Introduction to Non-Linear Optics and it is Applications
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Lecture - 11
Nonlinear Optics: An Introduction (Contd.)

So, welcome student to the next class of Non-Linear Optics and it is Application. So, in the last class; this is the lecture number 11. So, in the last class we learned the overview of 2 important effects in non-linear optics one is called the chi 2 effect and another is called the chi 3 effect. We just give you a very brief outline of what are the topics are there in non-linear optics, that we will going to cover in our future classes.

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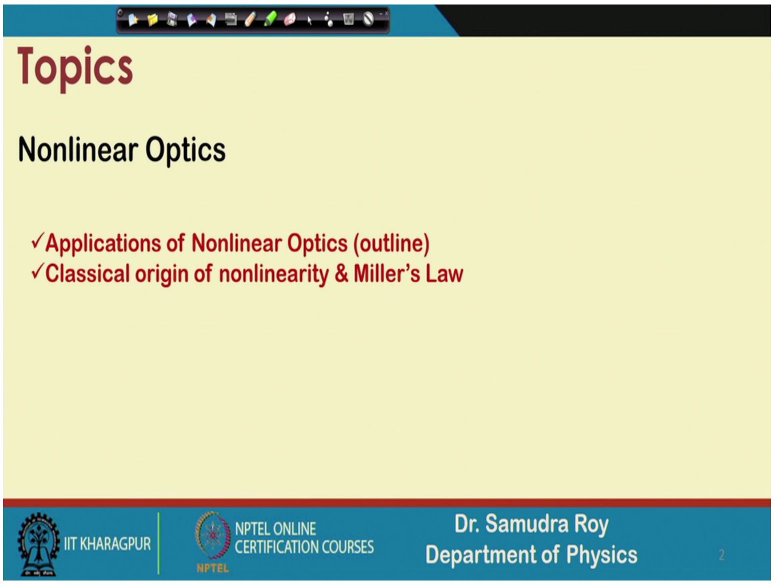

So, today we will very briefly say something about the application of non-linear optics. There are vast applications are there, but I try to make it as concise as possible. We will discuss it in our future classes and then classical origin of nonlinearity and millers law that will be our goal, but this classical nonlinearity and Miller's law. It may not cover to a single class may be in the next class also, we will do the same thing the part 2 of that.

So, let us start with the application of non-linear optics.

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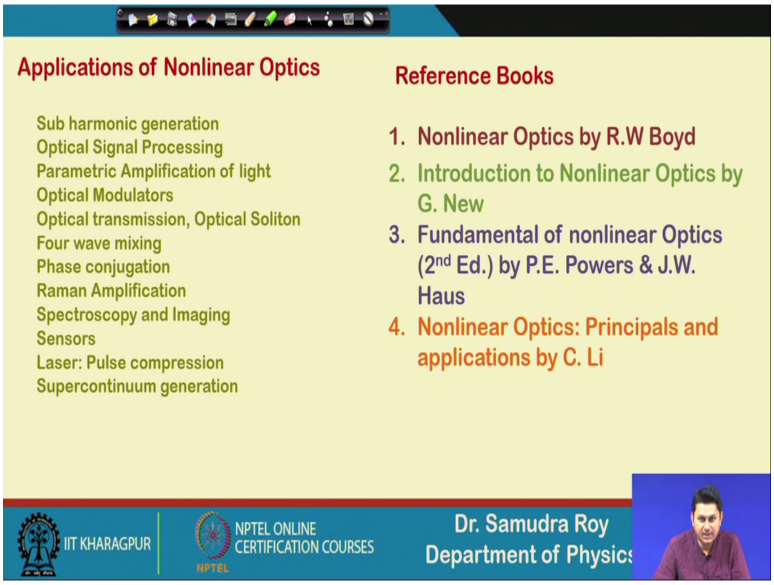

So, there are vast applications are there. Already, we have mentioned that in non-linear optics different frequency components are generated. So, that is an very important aspects of non-linear effects and some harmonics are generated. We discussed in the last class that if I launch a particular wavelength, then I can get a different wavelength or frequency light; when it is passing through a non-linear medium or some non-linear crystals then in optical signal processing.

It is also useful parametric amplification of light; that means, we want to launch, we want to amplify some light we launch the light and then due to some non-linear effect, what happen some kind of energy will be exchanged between another way? We called it pump. So, this pump wave will give some kind of energy to the signal wave and the signal wave will get amplified and all this process are non-linear in nature and this is called the parametric optical amplification.

Also optical modulations are there optical transmission are very important where very important field optical ah nonlinearities applied. Optical soliton is some kind of stable structure, which can be used in optical communication system nowadays. So, a very important concept soliton we know, but optical soliton we will going to cover in this course if our time permits.

Then 4 wave mixing that we also discussed in our last class, that we can generate different kind of wavelengths by mixing 2 or 3 waves through this non-linear process, is a essentially a chi 3 kind of nonlinearity. The Raman amplification will launch a light and in Raman active medium and we get generate the stokes and antistokes wave. So, that some sort of amplification of light is still there.

So, amplification and different frequency generation are the most important aspects of non-linear optics and in different applications. These few features are always use Spectroscopy and imaging sensors laser pulse compression and finally, super continuum generation. These are the similar kind of applications that we have already mentioned, but super continuum generation, the last one is quite important in nowadays.

Because, super continuum generation means you are generating not a single wave length, but a span of wavelength through non-linear process, you launch a single wavelength light when you are getting in the output; you are getting something a very wide spectrum of light whose bandwidth is very, very large. So, different frequencies discrete frequencies are you are generating through non-linear process in second and third harmonic generation or some sub harmonic generation process, but at the same time there is a possibility that you can generate continuous waves or continuous lights in under this non-linear process called super continuum generation.

