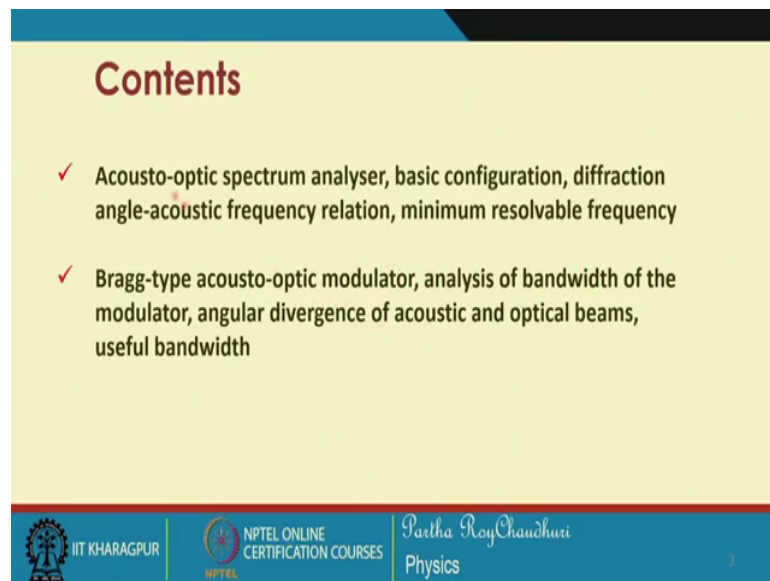


**Modern Optics**  
**Prof. Partha Roy Chaudhuri**  
**Department of Physics**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 57**  
**Asousto-optic Modulators and Devices (Contd.)**

So, we were discussing Bragg-type acousto-optic modulators.

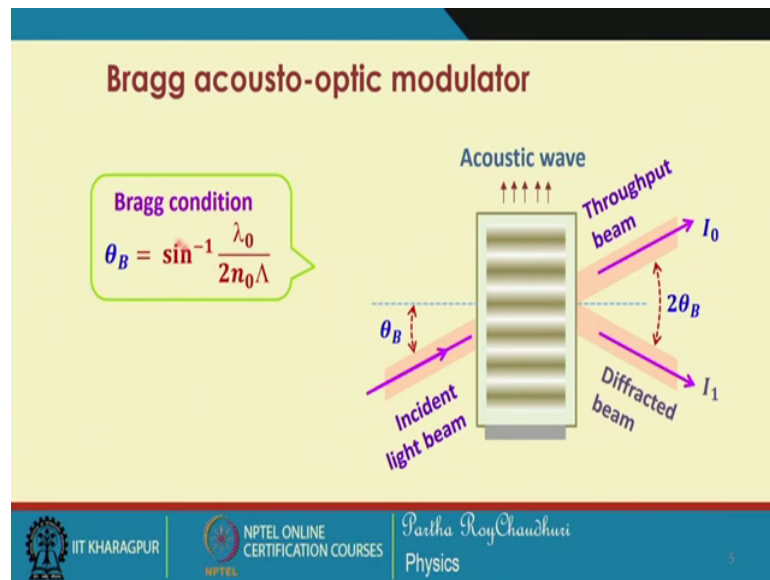
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And now, we will discuss this acousto-optic spectrum analyser, which is a very useful application of this Bragg-type acousto-optic modulator. We will look at the basic configuration of this spectrum analyser, we will try to understand how this modulator can be used for this purpose; considering the diffraction angle and acoustic frequency dependence. Then we will look at the minimum resolvable frequency.

In the earlier occasion, we in the case of deflector we looked at the minimum resolvable spots and here it is the individual frequencies that can be resolved by this instrument, the limit of that then, this acoust Bragg-type modulators analysis of the bandwidth, that we will understand looking at the angular divergence of the acoustic wave and the optical beams and from there we will locate the useful bandwidth.

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So, therefore, this let us begin with this Bragg condition; that is, this theta B equal to sin inverse of lambda 0 by twice n 0 capital lambda. And the basic configuration is the same that you have an optical beam incident at Bragg angle. There will be a through beam which is in absence of any acoustic field and there will be a reflected beam when this acoustic frequency is there and that condition satisfies the Bragg condition of this angle theta B.

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**Principle: Bragg acousto-optic modulators**

- ✓ The interaction length between optical and the acoustic beams is long  $L \gg \frac{\Lambda^2}{\lambda}$
- ✓ Input angle of the optical beam should be optimally the Bragg angle  $\theta_B$   $\sin\theta_B = \frac{\lambda}{2\Lambda}$
- ✓ The zero-th order beam is taken as the output beam of the modulator
- ✓ The modulation depth is given as  $\eta_B = \frac{I_0 - I}{I_0} = \sin^2\left(\frac{\Delta\phi}{2}\right)$

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Again this interaction length for this Bragg type of modulator is one of the basic requirement that the optical acoustic beam width should be large compared to this quantity. The input angle of the optical beam should satisfy the Bragg condition for all situations, that is the  $\sin \theta_B$  will be equal to  $\lambda_0$  upon twice of capital lambda. Here also the zero-th order diffracted beam will be taken as the modulator output and the modulation depth is given by this.

