Introduction to Non-Linear Optics and Its Applications
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture - 15
Sum & Difference Frequency Generation

So, welcome back student to this course of Non-Linear Optics and its Applications. So, so far we are dealing so, today is the lecture number – 15. So, today we will going to learn something again which is related to this frequency mixing. So, in the previous classes we learnt very important concept like second harmonic generation, the physics behind that and also two very important thing which is, one is optical rectification and another is linear electro optic effect.

In optical rectification we find that if I launch an electric field having a frequency component omega, because of the non-linear effect what happened that the dipole will going to vibrate at two different frequencies, but also the polarisation should have one component which does not contain any kind of frequency which is called the null frequency component or zero frequency component. This term basically changes the average location of the dipole and because of that we have optical rectification.

In the second case, in linear electro optic effect what happened we are launching the electric field e omega and we launch the electric field when you launch the electric field E omega; and then with that if I launch another electric field which is E dc or the dc electric field which have a amplitude only, no frequency component then these two going to mix up because of this E square term which is very very important. And as a result we have something which can change the refractive index.

So, this electric field external electric field E dc basically is responsible for changing the electric field and it can change the electric field in a linear fashion, if I launch the electric E dc field. And if I increase it is amplitude what happen this delta n which is a change of electric field will going to change linearly.

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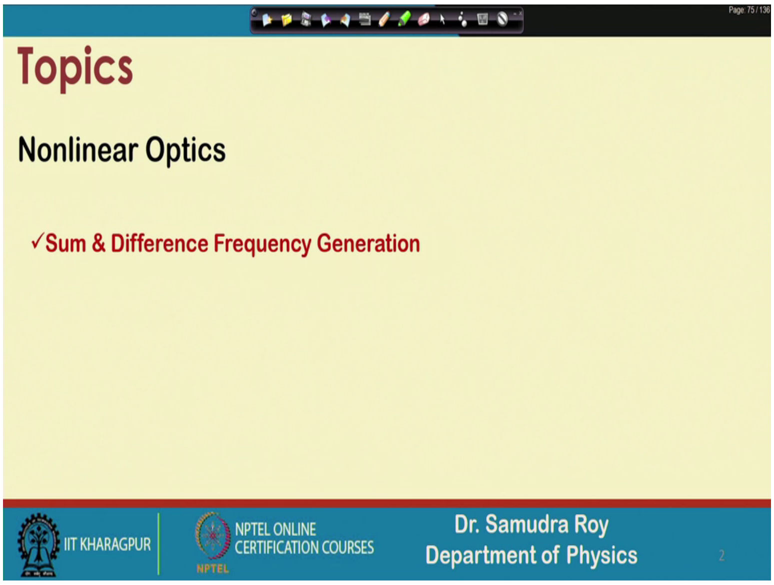

So, today we will going to learn another important concept, which is sum frequency generation and also, if I have sum frequency generation. Then also there is a possibility that we will have something called different frequency generation. So, in compact we can say in compact form we can say sum and different frequency generation.

So, the question arises here what we do we mean by sum and difference? Sum means we are adding something and difference mean we are subtracting something. So, when we say adding frequency; that means, what is going on? So, there are only one frequency so far, that we are launching that is omega and in the output we are getting another frequency which is two omega. Obviously, some kind of add up is here; that means, omega plus omega is getting 2 omega, but what else if I say I am launching two different frequency omega 1 and omega 2 and what will happen?

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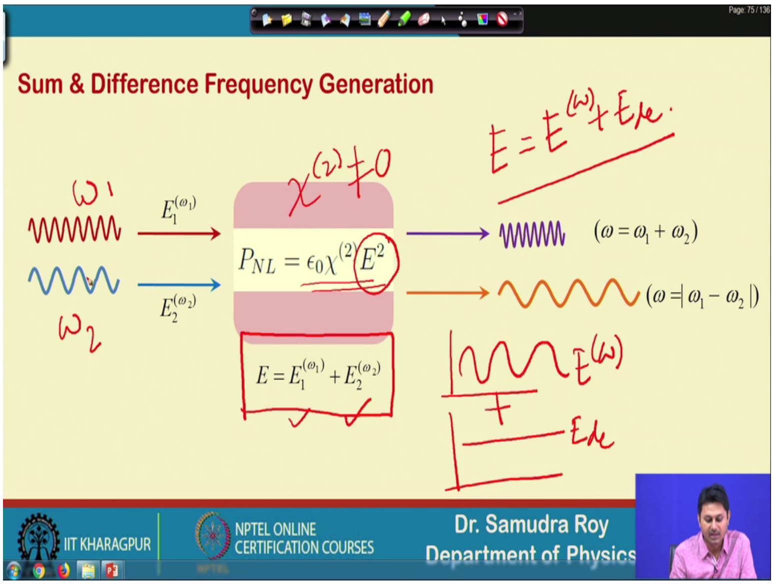

