Introduction to Non-Linear Optics and its Applications
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Lecture – 18
Phase Matching

So, welcome student, to the next class of Introduction to Non-Linear Optics and its Application. In the previous class, we learned from the non-linear Maxwell's equation how one can derive the equation for the second harmonic and fundamental wave and how the amplitude of the second harmonic will going to evolve we figure out from the non-linear Maxwell's equation. So, today we will going to use this two coupled equation that we have derived in the last class and try to solve it.

So, solution is important, but we need to find out whether we can solve these things properly or not.

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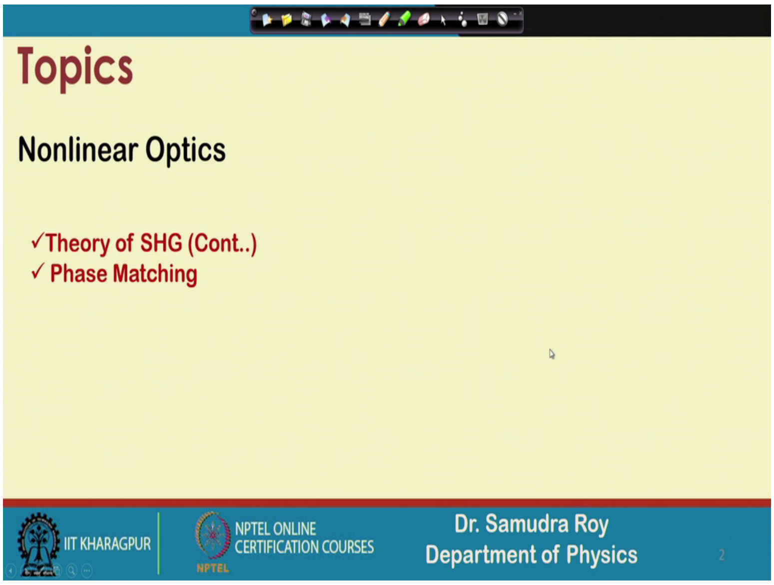

So, today we have the topic and the topic is theory of second harmonic generation which we have already started in the previous class that we will going to continue and very important thing which is phase matching. What is a phase matching and what is implication of second harmonic generation we will going to learn in this class. So far, all the treatment is classical. So, I believe it will be easier for you to understand. One can also calculate this things the susceptibility and other calculation in quantum mechanical way for this particular course we basically do not use any kind of quantum approach.

So, classical approach will be our main emphasis and I believe it will be useful for you also because you need to understand the non-linear the physics of this non-linear optics and classical treatment is easier for you to understand so, ok. So, let us try to find out the theory of non-linear this second harmonic generation and a non-linear optics and how this theory basically developed we already we have shown you in the previous class. So, today we will going to solve so, that two equations. So, let us go back to our previous slides.

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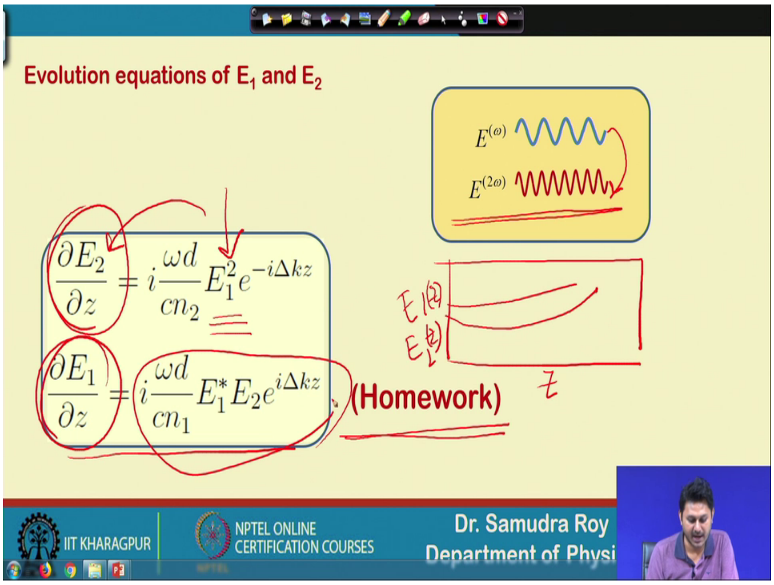

So, this slides already we have shown in our previous class that eventually we have two coupled equation and this two coupled equation basically suggest how E 1 and E 2 will going to evolve. One interesting thing if you look which is quite obvious, but still I will going to emphasis you to understand that what is going on. So, this is the evolution of the second harmonic. So, E 2 is corresponds to the field, which is containing the electric the frequency 2 omega.

For example, this is the wave that is inside the material. Now, in the right hand side we have a source term. So, basically the source terms suggest in order to excite the second harmonic, you have some sort of source and this source is nothing, but the fundamental wave. So, fundamental wed wave basically pump, it is using as a pump that basically give raise to the wave which is having a frequency 2 omega. So, from here we have the energy, so that I will have some kind of electric field in 2 omega wave.

Also, we have another expression in our hand where we can see that we have the evolution of E 1. That means, if this is my system E 1 will also going to evolve some way and E 2 is also going to evolve some way, we to need to find out. So, this is over Z this is E 1 or E 2, but from this coupled equation is very difficult to say because we need to solve this carefully that how this E 1 and E 2 both or function of Z Z will going to evolve, but it is for sure that in order to evaluate E 2 we need to make some kind of approximation.

And, in the next slide we will show what kind of approximation I am talking about, but here it is important to note that E 2 will going to generate in presence of E 1; that means, E 1 is a basically the same sort of driving force that generate E 2. Also, in the previous class it was a homework, I believe all of you have done this homework and you have this expression in your hand. So, with this confidence, let us go to the next slide, ok.