Well we will discuss all this feature time to time during our course ah. So, today we will like to since our non-linear optics course is officially started from last class. So, it is better that I should mention some reference book to the students and these books are given here. So, 4 books I have given, but your study should be restricted to this 4 books there are ample amount of non-linear optics books are there in the market. So, whatever is preferable what whatever you like you can go with that, but I find non-linear optics by Robert W Boyd is very useful.

So, most of the people like this book because of this simple approach also apart from their that book there are introduction to non-linear optics by Geoffrey new fundamental of non-linear optics by P Powers and JW Haus. This is a very important book, book number 3 normally, I prefer this book because the second edition is very, very nicely written and very elaborately present a different non-linear effects. So, I really suggest book number 3 for the students.

But also you can look introduction to non-linear optics by G New and you must look the RW Boyd by non-linear optics another book is still there. Non-linear optics principle and application by C Li this is also a very interesting book if you want to read. You can read these are the 4 reference books uh. You can follow during the course and all most of the most of our lectures and all these things the concepts and the topics are taken from this 4 books.

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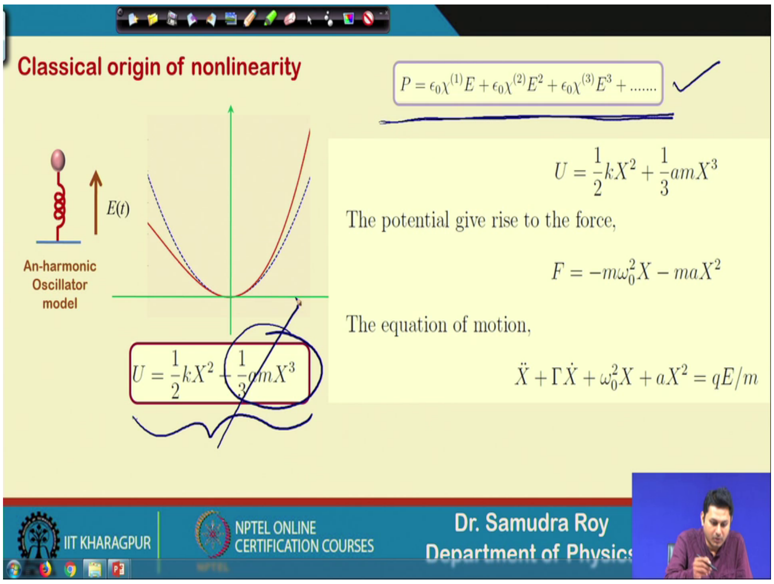

Well let us go to our next important topic which is classical origin of nonlinearity. So, today we will learn in this course or in this class rather that how the nonlinearity is generated inside the system. And how classically you can explain these phenomenon which are very interesting.

So, first we will we will write our non-linear polarisation equation, which is this is our major equation that we have been using for last few classes. Thus, polarisation is a function of electric field, when the electric field is very high then the polarisation can be written in terms of electric field in this fashion, So E square E cube all this term will be there inside the polarisation term. So, these are the higher order terms So when you have a very high electric field, what happen? Let us go back to our classical model, the Lawrence model. In Lawrence model, what happen that in the electrons are considered as a vibration of under harmonic oscillator by some electric field.

So, if the amount of electric field is very high, what happen the restoring force that was followed by Hookes law will not be true anymore? So, that is why some kind of an harmonic term will be there. So, as it now the electrons are vibrating under some an harmonic oscillation. In that case, the potential you can see the potential of the system can be written only half k X square rather you should add another term which is this one. This is the next higher order term that you should add when our system is an harmonic in nature.

So, our goal is to find out if the electric field is very high and system is an harmonic then, how the Lawrence model will be modified? So, so far the Lawrence model take care of everything by removing these term. So now, what we are doing? We just include this term in the potential and try to find out what will going to happen, when this extra term is there and how the vibration of the electron will be modified. Well under this potential the very next thing is the force.

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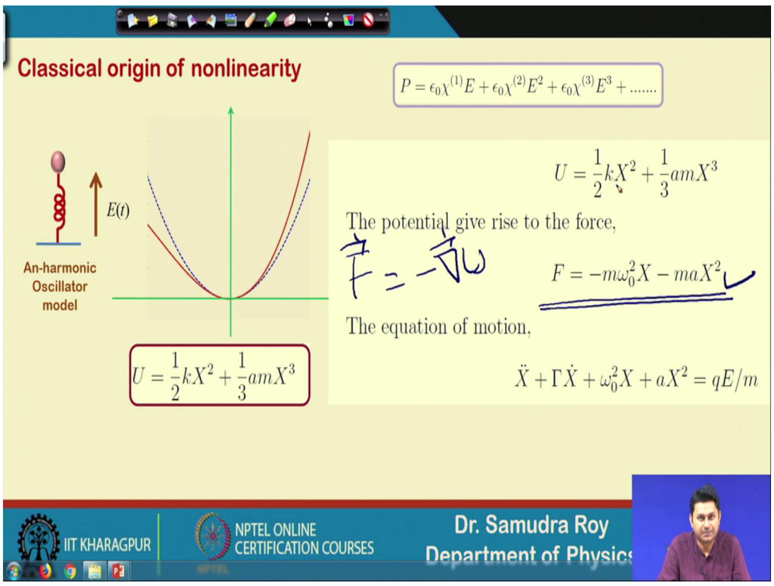

The force is nothing but if I write force in terms of potential it is nothing but this if I write a vector sign it will be grad of U. Since, it is a one dimensional just to you make a derivative with respect to it, both the side with negative sign and we will have this kind of term this 1 by 3. So, this is X cube so these 3 1 by 3 will be absorbed and we will eventually have a m then X square. Also the first I will have kX, and this k will be replaced by m omega 0 square. Where, m omega 0 is nothing but the resonance frequency of the system.

Now, the important thing after modifying this force term. Mainly, this force term is modified because of this an harmonicity. We have the total equation of motion in our hand.

