Introduction to Non-Linear Optics and it is Applications
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Lecture – 01
Basic Linear Optics

So, welcome student to this Introduction to Non-Linear Optics and Application Course. In this course, we will learn about the non-linear interaction of material when an incident light is on that material. So, the material response here will be non-linear and we will like to learn how because of this non-linear response of the material, how light will behave inside such systems? So, in non-linear optics, we know that the intensity of the light should be very high. So, normally, we deal with laser which is nothing but an electromagnetic field.

So, electromagnetic theory is essential part of this particular subject. So, before going to the non-linear optics part, so let us try to learn about some basic optics which will be very important here in the initial stage.

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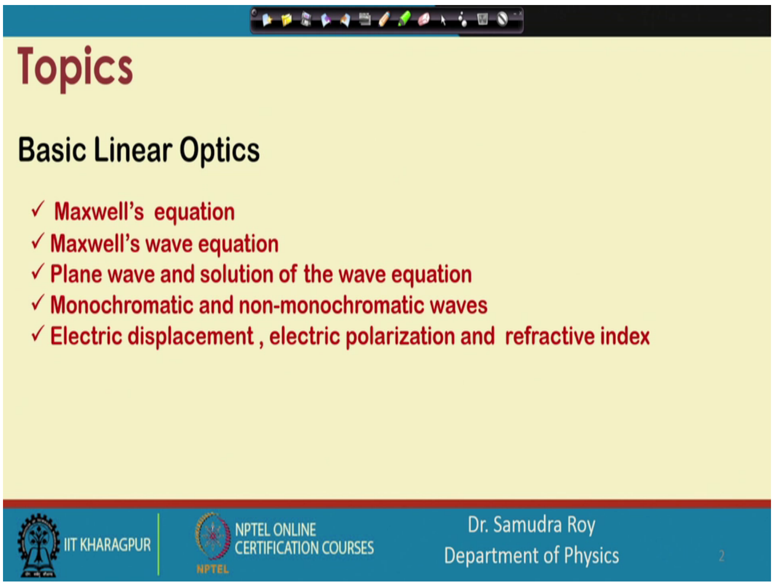

So, let us let us go back to these slides. So, in the first part, we will like to learn the Basic linear optics as mentioned in this slides. We like to learn the Maxwell's Equation, then Maxwell's Wave Equations. We will like to learn; what is wave equation and then plane wave and solution of this wave equation. So, plane wave is a very important thing and we need to know what is actually mean the plane wave?

Then monochromatic and non monochromatic waves and finally electric displacement, electric polarization and refractive index; these are the few things that we will like to learn in this particular basic courses ok.

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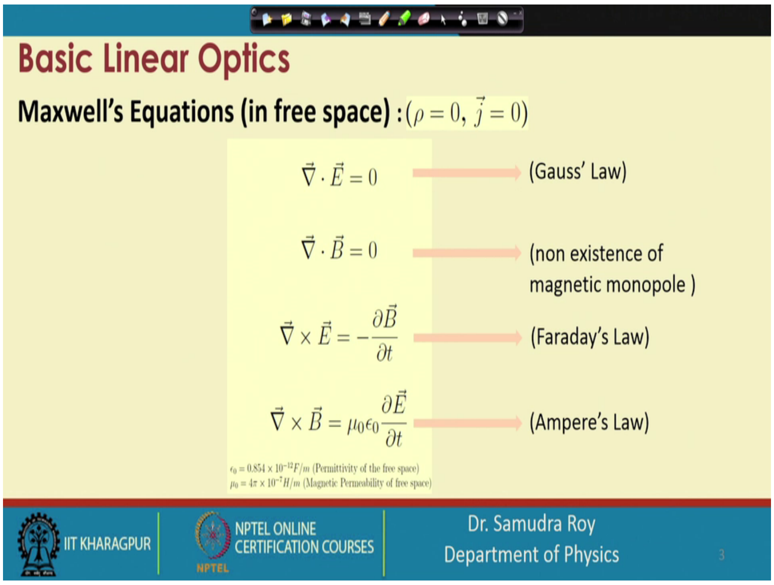

So, our first thing is Maxwell's Equation; so, in Maxwell's Equation, we know there are four different equations are there. This four equations are individually called Maxwell's Equation but this four equation contain four different laws. The first equation, grad dot E equal to 0 here, because this is written in the free space and we know that when I, when we write these things in the free space, the charge density rho and the current density j will be 0.

So, the first equation here is grad dot E equal to 0 stands for the Gauss's law. Second equation, grad dot B equal to 0 is something which also caught the Gauss law in magnetic system but these as a physical significance, this laws has a physical significance and the physical significance is magnetic monopole does not exist. And then, we have a couple of equation with cross term; that means, curl of these E will be equal to minus of del B del t and curl of B is equal to mu 0 epsilon 0 del E del t. By the way, mu 0 is a permittivity of the free space and epsilon 0 is a permittivity of the free space and mu 0 is the magnetic permeability on the free space. These twos are constants of the systems.