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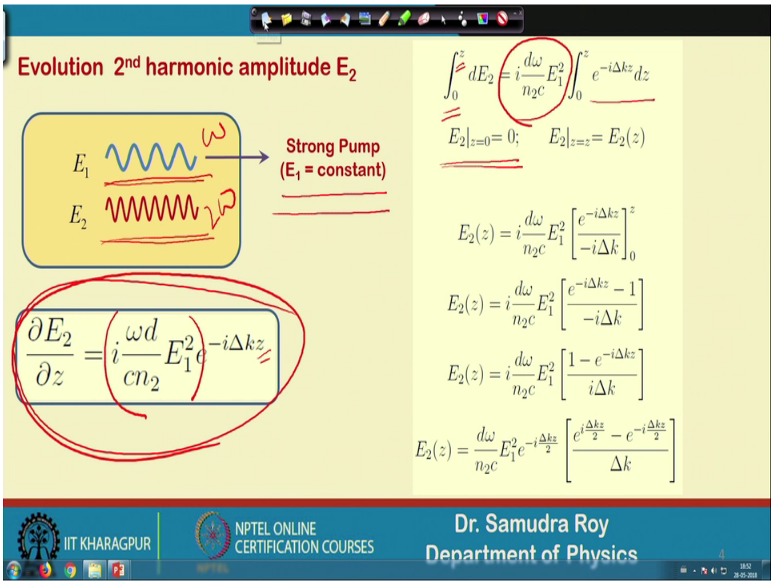

So, our goal here is as I mentioned our goal here is to find out the solution of this coupled equation. So, physically what is going on let us understand. We have a E 1 wave this is a frequency of E 1 and this is a frequency of E 2, which is twice of the omega, if the frequency is omega this is 2 omega and we say that E 1 is a strong pump; that means, we are launching some kind of strong E 1 and try to find out whether because of the launching of this strong pump E 2 will going to evolve or not and this is the equation which basically guide us how it will going to evolve.

Now, since E 1 is a strong pump I can consider E 1 as a constant. So, that is a very important approximation here. So, this is not a very physical approximation for the time being, but this is not a physical approximation, but for the time being we can consider that E 1 is a constant. For the strong pump we can consider E 1 is a constant and try to find out what should be the value of E 2 or what is the functional form of E 2.

So, now this differential equation is solvable, because we have E 2 dz and the entire term is now constant. If this entire term is constant we can take it out from the integration. So, it is basically taken out. So, in the integration we have only the exponential term which contain this Z and integrate this both the side with respect to z, where we will do that for 0 to z. So, we integrate this differential equation. When we integrate this differential equation we need to also consider about the boundary condition and the boundary condition is that, there is no E 2 at the beginning that is a quite justified approximation.

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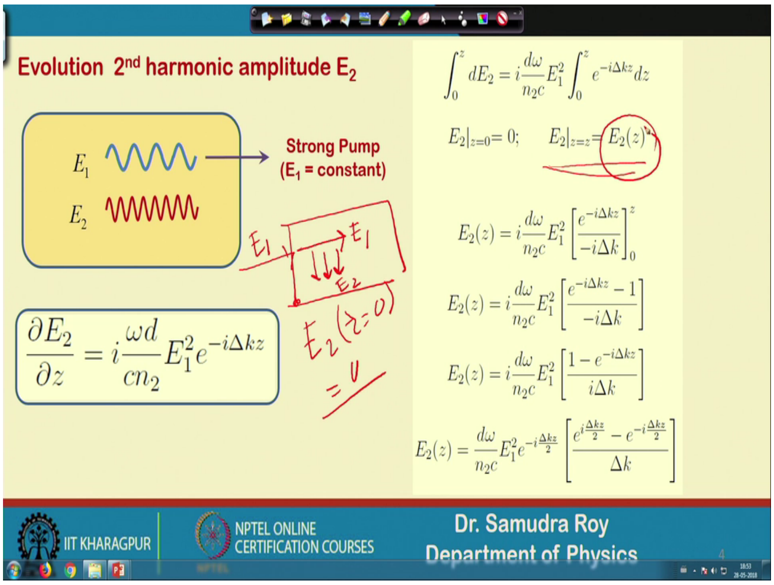

Because, if this is my system and if I launch a strong pump E 1 here so, E 2 will 0 here, there will be no E 2. Once the pulse is moving the fundamental field is moving what happened that it will basically feed the system, so that E 2 will going to generate gradually. So, the boundary condition is E 2 at Z equal to 0 is 0. So, there is no E 2 at Z equal to 0 is 0. On the other hand E 2 are Z equal to Z is E Z and this E Z basically we try to find out what value of E 2 one can have at some Z point, ok.

So, now the integration part we know this integration and this is a very straightforward integration. So, this will be E 2 and this is a constant term as I mentioned earlier it will come outside and this integration is e to the power ikz. E to the power ikz and ikz will come minus ikz will come here in the denominator with limit 0 to 1. Once I put this limit 0 to 1, what we have is d i d omega into c multiplied by E 1 square and this boundary condition if I put the first term will be e to the power i delta kz minus 1 and the denominator will be as usual minus i delta k.

We further simplify that and we will get something like if I absorb this minus sign so, it will be 1 minus of this. So, I will have 1 minus of e to the power minus of i delta kz, so that we can absorb this minus sign. So, this minus sign it was here, it was not here, so, it is plus.

