Introduction to Non-Linear Optics and it is Applications

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Lecture – 17
Theory of SHG

So, welcome student to the next class of introduction to non-linear optics and it is application course. So, in the previous class, if you remember we started a very important concept which is the Maxwell's equation, but the Maxwell's equation is not in the regular form. We introduce a non-linear polarization into the Maxwell's equation and finally, have an equation, which contain a non-linear term; that is why we called the entire equation as Maxwell's non-linear equations or non-linear Maxwell's equations.

So, this nonlinearity is coming because of the fact that now we introduce the non-linear polarization term into the system and it behaves a source term and that basically give raise to the new component frequency component like second harmonic generation. So, we continue with our second harmonic generation topic. So, today also we will going to introduce the same topic theory of second harmonic generation, but we do all the calculation in detail. So, let us see what we have today.

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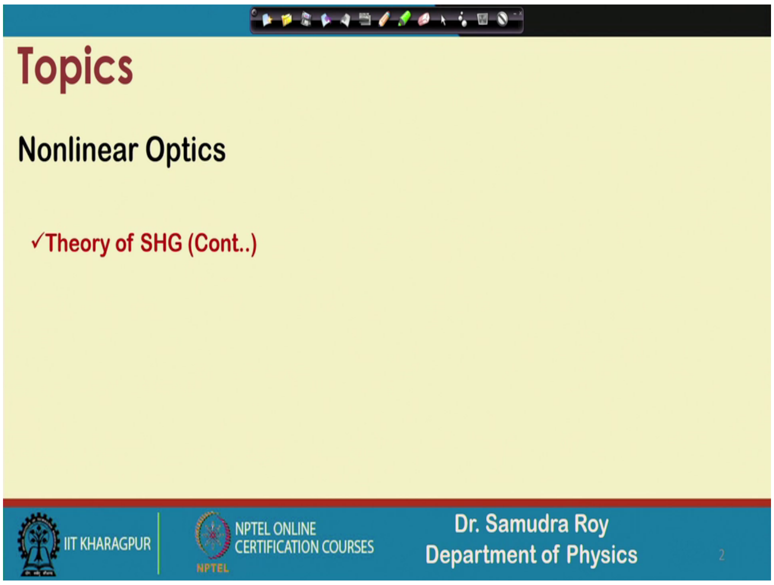

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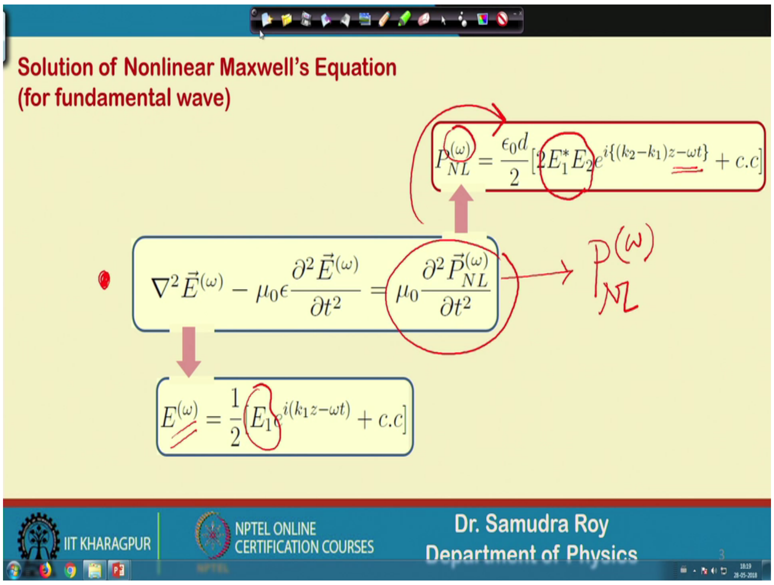

So, this is our expression that we have derived in our previous class. So, in order to have the solution of the amplitude of the electric field. So, let us first once again remind what we have. So, these was our first expression this which basically Maxwell's equation, but here we have a term related to P Non-linear P non-linear frequency omega say. So, this is the term which we called a source term. So, this source term what it will do it will start generating electric field which is vibrating as a frequency omega.

Now, what is the explicit form of the P non-linear omega is represented here that we also derived in our last class. You should know that here, in P non-linear omega, the frequency component is written here in the suffix. So, this is the frequency component and if you calculate that which we have done in the previous class the frequency component here you can see, it is omega.

So, we just extract there are many frequency component that is possible, but we just extract the frequency component omega to generate a frequency omega on the field omega a field which is associated with a frequency omega. So, P N L is nothing but the source term. Here. This amplitude is very important because, this amplitude will going to use and at the end of the day, we try to find out the amplitude of the field E which is E 1.

So, evaluation of the amplitude is our goal. In order to find out evaluation of the amplitude for electric field E omega in this case; that means, the fundamental field we just plot this solutions to our fundamental Maxwell's equation and then find out what should be the effect or what solution we have certain approximation we will do. So, gradually we will going to understand which solution I am talking about or which approximation I am talking about.

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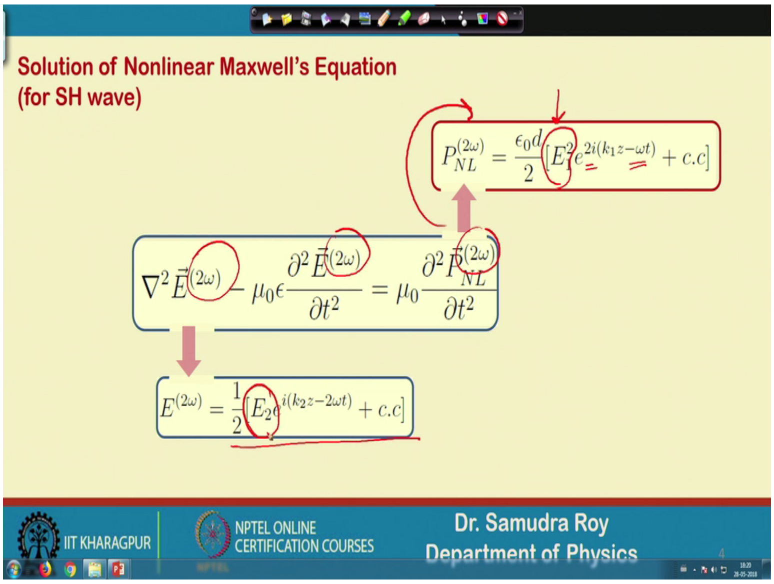

So, this is another equation and this is for E 2 omega. So, there should be 2 equation one for the fundamental wave and another for the second harmonic wave. So, this is the second harmonic wave. So, for second harmonic wave the non-linear Maxwell's equation will have almost of the same form that we have only thing you should note that now we are dealing with a frequency 2 omega. So, in all cases we have 2 omega components here.