So, let us see the condition. So, this basically the schematic figure of sum and difference frequency generation. So, let us understand this figure. If you understand in non-linear optics, if you understand the figure clearly then the physics will be very clear to your mind. So, that is that is why I am going I emphasis that you should understand this figure the meaning of this figure which is very very simple to understand.

So, here what we have done is we have launched the electric field e omega 1. So, this is a frequency components a omega 1 and this is a frequency component of omega 2 of two electric field E 1 and E 2. So, that means, E 1 is having an electric field a frequency E omega 1 and apart from E 1, I am also launching another electric field E 2 which is a frequency which is having a frequency component omega 2 and these two frequencies now there into the system.

Now, it is passing through a medium where we have non-linear polarisation; that means, chi 2 in the system is not equal to 0; if chi 2 is not equal to 0; that means, my non-linear polarisation will evolve like E 0 epsilon 0 chi 2 E square as usual in this way and here this E term is interesting. When I write E that essentially means this is the sum over two field. In the previous case when we are learning the electro optic effect you remember if you remember the figure that I am launching one electric field having a frequency component like this and also I am launching an electric field with a frequency component of this. That means, this is a dc electric field and this electric field was having a frequency component.

So, my total electric field component in the previous case was E omega plus E dc. So, that was my total electric field here we are doing the similar kind of stuff we are launching two electric field, but this two electric field is having two different frequency components. So, that is why this is one frequency and that is one frequency, there is no dc term and I am launching these two together. So, that is why what happened the two field is now add up. So, total electric field is nothing but E 1 plus E 2 as written here. Now, if these two frequencies or these two electric field having two different frequency components are now inside the medium. When it is inside the medium they will interact with them self, they will coupled by dc square term and this coupling basically give rise to two another two different frequencies.

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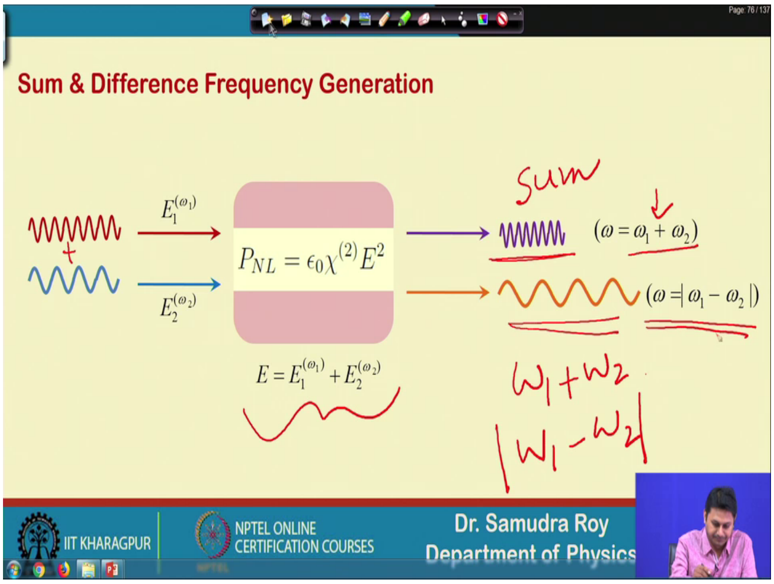

One is called the sum frequency, this is the sum frequency, this sum frequency is nothing but the combination of two frequency omega 1 plus omega 2 and this omega 1 plus omega 2 the plus sign suggest that they are add up and the figure also suggest that when the these two frequency are adding up. That means, the total frequency is higher. So, that is why I make this frequency like that the figure of this frequency is that it has more frequency the frequency is high rather. And also we have another frequency component omega which the difference between two frequencies.

So, in the first case we have omega 1 plus omega 2 and the second case we have omega 1 minus omega 2. So, now, one important thing you should note that here we put a mod sign because we know that which one is greater, omega 1 or omega 2? So, that means, the difference between these two frequency is important. It might be omega 1 minus omega 2, it might be omega 2 minus omega 1, but the difference is important. So, that is why I put a mod sign.