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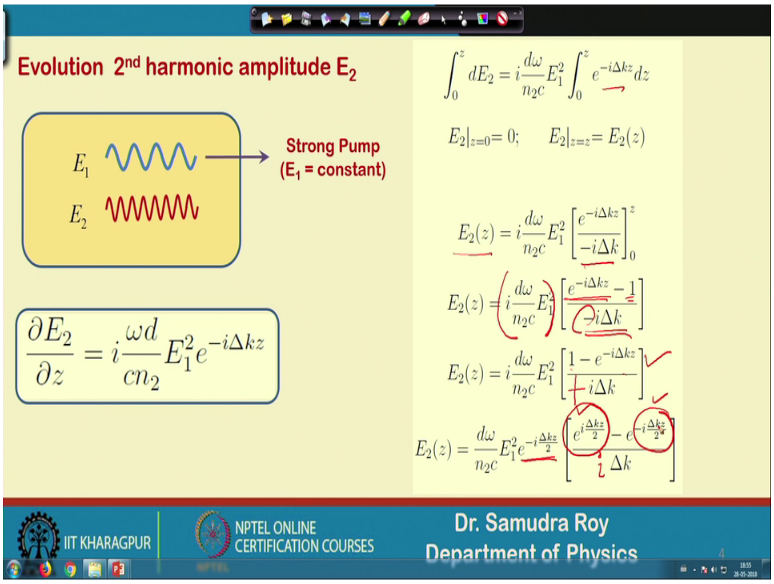

And, finally, what we do that this looks quite a similar term or this is one can make some symmetric kind of term using that. So, in order to do that what we do that we multiply and the entire stuff with e to the power minus of i delta kz by 2. When I multiply by 2, then one term we have here which it should be e to the power plus i delta kz by 2. So, that when we multiply with e to the power minus i delta kz by 2, we will have 1 here and since I am taking one e to the power i delta kz by 2 term, one e to the power i delta kz term will be still sitting here.

So, we will have one term here and one term here with opposite sign and delta k will be sitting here in the down stair. So, with i term. So, I think the i term is missed here somehow. So, it will be i delta k, here also we have some i delta k term with this in the numerator, ok.

So, after having this term we can simplify and we will get some term which is which is some sort of ok. So, here we have, ok, let me go back. So, let me go back to the previous term, so that we can understand. So, we integrate that when we integrate we will have this and basically, ok. So, this i will not be here because 1 i was sitting here, i was, forget about this i. So, this i and this i will be cancel out. So, one once this i will be cancel out, there will be no i term here and also here there will be no i term. So, the expression whatever is written is correct.

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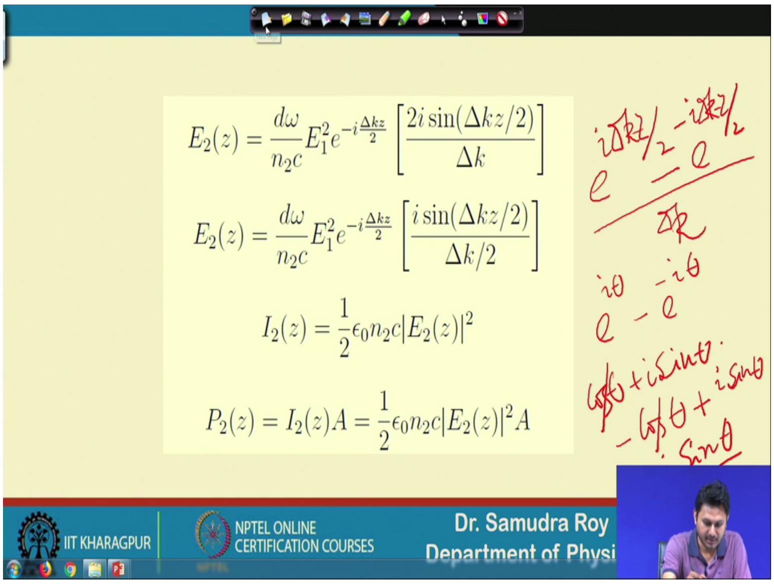

So, go back to the next slide yeah. So, we have one important term here there which is e to the power of i delta kz by 2 minus e to the power of i delta kz by 2 with a negative sign divided by delta k. This term is nothing, but e to the power of i theta minus e to the power of minus i theta. If I say delta kz divided by 2 is theta.

So, once we have e to the power i theta minus e to the power of minus i theta. So, we will have like cos theta plus i sine theta, one term; second term is minus cos theta minus or there is a one minus so, it should be plus of i of sin theta. So, this cos theta, cos theta term will cancel out. We will eventually have 2 i of sine theta 2 i sine theta term. So, here we have 2 i theta is in this notation it is delta kz by, so, we will have 2 of i of sine delta k Z divided by 2 divided whole divided by delta k. That is a term that we have when we derive this things. We further simplify since we have delta k divided by 2 and 2 term is sitting here. So, I will put this 2 term here, so that I have delta k divided by 2 here, in the numerator also I have in the argument of sine delta kz divided by 2. So, some sort of sinc function we have.