In the similar way, in the P non-linear term, we should have a 2 omega term and here if you look carefully, we have the frequency component 2 omega 2 multiplied by entire phase. In the phase we have omega term. So, entire thing will be 2 omega, but the amplitude is slightly different from the previous one. Here, we have E 1 square as amplitude which is amplitude of the P non-linear 2 omega term in the similar way E 2 omega is defined as this and we need to find out the evaluation of this quantity which is the amplitude of the second harmonic.

So, second harmonic wave amplitude and the fundamental wave amplitude there should be 2 amplitude and evaluation of the 2 amplitude give raise to 2 different equation and that equation basically the equation of second harmonic generation. So, we will going to derive this 2 equation which will be a coupled equation and this coupled equation we need to solve under different condition. So, let us start how to do that let us start ok.

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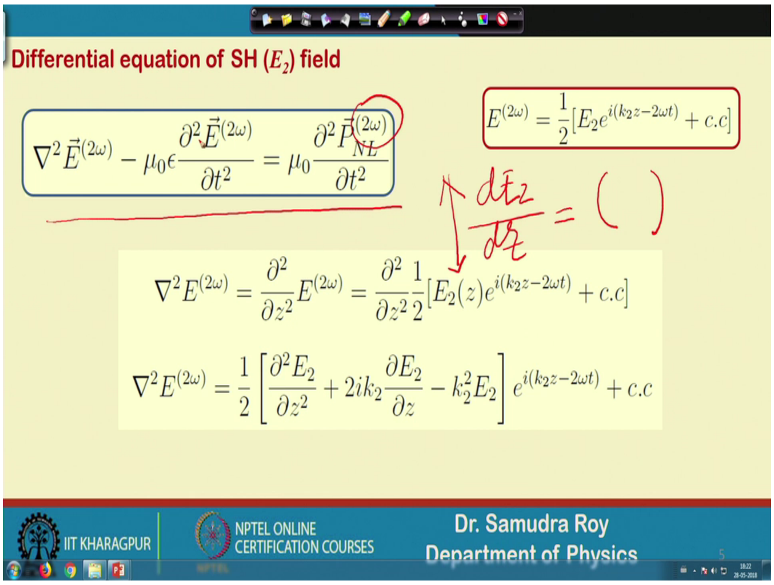

So, first this is our non-linear Maxwell's equation and this non-linear Maxwell's equation written for 2 omega term. Why we are using this particular equation because, our goal here is to find out the expression like this d E 2 d z which is something this something we will going to find out. But the aim here is to find out the second harmonic field of the second harmonic and how the second harmonic field will evolve. That is our goal. So, the second harmonic for the differential equation for the second harmonic if I want to find out, I need to use this Maxwell's equation where 2 omega components are there E omega we have already defined that this is my E omega.

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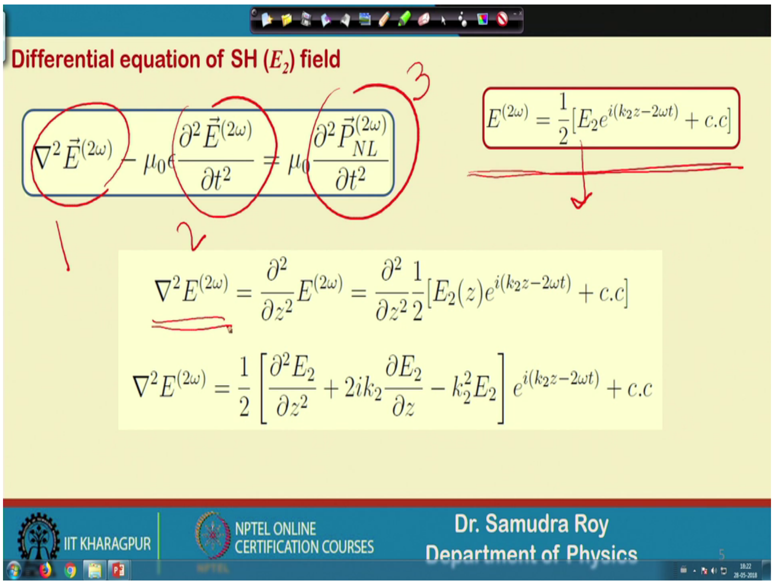

And E 2 is here P non-linear also we know what is P non-linear. So, we will put all this together. So, let us first try to find out what is this first term. There are how many terms this is one term; this is 2 term and this is 3 term. So, all this 3 term we will going to evolve evaluate using whatever the value we have and then find out the final expression. So, this 1 2 3, if I say this 3 term we need to evaluate. So, first in this section we evaluate the first term that is grad square E 2 omega. So, it is quite straightforward because E is known; this is my E 2 E 2 omega.