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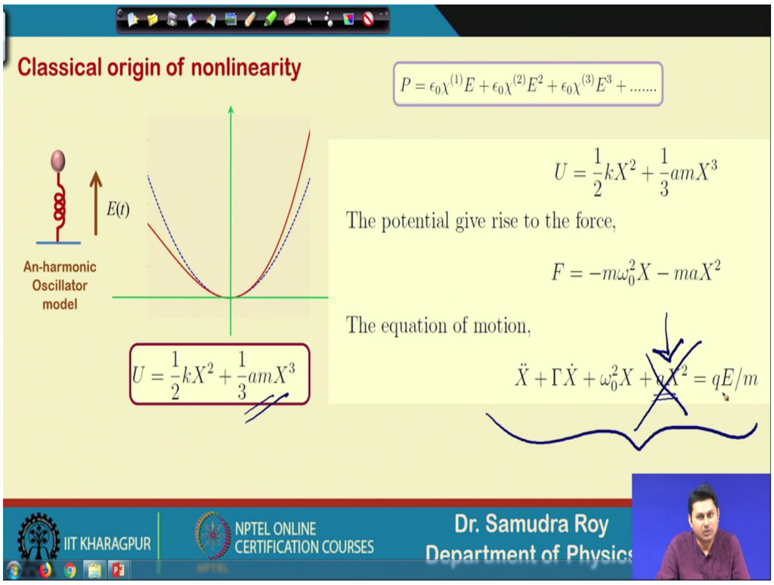

And if you see that this is the additional term we have the term that is depending on a is the additional term because of these quantity. This an harmonic term in the potential basically gives me the extra term in the equation of motion.

Now, we know that in absence of these when a equal to 0; that means, there is no an harmonicity in the oscillation or my model, my oscillation model is harmonic in nature this term was not there. So, we already have solve this equation. So, let us go back to that the question is what happen when this kind of terms are there. So, let us go back to the next slide well.

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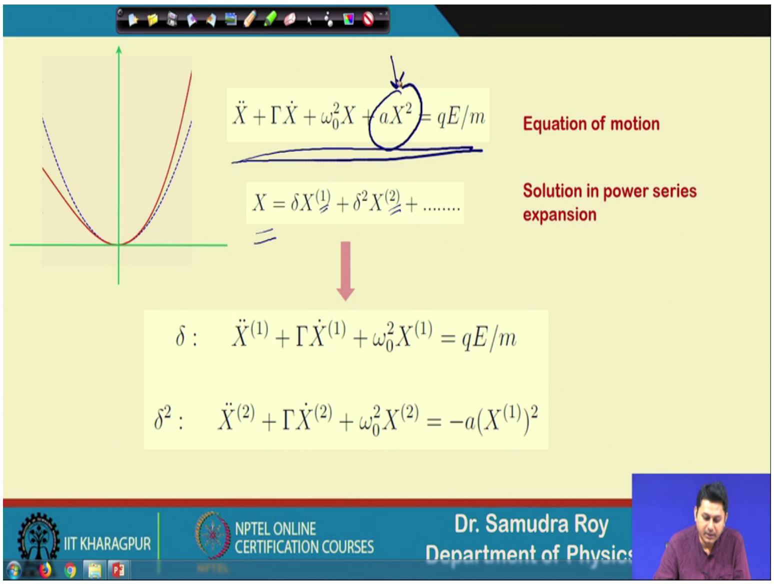

So, this is basically the equation is written here. This is our governing equation, equation of motion and in order to solve what is the value of X? Under this additional term, what we will do this is a very important step please concentrate on that. We write our X as a power series solution we occasionally do that. So, in power series solution delta is a term there is a weightage factor and the first term is delta X1 X1 X bracket 1 gives me the first order solution and then the additional stuff is a second order addition of in in order to delta square and so on.

So, we make a series of X with the weightage of delta, delta square and try to find out what is happening, when these additional terms are there. So, we will find that when a equal to 0, we will have only our fundamental equation and when a is not equal to 0 we have we should have our next term.

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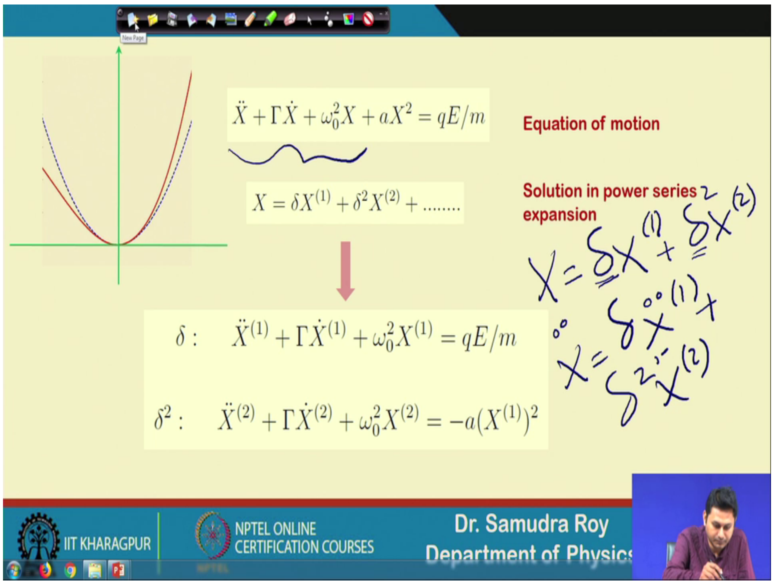

So now, if you put this solution. Next step is if you put this solution here, this is my solution. I will going to put this here. So, then what happened? There should be some delta square dependent term. So, forget about the higher order terms. So now, what we are doing my X is now replaced by only 2 first order term 1 is this and other this ok. Let me write once again. So, my big X is equal to delta big X1 plus delta square big X2.

