides of delta k around delta k equal to 0, but the thing is that delta k equal to 0 is not the only condition to have the efficiency. Efficiency still is there when delta k not equal to 0 and the reason behind that the un uncertainty in the momentum basically gives the efficiency even delta k not equal to 0, because there is quantum mechanics there is a uncertainty.

So, delta k equal to 0 condition is not the prime condition to have a second harmonic you can still have second harmonic when delta k not equal to 0. So, that means, there is a band width of this efficiency curve. So, we have a certain amount of the range of delta k. So, if I say delta k is less than equal to some value A and minus A so, in between this two values which is 2A. So, in between this two values if I say this is this quantity where we have the 50 percent efficiency, I have delta k equal to say some value called A.

So, now the question is how do we find out this bandwidth and why this bandwidth is generated as I mentioned quantum mechanically if delta k not equal to 0, then still we have some kind of efficiency, but classically also we find out the functional form in such that even delta k not equal to 0, give us some kind of efficiency. So, now, we measure this bandwidth, now width can be measure in a different way. So, we consider full with half maxima.

So, what is a full with half maxima? So, we have a bell kind of curve, this is a bell shaped some sort of bell shaped curve and here if this is the maxima then the half of the maxima is around this point. So, half of the maxima we have, now try to find out what is a width of that point that particular point. So, that is why it is called full width at half maxima. This is the maxima that means, the 50 percent efficiency and try to find out what is the width of this thing. So, we define fully the half maxima in such a way that we have some kind of width of delta k.

So, delta k is a width here in this point we have some amount of width for which we are getting the efficiency and this efficiency is around 50 percent of the main peak efficiency.

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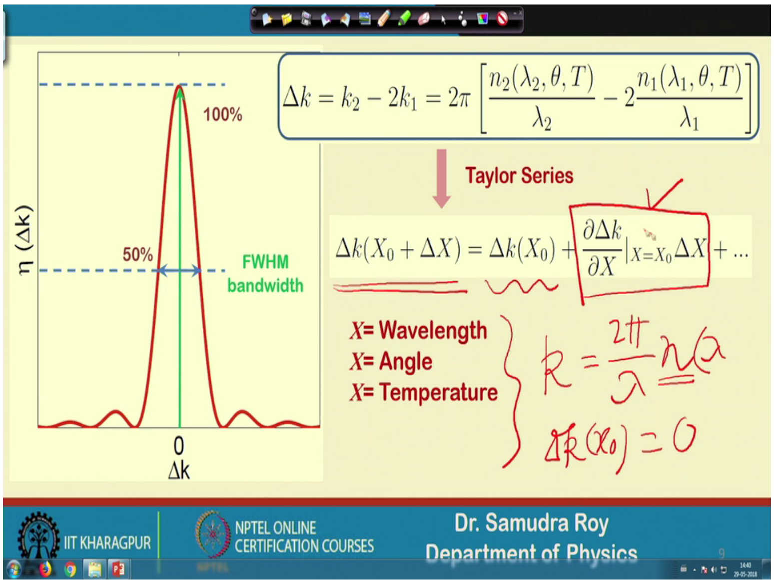

Well, which of the parameters on which this things is depend on? That is a important quantity, that is a important thing to understand. So, let us first see what is delta k? Delta k is k 2 minus k 1; k 2 is the propagation constant. If I write this propagation constant in terms of refractive index then it should be simply. So, k is 2 pi by lambda into refractive index, in free space n is 1. So, we have k equal to 2 pi by lambda, but if the space has some kind of refractive index if the medium has some kind of refractive index then we can write k equal to 2 pi by lambda into n.

Now, this refractive index n may be function of many things. First of all it is a function of lambda no doubt about that. So, whatever the lambda I am launching, so that means, I have to launch very precise lambda. So, lambda 2 and lambda 1 if I say these are the fixed quantity still n 2 can be varied with temperature and the angle at which I am launching. So, theta here is a angle and temperature which is a very popular parameter to change the refractive in the specially in the crystal.

So, these are the parameters which are depending I mean which basically because on I mean this parameter basically n 2 is depending on. So, now, if I want to change want to find out what is the variation based on this parameter we can say delta k will vary when this one of this parameter will vary. So, this x can be any parameter, may be it is a wavelength, may be it is a angle or temperature.

So, now delta k will going to vary with a small change of delta x, then what happened? If I now make a Taylor series we call this is a small amount delta x is a small amount if I make a Taylor series we have k X 0; X 0 is exact value for which the phase matching is there. So, delta k X 0 we can safely say this is a 0, because this is the absolute phase matching condition. All the condition is valid here so, it is a absolute phase matching condition or x is 0 is a value for which the phase matching is there.

But, if we have some kind of deviation from x 0 then we have a next higher order term which is this delta k X d delta k d X at X equal to 0, multiplied by delta X; that means, this is the term which basically the measurement of the amount of deviation of the delta k. So, how to find this term so, we need to know how the delta k is changing as a function of X? X may be wavelength angle or temperature as I say. So, how this things is changing and when this things are changing this rate and if I multiplied with delta X, we can find the total amount of change delta k because of the change of delta X.