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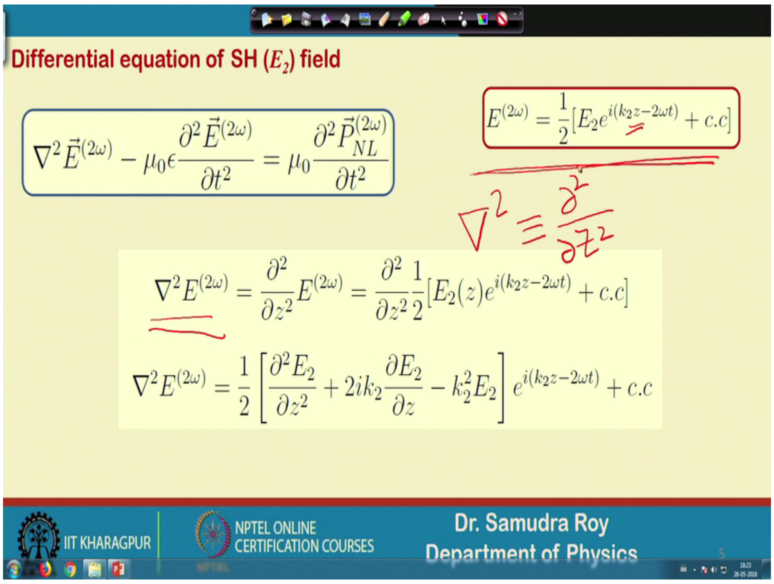

So, if I now put it here. So, this is the term I need to find out. So, this is in one dimension. If I consider this is one dimension. So, this operator will be simply this. So, we are considering that the way which propagating along z direction in order to reduce all the complexity basically we always choose that the electric field is with one coordinate. Here, this coordinate is z; that means, electric field is propagating in one direction which is z.

So, this operator significantly now reduces to del 2 del z square. So, if I now try to find out what is the value of del 2 del square for this entire quantity, we will find 2 important things that there are 2 terms where we have the z dependency one term is this.

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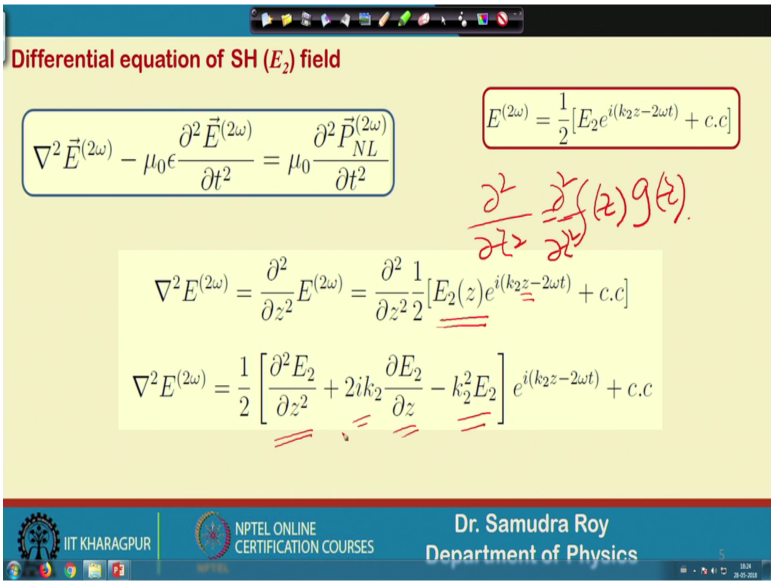

The amplitude and all the terms sitting here as, if we are doing the calculation for 2 functions like f z d z g z. So, we know that when we calculate with the derivative with respect to z. So, we know that when we calculate the second order derivative of 2 functions which are multiplied to each other, then there is a standard rule that we need to derive the first function twice, second function twice and 2 multiplied the derivation of the first function and second function. So, if I apply this standard rule, then we will find something like this. So, let us see what happen.

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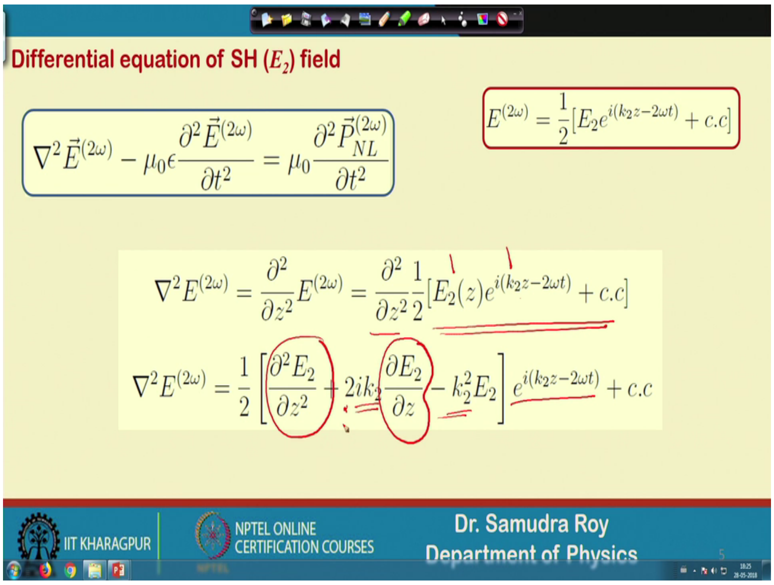

So, this derivation derivative with over this term. So, what is my first term? First term is a double derivative from the first function. So, this is the double derivative of the first function second the double derivative of the second function. So, when we make the double derivative of this function e to the power i k z. So, so 1 k i k will come both the cases because, there are 2 times of derivative. So, eventually we have minus of k k 2 square. So, minus of k 2 square here and 1 E 2 term will be there because it is already multiplied with this term.

But in all the cases, we have e to the power i k z 2 omega term since we have e to the power i k z omega term 2 omega term all the times. So, I take this term common what are the other terms the other term is 2 time the derivative of individual terms. So, derivative of this term; that means, this and derivative of this term when we make the first order derivative of this term i k 2 term will be here. So, I will have i k 2 and 2 will be multiplied because, they are 2 times we will get this term. So, eventually we will have an expression something like this.