Now, if I see this last week equation, they are very important because these two equations directly relate electric field and magnetic field E and B is related through this two equations. So, that is why this two equations are very important. The third equation is related to Faradays law are already shown in the slides and the last equation also known as Amperes law, this is a one form of amperes law where magnetic field and electric field if again related. In the first equation, in Faradays law, we can see that the change of magnetic field can generate some kind of electric field or vice versa. In the second case also, we have a similar kind of effect.

I believe the students are aware of these four equations, I will not going to say much about that because this is not that course. This is a basic course of non-linear optics. So, these are the few things we need to know and Maxwell's Equation is one of them.

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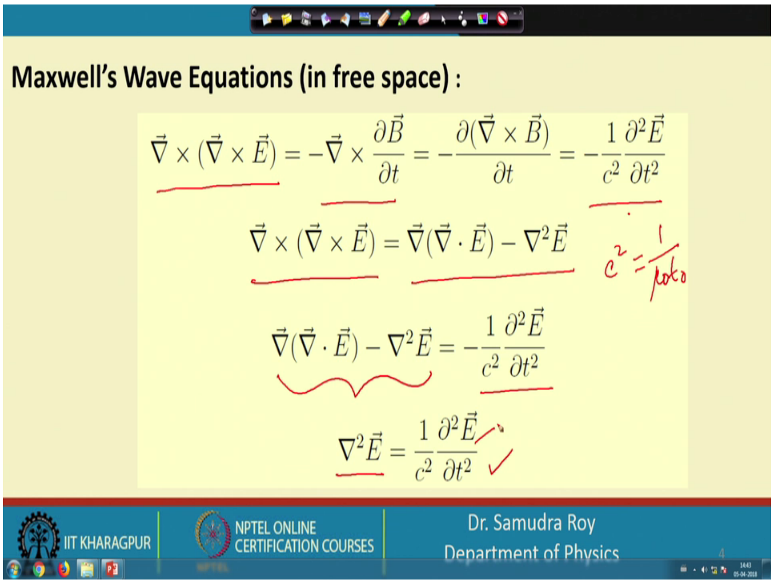

So, let us now go back to the next thing that is Maxwell's Wave Equation in free space. So, when I talk about the Maxwell's Wave Equations, so that means, we are try to derive some kind of equation that produce or the solution of which produce some kind of wave.

Here we are doing the thing in a very straight forward way. I believe, most of the students are aware of that but still let me explain how this things one can generate this wave equation one can generate using the four basic Maxwell's equation. In the first line, I can see that curl of curl of E; that means, curl of E was already there and I make a curl over that and the right hand side we know that is curl of E is del B del t. So, since I operate curl both the side, so it should be curl of del B del t with the negative sign. This is the first step.

After that, we just use curl cross del B del t as del del t of curl cross B. So that means, the operator here is now exchanged. If you look here, you will find that first the curl operator was there and then the derivative operator or del del t operator was there. So now, what we have done here, we just exchange this operator. So now, one can exchange this kind of operator; that means, first I will make the curl and then derivative. Here what we are doing, we are making the curl first over b and then make the derivative; one can do that when these curl operator and t operator are independent to each other.

In this particular case they are independent to each other. So, it does not matter that which operate operator I operate first. So, that is why I can do these things and when I do this things, so curl cross B is now replaced by our next equation or the fourth equation. If I write these things to del 2 E del t square, then there is a term mu 0 epsilon 0 should be here. So, mu 0 epsilon epsilon 0 term is now replaced by 1 by c square. Now the next thing is, now the next thing is this curl cross curl cross E, I will like to replace this or expand this in this form. This is a well known expansion that how a curl cross curl cross vector is expanded. The first term is grad dot this minus grad square E.

So, this is a well known vectorial equation we use in almost all the cases. So, we are doing these things here. After doing that, so left hand side I replace with this whatever we have and in the right hand side we can write minus of 1 by c square del 2 E del t square which was already we figure out. So, what is the thing, we have started with equation curl cross E equal to minus of del B del t, then make a operation curl operation both the side. After making a curl operation both the side, I use curl cross B equation. So, curl cross B is nothing but mu 0 epsilon 0 del 2 del E del t, that is the fourth Maxwell's equation. So, I use this equation and replace mu 0 epsilon 0 by 1 by c square.

Because I know that mu 0 c square is nothing but 1 by mu 0 epsilon 0. So, I replace this. After replacing that, I will get this equation. So, finally, I have the equation grad square E is equal to this. So, this is nothing but the wave equation, this is nothing but the wave equation. The form of wave equation is something which is same. So, from Maxwell's four equations, I derive Maxwell's wave equation and when I do that we find that this 4 equation can be adjusted in such a way that I can have a wave kind of thing, so that means, here the solution is nothing but the E. So, this E should behave like a wave. That is the most important thing because this wave equation suggest that electromagnetic radiation or whatever we are aware of is nothing but a electromagnetic wave. So, light is nothing but the electromagnetic wave ok.

So, let us go back to the next slide yeah.

