Modern Optics
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Lecture – 07
Maxwell's equations and electromagnetic waves (Contd.)

In the preceding discussions we have extensively used this plane waves particularly in the discussion of electromagnetic waves, in isotropic medium, an isotropic medium and also mediums like absorbing medium, conducting mediums.

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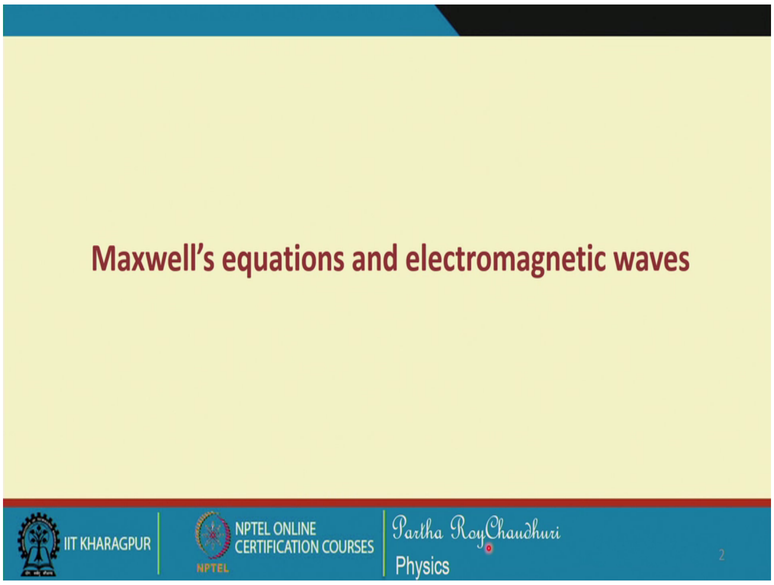

And in almost every part of optics and photonics, it is the plane waves which is mostly used and the formally formulation is also very useful in making analytical formulation for the various propagation characteristics of electromagnetic waves.

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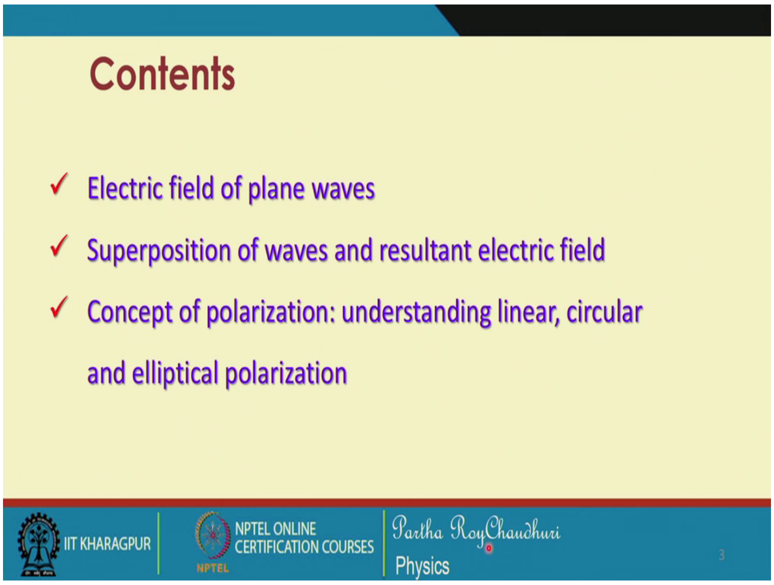

So, the content of these discussion is the electric field of plane waves. We will try to learn how simply and lucidly we can express electromagnetic plane electromagnetic waves. And then, we will take the superposition of waves which will give the resultant field in terms of the electric field, we will take a concept of this how the superposition gives rise to interference. Then making use of the plane wave formulation of using this electric field, we will bring out the concept of polarization. Then we will talk about the linear circular and elliptical polarization.

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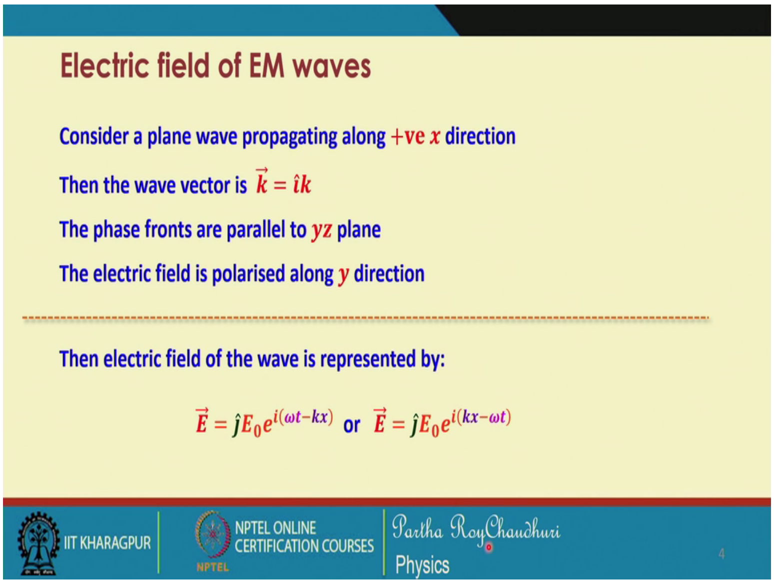

So, let us first consider a plane wave which is propagating along positive x direction. Then the wave vector is k equal to i k. The phase fronts are parallel to the yz plane. The electric field is polarized along y direction. Let us suppose that the electric field is polarized along y direction in that case, then we can write the electric field as E equal to unit vector j then E naught, which is the amplitude of the electric field and e to the power of i omega t minus kx.

Because, the propagation direction is along the x positive x, you can write this expression E as j E naught e to the power of i kx minus omega t as well. You can see that kx minus omega t or omega t minus kx both represent electromagnetic waves propagating in the positive x direction.

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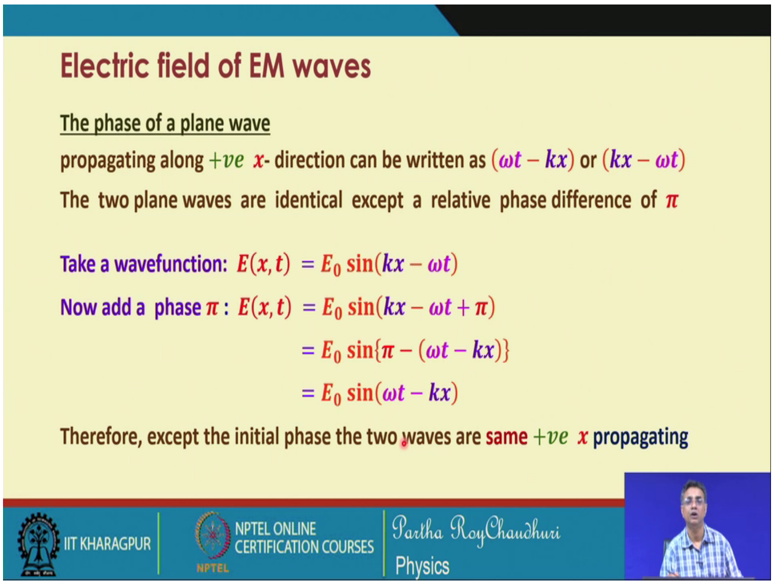

Let us see that let us assume that a plane phase of the wave plane wave which is propagating along positive x direction, that can be represented by omega t minus kx. We can also represent this as kx minus omega t, as I said before and we will see that they will actually a point to the same electromagnetic wave with an initial phase; which is which is not very important and relevant.