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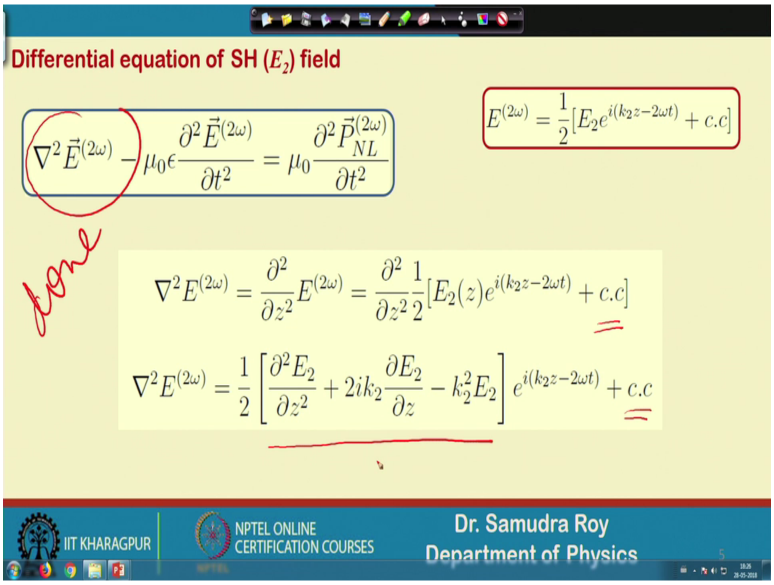

And plus complex conjugate because complex conjugate is always there. So, if I even if I make the second order derivative, we will have a complex conjugate term. So, first part is done. So, it is done. So, we derive the grad operator over E 2 omega. When E 2 omega is given like that we have these term in our hand. So, now, the second thing is to calculate the second term.

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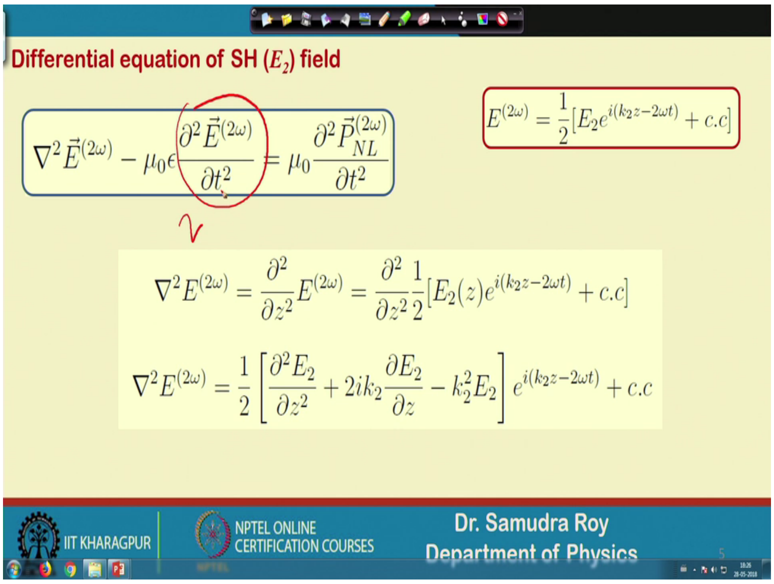

So, this is our second terms. So, we will going to calculate that in the first term we derive this with respect to z and now we need to derive the entire term with respect to t again we have a second order derivative. So, let us check what we have or let us do the calculation which is simple.

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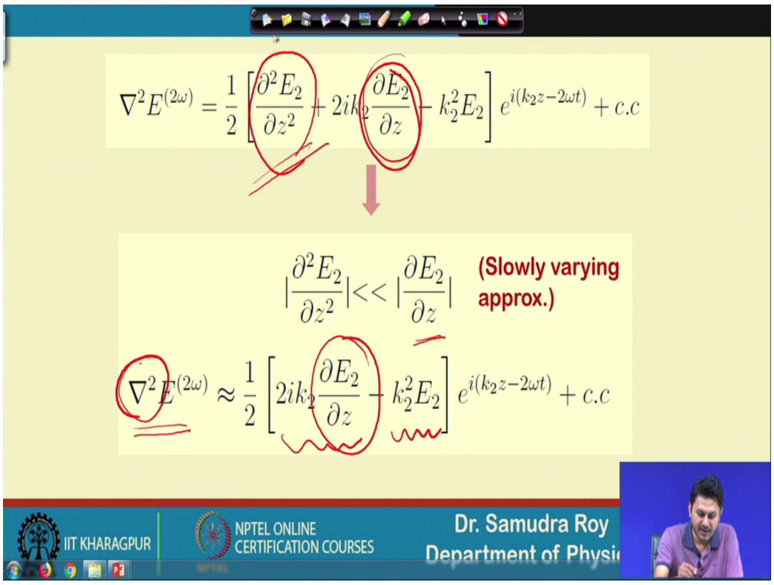

Before going to the second order derivative, now we put some sort of approximation and it is called slowly varying approximation. This a very important approximation slowly varying approximation where we approximate that the second order derivative of the field will vary much slowly compare vary much first this term is very the amplitude is varying slowly.

So, we will going to have only the first order derivative term of that the second order derivative will going not going to take into account after putting this slowly varying approximation. The equation significantly simplified because, we basically remove the second order derivative term from the entire system, but this is a justified approximation. Normally, the second order second order derivative of this E 2 is very small compared to the first order derivative term. So, first order derivative is sufficient because, it is a much bigger term.

So, we basically approximate that when we approximate under slowly varying envelope then these grad square E operator has 2 term; one is this and another already is there. So, only we have a first order derivative term note that we start with a second order derivative. So, this is basically a second order derivative, but we just remove the second order derivative term considering the fact this is varying slowly. So, we will only have the first order derivative, after having this approximation, which is valid approximation.

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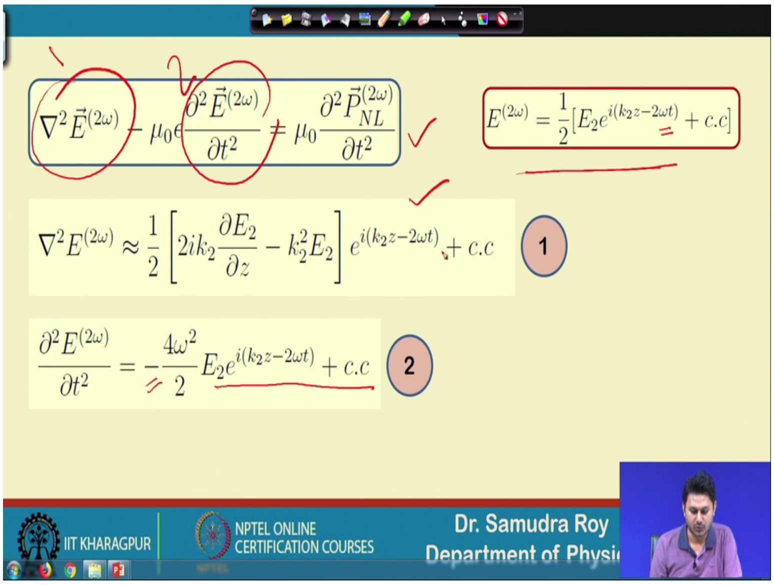

Next we will try to find out we will try to find out what value we are having. So, ok. So, let us see.