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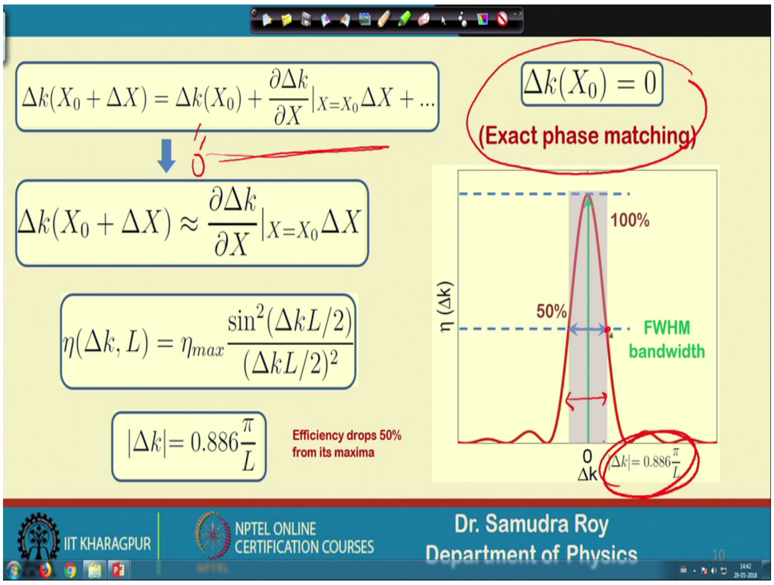

So, next we will going to find that what is basically the value. So, this is the amount of value we have for delta k expression the value of delta k as a function of X 0 plus delta X by expanding this as a Taylor series. As I mentioned for exact phase matching condition delta k X 0 has to be 0, see if I put this term equal to 0, then the rest term is something like this.

Now, we know there is a bandwidth of the efficiency. So, I can figure out what is my delta k in this shaded region. If I calculate we find that my delta k is this value. At this value it is not equal to 0, mind it this delta k is not equal to 0, but this delta k is 0.886 for sinc function this is the value you can, where the efficiency drops 50 percent from its maxima. So, if I change delta k from 0 to some value this value this efficiency reach to that point, in the negative side also it will reach to this point. So, that means, my delta k here is some value where my width is something.

So, in order to find the width I need to calculate, what is this delta k. So, delta k is this value. So, what we will do? We now have the delta k in our hand. So, delta k is the amount that is changing. So, I can put this delta k here. So, let us go to the next slide may be that will be easier for understand.

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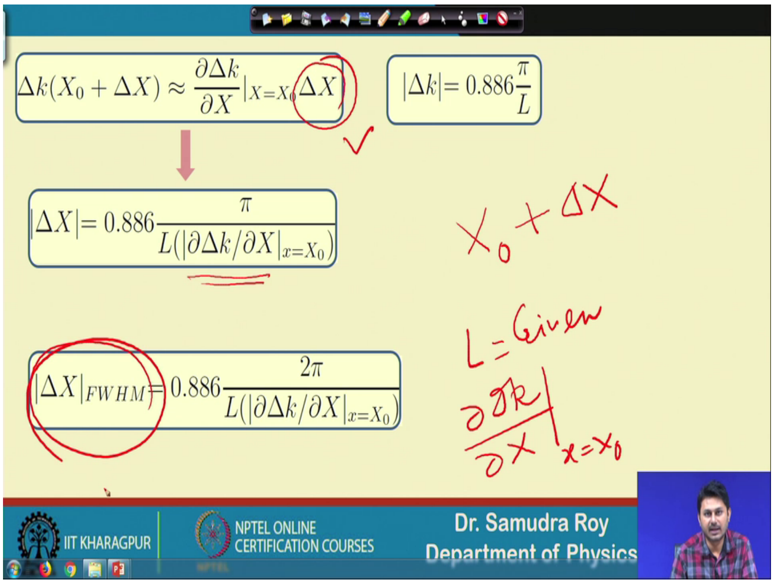

Well, this is the slide we have. So, this is the quantity I want to find out this is basically the tolerance that up to which delta X, how far the parameter X can be change. So, X 0 is the exact condition for phase matching. Now, we change this parameter to delta X amount. Now, the question is how far we can extend this delta X. So, if I now calculate from this. So, delta X has to be delta k is this amount. If I put delta k equal to 0.006 pi by L then 0.006 pi by L divided by this quantity basically gives you my delta X.

So, delta X tolerance or delta X 4 with half maxima is this amount. Now, if L value is given delta k, the rate of change of delta k with respect to the given parameter X at X equal to X 0 point is given if this two terms are given, then we can readily find out what is the tolerance limit of delta X. Delta X is the value which can which can change the amount and now, we can find exactly if this two values are given that what should be the tolerance of delta X parameter which may be angle, which may be temperature and which may be wavelength.

So, with this note so, let me conclude this class. So, in the next class we will extend this concept and try to find out more on second harmonic generation and how to increase the efficiency and all this issues. So, with that thank you very much and see you in the next class.