Take a wave function E of x t E 0 sin kx minus omega t. Then add a phase pi, then we will get the phase factor kx minus omega t plus pi. You can write this respected as E 0 sin pi minus omega t minus kx. Therefore, this will give you E 0 sin omega t minus kx. You can see that omega t minus kx which started with kx minus kx minus omega t, but you get omega t minus kx. So that means, they at the same, but there is an initial phase which we have added constant phase pi; expect the initial phase the two waves are same and propagating in the positive x direction.

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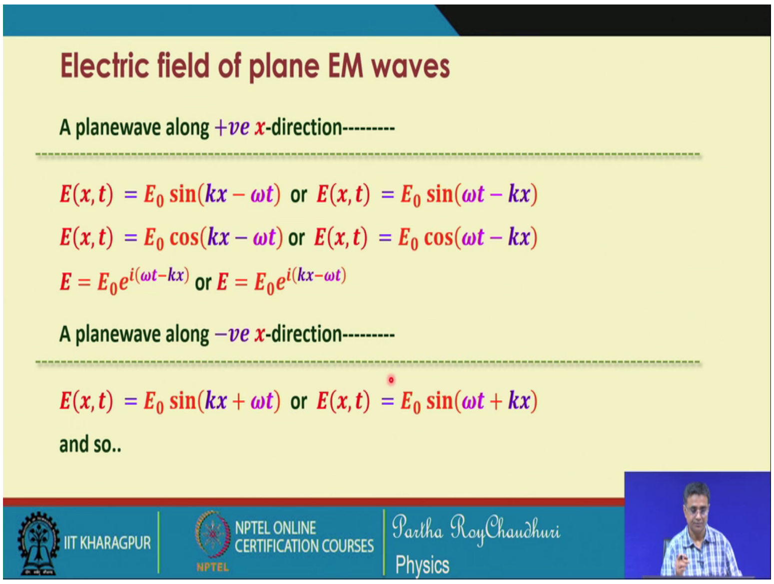

So, this is very useful to understand that whether we write kx minus omega t or omega t minus kx, in general when it is a three-dimensional wave three-dimensionally, three-dimensional propagation then we write k dot r minus omega t or omega t minus k dot r as well. So, this is one very useful aspect whenever we rewrite the plane waves. So, the plane wave E of x t E 0 sin kx minus omega t, we are started from here at x equal to 0. The amplitude is the wave is 0, field is 0 and then it is increasing at x equal to at t equal to 0, but we have plotted at against x.

So, this is this is the one which represent that there is a phase of pi initial phase, but both of them are representing the same wave. But, you can write this wave in terms of cosine because, you can translate sin omega t minus kx as E 0 cosine omega t minus kx plus pi by 2; in that case it will also represent the same wave. So, these are the various ways of writing a plane wave and this plane wave formulation is very useful almost everywhere in the in the branch of optics and photonics.

The conclusions are the initial phase is just a constant contribution. Maybe this initial phase may be attached to the phase at the source or the generator where there is already because, you start counting the phase when it is already pi and this initial phase is independent of the propagation in space and time.

Now, electric field of a plane electromagnetic waves electromagnetic wave which is propagating along positive x direction, we can write that is equal to E 0 sin kx minus omega t or you can write E of x t equal to E 0 sin omega t minus kx. These are the various ways of writing the electric field of a plane wave. You can write as E 0 cosine kx minus omega t or you can write E 0 cosine omega t minus kx. These are the various ways of writing this and E 0 E to the power of this if you write as the complex ways the plane wave along negative x direction which is propagating along the negative x direction you can write that.

In place of minus it will be plus kx plus omega t and all other things will remain the same. So, this is very useful and very interesting to know if you have kx plus omega t or omega t plus kx, this is a positive negative x propagating wave whereas, if it is kx minus omega t or omega t minus kx then it is a positive x propagating electromagnetic wave.

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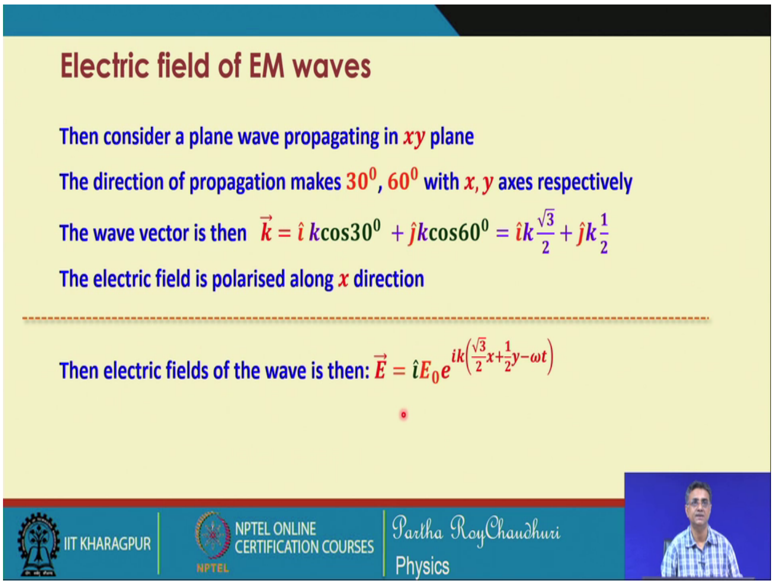

Now, let us consider a plane wave which is propagating in the xy plane and the direction of propagation makes 30 degree, 60 degree with x and y axis. Then we can represent the wave vector k equal to i k cosine 30 degree, j k cosine 60 degree because, these are the angles which are made with the x and y axis. So, you can write this k vector in this form.

The electric field is let us suppose that it is polarized along the y direction x direction, then you have to use this i cap E 0 e to the power of i k and this phase factor.

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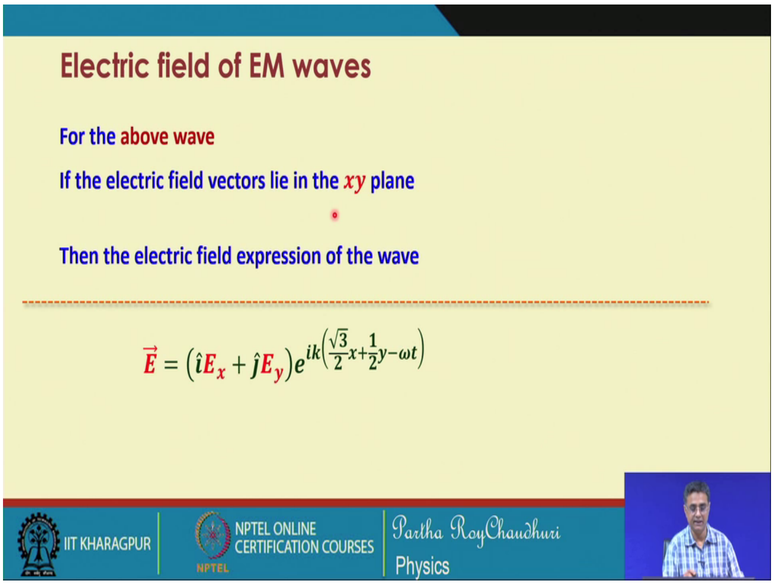

For the above wave that is for this wave, if the electric field vector lies in the x y plane then the field expressions because, it can have two polarizations E x and E y. So, we will have to modify this equation in this from. So, we are trying to write the electric field in different configurations different situations.