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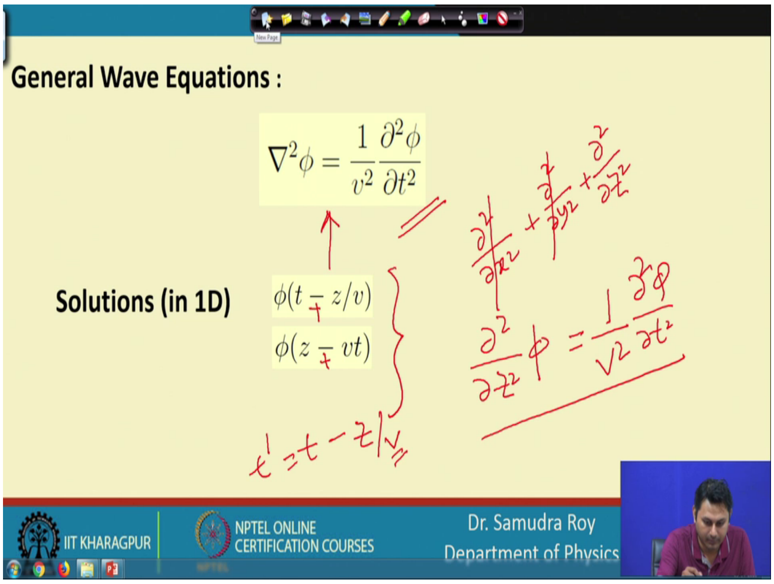

We were mentioning about the wave equation. So, this is the generalized wave equation. So, this is the generalized wave equation.

This equation is the generalized wave equation. This equation is a generalized wave equation, grad cross phi, this is a generalized form and v here is nothing but the velocity, t is a time and this operator grad square operator, we know that in 3 dimension, this operator is something like this. This is the operator, the operator form and it will operate over some function phi which is a function of x y z and t and if we have this kind of expression in our hand, then we called phi is a wave. Why phi is a wave? If I make this equation one dimensional; that means, if I remove this term and this term then this particular equation whatever is written here, this general wave equation will have a form like this.

This particular form has a specific solution. So, here two solution is written. These two are one of the solution. There are also many solutions in terms of this minus sign. I can also put a plus sign which is also another potential solution but this minus sign suggest that the wave is put moving in the positive direction, anyway. But the important thing is that, if we have this kind of equation, then we have a solution of the form like this. So, you can note that we have a variable t, we have a variable z but inside the solution one can have a new variable t and z is associated with new variable.

So, you can find that t prime is a new variable which is related to this and also velocity is associated. So, I left this problem as a homework to those were doing this particular course that using this solution, you just check whether this is really a solution, using this form of solution, just check whether you are really getting this particular solution or not. So, just change the variable t prime is a one one variable and just change the variable and readily you figure out that you will get the solution. So, wave equation has a specific solution and the solution t minus z by v or z minus v t, both are the solution of this system and which suggest that this is really your wave ok.

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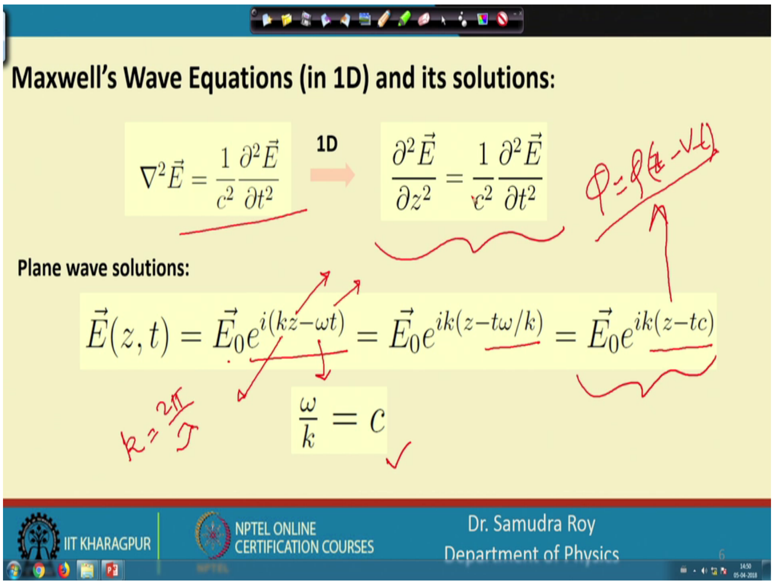

So, now we will like to have the solution in our case which is Maxwell's equations. So, we have already derive the Maxwell's equation in free space which is already shown here that this is our Maxwell's equation in free space. If I make it 1 D form as I already do in the did in the previous slides, that replacing this grad square operator to del 2 del z square in 1 D means I have use just one variable x y z is a 3 variable and 3 D system normally we used that. But here what we are doing that we use one variable and this one variable is z and when we do that we find this equation exactly looks like the general wave equation and the previous case, we find that any solution of this general wave equation have a solution, general solution like phi is equal to phi of z minus v t. This is the general kind of solution, one of the solution.