So, this our original equation this our original equation and this is the corresponding electric field that we have equation 1 is something which we have already calculated by slowly varying approximation we get this equation. And now, we will like to get another equation that is this part this is number 2. So, here we calculate since it is a derivative with respect to t so, that means, we will going to make a derivative of this term here only one t is here E 2 is not a function of t. So, when we will make a derivative double derivative we will eventually have 2 omega term here twice.

So, eventually we have since i is here. So, we have a minus sign and 4 omega square divided by 2 and then the rest of the term as usual. So, this is our equation 2. So, first equation, second equation both this is the first term and this is the second term. So, first and second term both we evaluate and when we evaluate, both the term we have the term in our hand which is written as 1 and 2 ok, next.

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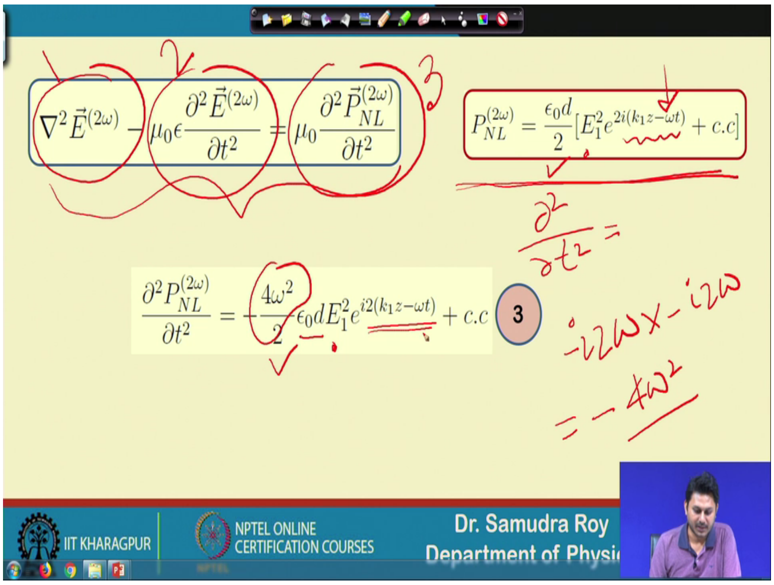

So, this is our non-linear Maxwell's equation. In non-linear Maxwell's equation, I have this term in our hand, I have this term in our hand and finally, I have a source term. One we have already derived 2 also we have derived, but finally, the source term we need to derive this is our third term. So, P non-linear omega we have already defined or already derived these things in our early classes.

So, P non-linear 2 omega is nothing, but epsilon 0 d multiplied by E 1 square E of 2 i k one z minus omega t. So, here we have a frequency component 2 omega plus complex conjugate. So, here we will be making the second derivative of this term. So, if I make a second derivative of this term; that means, we like to derive the entire thing with respect to t and this derivation is twice.

When this derivation is twice, then what happened that E 1 is not a function of t only the function of t is sitting here like the previous case for E. And in this case also, we will have 2 omega and 2 omega twice with a i term so; that means, we will have minus of i of 2 omega multiplied by minus of i of 2 omega both the time it will come out and eventually we will have minus of 4 of omega square these term.

So, here, we have minus of 4 omega square and 2 term is already there. So, I will put 2 term here epsilon d epsilon d is here. E square is amplitude term we will have E 1 square and also 2 of a 1 z omega 2 of k 1 z omega. So, finally, finally, we basically get all the 3 terms.

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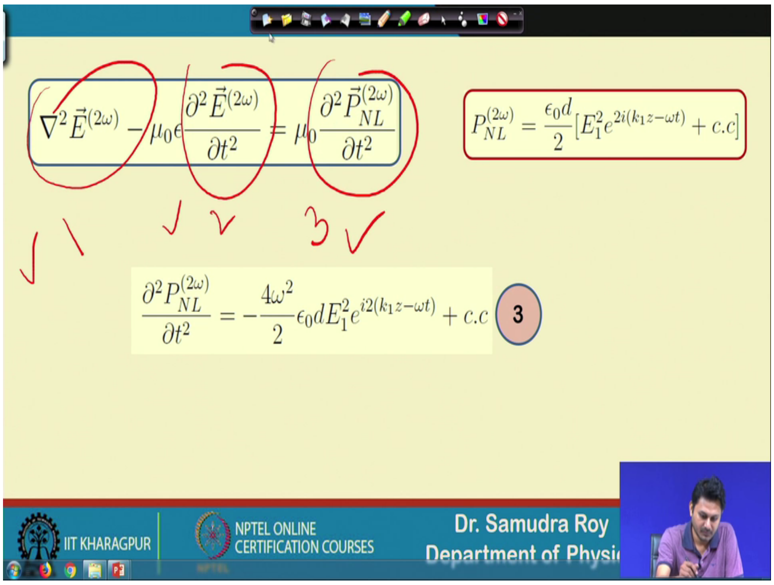

So, this is one term is done, second term is done and the third term is done. This is second term and third term this is also done. Once we have 1 2 3 all the terms, the next thing is to just put this term here into the equation and try to find out whether we are getting something extra or not or some simplification is there or not. Let us what happened when you put all the term together.