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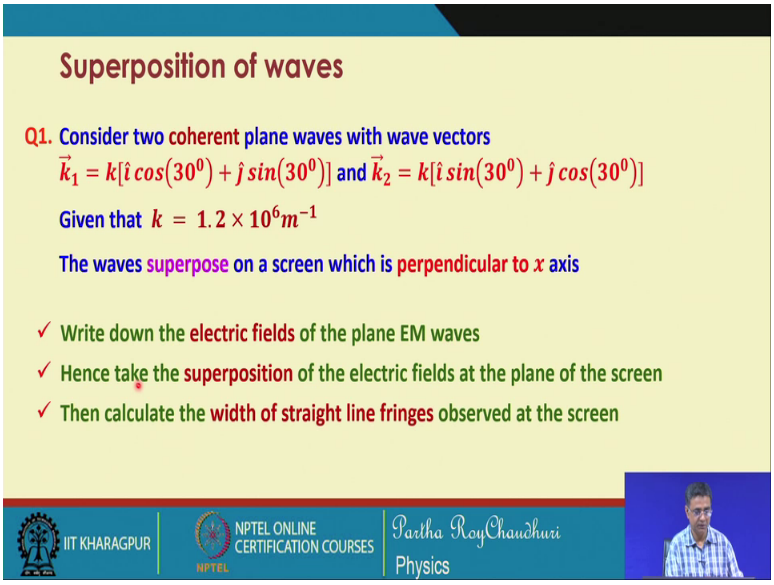

Now, we will take up one example that, let us consider two coherent plane waves with the wave vectors k 1 equal to this i cos 30 degree and j sin 30 degree whereas, this k 2 is k i sin 30 degree j cos 30 degree. So, these are the waves then given that k equal to 1.2 10 power minus 6, this is the propagation vector. The waves superpose on a screen which is perpendicular to the axis. So, under this condition under this condition we see that this is i cos 30 degree, this is i sin 30 degree, j sin 30 degree, j cos 30 degree.

So, they are just opposite. So, we will have to look at the configuration how the electric fields, the waves are incident on the screen. Write down the electric field of the plane electromagnetic waves. Hence, take the superposition of the electric fields at the plane of the screen. Then let us, calculate the width of the straight line fringes which will be observed at the screen.

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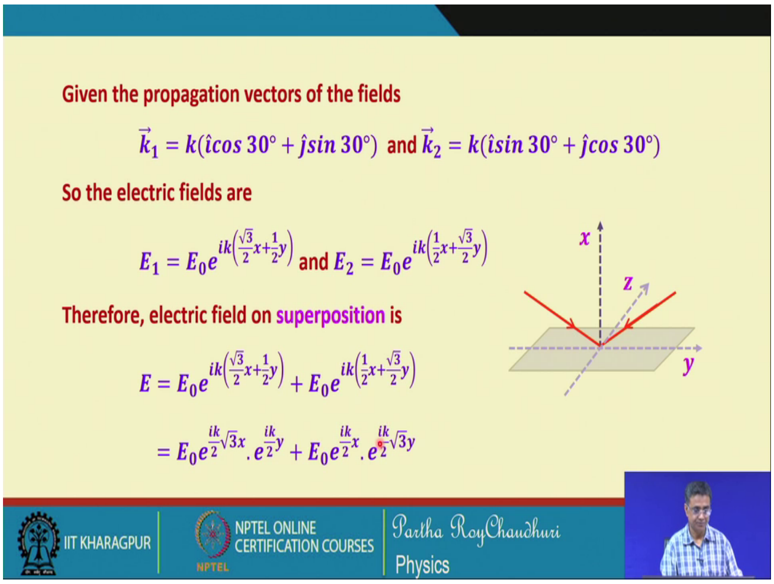

So, to do this let us look at the configuration of the electromagnetic waves. So, k 1 will constitute this because, this is i cosine 30 degree and j sin 30 degree and k 2 so, they will actually look like this. These are the two waves which are meeting on the on the screen at this point and let us consider this is the origin of the coordinate system. Then the electric fields are E 1 will be E 0 e to the power of i, this phase factor and for E 2 this will be the phase factor. We have just replaced this i cosine 30 degree and j sin 30 degree in terms of the numerical values. Here E 2 also we have replaced in terms of the numerical values.

Therefore, electric field on superposition we just have to add this because, these are the figure addition, the vector addition. So, E 0 e to the power of i k this phase and so, this is E 1 and E 2. If we add this because, e to the power of i k under root 3 by 3 x by 2 this quantity, we just break them into their individual index components; then considered that the screen is perpendicular to the x-axis.

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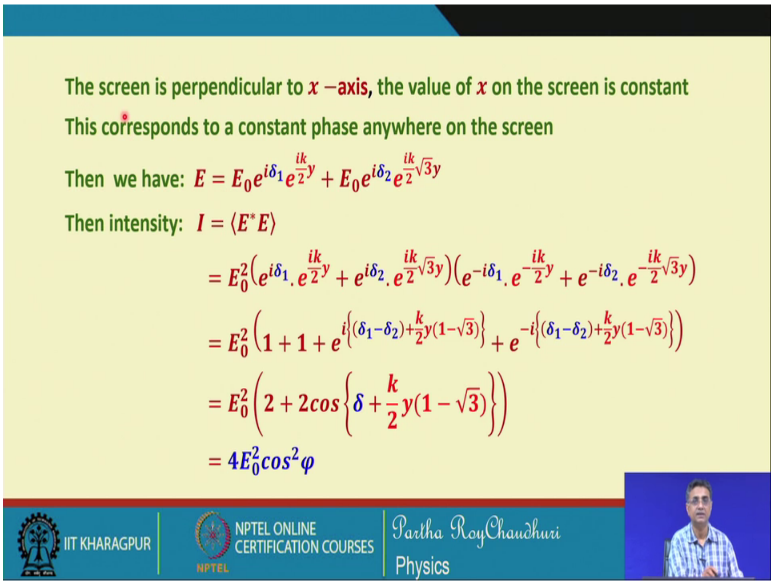

As I have mentioned this the screen is perpendicular to the x-axis and therefore, at this point because this is the x y plane so, the value of x at this point the value of x on the screen is constant. If it is if it is the origin then it is 0, but otherwise if it is x equal to a x equal to b some constant, with this constant will be throughout the screen. So, we take the value of x as constant and which will add to a phase in the case if it is the origin x equal to 0, this delta 1 and delta 2 will also become 0. These are actually come from this part of this, from this part of this

So, the I take out this constant part and then add them together which will which will give you that the intensity will be E and mode of E square which is E star E. So, I get E 0 square this quantity into this quantity. So, on multiplication I get 1 plus 1 into this quantity and this quantity. Therefore, if I therefore, if I summarize this E 0 square 2 plus 2 cosine delta plus k by 2 y 1 minus under root 3 which will give you that which will give you if you take 2 out outside because, this will be 1 plus cosine 2 theta.