So, whatever the frequency we will have that will be the difference between these two. So, since it is difference we have a frequency structure which is much less than whatever the frequency we have. So, omega 1 omega 2 these are the two frequencies, omega 1 plus omega 2 is another frequency and omega 1 minus omega 2 is another frequency. So, these two frequency will get generated under sum and difference frequency generation due to this non-linear interaction, ok.

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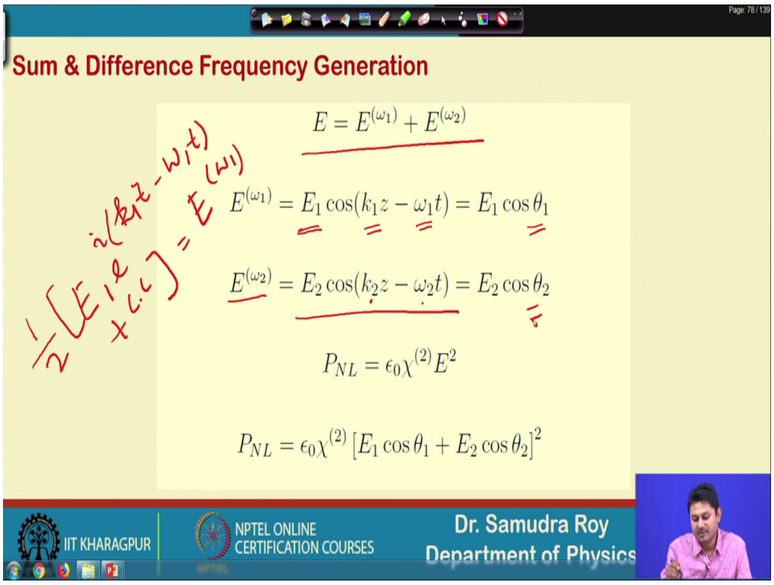

So, now go back to the mathematics which is again quite straightforward and we have already done this things in our previous classes as I mentioned. That the total electric field E omega 1 plus E omega 2 is there. So, this is E omega 1 and E omega 2, E omega 1 and E omega 2. E omega 1 is nothing but a frequency component of omega 1. So, I write E omega 1 is amplitude I write E 1 propagation constant k 1 and frequency component omega 1 E omega 1 real electric field; so E 1 cos k 1 z minus omega 1 t. So, if I write in complex conjugate it will be exactly the way we have written previously, it will be half of E 1 e to the power of I k 1 z minus omega 1 t plus complex conjugate this is nothing but E of omega 1.

In the similar way, I can write E of omega 2; only change is k 2, omega 2 and E 2 amplitude will going to change the propagation constant and the frequency components change k 1 z minus omega 1 t, k 2 z minus omega 2 t I just reduce this term as theta 1 and theta 2 because we are doing some kind of calculation one when we need to make a square of that and all these things.

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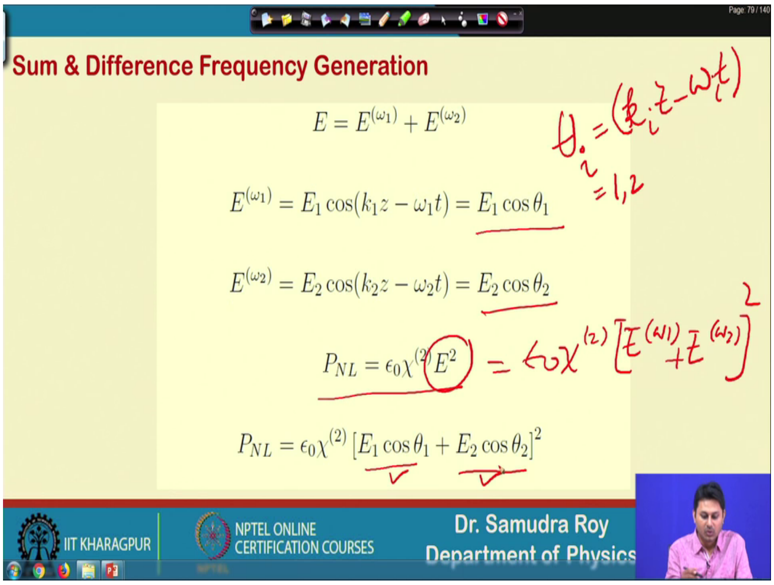

So, this term will always be there. So, I just replace to make this thing simple I just replace that. So, according to my notation theta i is in general k i z minus omega i t, where i belongs to 1 and 2, ok.