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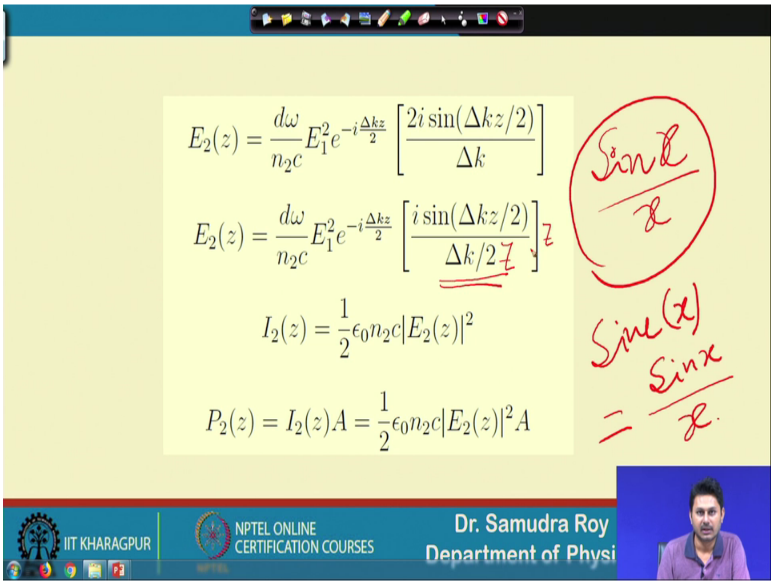

So, this is the function which looks like this. So, this is important you should correlate this things sin x divided by x. We know this is called the sinc function. So, this is sometime it is written sin c of x which is nothing, but which is nothing, but sin of x divided by x. So, sin of x divided by x term is something like this, but you should also note one thing that one Z is missing. So, if I multiply Z here and multiply Z outside so, we can write this as a sinc function, but right now it is not exactly the sinc function because the argument here and this term is not same, but I can always do that by multiplying one Z and in the numerator and one Z in the denominator, that we will eventually do.

So, after having E 2, the next thing is that if I know what is my field, second harmonic field which is E 2 what should be the value of the intensity of that?

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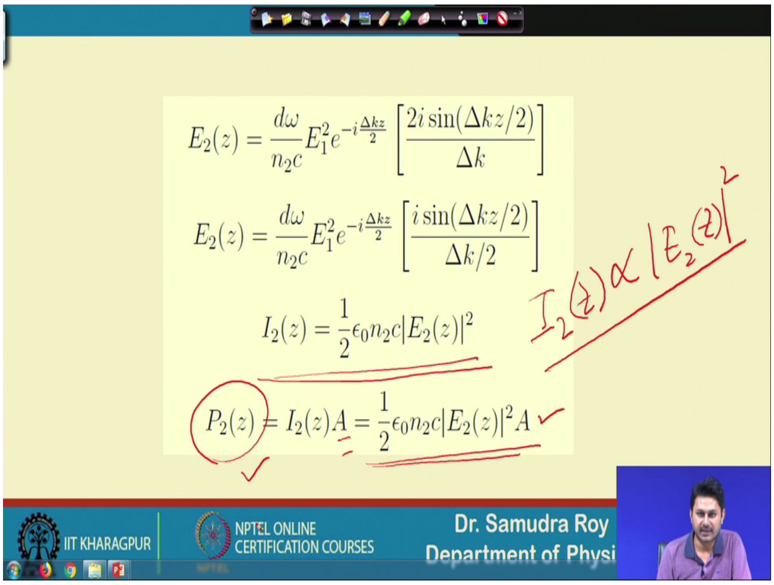

So, intensity is nothing, but mod square is a half of epsilon 0 into c mod square of the field. So, this expression we are using several several times. So, we should remember the relationship. So, we know that I 2 which is a function of Z is proportional to mod of E 2 function of Z square. So, once it is proportional to mod of this square so, we can readily find out what is the corresponding intensity.

So, now this intensity can be represented in terms of power also by multiplying the area. See if I do that we will have the same expression multiplied by area. Since the value of E 2 is known, I can try to find out what is the corresponding power and at the end of the day we try to know that, what is the second of power of the second harmonic. So, that is why we can convert everything into power, ok.

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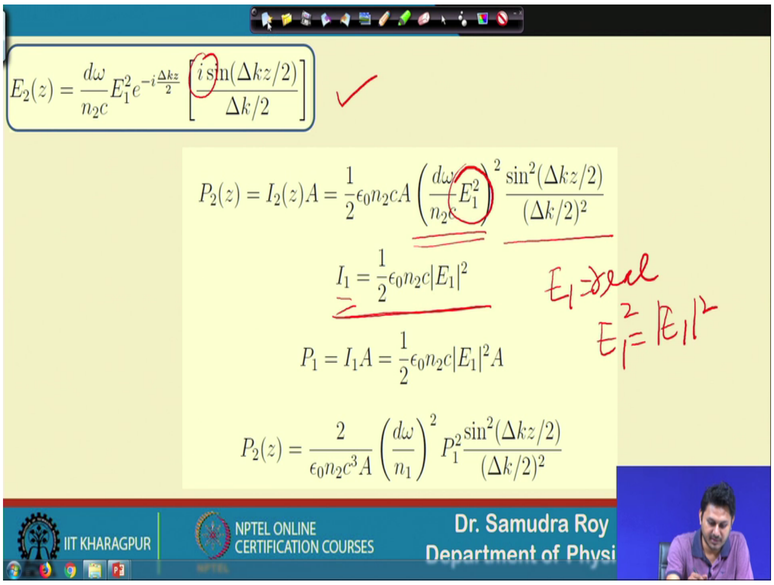

So, if I do so, one value we are getting let us find. So, E 2 is this, that we have already calculated and we just put this value here. So, since it is a mod square this i will going to absorb. So, there will be no i and we will have some sort of sin of delta kz by 2 and then delta k by 2 whole square term and this term is already there, so, we will have a square of that.

So, intensity of one if I try to write in terms of intensity and power so, the relationship between the power of the second harmonic and the first harmonic so, E 1 is sitting here. So, this E 1 I can replace by i 1 considering the fact that this E 1 is a constant, since it is a constant we can say that it should not have any kind of phase we consider this is a real. So, E 1 is real. If it is real so, there is no phase term. So, E 1 square is nothing, but mod of E 1 square, since it is a real that does not make any difference. So, i 1 if I write. So, P 1 can be represented in this term.