So, when I put this things here So first we need to make a X double dot when you make the X double dot nothing will happen here. It will be just X double dot of 1 plus delta square X double dot of 2. In the similar way, in the next term, in the similar way, in the third term, all these things are only just add we will just going to add these things one is delta dependent term. Another will be delta square dependent term. I emphasise that, this is the very important exercise. So, I will like to I will like to say the student that please do this things by your own hand this is a very simple calculation, but very important one ok.

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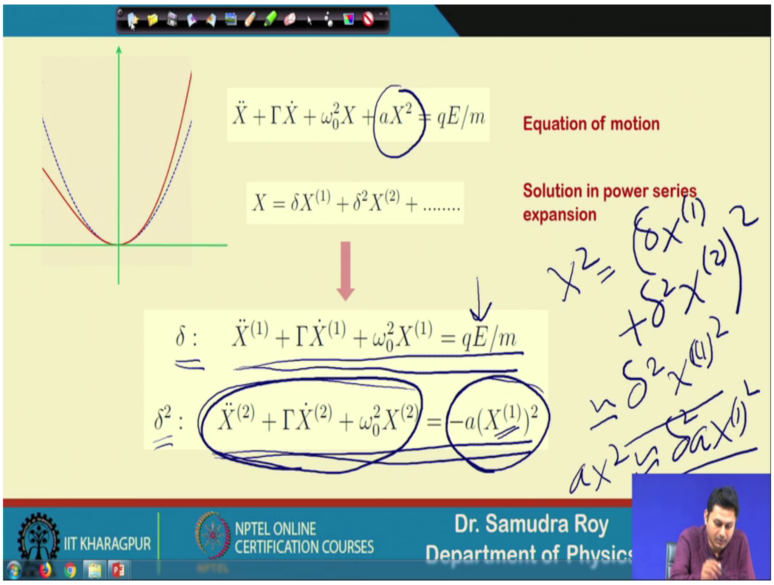

But the important thing lies here. What will be the X square term? So now, you can see that when I write X square, X square is nothing but delta X of 1 plus delta square X of 2 square of that. So, this X square term basically give rise to delta square then delta to the power 4 and delta cube and so on.

So, if I only restrict to delta square term then I will have one term which is delta square and then X1 X1 square. So, this is my first term and all the term is higher of delta square. So, I will not going to take this. So, this a X square term basically gives me delta square into X1 square of term

So now the next thing is that to separate out the value of the solutions and with the weightage. So, weightage of delta will give me the first order equation which is nothing but the solution, without any kind of an harmonicity; that means, the a term is not there. So, we have our old equation that we are using when our system is totally harmonic, but delta square term we find that we have the second order solution or X2 solution, but in the source we have a new term sitting here.

this is very, very important here, the source term is externally electric field, but because of this an harmonicity what happened we have an additional source which is depending on the value of X1 and acts as a source which basically govern the rest of the part of X2.

So, X2 will be governed by X1 so, this is very important that we have.

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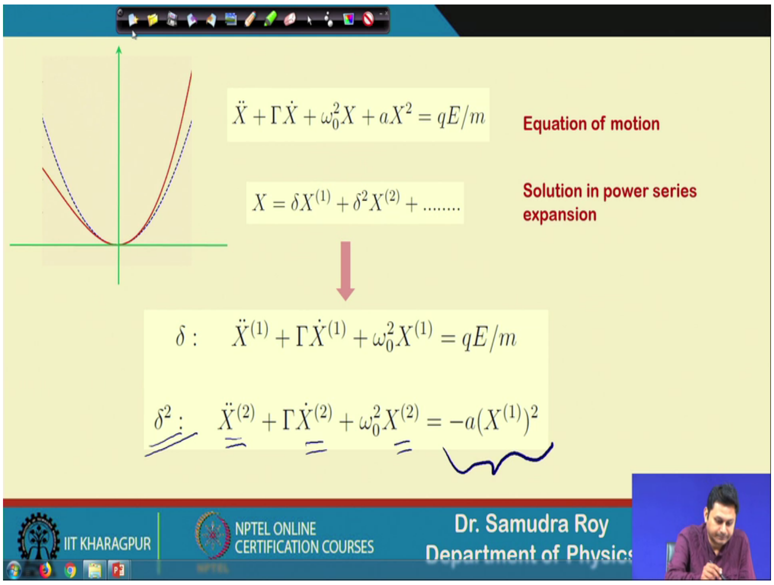

A source term which is in terms of X1 square and that is a governing term or the source term to find out X2 and that is coming when we take all the values that is related to delta 2 ok. So, next what we will do let see.

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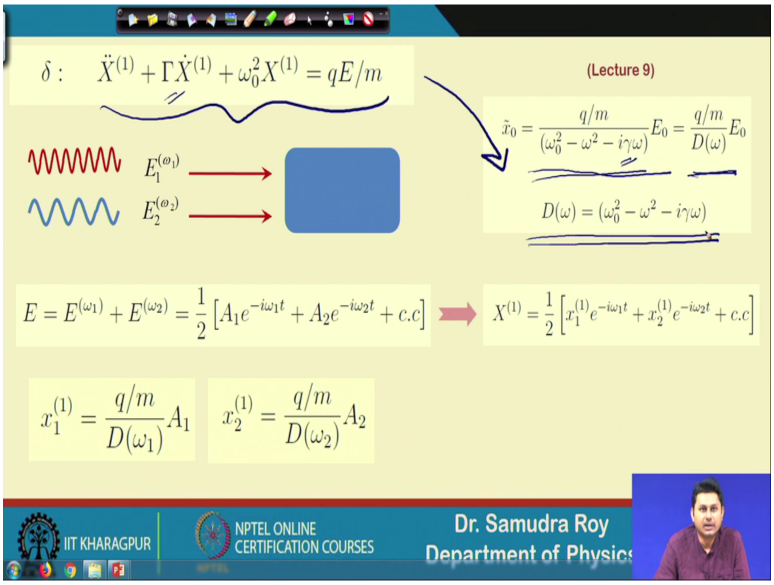

So, next I try to find out the solutions. So, the solutions of related to delta equation related to delta is straightforward because we have already have this kind of solution before in lecture 9.