Now, if we if this is the case that these has a solution like this, then we can assume a solution in this form; that means, by variable should be something like this. So, I assume a plane wave solution. This is called a plane wave solution. E is a function of z and t, E 0 is amplitude and we have e to the power a term k z minus omega, t very important term. Why it is called plane wave, we will discuss shortly, but here, first we need to understand why this things is a solution of this Maxwell's wave equation. Now, what we have done here k z minus omega t, this is a form we are using, this a very important form. Here k is a propagation vector, z is a distance is a variable, omega is a frequency of the system and t is a time.

These four things are there and if you look carefully, you will find that this is a exponential term. So, this entire thing should be dimensionless. This is a frequency whose dimension is 1 by t. So, that should be multiplied with t, so that these things become dimensionless; z as a dimension of centimeter, meter or kilometer or the length dimension and k is a propagation constant and normally represented by 2 pi divided by lambda. So, lambda is a unit, has a unit of length and if I multiply k with z, again we have a dimensionless quantity. So, this is something important, quite trivial but important. Next what we did?

We take k common and then my rest equation become z minus t divided by t omega and K, omega divided by k. So, omega divided by K, we know that this is nothing but the C. Omega divided by k is C. So, I just replaced here z and omega divided by k replaced that with C. So, you can see that this function, e z t can be represented as a function of z minus, in this case t c. If you compare that, so you will find both the things are same; that means, this particular expression should be a solution of this equation. This particular form of solution or form of expression will be a solution of our Maxwell's equation. So, in the next slide, we will directly try to solve these things and let us see that whether really we will have a solution or not, this is really have a solution or not.

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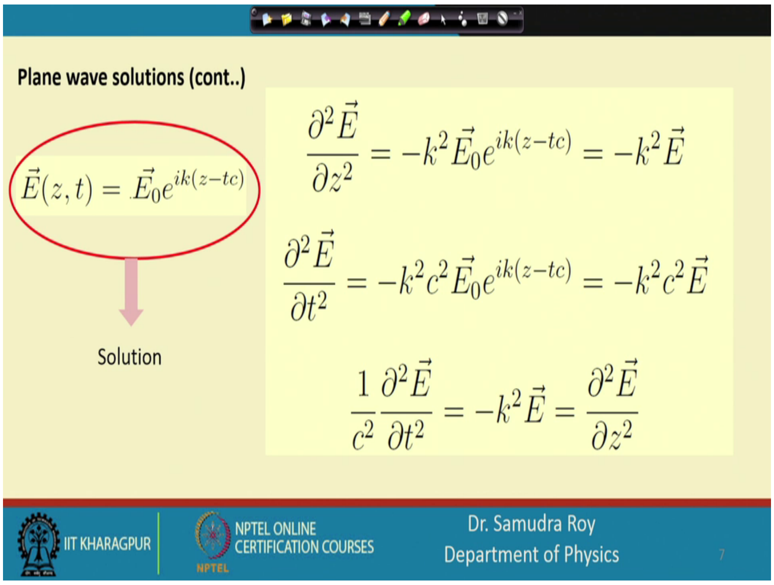

So, plane wave solution is E equal to E 0 k z minus c t, as I mentioned here. So, this is the solution I am looking for, as I it is mentioned that this is a solution and if this is a solution, then what we will do, we will put that into the equation. So, in the equation, we have del 2 E del z square term. So, if I make a derivative over that, then we will find that it is k square, if I make a derivative with respect to E with respect to z. So, when we make a derivative with respect to z, then you will have a i k term and when we make a again make a derivative with respect to z, then we will have another i k term. So, this i k, i k will give us a minus of k square term here. And the rest of the part k 0 E to the power i k z minus c t will remain intact.

So eventually, del 2 k del z square will be minus k square E. In the similar way, if I do del 2 E del t square. So, I will have a term i k c with a negative sign. So, i k c will come outside. Again, if I make a derivative over t, again another i k c term will come. So eventually, I will have minus of k square c term. Then I will have minus of k square into E, in this case. So now, what we will do; I have del 2 E del z square, del 2 e del t square and if I put that into the equation whatever we have, we readily find that these basically gives, this basically satisfied the left hand side and right hand side. In the left hand side, whatever the value I have; in the right hand side of the equation, I will have the same value.

So, 1 by c squared del 2 E del t square is minus k square c which is nothing but del 2 E del j square. So, with this simple treatment, we can say that E is equal to E 0 e to the power i k z minus t c is a solution of Maxwell's wave equation. This solution is called the plane wave solution. So, since it is the plane wave solution, so that means, some kind of plane is associated with that. So, we need to know that what is plane and how this is there. So, in the next slide, we will like to learn more about the plane wave. What is the meaning of plane wave ok.