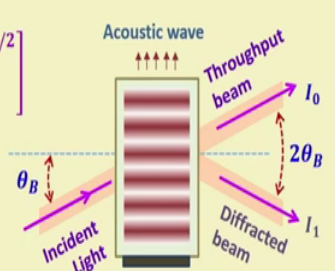
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**Bragg modulator: diffraction efficiency**

**Diffraction efficiency**

$$\eta_B = \sin^2 \left[ \frac{\pi}{\lambda_0 \cos \theta_B} \left( \frac{M_2 L}{2 H} P_a \right)^{1/2} \right]$$

**For small acoustic power**

$$\eta_B \approx \frac{\pi^2 M_2}{2 \lambda_0^2 \cos^2 \theta_B} \left( \frac{L}{H} \right) P_a$$


The diagram illustrates a Bragg modulator. An acoustic wave, represented by horizontal lines with upward-pointing arrows, propagates through a medium. Incident light enters from the left at an angle  $\theta_B$  relative to the normal. The light is split into two beams: a 'Throughput beam' with intensity  $I_0$  and a 'Diffracted beam' with intensity  $I_1$ . The angle between the two beams is  $2\theta_B$ .

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Diffraction efficiency this we have seen and for small acoustic power this is directly proportional to the acoustic power, it is also proportional to the length of the width of the acoustic beam and the height of the acoustic beam.

(Refer Slide Time: 03:00)

**Acousto-optic spectrum analyser**

Acousto-optic spectrum analyser  
devised using the principle of acousto-optic beam deflector

- ✓ In acousto-optic beam deflector, the angle at which the diffracted beam appears is proportional to the acoustic frequency  $\theta_B \approx \frac{\lambda_0 f}{2n_0 v_a}$
- ✓ If the transducer is fed simultaneously with different frequencies  $f_1, f_2, f_3, \dots$ , etc., then the Bragg diffracted light will propagate along different angles  $\theta_{B1} \approx \frac{\lambda_0}{2n_0 v_a} f_1, \theta_{B2} \approx \frac{\lambda_0}{2n_0 v_a} f_2 \dots$  etc.

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So, as a spectrum analyzer which is devised using the principle of acousto-optic beam deflector, so, is now very obvious that we have seen that the you know this Bragg angle condition is proportional to the frequency of the acoustic wave. For example, if we have acoustic frequencies  $f_1, f_2, f_3$ , then for these 3 frequencies the Bragg angle will be  $\theta_{B1}$  is equal to some constant which is all known parameters;  $f_1, f_2$ , etcetera. So that means, if you have different frequencies, the different Bragg angle condition will be.

So, Bragg diffracted light will propagate along different directions. This tells you that because there for different frequencies the waves optical beams are traveling in different directions, they will not overlap.

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**Diffraction angle ~ acoustic frequency**

$$\sin \theta_B = \frac{\lambda_0 f}{2n_0 v_a}$$
$$\cos \theta_B \Delta\theta = \frac{\lambda_0}{2n_0 v_a} \Delta f$$
$$\Delta\theta = \frac{\lambda_0}{2n_0 v_a \cos \theta_B} \Delta f$$
$$\theta_B \ll 1: \Delta\theta = \frac{\lambda_0}{2n_0 v_a} \Delta f$$

Incident light beam

Variable frequency sound source

$v_a$

Diffracted light beam

$\Delta\theta \propto \Delta f$

$\theta \propto f$

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And then they will not give you any modulation. So, looking at starting from the basic equation, that is  $\sin \theta_B = \frac{\lambda_0 f}{2n_0 v_a}$ . So, we did this before also to show that this the optical beam angular width of the optical beam will be proportional to the; that is the angular direction, the deflection of the optical beam will be proportional to the acoustic frequency. [FL].

So, starting from the basic Bragg diffraction condition that is  $\sin \theta_B$  is equal to this quantity, we have seen that the angle of the deflection is proportional to the acoustic frequency; that is  $\Delta\theta$  that is also for small Bragg diffraction condition.

So,  $\Delta\theta$  is proportional to  $\Delta f$ ; that means, if you have different frequencies  $f_1, f_2, f_3$ , the angle  $\theta_1, \theta_2, \theta_3$ , they will be also different. So that means, for different acoustic frequencies, the diffracted light will travel along different directions.