So, in lecture 9 if you go back to your lecture 9 what you will find that, the solution is simply if I place E as E 0 E to the power I minus omega t the solution will be order of X0 e to the power I minus omega t and if you put these things then X0 will come out and this X0 is nothing but q divided by m. And in the denominator, we have this omega 0 omega square and this damping term and here there in that case the damping coefficient was written by gamma. Here it is written by big gamma, but that does not make any difference.

So what happened that, we will have a solution in the form of e q m and a factor D omega like this D omega is nothing but this quantity that we have already explained earlier. So, that was the solution we know. So now, what we will do? We will change the systems slightly instead of launching one electric field. Now, I am launching 2 electric field with 2 different frequencies.

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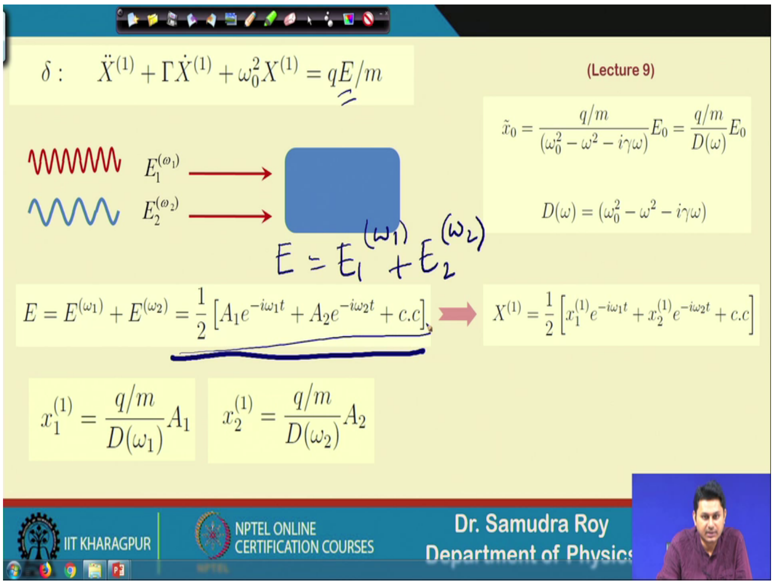

So now, my total E here the total E is nothing but E1 of omega 1 plus E2 of omega 2;

that means, 2 electric field with 2 different frequencies is there.

So in this figure, we have shown how these 2 frequencies should look like. So, omega 1 is the frequency which is greater than omega 2 as far the figure suggests, but both the things are there. So, total electric field is written in this form as I already mentioned it is written in this particular form. So, this is the total electric field and this total electric field has a amplitude part a and A1 and A2, which corresponds to the frequency omega one and omega 2 and the corresponding complex conjugate.

So, what happen if I launch the electric field with 2 different frequencies? I should have the solution in the similar way that we have done here in the previous case. So, my solution here is now.

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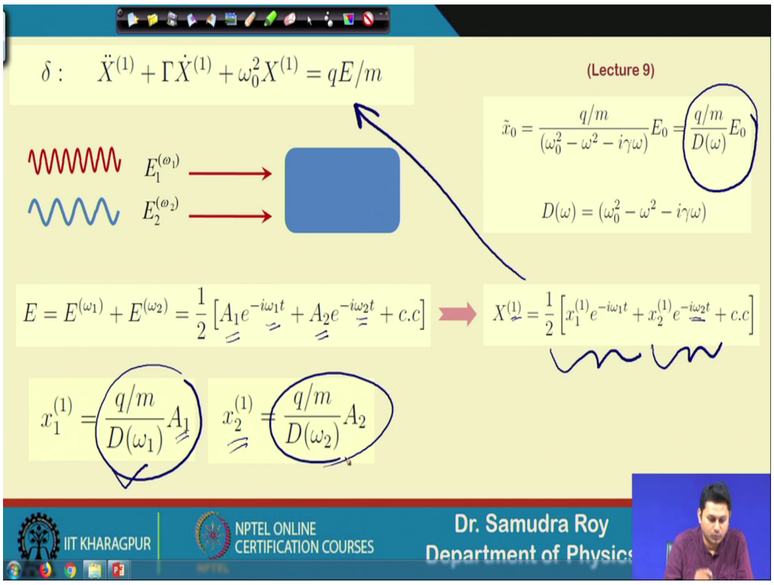

Not a single X, but 2 Xs correspond to 2 different frequencies. So, the first omega1 frequency I write according to my nomenclature X1 and then someone here to ensure that this is the solution of x big X1 and X2 is 1 and this 2 is suggest that it is related to the frequency component omega 2.

Now, if I put this solution here again and extract the 2 frequencies. Then separately it is possible to find out my X1 and X2 and this X1 I can write in the similar form that we have already figure out here. So, we have already figure out this things that X0 should be the form of q by m divided by D omega E0.

E0 is the amplitude of the field here we have exactly the similar form q divided by m d of omega one here the frequency was omega, but here the frequency is omega one because X1 is related to the frequency omega one it should be there and also omega one amplitude is a one. So, I should put a one in a similar way I should have X2 with the same notation just one should be replaced by 2 ok

So now we have 2 solution in our hand when I am launching 2 electric field and X1 is represented by this. So, I have all my solution in our hand. So, X1 X2 is known X1 is the component of frequency omega one X2 is corresponds to the frequency component X2 omega 2. So, here it is very important that you should appreciate how the frequency components are separated out and what is the corresponding frequency component and so on.