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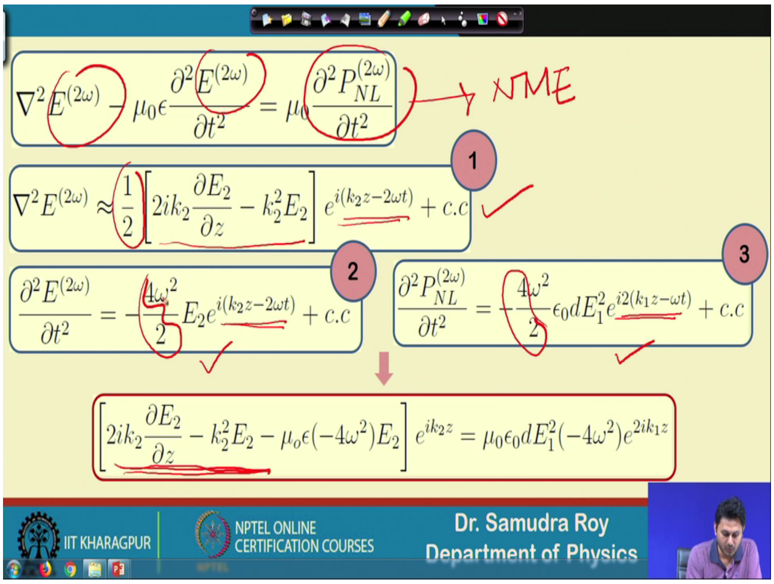

This is a very important slide where we have all the terms and if we have all the terms. Then it is necessary that we put this term together ok. This is our equation. This is a very important slide I must say because, all the information are there. So, this is our non-linear Maxwell's equation this is our source term and we need to find out how the E 2 omega is evolving any 2 omega we have amplitudes. Eventually, we try to find out how the amplitude you going to evolve this is equation 1 that we have already derived this is equation 2 we derived and this is equation 3 we derived.

Now, we put one by one. So, when we put this term one thing you should note that we have k 2 2 omega term. Here we have k 2 z 2 omega term here, but here we have 2 k 1 to omega term. So, when we put this term and if I take only one part, not the complex conjugate part complex; if I take the complex conjugate part, we will get a similar equation with complex conjugate. So, we can remove this term. So, only for complex conjugate, removing the complex conjugate term we will have this half of 2 i k 2 d 2 E d z z minus k 2 square E.

So, this is the first term. We will have also you can note that the half term is here; one half term we can have from here and also one half term we can have from here and when we remove this half term. So, this half term will gone; this half term will gone and this half term will gone. So, I will have only these expression here and for d 2 E d omega square, this things I will have 4 omega square with a negative sign.

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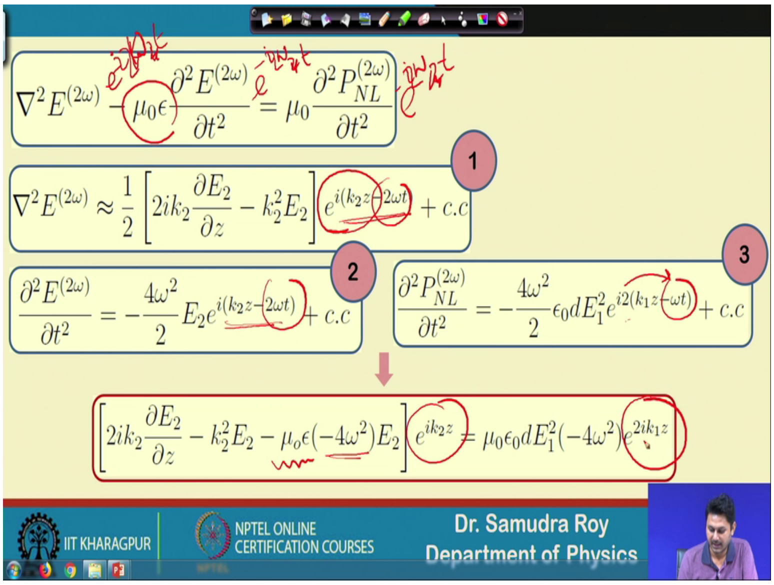

So, I will have 4 omega square by 2 here 4 omega square and 2 term is already removed and mu 0 epsilon 0 term is here. So, I will multiply mu 0 epsilon 0 then entire thing and

E 2. So, one thing I should mention here about this exponential term. As I already mentioned, actually so, these term for all the cases we have 2 omega term common. So, E 2 power i 2 omega is same for all the cases so, that means, e to the power i minus of 2 omega t term 1 can remove from all the sides.

Here, we have e to the power 1 omega 2 t term with a negative here we have e to the power minus of i omega 2 t term here we have e to the power i omega 2 t term with a negative sign. So, once we have all the omega 2 term because, we are taking only it is not omega 2 sorry there this is 2 omega 2 omega and 2 omega.

So, all this 2 omega term will be cancelled out from both the sides. So, all this 2 omega time will 2 omega term will cancel out. So, only we have the exponential term having k 2 z. So, here we have e to the power i k 2 z, but here we have e to the power 2 i k 1 z. So, here this is a very important distinction that in one case, we have e to the power i k 2 z, but here we have e to the power 2 i k 1 z ok. What about the other terms? Let us try to find these term is already there as I mentioned.

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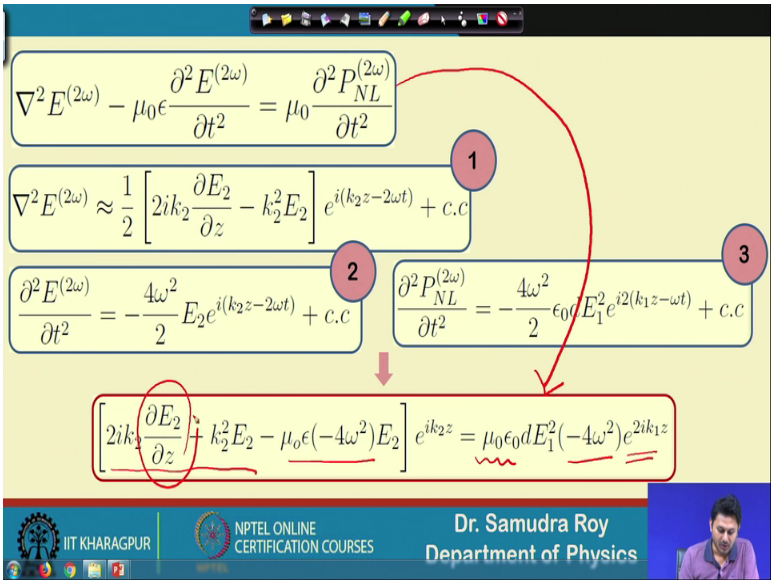

From here, I replace this find in the right hand side I have the entire term 4 omega square this 2 will cancel out. So, epsilon 0 and then this mu 0 is there. So, mu 0 epsilon 0 is there, d is there E 1 square is there and then minus 4 omega square term will be there with exponential 2 i k z. So, from here actually we have an equation where the differential equation is on over the amplitude the phase term is there, but the differential equation we have over the amplitude. So, once we have that. So, next see let us see what happened next ok.