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### Acousto-optic spectrum analyser

Intensity distribution of diffracted wave on plane P is proportional to the power spectrum of the signal fed to the acoustic transducer.

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For example, if you feed this acousto-optic modulator with frequency acoustic frequencies  $f_1, f_2, f_3$ , then they will be focused at different points on this plane P.

So, the intensity distribution of the diffracted wave on a plane P is then proportional to the power spectrum of the signal fed to the acoustic transducer. So, you have  $f_1, f_2, f_3$ , they will appear the deflected beam corresponding to these frequencies will appear at different points they will be collected by a lens.

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### Acousto-optic spectrum analyser

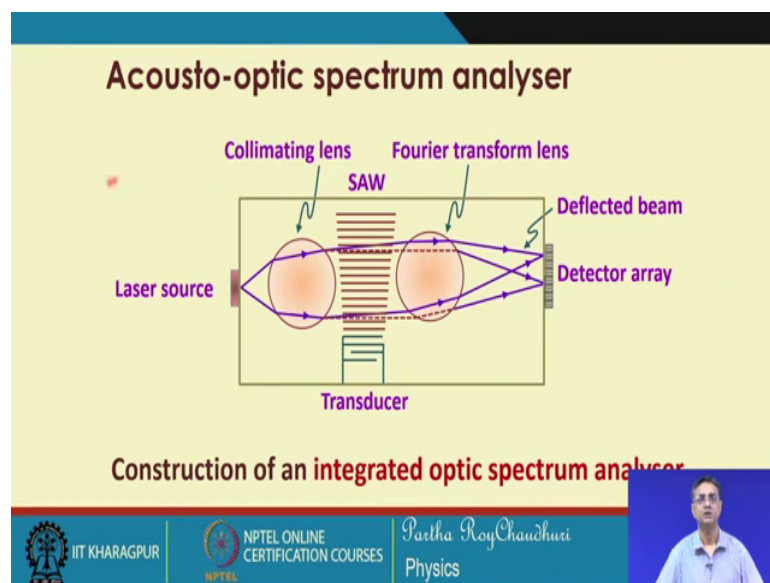
- ✓ In this configuration, different diffracted light beams focus at different points on the focal plane of the lens
- ✓ Since **diffraction efficiency** is approximately proportional to **power** in the corresponding **acoustic wave** component, diffracted light intensity on the output plane P gives the **Fourier spectrum** of the signal feeding the acoustic transducer
- ✓ This setup works as a real time **spectral analyser** for wide band signals

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So, in this configuration, different diffracted light beams focus, they will focus at different points on the optical plane of the lens. Since, diffraction efficiency is approximately proportional to the power in the corresponding acoustic wave. This we have seen from the diffraction efficiency which is proportional to the acoustic power, then reflected light intensity on the plane P gives the Fourier spectrum of the signal that is being fed to the acoustic transducer.

This setup works as a real time spectral analyser for the wide band signals.

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So, this is an integrated optics spectrum analyser. Configuration you have a laser source and you have a collimating lens which will make the optical beam approximately parallel by this lens. And then this is surface acoustic waves which is generated by a piezo transducer, then through this surface acoustic waves the optical beam will pass through and again there will be a Fourier transforming lens which will collect the beam at the detector array, the deflected beam will be collected at the detector array.

So, depending on the frequency components of the acoustic wave, they will be deflected at they will be collected at different points on the detector array. So, this actually works as an optical. So, you have the frequency for a frequency there will be one position in the detector, for another frequency there will be another position of the detector. Therefore, looking at the position of the director of the position of the deflected spot of the detector,

we can calibrate and make one to one correspondence of the acoustic frequency with the detected spot. So, that is the basic principle.

(Refer Slide Time: 09:30)

**Minimum resolvable frequency**

The angle of deflection  $\theta_d \approx \frac{\lambda_0 f}{n_0 v_a}$   
for a change in acoustic frequency, change in deflection angle

$$\Delta\theta_d = \frac{\lambda_0}{n_0 v_a} \Delta f$$

Angular divergence of optical beam

$$\Delta\theta_o \approx \frac{\lambda_0}{n_0 w_0}$$

Minimum resolvable frequency will be  $\Delta f$  for which  $\Delta\theta_d = \Delta\theta_o$

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So, the angle of deflection  $\theta_d$  we have seen is proportional to  $f$ . And for a change in the acoustic frequency  $\Delta f$ , the change in deflection angle will be  $\Delta\theta_d$ . So, angular divergence of the optical beam  $\Delta\theta_o$  of the optical beam which we have seen from the fundamental diffraction theory for Gaussian optical beam, this we have seen. And therefore, the minimum resolvable frequency will be  $\Delta f$  for which  $\Delta\theta_d$  is equal to  $\Delta\theta_o$ .