So, now, if I if I put this E 1 in terms of E 1 here in this equation, we will have an expression like this. So, this expression is important because in this expression we try to find out what is the power of the second harmonic and in the right hand side we find that the power of second harmonic basically, basically depends on few things. So, this is important physically you need to understand very carefully that what are the term for which this power is depending on.

So, we will find that there are the term d sitting here. So, d square; that means, if the non-linear coefficient is high then P 2 will be high that is one thing second thing is that P 1. P 1 is the pump and it depends on the square of the pump. So, if the pump is high, so, there is a possibility that we have a very high second harmonic the power of second harmonics. So, that is the term.

Apart from that one very important term is sitting here, which is sin square delta kz divided by 2 by delta k divided by 2 whole square. So, this is sinc function and we know that this function can reach to 1, when delta k tends to 0.

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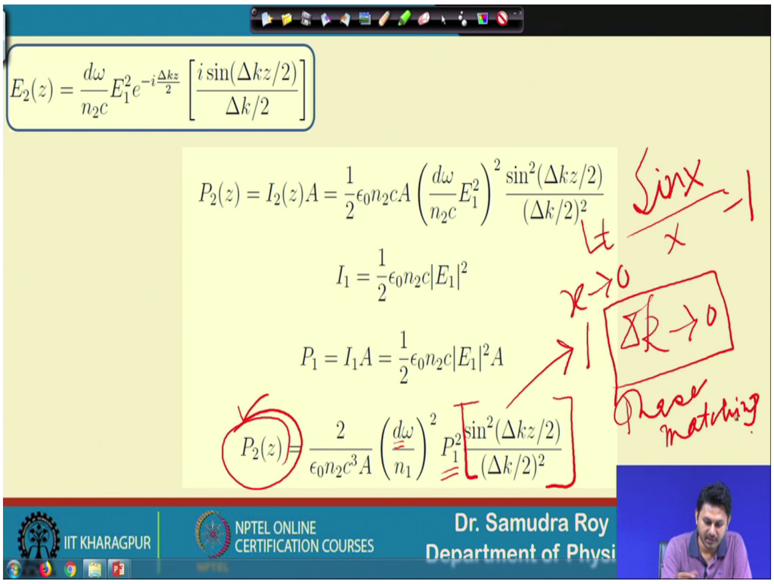

So, we know that sin of x divided by x if I make a limit x tends to 0, then this value is 1 and that is a maximum value. So, in order to maximize the right hand side one very important condition is that delta k has to be 0. This is basically called the phase matching, phase matching.

So, in the next slide we will be give you more information about the phase matching. So, let us go to the next slide.

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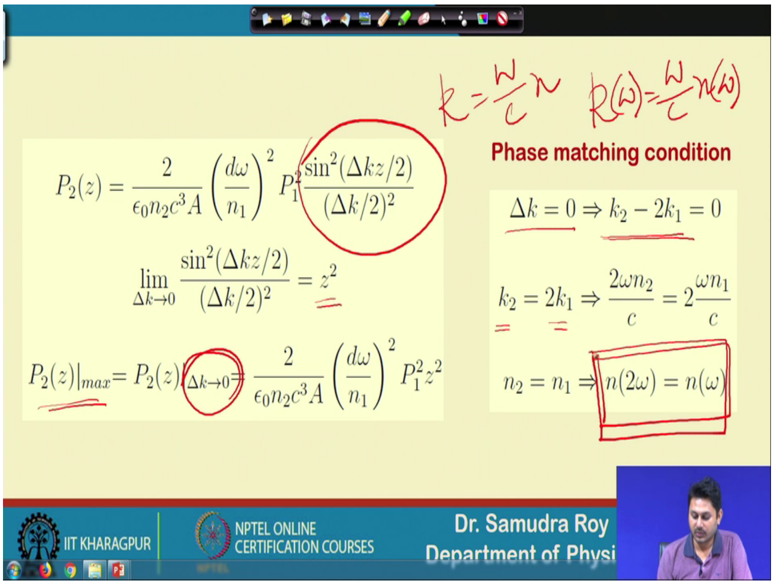

So, here that is the thing we try to emphasize that we have a sinc function, sorry we have a sinc function like the, we have a sinc function. So, so, we have a sinc function sin square delta k Z divided by 2 and this things. So, in order to have this sin x by x form when to multiply one Z and after multiplying one z, if I make a limit it will come as Z square as simple as that. So, P maximum 2 maxima is the maximum field one can have or maximum power one can have from launching a wave in non-linear medium, I want to find out the second harmonic wave and this second harmonic wave will be maximized on the condition that delta k tends to 0, that is the condition very important condition to excel this quantity.

So, delta k tends to 0, means what? So, in the right hand side basically we show the condition under which we have delta k equal to 0. As I mentioned this is the phase matching condition. So, delta k equal to 0 is essentially k 2 is equal to 2k 1. k 2 is equal to 2k 1 means k 2 is 2 omega divided by c multiplied by n 2 which is equal to k 1 means it will be 2 of multiplied by omega n 1 c because we know in general k is omega divided by c multiplied by n or in general k of omega is omega divided by c n of omega.

So, now if k is 2 omega we will have 2 omega here and n of 2 omega which is n 2. So, eventually we will have one expression which suggests that it can be possible if the refractive index of 2 omega is equal to the refractive index of omega. So, this is the condition, this is a very critical condition and normally we do not have this condition because of the dispersion property, so, we will discuss it later. But, at this point you should know that if I achieve somehow if we achieve somehow this condition we can maximize the power conversion. So, the second harmonic power will be converted more efficiently under this condition.