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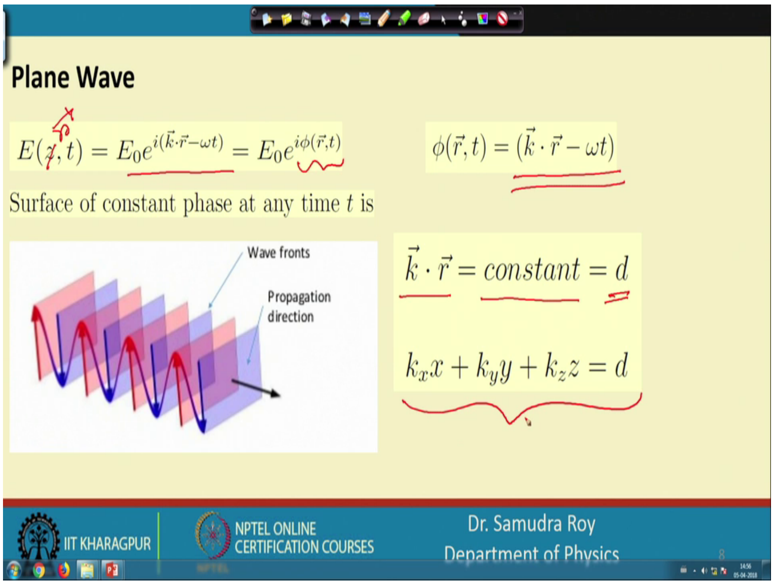

So, this is a very important slides where we like to learn about the plane. What is the meaning of plane? From where this plane is coming? Why we are calling this is a plane wave?

So, we know that this is the solution; E z t is k z minus omega t. This is the solution. If k z minus omega t is a solution, I can also write it in three dimension. And here, I write z, this is wrong; it should be in general, it should be vector r because in the right hand side, it is a function of r. If I write E z t, then I should write k z but anyway, this is a general form of the solution and when I write the general form of the solution, I should write k dot r minus omega t. So, in previous case, we use a solution which is one dimensional. So, one dimensional solution wave equation was one dimensional. So, the solution was also one dimensional. So, for one dimensional solution, we wrote k z minus omega t but here, we replace k dot r minus omega t. So, now, this is a phase term, phi r t.

This phase term is nothing but k dot r minus omega t. So, what is the meaning of plane here, because the surface. So, this is a wave that is moving and the surface of a constant phase at any time t is the phase front. We call is a wave front. So, this wave front is nothing but the surface of the constant phase at any time t. A figure is shown here. In this figure, we can see the wave is propagating. When the wave is propagating, we have a wave front and this wave front is nothing but the surface of constant phase; that means, when the phase is constant, we have a surface. So now, according to equation if I make k dot r equal to constant that is the phase at particular time t, then we have equation k dot r equal to constant, this constant say d.

So that means, k dot r equal to d is equation of the surface we are talking about. Now k dot r, if I expand, it should be k x x plus k y y plus k z z equal to d; so, this is nothing but the equation of a plane, so that means, the phase which is constant at particular time t, if I write this concept, then this equation gives me a plane. So, that is why this particular form is called the Plane Wave which is important. And also, you need to appreciate that this is a solution of Maxwell's equation, Maxwell's wave equation. So, that is a additional thing you know, ok.

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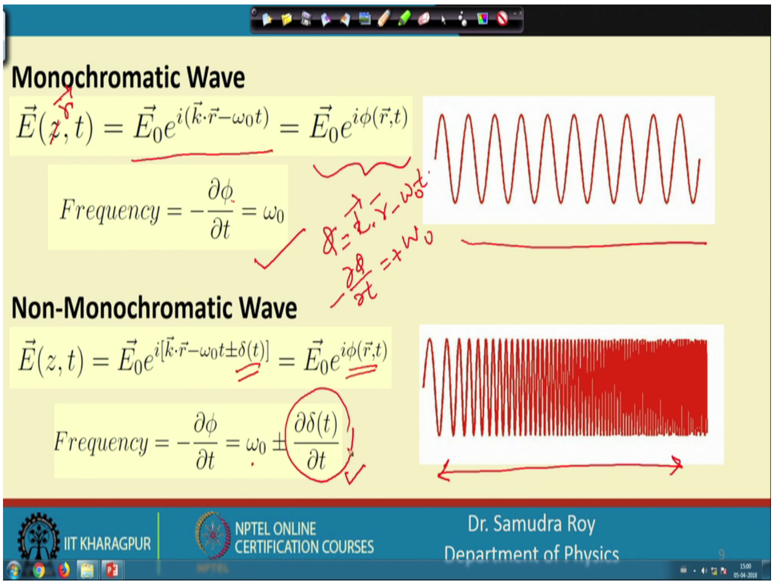

Next we need to know about the monochromatic wave. What is the meaning of monochromatic wave? We need to learn.

So, this is the solution of the wave we have, E 0 is equal to E z t is equal to, here again I write z it should be r, is the vector r. So, E is a solution of E 0, the plane wave kind of solution, k dot r minus omega 0 t. I write this as omega 0 because omega 0 is a frequency and I like to mention this frequency very specific, that is why I put 0 here and if I write this in the phase from this is the phase. So, phi r t is a phase. So, this is e to the power I this term. Now how the frequency is defined, we know the rate of change of phase with respect to time with a negative sign is frequency. So, this is the expression of the frequency we have. Now if I make a derivative with respect to phi; so, phi here is how much? Phi in one dimension, phi here in this case, phi here is k dot r minus omega 0 t.