So, this gives you the minimum resolvable frequency in this case.



(Refer Slide Time: 10:11)

**Minimum resolvable frequency**

Call this frequency as  $\delta f$  we obtain  $\frac{\lambda_0}{n_0 v_a} \delta f = \frac{\lambda_0}{n_0 w_0}$

That gives the frequency resolution  $\delta f = \frac{v_a}{w_0} = \frac{1}{\tau}$

**Frequency resolution** of the spectrum analyser is the inverse of acoustic transit time through the optical beam

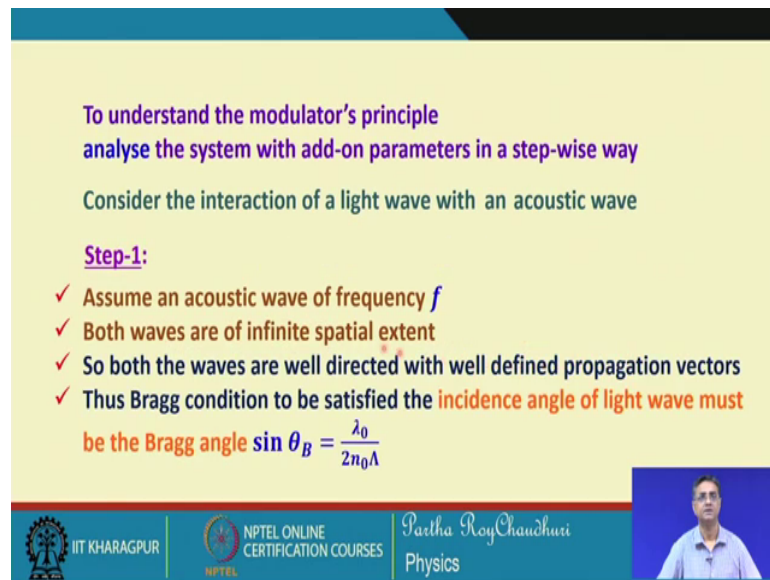
**The bandwidth** of the device is the acoustic frequency range  $\Delta f$  over which Bragg condition will be satisfied

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Now, call this minimum resolvable frequency as Kronecker delta f. Therefore, this  $\lambda_0 / n_0 v_a \delta f = \lambda_0 / n_0 w_0$  is equal to this. So, this gives you the frequency resolution. So, delta f because from here, if you bring this to the right hand side, then  $\lambda_0 / n_0$  all those things will cancel will leave just acoustic wave velocity by the optical beam width which is equal to 1 upon tau the transit time, the access time of the acoustic beam for the optical wave, optical beam.

Frequency resolution of the optical spectrum analyser is therefore, the inverse of the transit time. The bandwidth of the device is the acoustic frequency range delta f over which this Bragg condition can be satisfied because if I just randomly change the frequency, for all frequencies the Bragg condition will not be satisfied. It will be satisfied only for a range of frequencies and we are going to analyze for which range of the frequencies, this Bragg condition will be satisfied and it can be used as a useful spectrum analyser.

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To understand the modulator's principle  
analyse the system with add-on parameters in a step-wise way

Consider the interaction of a light wave with an acoustic wave

**Step-1:**

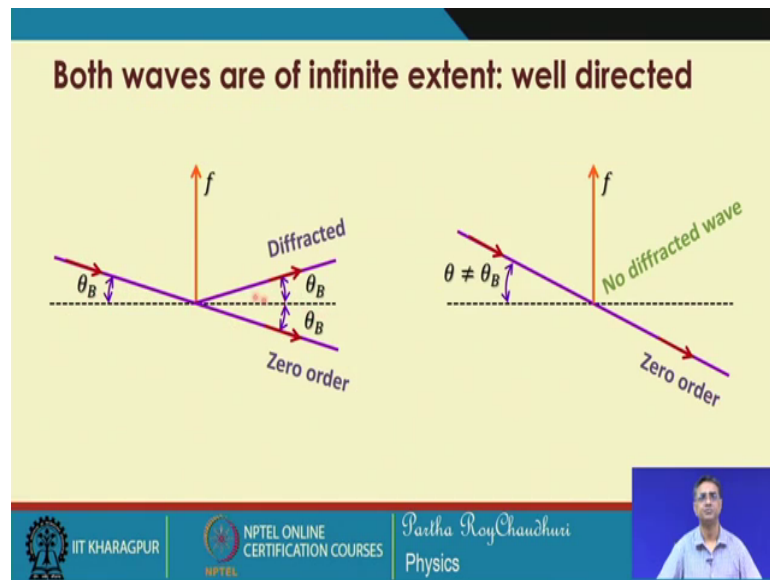
- ✓ Assume an acoustic wave of frequency  $f$
- ✓ Both waves are of infinite spatial extent
- ✓ So both the waves are well directed with well defined propagation vectors
- ✓ Thus Bragg condition to be satisfied the incidence angle of light wave must be the Bragg angle  $\sin \theta_B = \frac{\lambda_0}{2n_0\lambda}$