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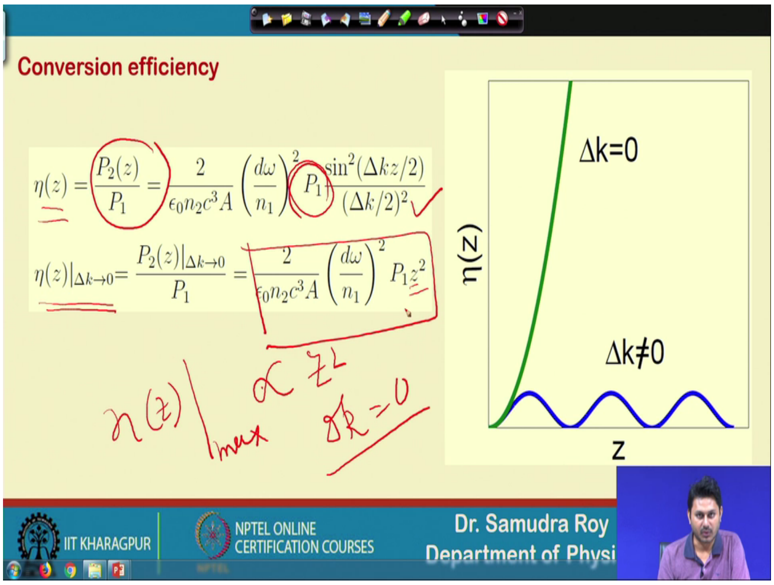

So, the next thing is to find out what is the efficiency. Efficiency is nothing, but the ration of the two powers. So, P Z is the generated power at second harmonic and P 1 is the launched power or pump power.

So, the ration between these two is called the efficiency. If I calculate the efficiency from the previous calculation we will find that again efficiency is a function of the power and this power is the power of the fundamental wave for the pump. So, it is proportional to pump power and this quantity this sinc value and again it will be maximized the efficiency is maximized if P 2 is maximized and this condition is already derived in the previous case and we will have some relationship here.

The most important thing is that this quantity will be maximized and if this is maximized, it will be proportional to Z square which is very important. Under delta k equal to 0, because if delta k equal to 0 do not then only we will have this quantity proportional to Z square. So, now, if I plot which is very important this plot is very important, then we will find that we are having a improvement of efficiency and this efficiency is improving with a function of Z square, it is a function of Z square which is improving very rapidly.

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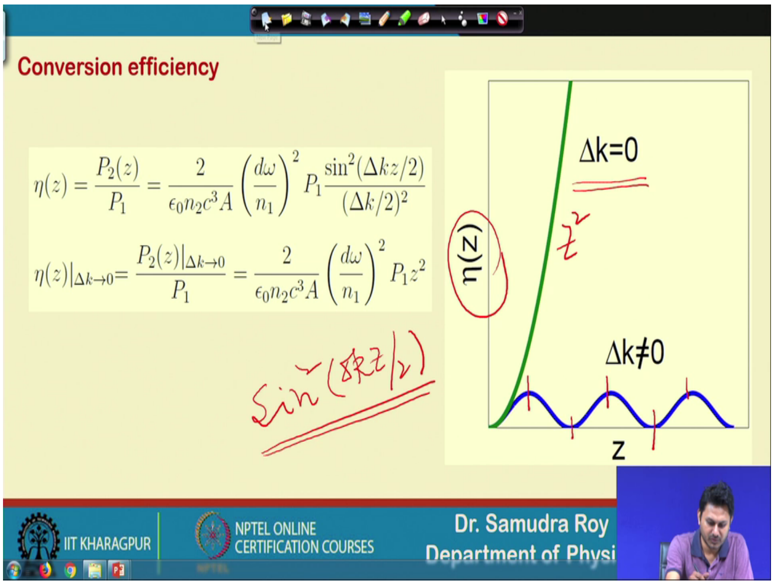

So, that means, I am able to able to generate the second harmonic very rapidly with respect to Z because it is improving in the form of Z square under delta k equal to 0 condition. However, when delta k is not equal to 0, the thing will be proportional to sin square of x. Please note, that if delta k is not equal to 0, then this condition will never be 1 and if this is not 1, we will have this as delta kz divided by 2. So, it is proportional to this quantity or the sin square. So, we will have sin square means we will have a sinusoidal variation.

So, in the initial few point the efficiency will increase, but after that it will go down to 0 value and then again go to increase and go to peak and go down to some value and peak and that means, the efficiency is periodically changing with respect to z, if delta k is not equal to 0. So, this is the interesting thing you should remember that when it delta k is achieved we have a efficiency is increasing like anything. So, it is proportional to Z square, but on the other hand, if delta k is not equal to 0 the efficiency is not that much it is getting some maxima and then go down and so on.

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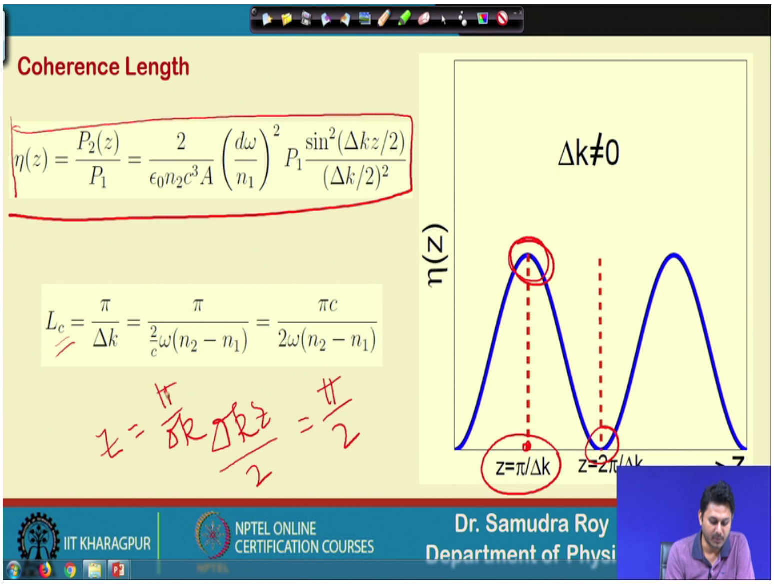