The next thing is now P non-linear. So, P non-linear is again epsilon 0 chi 2 E square. So, when it is epsilon 0 chi 2 E square. So, this square term basically nothing but E 1 plus E 2 square of that thing. So, P non-linear is epsilon 0 chi 2 and then the sum of two electric field and square of that. So, here it is epsilon 0 chi 2 E of omega 1. If I write in compact way plus E of omega 2 square of that; so, E of E of omega 1 is nothing but E 1 cos theta 1 see E 1 cos theta 1 E of omega 2 is nothing but E 2 cos theta 2; so E 2 cos theta 2.

So, when we have E 1 and E 2 cos theta 1 and cos theta 2 and square of that we have some terms which is not a new thing because again we are making a square and when you make a square you will going to manipulate few things, but here since I am launching two different field with two different frequencies. The calculation will be little lengthy, but very straightforward I mean very simple and very straightforward, but a bit lengthy.

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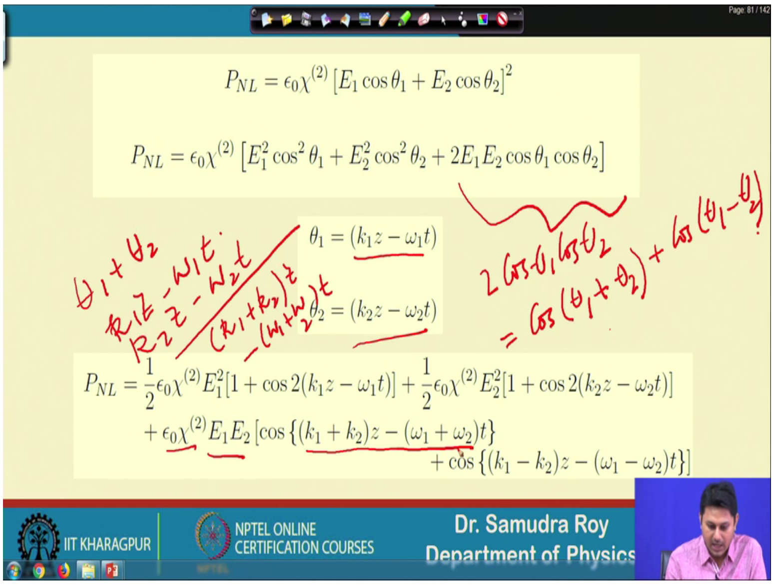

So, P non-linear here as I mentioned it is this. So, now, if I make a square of these these two terms E 1 cos theta 1 plus E 2 cos theta 2 it will be E 1 cos square theta E 2 cos square theta 2 and then the cross term, the famous cross term 2 E 1 E 2 cos theta 1 cos theta 2 theta 1 and theta 2 is represented by these we have already mentioned that.

Now, what we do that this square term in the previous way which is reduced to 1 plus cos 2 theta. So, again we will do the same thing half 1 plus cos 2 theta is basically cos square theta. So, we will do the same thing. So, half epsilon 0 chi 2 will be there because I am taking half comma let us consider only first term. So, for first term half epsilon 0 chi 2 E 1 square I will take and cos square theta is represented by 1 plus cos 2 theta and this 2 theta 3 theta term is now represented by k 1 z minus omega t omega 1 t.

In the similar exactly in the similar way the second term can be represent in this way that half chi 2 E 2 square and then 1 plus cos 2 of this term. So, we have one term here having frequency 2 omega 1, another term here having frequency 2 omega 2. And then after having these two term we have the term having cos theta 1 and cos theta 2. We know the 2 cos theta 1 and cos theta 2 can be represented in terms of cos theta 1 plus theta 2 and cos theta 1 minus theta 2 and we will going to do exactly the same thing. So, this multiplication term theta 1 and theta 2, we will separate it out.

So, if I do then this term can be represented like E 1 E 2 epsilon 0 chi 2 and theta 1 plus theta 2 can be represented as cos theta 1 plus theta 1 minus theta 1 plus theta 2 and then theta 1 minus plus theta 1 minus theta 2. So, if I do, so, let us write may be then it will be clear; so cos 2 cos theta 1 cos theta 2 will be cos of theta 1 minus theta 2 cos of theta 1 plus theta 2 plus cos of theta 1 minus theta 2.