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So, the bandwidth of modulator, this analysis to understand this modulators principle, because it will involve several parameters; like, the width of the optical beam, wavelength of the optical beam, acoustic beam width, acoustic frequency, range of frequencies. Therefore, we make a stepwise analysis to understand very clearly how this bandwidth can be optimized. So, for step 1, let us assume an acoustic wave of frequency  $f$ . So, it has only one frequency and the optical wave and the acoustic wave both are of infinite spatial extent. They are infinitely extended and there is an interaction.

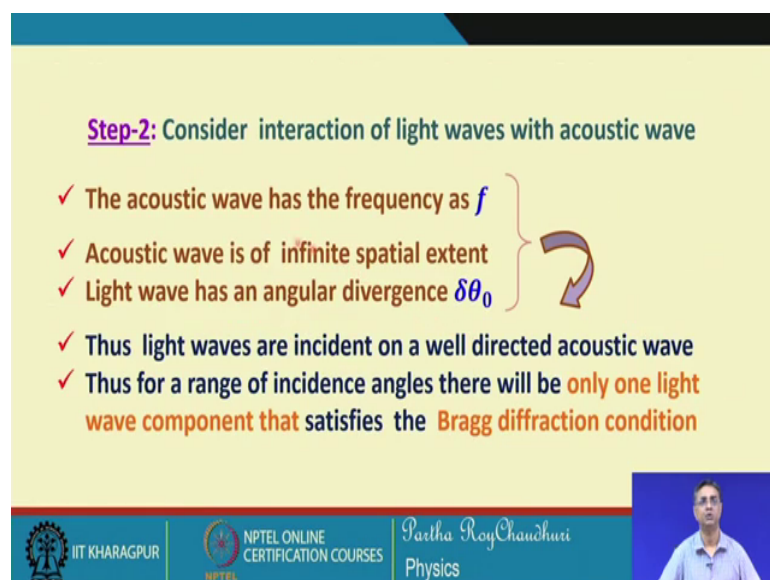
So, both the waves are well directed with well-defined propagation vectors because they are infinitely extended. So, both of them have only one propagation constant, each attached to the acoustic wave and the optical wave respectively. Thus the Bragg condition to be satisfied, the incident angle of light must be the Bragg angle; that is,  $\sin \theta_B = \frac{\lambda_0}{2n_0\lambda}$ . This is the condition that has to be satisfied. So and that is very clear here because you have only one frequency, you have one optical beam propagation direction.

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So, this is pictorially what I wanted to mean, you have one direction of the optical beam, you have only one direction of the acoustic beam. So, the interaction is one to one, you have a diffracted beam which is given by this theta B. And if this does not satisfy the Bragg condition, then there is no diffracted wave there is no diffracted wave, only you will get the zero-th order, the undiffracted direct beam through beam.

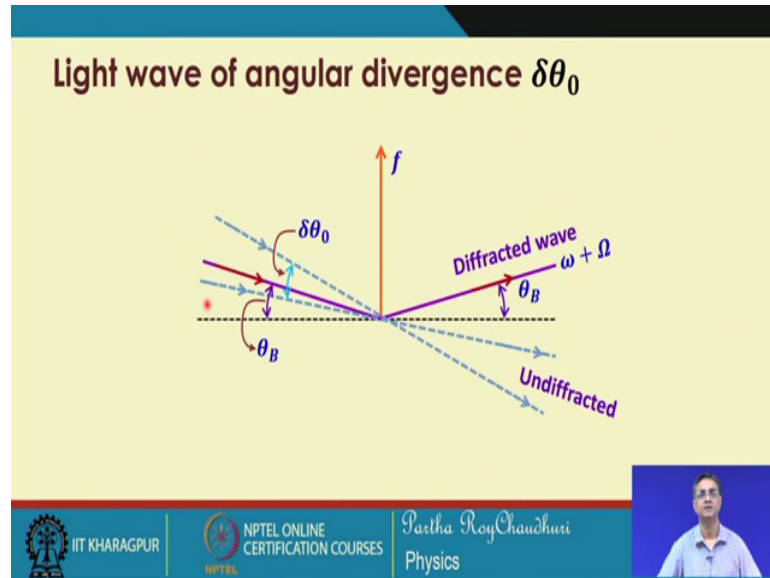
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Next step we consider the interaction of the light waves with acoustic frequency, which is  $f$  again.

But now, the acoustic wave is of the light wave has an angular divergence. Acoustic wave it remains the same, it has only one frequency, one propagation direction, but the light wave has an angular direction divergence of  $\delta\theta_0$ .