Now, if I make a derivative with this term, then whatever I have in the right hand side should be the frequency. Here k dot r is not a function of time. So, this term will be with 0, minus omega 0 t. So, t del del t t will be absorbed and eventually we will have plus of omega 0 because minus of this things, this minus in going to absorb here. We have already a negative sign here.

So that means, we will have a specific frequency omega 0 when the form of Monochromatic Wave is k dot r minus omega t. Now, if I plot this things, we will have a wave like this. This is a picture of a Monochromatic Wave. The frequency distribution is uniform as simple as that. The frequency distribution inside the wave is uniform. Now, what is Non-Monochromatic Wave? So, Non-Monochromatic Wave is nothing but the same thing with a additional delta t term. This delta t is a additional phase into the system.

So, again my phase is something where this delta t is there. Now if I make a derivative over that, then we will have omega 0 plus minus of this things. So, if delta t is giving to me which is a function of time. So, this I can calculate. So, we will find, we will be find that this frequency is not omega 0 here but with some additional term. If that is the case, then the wave should have a different frequency component over over inside the system. So, initial we have one frequency and then find that the frequency is more and more and that is because of this particular term, delta t del t.

So, because of this particular term, we find the frequency is more ok. So, let us try to find out what next we have.

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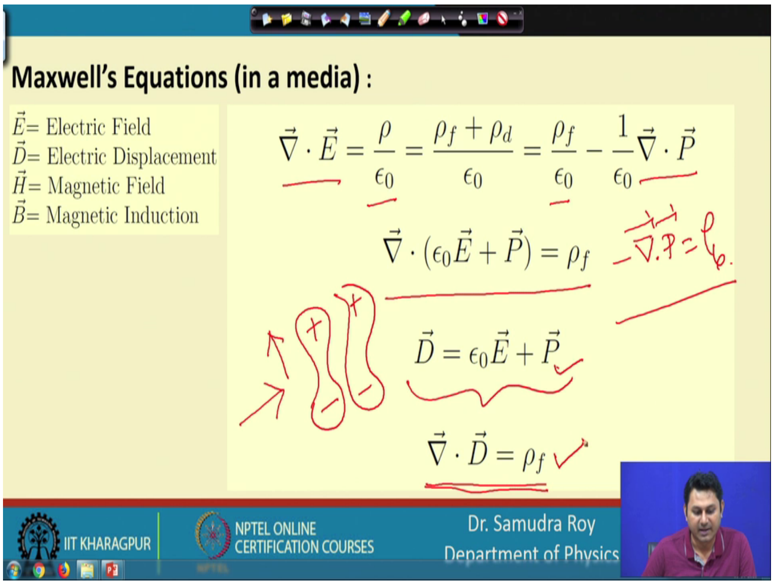

So, Maxwell's equation is in medium we have. So quickly, I will explain what is Maxwell's equation in medium. So, this is the same thing that we derive in the previous case but I will not going to derive in today's class. I am just giving you the idea what is Maxwell's equation in the medium.

So, Maxwell's equation in medium is grad dot E. So, Maxwell's equation in a medium is grad dot E is now rho by epsilon 0. So, rho by epsilon 0 term is appeared here because rho is not equal to 0 inside the medium. It has 2 components rho f and rho d. This rho f stands for free charge density and rho d is bound charge density. So, this can be represented as rho f divided by E 0 and 1 by epsilon 0, this term; this term minus of grad dot P is equal to rho d. This is a well known thing.

So, minus of grad dot polarization; this is the polarization is equivalent to the bound charge density. So, we just use this; after using what we do just we write these things, we rearrange this things and we find this expression. After having this expression, I just now write the displacement as epsilon 0 E plus P. This is a very important expression that now we are introducing some concept that polarization that appears when we have the electric field into the system electric; due to the due to the presence of electric field, the dipoles like these are arranged, this dipoles are arranged because of the application of the electric field and we have something called polarization.

Polarization is dipole moment or dipole per unit volume. So, dipole moment per unit volume is called the polarization. So, this term will appear; this is well known think. So, when we have this term, then my expression grad dot E is now replaced by a new equation grad dot D equal to rho f. So, inside the medium, instead of using grad dot E equal to rho by epsilon 0, it is better to use this equation. So, this is the first equation that is modified in this particular case; that means, Maxwell's equation, this particular form, we can see that grad dot D is now represented rho with rho f.

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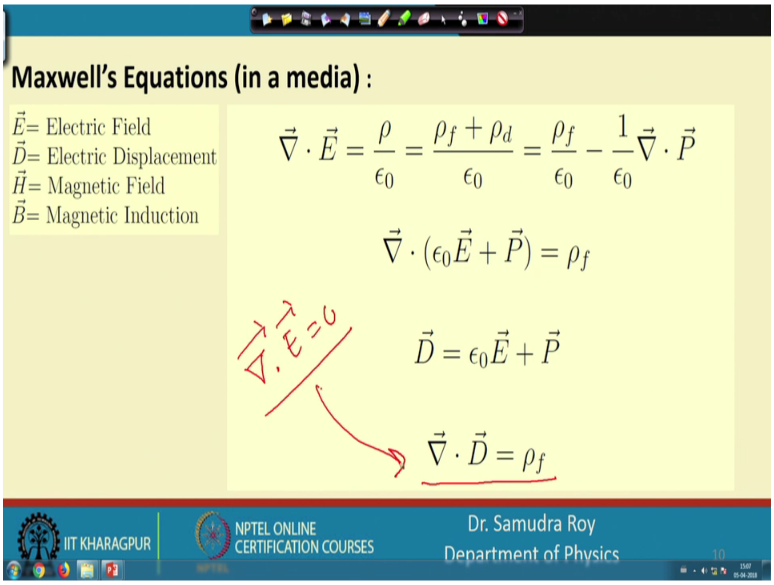