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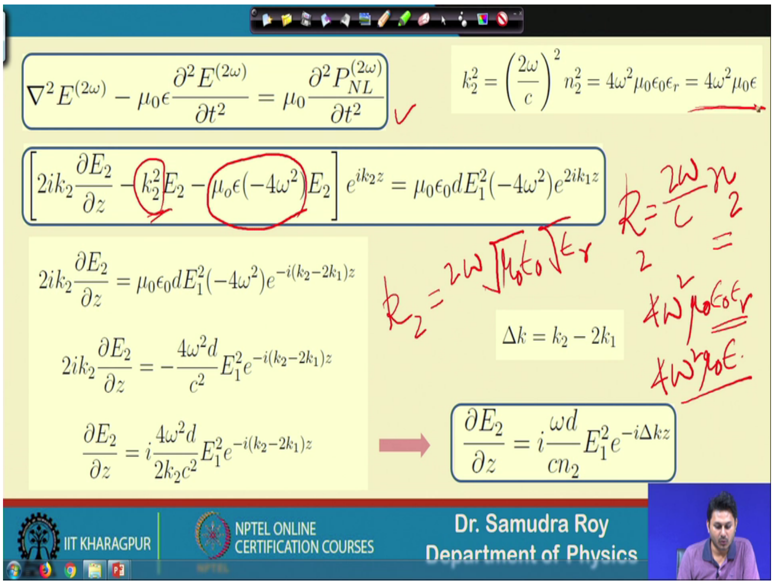

So, let us see what is happening here. So, this is my non-linear Maxwell's equation. So, before going to the next step, now what we will do? We will try to find out what is the value of k 2. So, we know that k 2 is the frequency 2 omega divided by c multiplied by refractive index at 2 omega or n 2 this is the definition of k 2. So, now, if I simplify if I simplify, I should write k 2 is equal to 2 omega and then 1 by c 1 by c, I can write root over of mu 0 epsilon 0 and this refractive index, I can write root over of epsilon r.

Now, after doing all this things if I make a square of that because we have k square term here. So, our goal here is to just put k square term here. So, k square will be coming out as 4 omega square mu 0 epsilon 0 epsilon r. Now, epsilon 0 epsilon r is nothing but 4 omega square mu 0 epsilon which is exactly the same term that we have here. So, k square is eventually mu 0 epsilon multiplied by 4 of omega square hair which is shown here in the corner. So, if I use the value of k and then put that since this is a negative sign this is a positive sign and all other terms are same.

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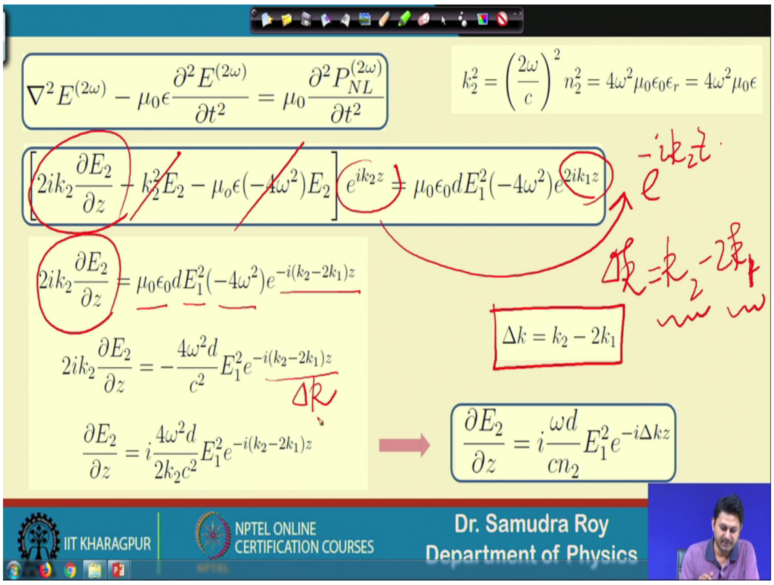

These 2 term will cancel out which basically simplify the equation very nicely. So, we remove this. If we remove this term, then we have only this term in the right hand side. In the left hand side and the right hand side we have some term which is mu 0 epsilon 0 which is mu 0 epsilon 0 d E 1 d 1 square minus of 4 omega square. This phase will going to modify, because we have this term here. So, what we will do we will put this here as e to the power of minus of i k 2 z.

Now if I write it here e to the power minus of k 2 minus 2 k 1 z. So, we have a phase term in the right hand side. So, we will further simplify our equation. So, 2 i k 2 d 2 E d z square mu 0 epsilon 0 is nothing, but 1 by c square 4 omega square term will be here d is as usual E 1 square and this term is there. So, now, we define a very important term here which is delta k delta k we called is some sort of phase mismatch term which is nothing but k 2 minus 2 k 1.

So, k 2 is a propagation constant or the wave vector for the second harmonic terms or the second harmonic wave and this is k 1 sorry and k 1 is the propagation constant of the fundamental wave. So, difference between k 2 minus 2 k 1 which is delta k is our phase term or phase mismatch. If delta k equal to 0 we have absolute phase match and this exponential term will going to vanish. So, we will come to this point later, but here we retain this term k 2 and try to find out the expression in a more compact form. So, here we have 2 i k 2.