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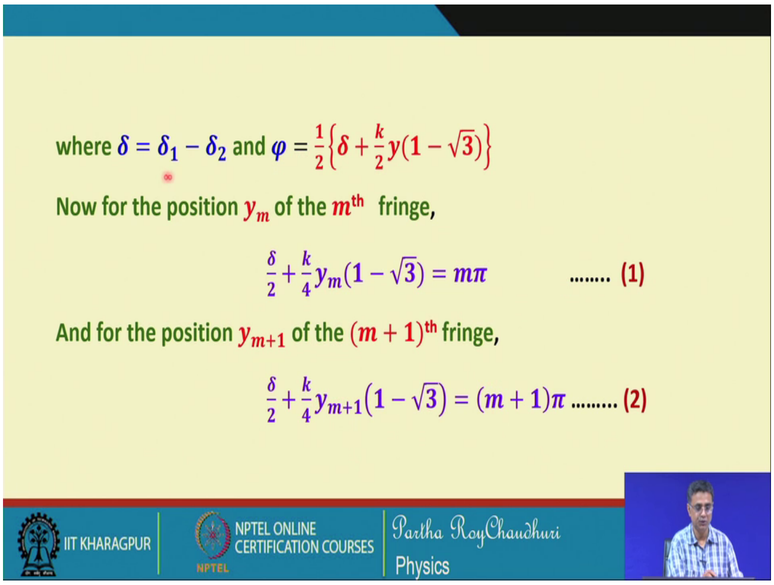

So, that gives you 4 E 0 square cosine square phi; where phi I have used to this value of phi equal to half delta plus k by 2 this which is actually the consequence of this, this quantity divide by 2 will be called phi. So, that is what half of this of this quantity is equal to phi. Now, for the position y m of the mth fringe, let us go back to this screen. So, this is the y direction any position any position, let us call this position is y mth position and for y mth position the free the for the y mth position of the fringe, you have this phase will be equal to delta by 2 plus k by 4 y m 1 minus under root 3 equal to m into pi.

This phase will be equal to m pi because, this will be the integral multiples of pi and for the position y m plus 1 of the m plus 1 th fringe, you will have this phase is equal to m plus 1 pi. Now, if I subtract these 2 quantities, then I should get the width of each of the fringes because, this is for mth fringe and this is m plus 1 th fringe which is very known and well understood.

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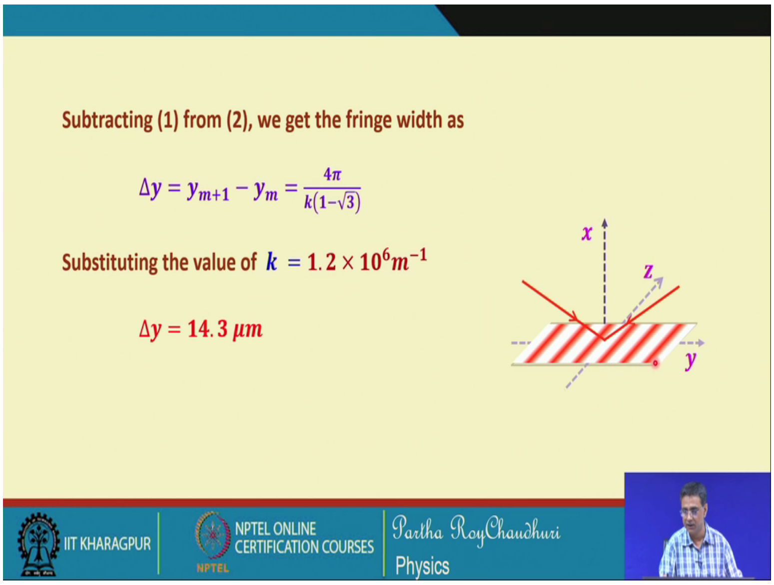

So, subtracting 1 from 2 1 from 2 so, we will end up with only 1 pi on the right hand side. So, this y m plus 1 minus y m which will be equal to the fringe with delta y and that is equal to 4 pi by this. So, if I substitute the value of k into this equation, then I will get delta y equal to 14.3 micrometer. So, and the fringes will look like this. So, this is the very beautiful example that we started with a plane wave which is incident here.

We have another plane wave which is incident here and these plane waves are meeting on the screen, they are overlapping on the screen and therefore, at different point the phases will be different. And therefore, there will be interference fringes on the screen the fringes will be along the y axis.

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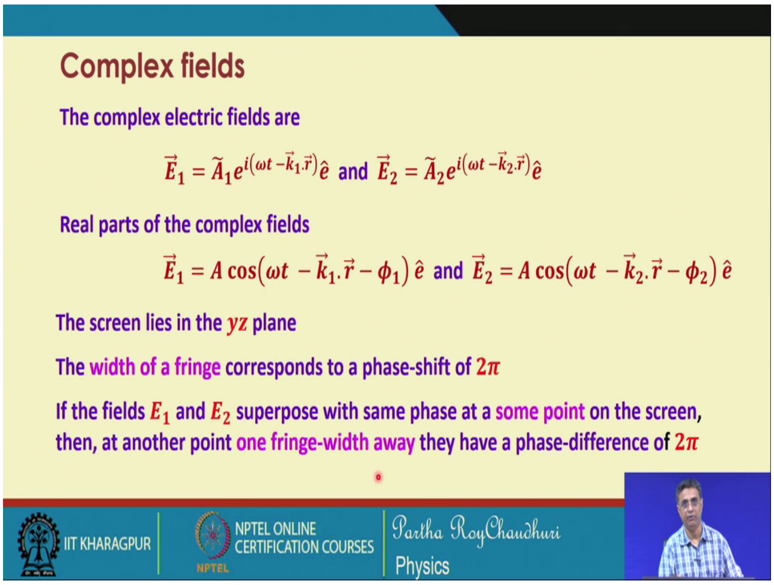

So, let us try to do this with some alternative way of writing the electric field; let us write the complex electric field as E 1 complex amplitude A 1 e to the power of i omega t minus k 1 dot r. This is for the wave k 1 with propagation vector k 1 and this is for the wave which is having a propagation vector k 2. And then, the real part of each of the complex phase complex a electric field can be written as A cosine omega t with a with an initial phase.

And here also, the real part of the wave can be written with an initial phase phi 2. The screen lies in the yz plane that we have seen. So, the width of a fringes corresponds to a phase-shift of 2 pi. If the fields E 1 and E 2 superpose with the same phase at some point on the screen, then at another point on the fringe on the ah on the screen one fringe-width away the phase difference will be twice pi.

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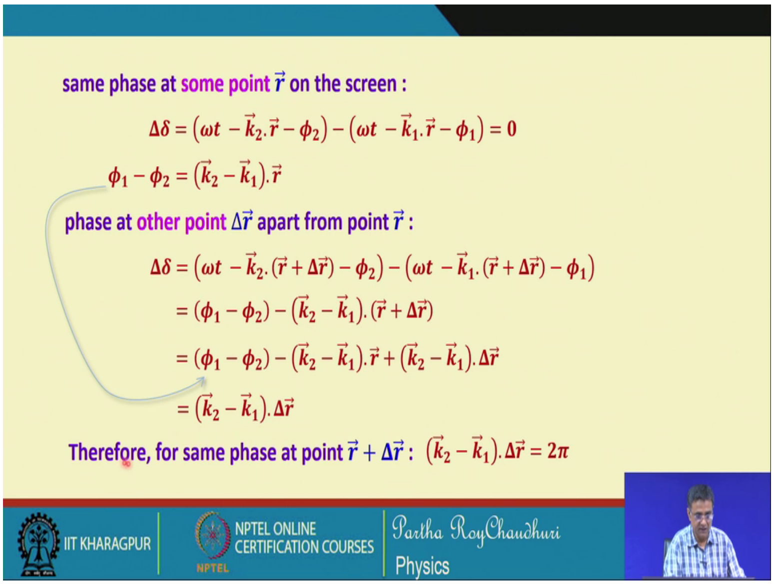

So, that is what we will use to see how the fringe width comes out. So, the delta the phase difference between the points where the phase difference is 0, then we can write this equation equal to this. So, it is the same phase at some point r on the screen and then will consider at some other point on screen which is delta r apart, but again it will have the same phase, but this is the minimum distance. So, this distance will correspond to one fringe width that is the concept.