So, previously in the free medium, we write grad dot E was 0 but in this medium, I mean if the medium is not free some medium is there then it is represented by grad dot D is equal to rho f. Accordingly, other equations will be modified slightly and all this modification will come because of this polarization term which is here.

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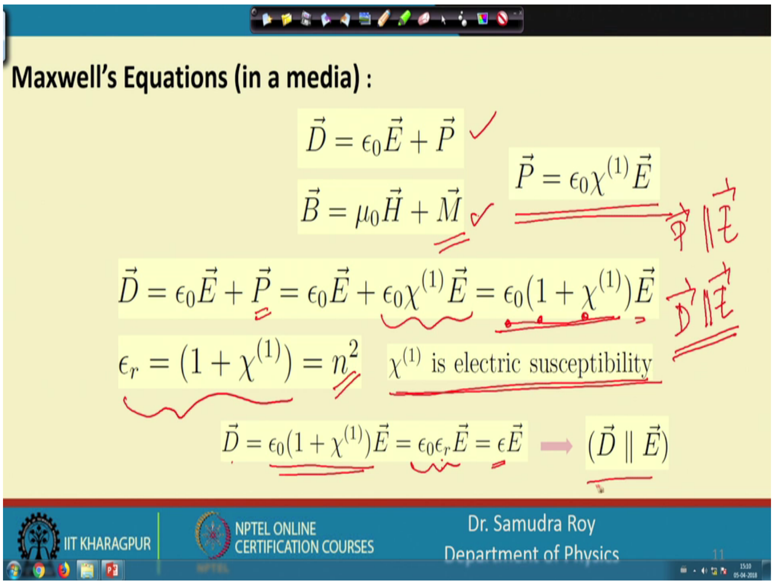

If I go to the next slide which is the continuation of Maxwell's equation in the media, so as it is written that d is epsilon 0 E plus P that is known. Also, B can be represented as mu 0 H plus M. This mu 0 H plus M is a term that is coming because of this magnetization. Like the polarization term in a magnetic material, we have something called magnetization.

But throughout our treatment, we will neglect this magnetization time because we will mainly deal with the material that does have any kind of magnetic property. So now, P is related to E with this very important equation where epsilon 0 and chi 1 is there. So, chi 1 is something called electric susceptibility which is very important and we will discuss that in detail in our future classes. But you can see that if chi 1 is a scalar quantity which not necessarily be a true thing but if I assume this is a scalar quantity, then P and E are parallel to each other.

So, in this particular case, I can write P is parallel to E, the vector P is parallel to E. So now, if I use this equation here replacing P with this condition, I will have epsilon 0 plus 1 plus chi 1 is equal to multiplied by E is equal to D. So, this is the relationship between D and E. Since I mentioned that chi 1 is a scalar quantity for the time being. So, this is a scalar, 1 is scalar also and this is a scalar so; that means, D is multiplied by some, D is some scalar quantity multiplied by E; so that means, E and D are also parallel; very important argument that E and D vectors are parallel to each other.

Again this is only true when the medium is isotropic. What is isotropic medium and all the things we will discuss in our later classes. But here epsilon 0 i this quantity can be represented as this; E r; E r is a relative permittivity. So, this relative permittivity is directly related to the refractive index in square and that is the form. So eventually, we have D equal to this quantity and then if I replace this thing to relative permittivity it is this and epsilon is the permittivity of the system and this things will be related to E and as I mentioned D and E are now parallel. Ok fine up to this it is ok.

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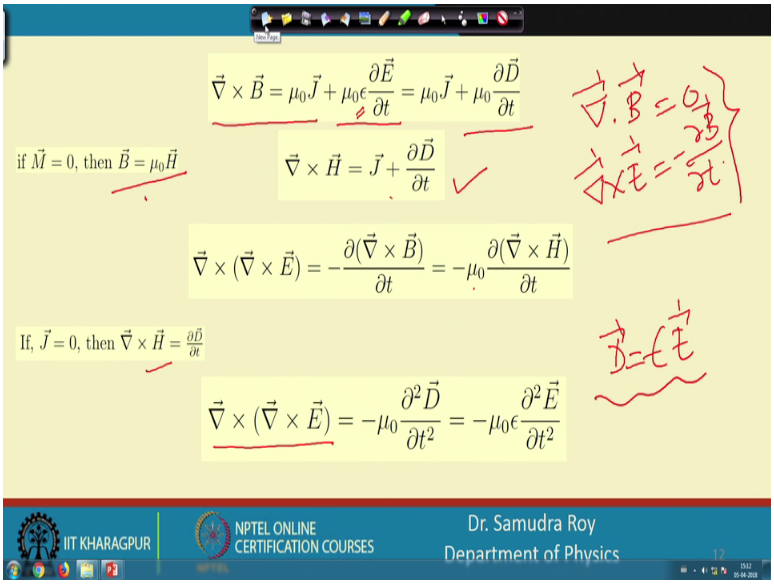