(Refer Slide Time: 13:43)



So, like this situation, you have an angular divergence of the light wave which is  $\delta\theta_0$ . Therefore, out of these different components along different angles, one of them will satisfy the Bragg condition, and this will be frequency accepted by  $\omega + \Omega$ .

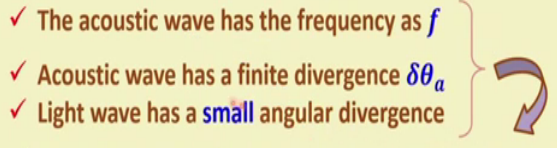
So, thus light waves are incident on a well directed acoustic wave just for a range of incidence angles, there will be only one light wave component that satisfies the Bragg diffraction condition. So, that is very easily that can be understood, that you have a range of optical waves falling on it with different angles, but only one of them will be diffracted which will satisfy the Bragg condition.

(Refer Slide Time: 14:34)


**Step-3:** Consider interaction of light waves with an acoustic wave

- ✓ The acoustic wave has the frequency as  $f$
- ✓ Acoustic wave has a finite divergence  $\delta\theta_a$
- ✓ Light wave has a **small** angular divergence

Thus there is a range of incidence angles for which the light wave satisfies **Bragg condition** by one acoustic plane wave component



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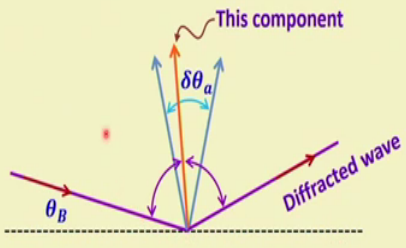


Next we consider that the acoustic wave has a frequency  $f$ , but now this time acoustic wave has a finite divergence,  $\delta\theta_a$ , light wave has a relatively small angular divergence. It is almost well directed, in that case what will happen? Because this time your acoustic wave has an angular divergence, but light wave is almost well directed.


So, in that case there is a range of incident angles for which the light wave satisfies the Bragg condition by one acoustic plane wave component, let us understand this.

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**Acoustic wave of angular divergence  $\delta\theta_a$**

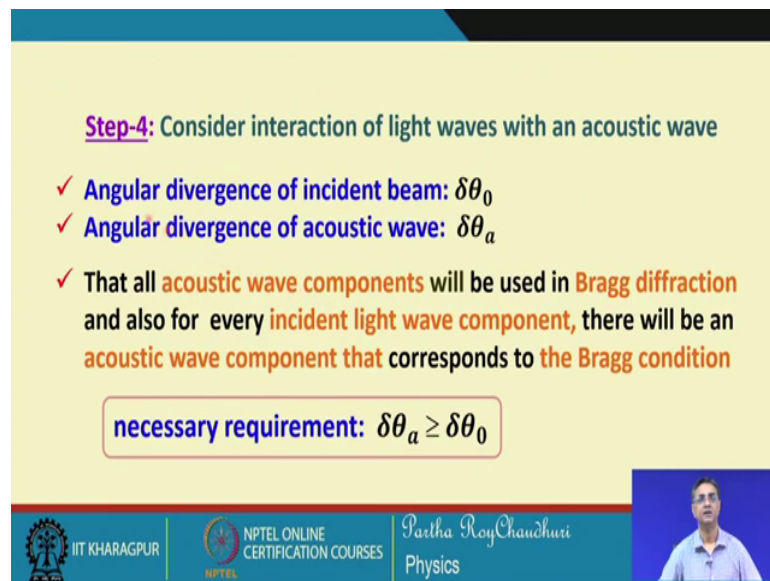


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Let us suppose you have one fixed direction of the optical beam, but the acoustic beam has a divergence of  $\delta\theta_a$ . So, there is one direction for which this beam will pick up this acoustic wave component and it will be diffracted. If you change this Bragg angle, if you change this angle at which the Bragg condition is satisfied, then this will pick up another acoustic wave component and it will be deflected. So, there will be a range of acoustic wave components for which this Bragg diffraction condition will be satisfied.