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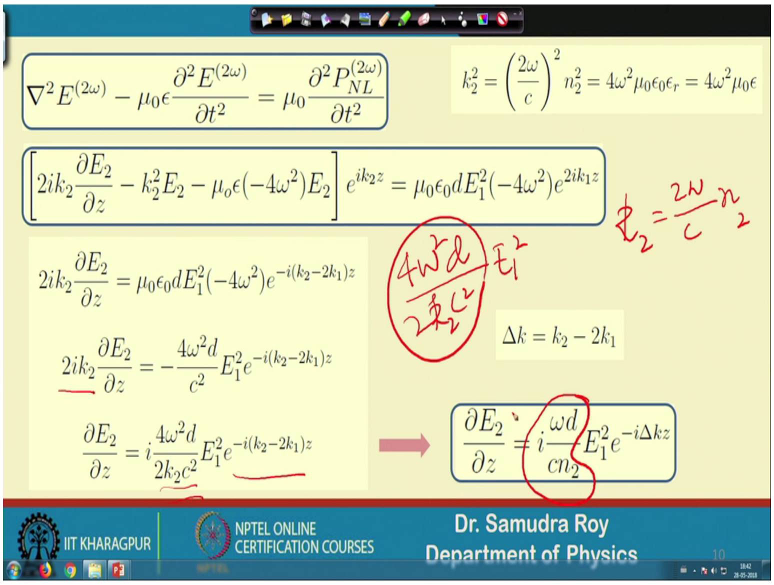

So, what we will do in the next step this 2 i k 2 we will put here. So, 1 by i is there. So, minus i is there. So, we have one i. So, 4 omega square term and d term will be as usual. So, 2 k 2 term will be there and then c 2 square and E 1 square and the other term.

So, now if I remove this k, so, we have 4 omega square d divided by 2 k 2 c square E 1 square. So, now, this term we can further simplify because k 2 is nothing but 2 omega divided by c multiplied by N 2. So, if I now put this term 1 omega and c will cancel out. We will have d and this 2 will cancel out with 2. So, eventually I have omega d divided by c n E 1 square e to the power i delta k z this is a very important expression.

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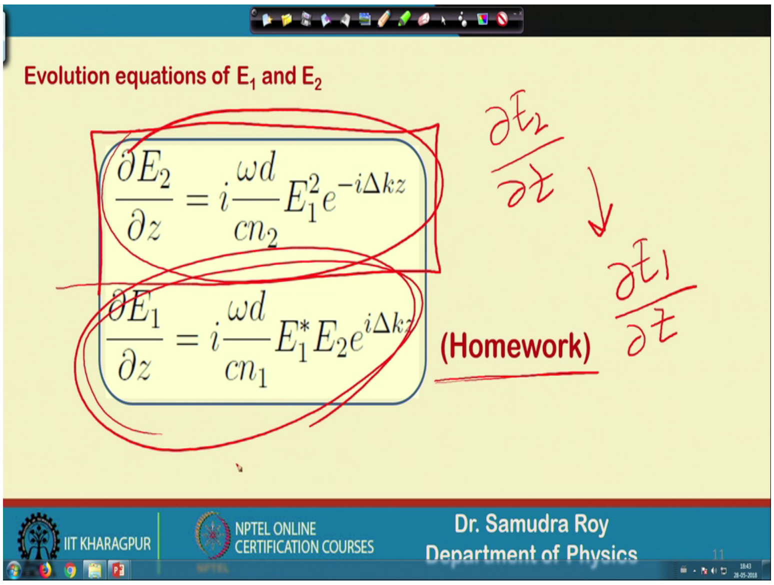

That we have this is a expression for the evolution of the field E 2 which is complex which is may be a complex amplitude, but the field of second harmonics. So, how the field of the second harmonic is evolving that is shown in this equation, now, today we will find a very important expression and if I write together because in the similar way it is possible. So, we have. So, we have this equation we derive this equation with detail calculation, but we did not derive this part.

So, we have one equation which is this, but also we can have another equation which tells us how the first harmonic or the fundamental wave will evolve. So, this is a very interesting expression also and you can calculate exactly in the similar way. So, that is why please note that I put it as a homework. So, I want the student to do the similar I mean the way we calculate this expression I want the student to use the similar treatment to find out this equation.

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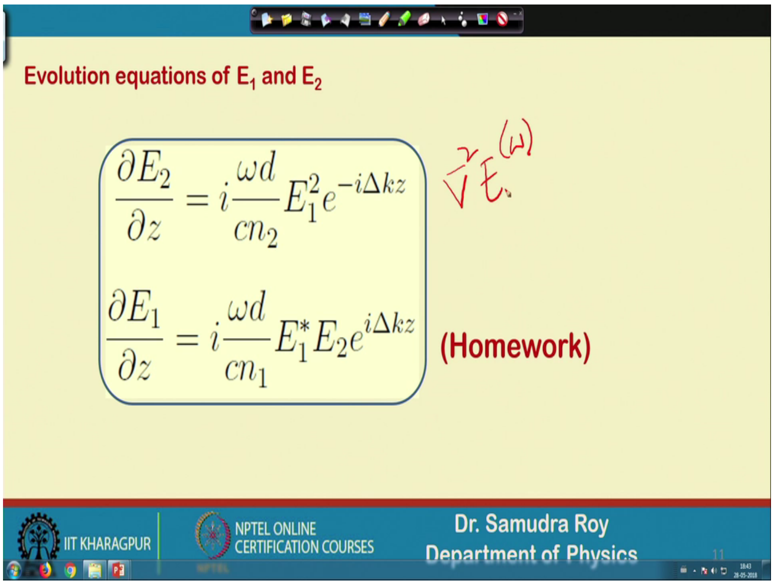

In order to find out this equation, remember you need to use the non-linear Maxwell's equation which is grad square, but you need to use E omega because you are try to evolve the expression for E 1. So, I believe all of you can do this homework very easily. So, I will not going to extend this class further because we do not have that much of time.

So, in the next class we will going to use this expression and try to find out some kind of solution, which is very important. So, we have 2 coupled equations in our hand and try to find out a general solution of this coupled equation by putting some condition we can simplify and this condition is how physical this condition is we will discuss in detail. But important thing that we derive today is the coupled equation of the second harmonic and first harmonic amplitude and this is very important to understand how second harmonic wave will going to evolve inside a non-linear material. So, with that note, let me finish the class.

Thank you for your attention. See you in the next class.