So, what is a reason behind that we need to find out. So, now, we need to find out what is a coherence length. So, the expression of the the expression of this efficiency we have already derived and we find that this is proportional to a sin square when delta k is not equal to 0. So, we find that there is a specific distance of k for which I will have the maximum value and these maxima will occur when delta kz by 2 become pi by 2. Because, we know that delta k Z by 2 pi by 2 means we will have sin square pi by 2 which is 1. So, this value will which maxima when we have these value. So, that means, Z has to be equal to pi by delta k. So, that is the point which is indicated here and at that point what happened we have a maxima.

On the other hand we will have a minima exactly twice of that length. So, this particular length is called the coherence length with is which is pi divided by delta k. Delta k can be defined in this way because delta k is k 2 minus 2 k one and if I simplified it it should be 2 c omega into n 1. So, let me do that.

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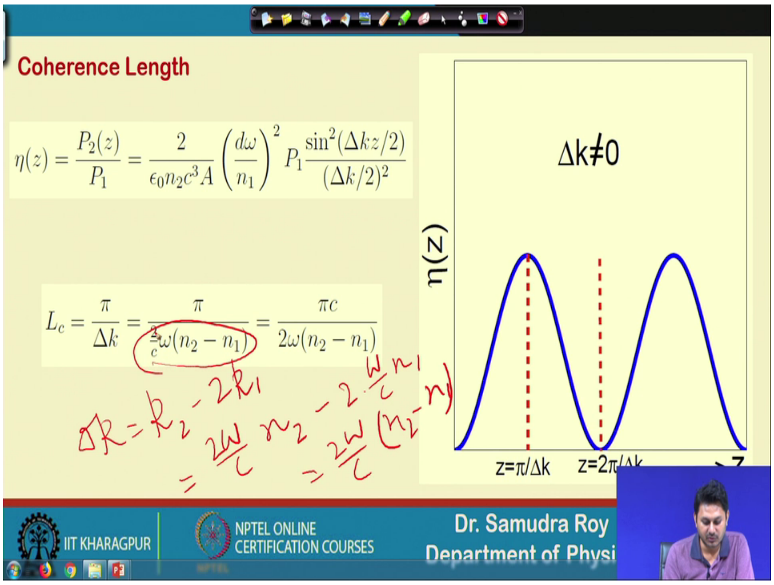

So, delta k is equal to k 2 minus 2k 1; k 2 is how much? It is 2 of omega of c n of 2 minus 2 of omega by c n of 1. So, if I take 2 omega c common then we will have n 2 minus n 1 which is nothing, but delta k. So, we just put this term here. So, when we put this term and then simplify this it will be pi c divide by omega into minus n 1. So, coherence length is important. Once we have the knowledge of n 2 and n 1 of a system then we can able to find out what is the coherence length before that coherence length we have the maximum efficiency, but after the coherence length what happened, the efficiency will drop very rapidly and eventually we will not going to have anything at some points, here the efficiencies go down to 0.

So, you should be very careful under the condition delta k. So, delta k is a condition for which we have some efficiency and we will not going to have anything when we have the distance 2 pi delta k.

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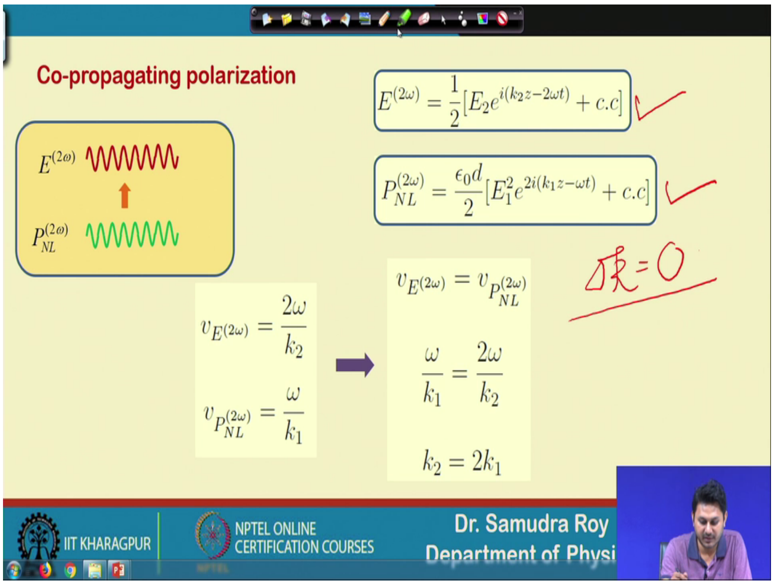

So, why this things are there, it is a important thing and we will going to discuss this thing here already that what is going on. So, we have the wave here, there are two waves. So, once we have the electric field E, so, this electric field is nothing, but the second harmonic field. Also, we have polarization term which is P non-linear 2 omega and which also have in our hand. The condition the phase matching condition is delta k equal to 0. So, delta k is equal to 0 this phase matching condition if I want to figure out in a different way then we can find one important thing. So, what is a velocity of E 2 omega from this plane wave expression we can readily say that the velocity of this E 2 omega is nothing, but 2 omega divided by k 2.