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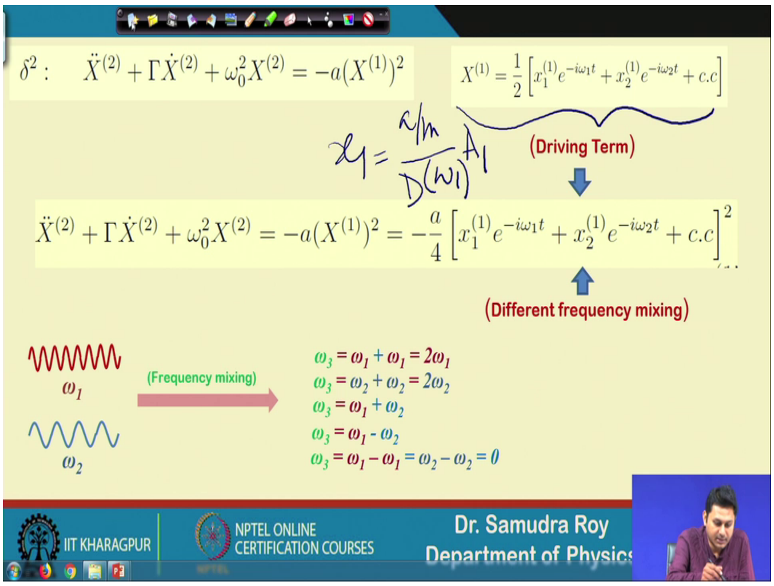

So in the next slide, what we do after having the knowledge of X1 we will going to find out what should be the frequency of what should be the equation for X2. X2 is the second order solution and the nonlinearities appearing because of these 2. Because, it is a fact that when there is no an harmonicity, the only solution is X1 and this X1 we have already figure out in our last slide.

So, this is my X1 that we have already figure out where X1 and X2 is of the form X1 is of the form q divided by m, divided by d of omega one multiplied by A1 and for X2 it is everything same except omega one should be replaced by 2. So, all the solutions are there.

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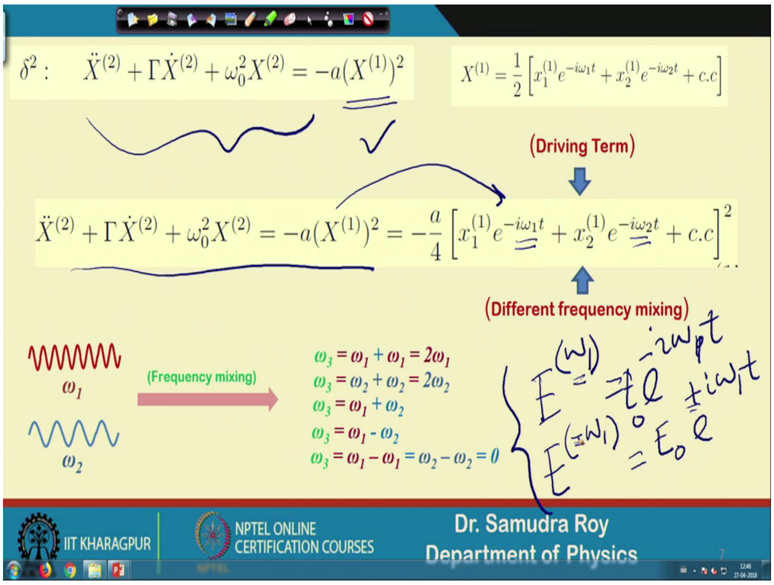

So what we will do we have these Value in our hand. This is the source term and now we need to find out if I put this X1 into the system. This is the differential equation for X2. What will happen? So, this is the differential equation now, what I do that, I put this here. So, this is my driving term, but inside the driving term we find that, there are different frequencies are there omega1 is there, omega 2 is there, also the complex conjugates are there; that means, minus omega 1 minus omega 2 are still there.

One thing you should know to that when I say omega frequency component I write minus of I omega this is just the notation when I write E as a frequency component of omega 1. It should be E to the power of I some e 0 will be multiplied here by notation minus omega 0 omega 1 t this is the frequency component of E omega 1. If I do the same thing and if I write what should be the component of E minus 1 then it should be the complex conjugate of that quantity. So, it will be plus of I omega one t this is the notation.

So when I write this, it essentially mean this is omega1 component, this is the omega 2 component and complex conjugates are minus omega and minus 2 component. But please do not confuse with this minus sign. With this minus sign, I write this is a omega component and when I write this is a plus this sign will be automatically minus ok.

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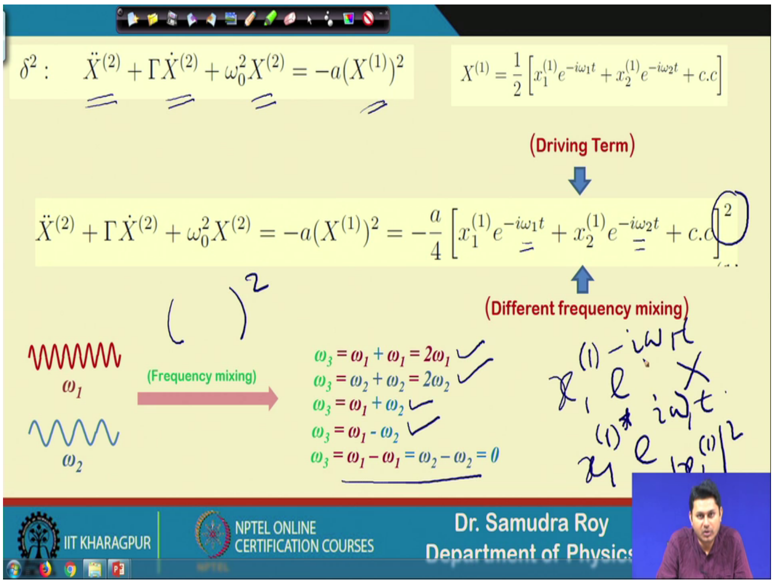

So, once we have the source term in our hand and you can see that there is a very important term. Sitting here which is square so; that means, different frequency components will be there in our source term because it is square of that. So, electric fields are now vibrating in. So, heavily that my potential is now become an harmony the harmonic part I calculate instead of launching one frequency now I launch 2 frequency to make the system more general because is a very important thing.