Now, theta 1 plus theta 2, if I do theta 1 plus theta 2 theta 1 is this theta 2 is this. So, it is simply k 1 z minus omega 1 t plus k 2 z minus omega 2 t, if I add these two thing then I will have k 1 plus k 2 z minus omega 1 minus omega 1 plus omega 2 into t, which is exactly this term the first one.

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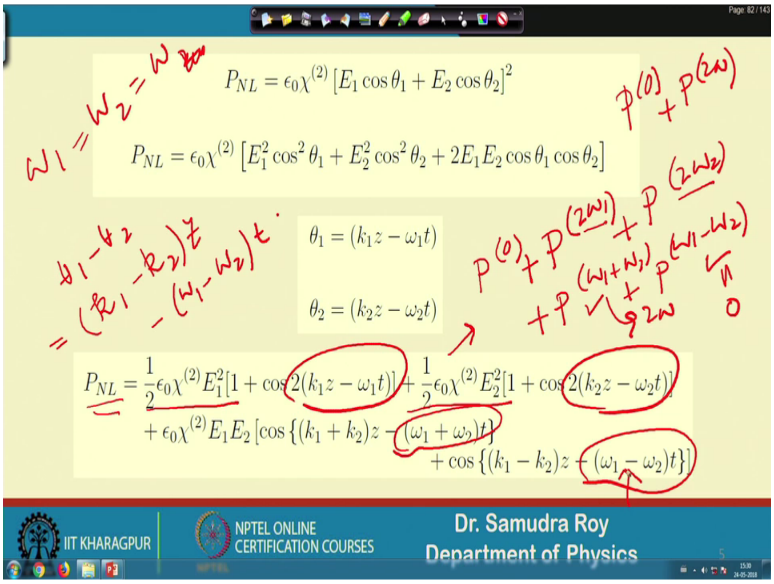

In the similar way we can do the other term minus. So, if I do this theta 1 minus theta 2, it eventually basically gives me k 1 minus k 2 z minus omega 1 minus omega 2 t. So, I now I am having in P non-linear now, I am having one frequency component which is omega 1 and omega 2 that is important, another frequency is also there which is omega 1 minus omega 2. See if I now write here how many frequency component terms are there in P it is interesting. So, one is P 0 which is the 0 frequency component like this first term and this term which is 0 frequency component.

Then, another term which has the frequency component of 2 omega 1, another frequency component we have which is 2 omega 2, because here the frequency component is 2 omega 1 here the frequency component is 2 omega 2 apart from that another 2 terms we are having which one omega 1 plus omega 2 P of plus P of omega 1 minus omega 2 assuming that omega 1 is greater than omega 2 I can write in this way.

Previously, we find that we have only frequency component omega 2 omega and 0 another component was there omega. So, here I am just writing the non-linear components, not the linear one. So, that is why P omega components are not there, but if I write the total frequency total polarisation component then two components will also going to add up. So, we will do that. But, important thing is that here omega 1 plus omega 2 and omega 1 minus omega 2 terms are there. That means, now the dipoles not going to vibrate in 2 omega 1 not going to vibrate in 2 omega 2, but also going to vibrate two different frequencies omega 1 plus omega 2 and omega 1 minus omega 2.

So, now one thing also one can understand that if omega 1 and omega 2 are same; that means, omega 1 is equal to omega 2 is equal to omega, sorry it is equal to omega then this term will become 2 omega and this term become 0. So, everything will collapse. So, 2 omega will become 2 omega 1 will become 2 omega 2 omega will become omega 2 become 2 omega. So, eventually we have P 0 plus P of 2 omega, that we have already figure out in our previous calculations.

So, this degeneration there will be a I mean if omega 1 and omega 2 are same we should have the same frequency components. So, the difference here does not make any sense because if the two frequencies are same then there will be no frequency component at all and the null frequency component is nothing but the difference of the two frequency component. And these two frequency if the two frequency are same we will have a frequency component which is 0. And, also if I add these two frequency omega 1 omega 2 we will find 2 omega if the omega 1 and omega 2 are same as omega. So, this is basically the second harmonic process the second harmonic generation. So, second harmonic generation is some sort of spatial condition of sum frequency generation or the difference frequency generation under the condition that all are same, ok.