So, when the when at point r when the phase difference is 0, we write in this form which is equal to 0; so, that gives you the initial phase difference is k 2 minus k 1 dot r and the other point which is r plus delta r away from this point, we will have this phase difference which will be equal to this minus this quantity. I just have replaced r by r plus delta r r by r plus delta r and then we write down this equation. But, phi 1 minus phi 2 we have already seen that this is equal to k 2 minus k 1 dot r.

Therefore, at this point the phase difference is also this, but this phase difference must correspond to twice pi and this, gives you the condition of the fringe width.

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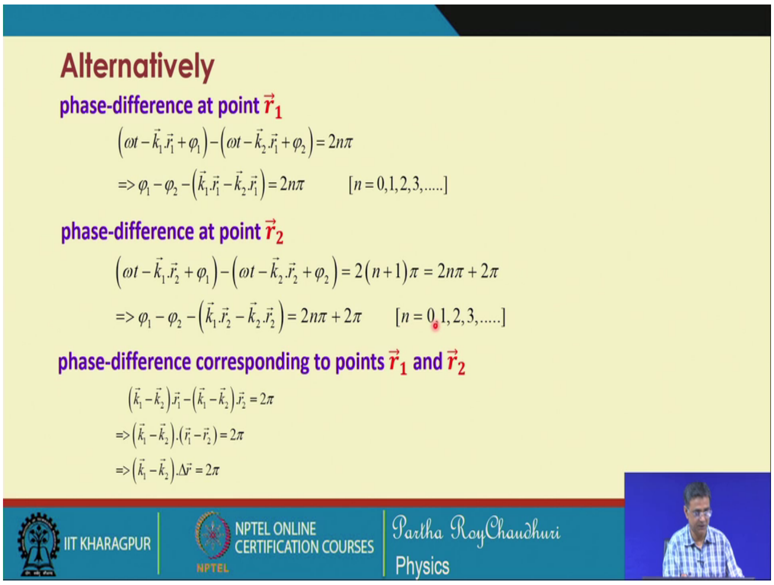

So, the phase difference at point r 1 will be this, phase difference at point r 2 will be this and the phase difference corresponding to the points r 1 and r 2 will come out to be the same. Because, we have done it in like three different ways that we considered the phase equal to 0 at some point, at some other point the phase will be twice pi. Then we look at the look at the condition between this distance and the difference of k 2 minus difference of k 1 and k 2.

The under this consideration the phase difference at point r 1, which could be anything and then the phase difference at point r 2 which could be anything. These differences if we make the condition equal to twice pi, this is twice n pi and this is twice n pi plus 2 pi. So, effectively the phase difference is only 2 pi, if I plug in that condition then again we will get k 1 minus k 2 dot delta r is equal to twice pi.

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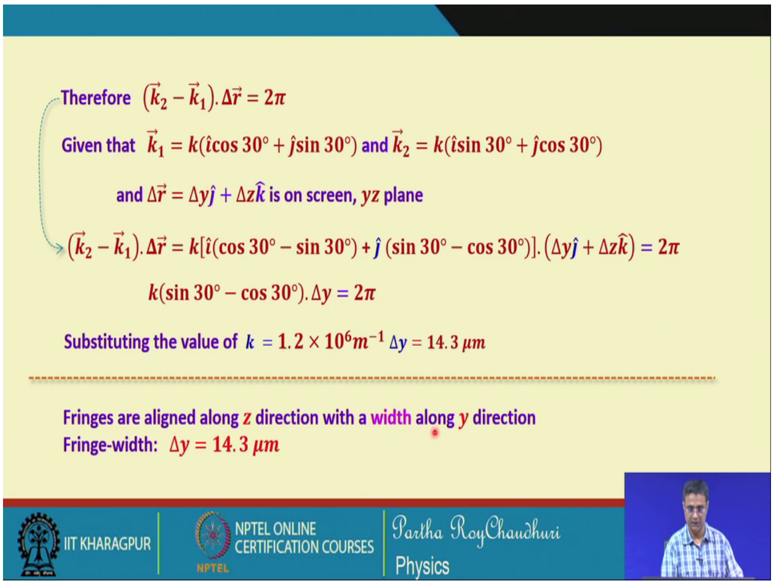

Now, we will use this condition to calculate the fringe width. So, we plug in the values of this cosine 30 degree, sin 30 degree and so on and delta r equal to delta y j plus delta z k because, it is in the yz plane. So, k sin 30 degree and putting all these things we will get and again if you substitute the value of k equal to this will get this value. So, effectively there are few ways of taking the superposition of this effectively, it gives you k 2 minus k 1 dot delta r which will be twice pi will give you the condition for the same phase points on the screen. The fringes are alound ah are aligned along the z direction with a width of y along the y direction.

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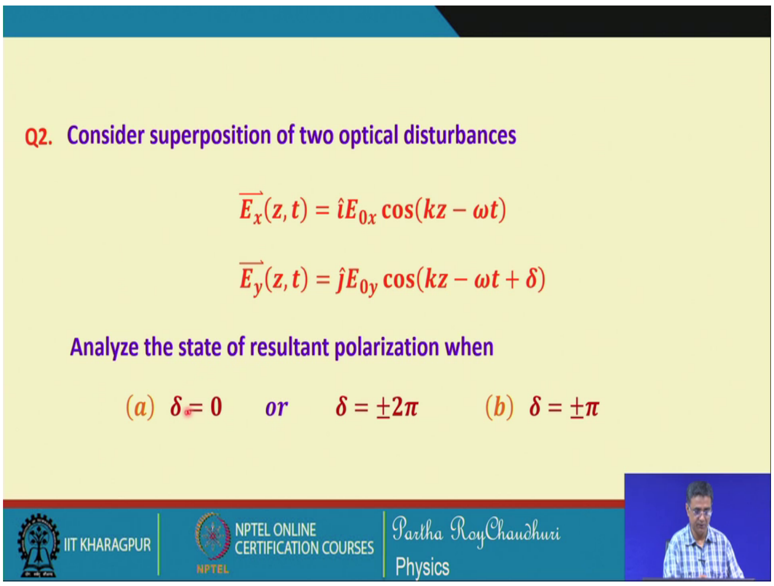

So, delta y equal to this and then we will consider another superposition of two optical disturbance. That is, given by E x equal to this quantity we have seen whether you write kz minus omega t or omega t minus kz; obviously, this wave is z propagating and the polarization is along x. This wave is also z propagating, but the polarization is along y. So, we have considered two waves which are which are orthogonal to each other, one is x polarized another is y polarized.

And we take the superposition of these, analyze the state of the resultant wave for this waves as it propagates along the z direction. So, the first case is when you take delta equal to 0 or delta equal to twice pi, what happened and when you take delta equal to plus minus pi then what happens.