So now, let us go to the next slide and in the next slide, we will just manipulate the well known equation for examples grad dot B is now mu 0 J plus mu 0 epsilon. Now I should not write epsilon 0 because now I find in the material, we have epsilon. It is not epsilon. So, in terms of D, I can write this in this form. So, when I write this, this mu 0 can be because B is mu 0 H. So, grad curl cross B is equal to mu 0 J plus mu 0 del d del t if I take mu 0 common and put it here, then it should be curl cross H J plus del D del t. Again this is equation, this equation is slightly modified; this is a forth Maxwell's equation that is slightly modified.

So, second and third equation which was this is intact. If you remember the second and third equation is intact. This was the second equation and this was the third equation. We are not writing any kind of modifications or any kind of modification is required for this two equation; however, the first equation is modified and the fourth equation is modified.

Now go back to our old technique to find out the wave equation. So, curl cross E is this equation. If I make a curl over both the sides, then curl cross curl cross E is equal to minus of del del t, curl cross B, curl cross B can be represented as H with a mu sign because they are related to this.

And if J is 0 and curl cross H is del d del t again for J equal to 0, this is a relationship. I can have curl cross curl cross E is equal to minus of mu 0 del 2 d del t square and again D is replaced by E with because D is epsilon E. So, I can replace this here. Once I replace this here, then I have a form which is a form look likes wave equation.

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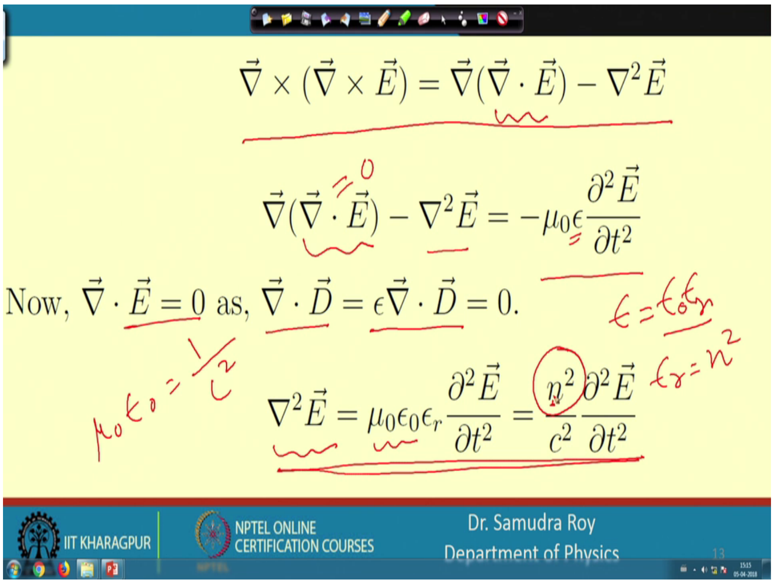

So, curl cross curl cross in the next case, I just remove this; I just remove curl cross curl cross with these identity. This is a well known identity again we are using. So, curl cross curl cross E is grad dot grad dot it is a gradient of this things and del square E.

In the right hand side is it was mu 0 epsilon is the exactly same treatment that we have using, that we have used in the previous calculation in the free free medium. In only thing is that there in this place it was epsilon 0. Now it is epsilon because of this the presence of the medium. So, again grad, this quantity is 0 because there is no free charge available. If there is no free charge, then these quantity is also 0.

And now if that is the case, then finally, if I write this is equal to 0 finally, I can have grad square E is equal to mu this; you may remember that epsilon was defined by epsilon

0 into epsilon r. So, I just replace epsilon 0 epsilon r and mu 0 epsilon 0 is nothing, but 1 divided by C square. So, if I use this identity 1 divided by C square, C is the velocity of light then I will have n square divided by C. Again in epsilon r is equal to n square; also we have done in the previous slide.

So, with these, eventually we find a wave equation which look exactly the same like the previous case, in the free medium case but only difference is this n square term. This n square is coming because now the medium is not free; that means, n is not 1. Since it is a medium, then we should have some value of n and that is appearing in our equation. So, this equation is a Maxwell's equation, wave equation but not in a free medium inside a medium we will going to use this equation extensively in our future classes because non-linear optics does not appear in free medium.

So, some medium is required. So, since some medium is required, this equation is very important for us. So, with that note let me conclude today's class. So, today we have learn the basics of Maxwell's equation, plane wave solution Maxwell's equation in free medium and Maxwell's equation in medium and other few relations. So, with that let me conclude that in the next class, we will start from this and try to find out few other issues also like isotropic system and anisotropic system. So, with that note, I would like to conclude.

So, thank you very much for your attention. See you in your in the next class.