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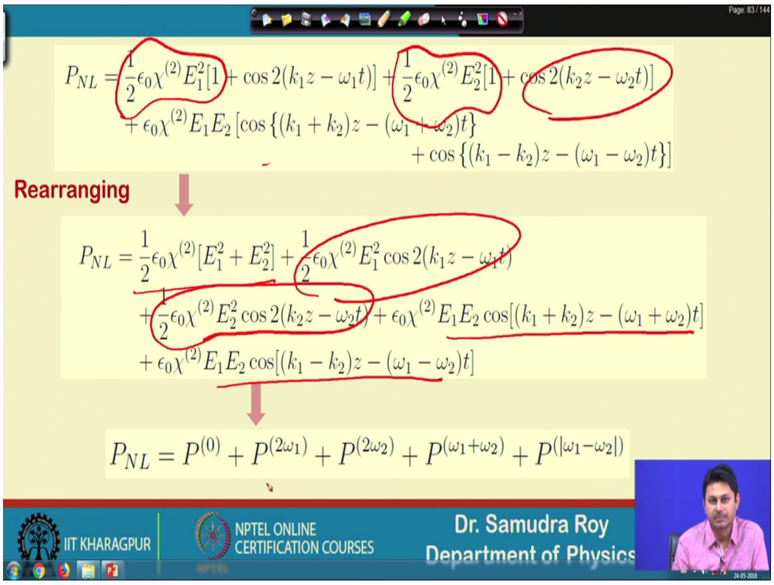

So, now we have already figured out this all these terms. And if I now rearranging; so this is the previous term which we have already mentioned; now I rearranging; so rearranging means nothing but I just take this constant term this constant term or the frequency independent term and add them up I am taking the other term this term and another term this term and it will be giving this term and also the two terms like this and this one is omega 1 plus omega 2 and 1 is omega minus omega 2.

So, if I again if I write these things we have already written in the in the previous slide that P non-linear is now containing a null frequency a frequency of 2 omega 1, a frequency of 2 omega 2, a frequency of omega 1 plus omega 2 and the frequency of omega 1 minus omega 2. So, there are four different kind of frequencies are there when we are dealing with sum and different frequency generation.

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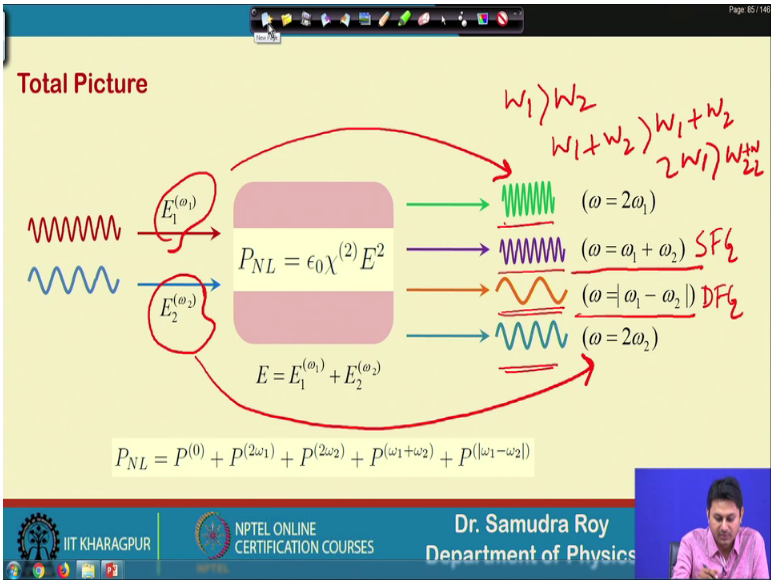

Well, let us try to understand the physics in detail, what is going on. So, here we have the physics. So, how many frequencies are there and we should appreciate that what is going on. So, in the if you remember the previous figure when we are launching just one frequencies you will have either 0 frequency that is optical rectification term or the fundamental frequency that is omega and the second harmonic frequency 2 omega.

So, here we are having something similar. Individually, if I consider the frequency omega 1, electric field corresponds to electric field E 1 we will find the second harmonic of this term. If we consider individually another electric field which is E 2 having a frequency component omega 2 we will going to have the second harmonic of this which is this one. So, omega 1 is a frequency whose second harmonic we will find. So, this is the total picture what is basically going on when two different frequencies are there. Omega 1 will contain omega 1 will give you second harmonic 2 omega 1 omega 2 will give you the second harmonic of 2 omega 2.