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**Step-4: Consider interaction of light waves with an acoustic wave**

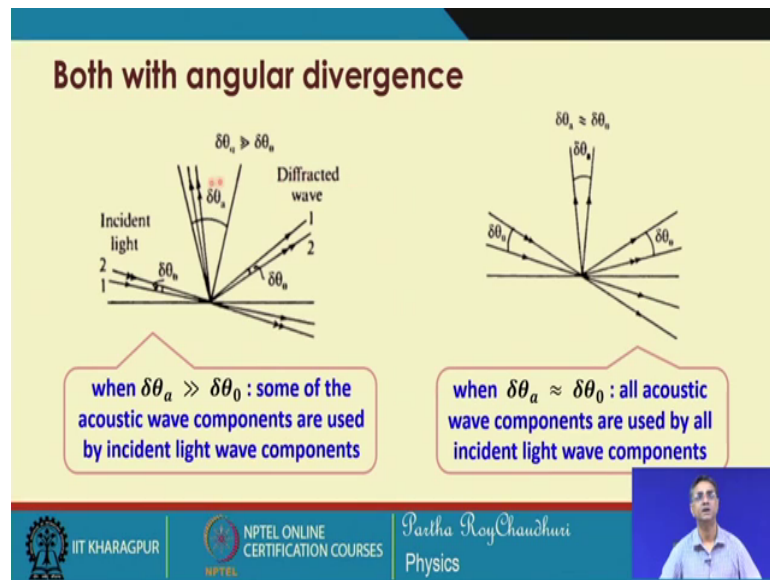
- ✓ Angular divergence of incident beam:  $\delta\theta_0$
- ✓ Angular divergence of acoustic wave:  $\delta\theta_a$
- ✓ That all acoustic wave components will be used in Bragg diffraction and also for every incident light wave component, there will be an acoustic wave component that corresponds to the Bragg condition

**necessary requirement:  $\delta\theta_a \geq \delta\theta_0$**

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Let us now look at the next step that angular divergence of the incident optical beam,  $\delta\theta_0$  is also there as well as the angular divergence of the acoustic wave that is also there, but frequency is the only one acoustic frequency. We will add on this frequency next time. So, that all acoustic wave components will be used in Bragg diffraction and also for every incident light wave component, there will be an acoustic wave component that corresponds to the Bragg condition. Then this has to satisfy.

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Let us see that, because if you look at this configuration you have 1 an incident wave for which you have a diffracted wave. This is the range of different acoustic wave components, for optical beam 1 you have a refracted wave which is a Bragg satisfied, then incident wave incident optical beam 2; for which there will be another diffracted beams under Bragg condition. So, these are coming from the various available components of the acoustic waves. So, for a range of optical angular width, or optical beams you will have this. So, at least this much of change in the angular width of the optical beam should be accommodated within this range of acoustic wave components.

Therefore, this should be more than this, so that all the optical beam components will be utilized by this acoustic plane wave components to give the Bragg satisfied Bragg condition diffraction of the beam. So, that tells you that this acoustic wave angular width must be equal to must be more or approximately equal to, it must be more or approximately equal to the angular divergence of the optical beam.

So far all that we have discussed is that to have if we have a rang range of optical waves making some angle delta theta, this range then there is a need of delta theta a, the angular width of the acoustic wave for all the optical beams to be utilized for Bragg diffraction.

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**Modulated acoustic wave**

- ✓ In discussion so far, we considered acoustic wave of single frequency  $f$
- ✓ Now consider a modulated acoustic wave for diffraction of light beam
- ✓ A modulated wave is a superposition of waves at different frequencies

---

when light wave interacts with an amplitude modulated acoustic wave,

- ✓ light wave interacts with each of the component acoustic frequencies
- ✓ Bragg diffraction occurs if the Bragg condition corresponding to various frequency components is satisfied

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*(A small video inset of the speaker is visible in the bottom right corner of the slide.)*

Now in this discussion, that is what we have said, we considered one of single frequency and for that we considered a range of optical beam, angular width and a range of acoustic wave. A modulated wave across is a superposition of waves at different frequencies. Therefore, when light wave interacts with an amplitude modulated acoustic wave, then the light wave interacts with each of the component of acoustic frequencies. So, far we have considered only one frequency.

Now, as an add-on if we increase more number of frequencies to the acoustic wave, then for each of the frequencies this story this condition will be satisfied. That is Bragg diffraction occurs if the Bragg condition corresponding to various frequency components is satisfied. So, for each of the frequency component of the acoustic wave, the case that we have discussed must be satisfied.



(Refer Slide Time: 19:29)

**Beating of waves: intensity modulation**

The diffracted light waves are **shifted in frequency** by corresponding acoustic wave frequencies

Thus when the diffracted light waves interfere, there will be **beating** between the waves which ultimately results in **intensity modulation** of the diffracted light wave

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So, therefore, the diffracted light waves are shifted in frequency by the corresponding acoustic wave frequencies. Thus when the diffracted light waves interfere, there will be a beating between the waves we know this. If we have 2 frequencies  $f_1$  and  $f_2$ , there will be a beat frequency  $\Delta f$  which is the difference of  $f_1$  and  $f_2$ .