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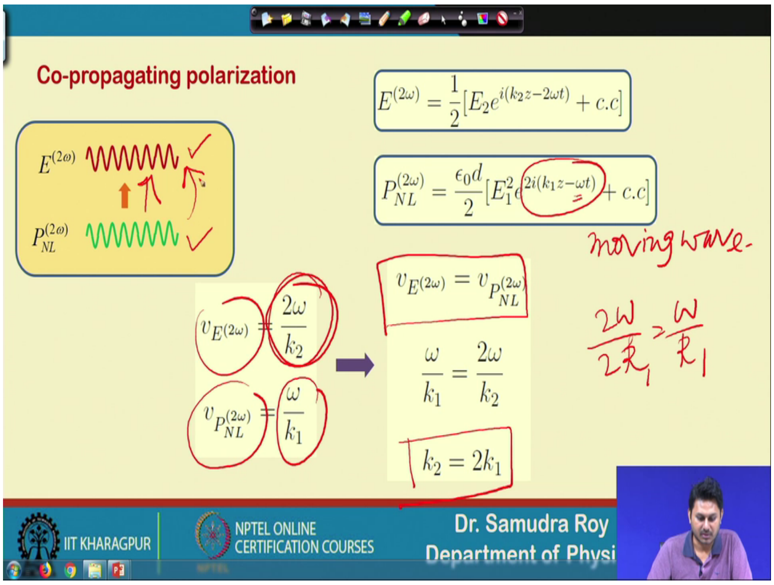

So, this is the velocity the phase velocity of the wave to E 2 omega. So, E 2 omega is also moving inside the medium and P non-linear 2 omega which is a source term of this basically these basically generate this at the same frequency. So, the point is P non-linear is also moving this is a moving wave some sort of moving wave because we have this plane wave structure. So, it is moving.

So, the question is what is the velocity of this and what is the velocity of this? So, we have already calculate that. So, this velocity and this velocity if I calculate we have it is 2 omega by k 2 and it is omega by k 1, very easily one can see that this is the velocity is 2 of omega divided by 2 of k 1 or it is simply omega by k 1, this is the velocity of the non-linear wave.

Now, if I equate this two waves, the phase velocity if I equate so, what essentially we have omega by k 1 is equal to 2 of omega by k 2 or finally, we have k 2 is equal to 2 k 1. It is very important outcome and this outcomes suggest that the phase matching is eventually suggest that the two waves, the electric field having frequency 2 omega component which is a moving wave and the non-linear polarization which is also having E 2 omega component these two are moving in a same velocity.

So, when these two are moving in a same velocity we will have a maximum energy transfer from here to here. So, maximum energy can be transformed under the condition that when the non-linear polarization of the frequency 2 omega is moving with the same velocity same phase velocity with the velocity of E 2.

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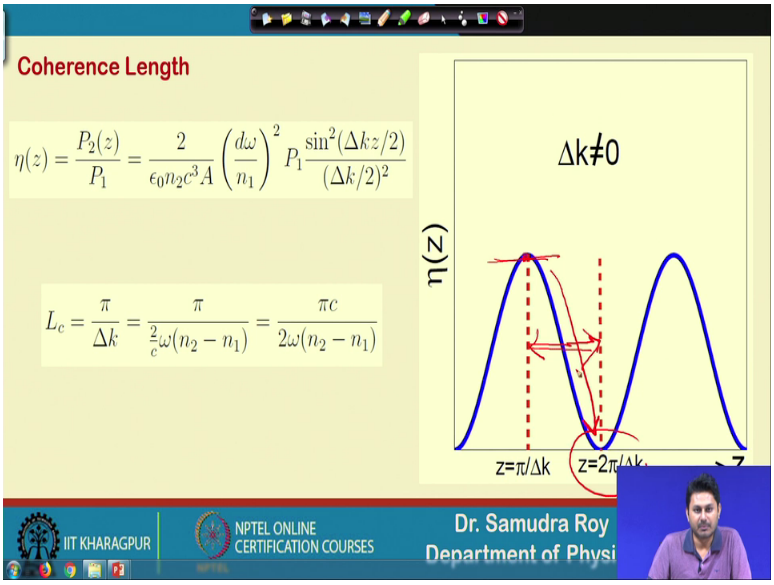

Well, we will not go into see much about this. So, today we will going to conclude here. And, in the next class basically we try to understand that what is the meaning of phase match in terms of other quantities that why the. So, that is why actually I try to show this figure. In the next class we try to understand that here we find that is the efficiency is going to a maxima and then go down to 0. Why this things is going down to 0 at this from here to here in this length, we try to find out in terms of the phase term.

So, well we will like to conclude here. So, today we will learn a very important thing that we have some kind of expression which gives us the value of the second harmonic. So, this second harmonic wave the evolution of the second harmonic wave we derived and we eventually solve that under the condition that there is a there is pump depletion. So, we solve this things that there is no pump depletion; that means, if I launch a very strong E 1 wave, this E 1 wave basically not going to change.

And, if I take it as a constant we find that there is no change at all in in E 1 and as a result we will have a evolution of E 2 proportional to Z square; that means, it is evaluating with respect to Z square and if delta k is equal to not equal to 0, then we find that it is periodically changing the evolution of efficiency is changing periodically. So, we try to understand the physics of this things.

So, in the next class we will start from here and try to find out how the phase is related to this phase matching condition and how from 2 omega, omega to 2 omega generation is there and also from 2 omega to omega generation is there and that is why we will find there is certain drop in the efficiency. So, with this note let me conclude here. Thank you very much for your attention. So, see you in the next class.