And then we find that the second order effect; that means, solution in the second order solutions which is X2 has a source term related to first order solution and this first order solution has a square term and the first order solution also have 2 frequency component when it is square then; obviously, there will be many frequencies are there.

So, let us understand with this schematic figure. So, omega one omega 2 this frequency was there and what we are doing we have something in there with the square term. So, if I make a square term of this quantity if you do that by your own hand, you will find there are 4 term associated with that. So, we will have the solution in the next slide, but you will have different frequency components.

So, omega 3 is the frequency that we will going to generate because of the squared term. So, there are different kind of frequencies one can generate. When you just square that you will find that omega 2 omega one will be there immediately. So, 2 omega term will be there 2 omega 2 will be there. This is the second harmonic terms also the an omega 1 plus omega 2 will be there.

When you multiplied this by the another term having the complex conjugate of this term we multiply then you will get this things. Also, we will get this term which is a complex conjugate of the previous 1. This is the difference term and also there is a possibility will not going to get any kind of frequency term at all. This is the optical rectification term.

So, here when you make a square the complex conjugate is there. So, the complex conjugate multiplied by this term will absorb these exponential term and as a result you will have. So, the term will be X1 1E to the power of minus I omega I t multiply there will be many terms, but this is one of the terms that you will have star of that E to the power of I omega1.

And t when you multiply these 2 things you will eventually have mode of omega1 square only these exponential I omega1 t exponential minus I omega1 t will cancel out. And you will not going to get any kind of frequency term associated with that ok. So, this is important that how to find the frequencies. So, let us try to find out how to find the frequencies.

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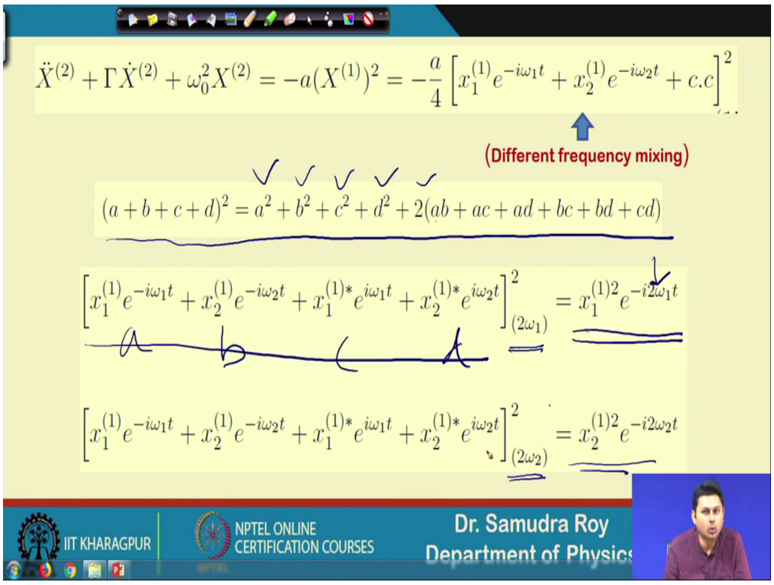

So, this is our total term total equation different frequency mixing will be there. It is a b c d complex conjugate of these. So, there will be 4 terms as written here this is will be the 4 term in order to use different frequencies. What I will do? That, I will use the simple formula you have already used earlier, a plus b plus c plus d it is exactly the same thing a plus b plus c plus d square of whole square of that. If you do, we have a square term, b square term, c square term, d square term and combination of term with 2 factor.

So, here we are doing the same thing a b c d we are making square of this term and try to find out what is the frequency which is related to 2 omega. So, if I extract the 2 omega frequency out of that you will find the 2 omega frequency will be the frequency that will come when I make a square of these things; because, a square term basically gives me 2 of omega1 any other term will not going to give this thing.

So, a square term is basically give me this quantity X1 whole square E 2 the power of 2 omega. So, here we can see the frequency component is 2 omega. Here in the similar way, you have 2 omega 2 term and 2 omega 2 is nothing but, the b square term. So, you will have d square here.

So, I am just taking out the different frequency component there are many terms you can see 1 2 3 4 5 6 7 8 9 10 there should be 10 different terms with different frequency components. So, here I am just extracting those terms well next.

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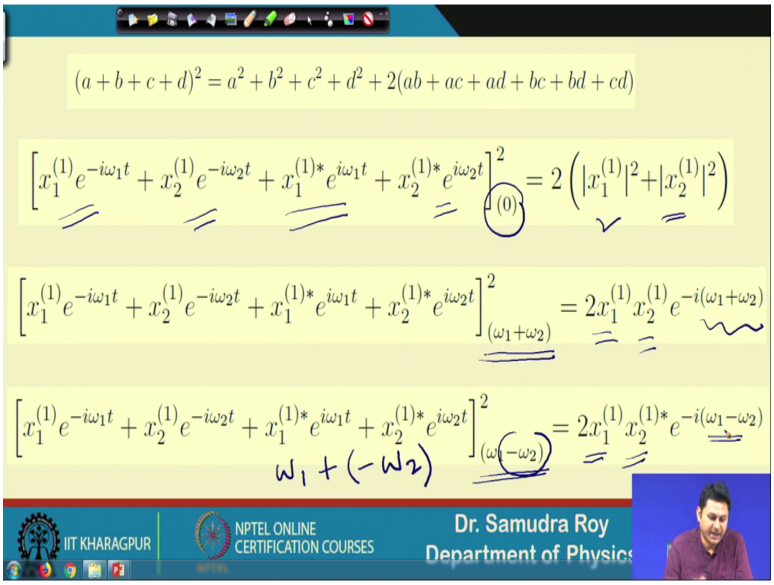

I will continue this process, so that you are familiar with this. So, if I want to find out the other terms that what should be the frequency component of 0. Then you can see that, if I multiply these multi into this then you have this term and if you multiply these with these you will have this term. So, these both these term should not have any kind of frequencies, because this plus I and minus I will going to this plus I and minus I going to absorb this minus omega 2 plus omega 2 I am going to absorb I am not going to get any extra term.