Apart from that, apart from that is the interesting thing apart from that if these two frequencies are different, omega 1 and omega 2 there is a possibility that we will have another frequency omega 1 plus omega 2 which is called the some frequency generation and also another frequency which is the omega 1 minus omega 2 which we call the different frequency generation.

If you look into the picture then the frequencies are represented according to their weight age; for example, if this is omega 1 and omega 2 omega 1 plus omega 2 give is will give you some high frequency like this 2 omega 1 will be some frequency. If omega 1 is greater than omega 2 then 2 omega 1 is greater than omega 1 plus omega 2. So, so omega 1 plus omega 2 is greater than omega 1 plus omega 2. So, 2 omega 1 is greater than omega 2. So, this frequency will be higher than omega 1 plus omega 2. So, this frequency will be higher than this. So, that is why the frequency is represented in this way. The difference frequency means the omega 1 and omega 2 and their difference which is small so, that is why the frequency component is shown in this way and 2 omega frequencies are double frequency of whatever we have, so this roughly double.

So, representation was also made in considering the fact that which frequencies high and which frequencies are low. So, this is the total picture, apart from that I can also mention I should mention that two fundamental frequency will be still there. So, this is not only the output.

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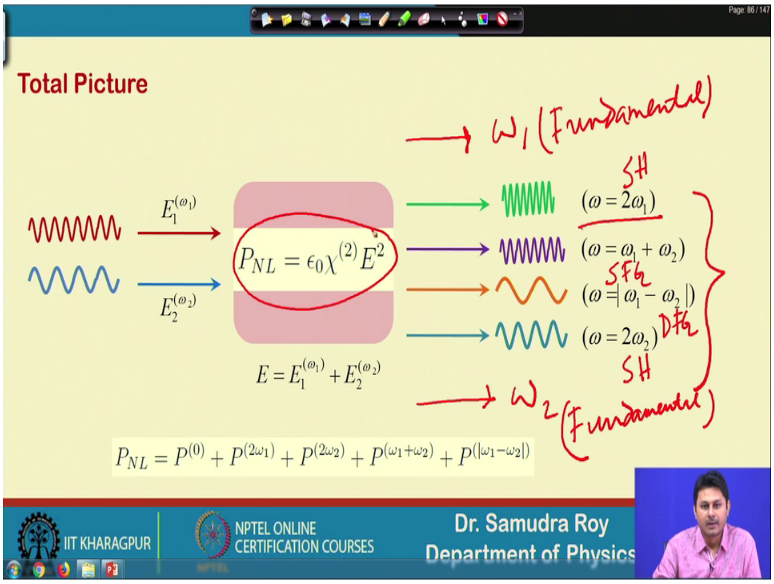

In the output I should also present two different frequencies; one is simply omega 1 that is the fundamental one and another is omega 2 which is also the fundamental one.

So, there should be six different frequency frequencies in principle there, but due to the non-linear effect this additional frequency will going to generate. These are the fundamental frequencies, these are the fundamental frequencies that is always there. But apart from that we have second harmonic of the fundamental second harmonic this is also second harmonic for the fundamental where the fundamental is represented by omega 2. This is the sum frequency generation and this is the difference frequency generation.

So, on total there should be six different frequency component in is there, when we deal with this non-linear polarisation and a very simple way in using very simple mathematics we can figure out which frequency components can be generated; and that is why as I mentioned in the previous class that is why the thing is important this scalar form is very very important. So, here in all the cases invariably I use the scalar form. So, this scalar form is very very important.

So, now the next thing is that once the frequency we know that all the frequencies are in general should be there, but the important thing is that it is not that easy that all frequency will going to generate. If I launched all the two things and readily it will going to generate it not that easy. So, which frequency we will going to generate and which not and how this frequency generation is activated and what should be the efficiency that I will going to generate the second harmonic is the next important thing that we will going to learn in the next class.

And, in order to do that we try to understand the evolution of the amplitude of the different frequency, how the amplitude will going to evolve, the amplitude of the second harmonic, amplitude of the sum and different frequency is going to evolve is very important. So, in order to find out the evolution of the different frequency field amplitude, we need to find out a suitable equation and this suitable equation is non-linear Maxwell's equation.

So, in the next class we will start the non-linear Maxwell's equation and try to understand, how the different components of the frequency or the different amplitude of the different amplitude of the different frequency is going to evolve. So, with this note let me conclude here.

Thank you for your attention. See you in the next class.