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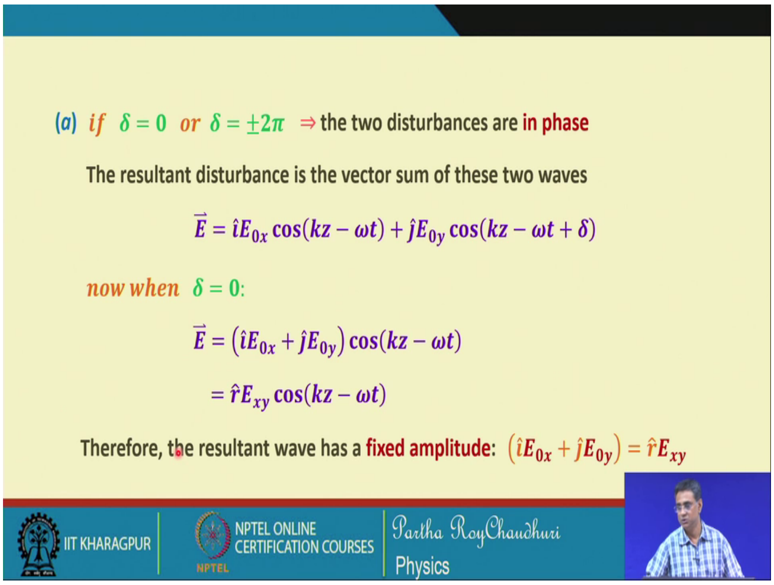

So, if delta equal to 0 or delta equal to twice pi the two disturbances are in phase. The resultant disturbance is the vector sum of these two waves. Therefore, therefore, therefore, you have this you have this E equal to E 0 cosine of kz minus; I just get the superposition of these two waves. Now, if you put delta equal to 0, then i of E 0 x plus j of E 0 y that quantity is the common quantity and because, delta equal to 0 so, we can write cosine kz minus omega t. Now, this i E 0 x and plus j E 0 y is nothing, but r E x y.

So, because this gives your fixed amplitude E x y and the variation is the same as the individual waves, the phase variation the phase is the same as the individual so, waves component waves. Therefore, the resultant wave has a fixed amplitude this and if I if I look at the amplitude is given by E of x y and the direction is tan theta which is equal to E y; you can look see this E 0 y equal to this, E 0 x is equal to this.

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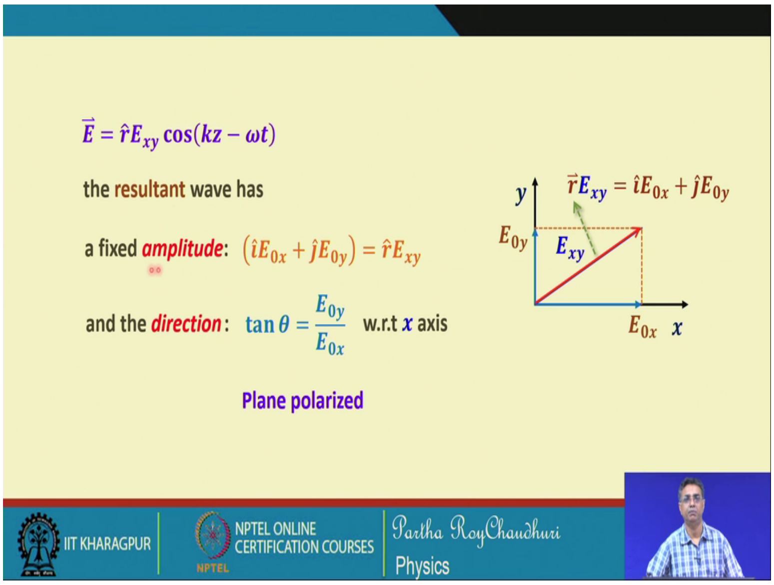

So, the angle the resultant wave makes an angle with this which is given by tan theta with the x axis with the x axis, this angle tan theta will be equal to E 0 y by E x. So, this wave is plane polarized. Let us take the other possibility that delta equal to plus pi or minus pi, the two waves are out of phase by pi. In that case what happens, we will write this E y E y becomes because you have taken the phase you have taken the phase delta equal to now plus pi or minus pi. In the in this case you have taken delta equal to plus pi so, you can write that sin will come out to be this, cosine there is a minus sin and the resultant wave will now be represented by this equation.

Now, because cosine kz minus omega t cosine kz minus omega t this factor appearing in both the terms so, you take them bracketed out and you can write the amplitude part like this i k 0. So, again you get a fixed amplitude which is again this r of r unit vector E x y. So, the resultant wave has a fixed amplitude r of E x y. The direction of this wave you can see this E x y E 0 x is along this direction, E 0 y is along this direction. So, the resultant wave E x y is along this direction. So, this is again a plane polarized.

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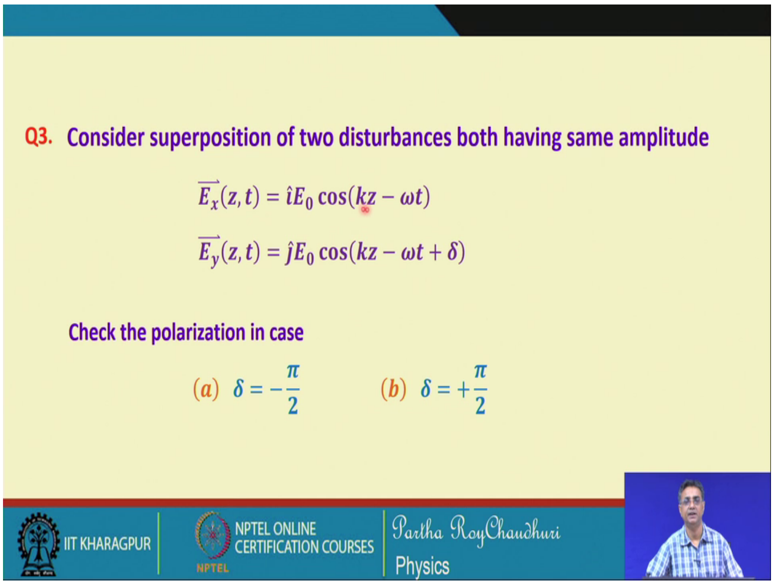

Now, we have another possibility that if the waves are out of phase by pi by 2 plus or minus, then what happens. In that case because, these are again the plane waves they are when they are superpose at some point on the space and both of them are travelling along the z axis. Then, we can write we can write the equation in this form with an initial phase difference of delta. If delta equal to minus pi by 2 then, what happens this putting this delta equal to minus pi by 2 will make this quantity sin this plus pi by 2.

So, sin in the second quadrant will be positive. So, I write this equation as positive j E 0 sin kz minus omega t, but the first quantity it remains the same as it is. So, cosine kz minus omega t and this becomes sin kz minus omega t so, that is what I have written here. Therefore, the resultant wave if I add these two waves i E 0 j E 0; one is sin another is cosine then it will have a fixed amplitude. I just have to square the amplitudes and add and it will be a time varying because, you cannot directly add them you have to look at the phase at various time and along various z.