Also there are few other terms which is omega 1 plus omega 2 as I mentioned earlier an D omega 1 minus omega 2. So, omega 1 plus omega 2 we will find simply by multiplying X1 and X2 and you will get omega1 omega 2 in the similar way omega 1 minus omega 2 you will need to multiply X1 and X2 complex conjugate of that because, you have a negative sign here. So, I can write it is omega 1 plus minus of omega 2. So, this minus sign will come because of the complex conjugate. So, you need to multiply the complex conjugate and you readily get omega 1 minus omega 2 out of that ok.

So, I believe you understand if there are 4 different kind of frequency term is like that. If you make a square or cube how to extract different frequency component because, when you make a square of that all the frequencies are there inside the system and you need to be very careful, what should be the corresponding amplitudes here the amplitude of omega1 omega 2 is X1 X2 amplitude of omega 1 minus omega 2 is X1 X2 star. So, this is very important.

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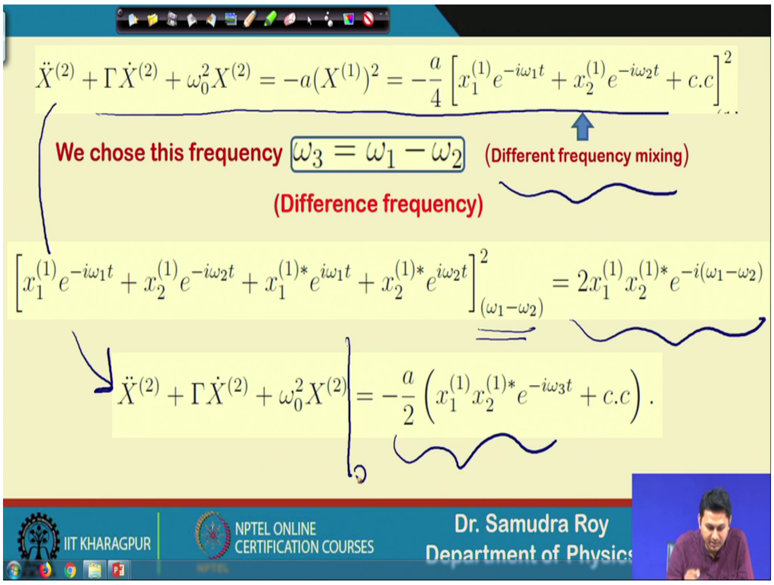

So, in the next slide what we will do, that we need to find out we just take a frequency, which is the different frequency we just do our calculation now we will do our calculation. There are several, this is our another equation I need to solve this equation and what happen that in order to solve that there are many frequency components are there.

As I written here different frequency components are there. So, I just need to choose one particular frequency and for example, just I choose omega 1 minus omega 2 which is the different frequency one can choose omega 1 plus omega 2 also. So, it depends on what frequency component you are choosing from the right hand side so that you can write only one term here.

So, omega 1 minus omega 2 frequency will give rise to this term that already shown in the previous slide. So now, my differential equation is having a form where we have these as a driving force there are many frequency terms are there. So, I basically try to find out the evolution equation of X 2 which is at the frequencies say omega 3 so, let me do that.

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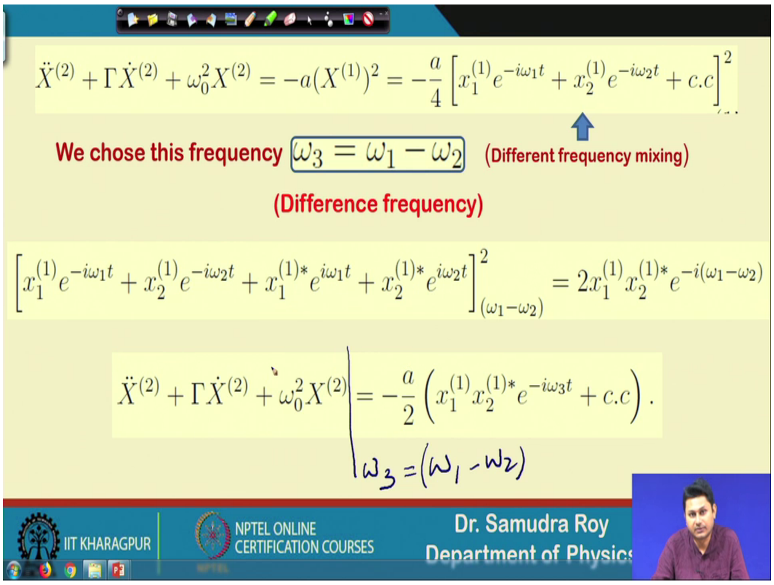

So, everything is at frequency omega 3. So, omega 3 is a frequency which is omega 1 minus omega 2 in our notation. So, I just extract what is the driving term and now try to find out what is the value of X2. The dependence of X2 of omega 3 since this is vibrating into omega 3. The X2 will start vibrating at omega 3 frequency it will also vibrate other frequencies, but my driving term is a vibrating at omega 3. So, X2 will also going to vibrate at omega 3.

So, this is the driving term. So, today I will like to conclude here because, the next calculation again we will take some time. So, today our time is restricted. So, in our next class what we will do we will start from this equation and try to find out how the solution. One can find and this solution is basically a different kind of the same kind of solution that we find in an harmonic. In harmonic case and how this harmonic part and an harmonic part can be correlated is important part that we will do in our next class. So, with that note, I will like to conclude here.

Thank you very much for attention. So, see you in the next class.