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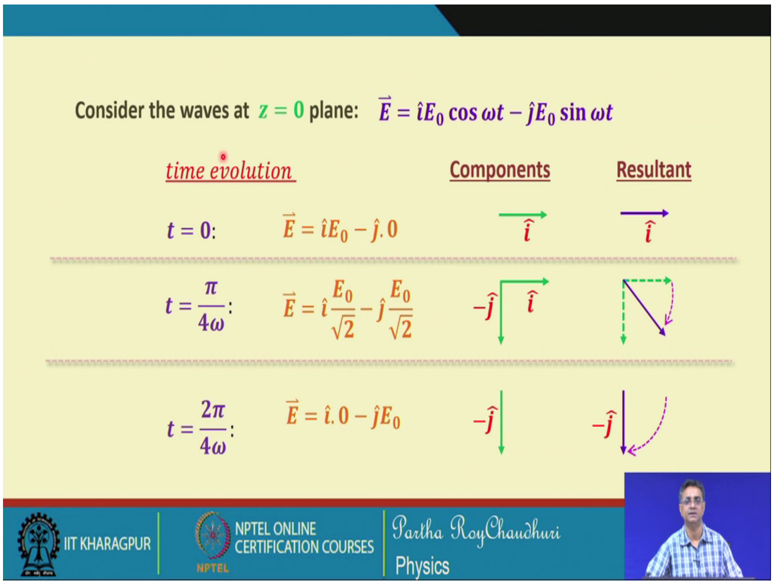

So, let us considered that at the point z equal to 0, at the point z equal to 0 what happens to this. So, this quantity become 0, this quantity also become 0 as a result it becomes i E 0 cosine of minus omega t which is equal to cosine omega t and this will become E 0 sin minus omega t. So, this will become minus E 0 sin omega t that is what I have written here, i equal to E 0 cosine omega t minus E 0 sin omega t.

Now, we look at the time evolution at z equal to 0 at z equal to 0 z equal to 0 and at all at time t equal to 0 this equation will give you E 0 minus j of 0 because, at time t equal to 0 this will give you 1, this will give you zero. So, that is what is written here. So, it gives you the entire field is now along the x axis along the x axis and the resultant is also x axis. But if you if you considered the time t equal to pi upon 4 omega, then E equal to i E 0 by under root 2 for this cosine omega t cosine omega t we will now corresponding to 45 degree and j this will also give you this 45 degree. As a result, both the amplitudes along x and y are now existing with a factor of 1 by root 2 this and this, but this is j is in the negative direction and i is along the positive direction.

So, the resultant vector will be along this that is what is here. So, you can see that initially the field was along this direction now, the resultant field has rotated through an angle of 45 degree and it is now along this direction. And, in this way if you considered twice pi by 4 omega t this time, then this quantity will become 0 and this quantity will become equal to E 0 because sin of pi by 2 will become 1. So, the entire amplitude is now along the minus y direction. You can see this component is here minus y direction and this is the resultant. So, we have the resultant field along this direction.

So, effectively we started at time t is equal to 0, when the resultant field was here then it has come down to this position and then it is moving in this direction. So that means, it represents in a wave whose electric field, the tip of the electric field vector is now rotating along this in the in the clockwise direction and as it is at the wave is propagating forward along the z axis.

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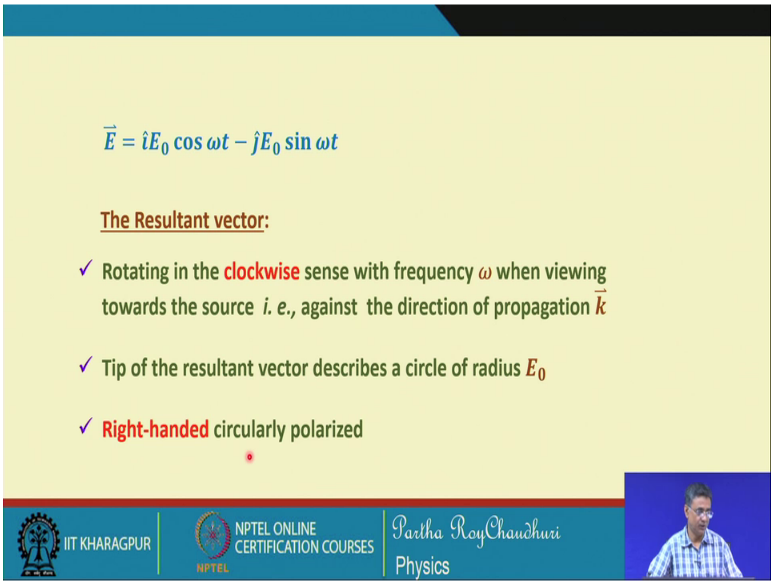

So, the resultant vector rotating in the clockwise sense with the frequency of omega which is the frequency of the component waves individual waves where those were under superposition; against the and when viewing towards the source this is in the clockwise sense. The tip of the resultant vector describes a circle of radius E 0. And this situation, we call the right-handed circularly polarized electric field.

Next, we consider this situation that is delta equal to plus by 2 plus pi by 2. The resultant disturbance is the vector sum of these two waves and in that case we can write this equation because, you had a you had a delta here. So, in place of delta I write a pi by 2 and it comes down it becomes this equation, this equation. Therefore, the resultant wave has a fixed amplitude again and the direction will be time varying. So, we have to again check the time evolution of the wave. So, the time evolution is at time t equal to 0, you can see cosine omega t will become 1, so you have entire amplitude which is along the x direction. So, the resultant is alone also along the x direction.

For time t equal to pi by 4 omega, then you have this resultant components are along this and this, but the amplitude is reduced by 1 by root 2 because of the 45 degree cosine 45 and sin 45 degree. So, the resultant amplitude is now along this direction and at time t equal to twice pi by 4 omega that is the next. So, this amplitude because this because of the pi by 2 phase factor this become 0, but this becomes equal to 1. So, the entire you know electric field amplitude is along the positive y direction. So, you have the resultant field is also along the positive y direction.

This means that we started at time t equal to 0, when the electric field was entirely along the x direction. Then it has rotated through an angle of 45 degree, find at this time it has rotated to an angle of 90 degree. So that means, if you continue with time then the tip of the electric field vector is rotating anticlockwise, as it is advancing along the z direction. So, this is how we see that the superposition of simple plane waves can give rise to various phenomena like interference and the polarization evolution along the z direction. So, this we have already discuss that the field is rotating in the anticlockwise sense. The tip of the resultant vector describes a circle of radius E 0 and this situation is called a left-handed circularly polarized right.

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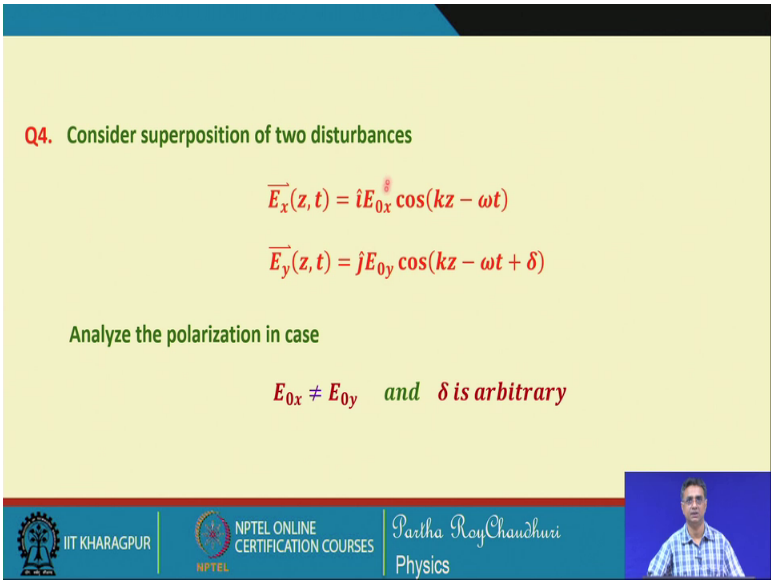

Now, we consider these optical disturbances that delta is arbitrary then you can write E 0 of x is not equal to so; that means, this initial amplitudes the component wave amplitude are not same and delta is also arbitrary.

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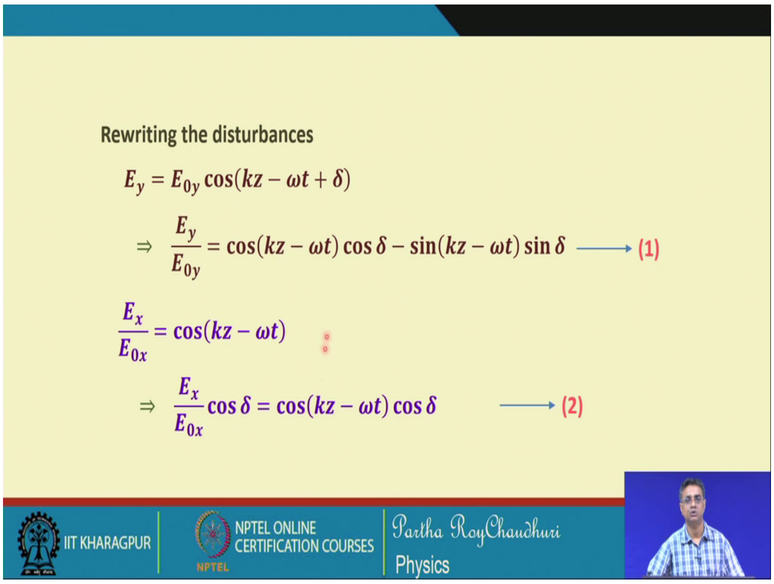

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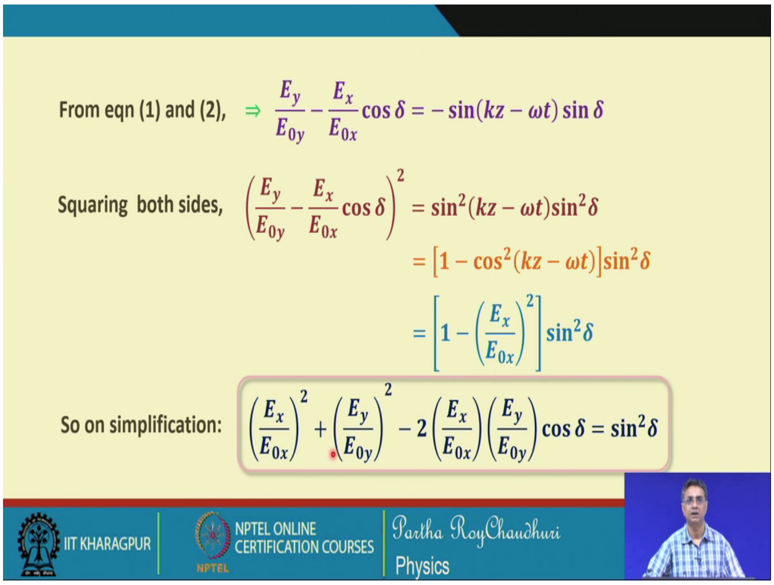

Then you can write E y by E 0 y and E x by E 0 x, if you square an add you will get this equation and now you square either sides, then you can write down this equation like this. So, you have the generally equation of an ellipse in that case. So, which the in the earlier case it was circularly polarized light.

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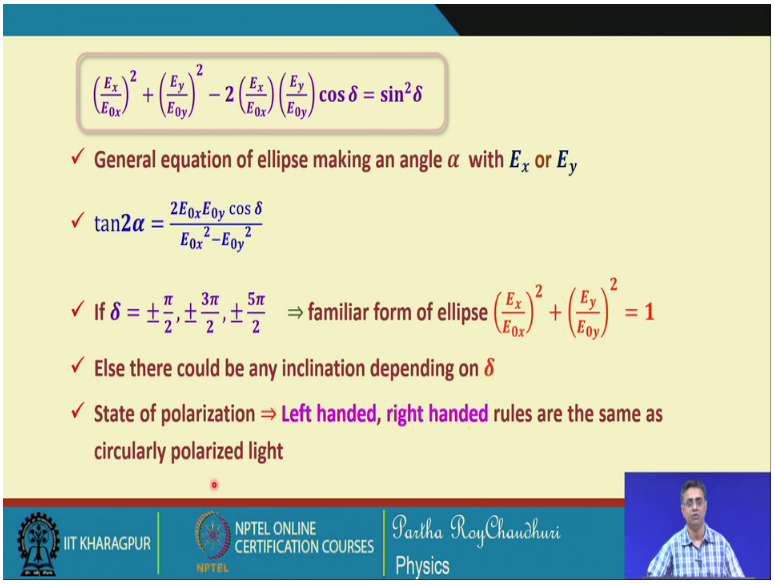

Now, this will correspond to elliptically polarized light, the generally equation of ellipse making an angle of alpha with E x and E y. So, and the angle will be equal to tan of 2 alpha which corresponds to this quantity; which is again a function of delta can assume plus minus pi by 2 plus minus 3 pi by 2 which is then you get the familiar form of the ellipse familiar form of the ellipse. Else there could be any inclination depending on the delta values.

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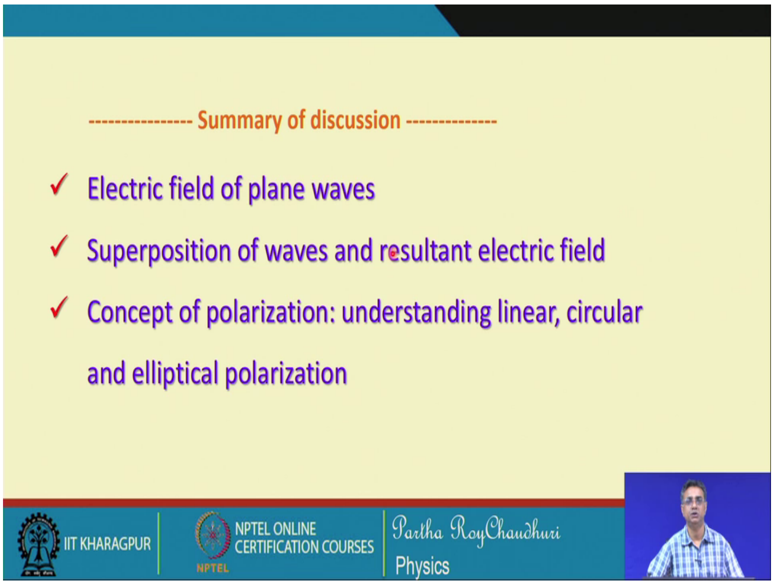

So, this is the delta initial phase that is going to decide the inclination of the orientation of the ellipse in the in the x y plane. The state of polarization can be left handed and right handed and these rules are the same as we have seen in the case of the circularly polarized light. So, through this discussion we try to understand how we can simply write the electric field for electromagnetic waves, the plane waves. And the plane wave is very useful because, throughout in optics and photonics we use this plane wave formulation.

Then to understand how the superposition of this plane waves can be mathematically tackled, we look at the resultant waves of the coherent source will give rise to interference. So, we try to understand that inclination factor that gives you the fringe width which is very interesting. Then we will look at the superposition of two orthogonal plane waves, the polarizations are orthogonal in that case how you encounter this linear, circular and elliptical polarization.

Thank you.