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**Step-5: Consider interaction of light waves with acoustic wave**

- ✓ Frequency of acoustic wave:  $f_a$
- ✓ Those of modulated acoustic wave:  $f_a \pm \Delta f_a$
- ✓ Bragg diffraction will occur if Bragg condition is satisfied for each of the acoustic frequency components
- ✓ Diffracted wave will be frequency shifted (up and down) by those respective acoustic wave frequencies

**Beating phenomena**

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So, in the fifth step we consider the interaction of light waves with acoustic wave, where the frequency of the acoustic wave is  $f_a$ . Those of the modulated acoustic wave will be  $f_a \pm \Delta f_a$ . So, this is the change in the frequency for the acoustic wave.

So, Bragg diffraction will occur if Bragg condition is satisfied for each of the acoustic frequency components, see you have a diffracted wave that will be frequency up shifted or down shifted by the respective acoustic wave component. So, if you have let us say  $f_a$  plus  $\Delta f_a$ , then this is a frequency which is more than  $f_a$ , and there will be another frequency which is less than  $f_a$ ; that is  $f_a$  minus  $\Delta f_a$ . So, these 2 will correspond to up shifted and downshift and frequency components.

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**Simplified discussion:**

Frequency of incident optical beam:  $\nu$

Frequencies of the acoustic waves:  $f_0, f'$

Equivalent to amplitude modulated acoustic wave having modulation frequency (the **beat frequency**)

$$\Delta f = f' - f_0$$

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Now, we will make a simplified discussion, because you have more number of parameters coming into play;  $\Delta \theta_0$ ,  $\Delta \theta_a$ , and  $\Delta \theta_f$ . So, frequency of the incident optical beam is  $\nu$ . Frequency of the acoustic waves is  $f_0$  and  $f'$  these are the 2 frequencies, let us consider that. Then this is equivalent to a beat frequency of  $\Delta f$  which is equal to  $f' - f_0$ .

So, this is the beat frequency which is the modulation frequency of the acoustic wave. So, acoustic wave is now modulated; which gives rise to a beat frequency of  $\Delta f$ .

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**Case-1:**

- ✓ small angular divergence of the incident optical beam
- ✓ but finite ( $\delta\theta_a$ ) divergence of acoustic wave at  $f'$  and  $f_0$

The diagram shows an incident optical beam (v) and two acoustic wave components (v + f' and v + f\_0) interacting to produce diffracted optical beams. The angle of divergence of the acoustic wave is labeled as  $\delta\theta_a$ .

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And then in that case, first let us consider that small angular divergence of the incident optical beam that is it is well directed this optical beam. But because it sees the angular divergence of the acoustic beam over a range of delta theta a, so, this optical beam can pick up 2 waves 2 acoustic plane wave components, one is along this another is along this.

So, as to get frequency shifted Bragg diffracted optical beam  $\nu + f_0$  and  $\nu + f'$  because in the acoustic wave you have 2 frequencies;  $f_0$  and  $f'$ . And they will correspond to different acoustic wave components for a given direction of the optical beam. You have one direction of the optical beam, for which there is one frequency of the acoustic wave, for one frequency component of the acoustic wave, which will give you the Bragg condition. And there is another plane wave component of the acoustic wave which will give you the Bragg condition.

So, up till here, this is now very clear. You have 1 frequency, 2 diffracted wave because of the 2 frequencies, 2 acoustic frequencies.

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**Case-1:**

- ✓ diffracted waves are travelling along different directions
- ✓ therefore, no overlap of the diffracted wave takes place

**So no intensity modulation!**

The diagram shows a horizontal dashed line representing a surface. A wave vector  $v$  (red arrow) is incident from the left. Two diffracted wave vectors,  $v + f'$  (green arrow) and  $v + f_0$  (blue arrow), are shown traveling away from the surface at different angles. The angle between these two diffracted waves is labeled  $\delta\theta_a$ . A small inset video of the speaker is visible in the bottom right corner.

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And now that because these 2 diffracted waves will travel in 2 different directions, very clear. So, they will not be superimposed, there is no overlap. And that is why even if you change the acoustic frequency, because they are not overlapping, so, they will not give rise to any amplitude modulation.

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**Case 2:**

- ✓ finite angular divergence ( $\delta\theta_0$ ) of light beam
- ✓ finite ( $\delta\theta_a$ ) of the acoustic wave at  $f'$  and  $f_0$

The diagram shows a horizontal dashed line representing a surface. A light beam with finite angular divergence  $\delta\theta_0$  is incident from the left, with rays labeled 1 and 2. Two acoustic wave vectors,  $f_0$  (red arrow) and  $f'$  (blue arrow), are shown. The angle between these two acoustic waves is labeled  $\delta\theta_a$ . The resulting wave vectors are  $v + f_0$  (red arrow) and  $v + f'$  (blue arrow). The diagram also shows the corresponding wave vectors in the frequency domain:  $f_0 \rightarrow 1'$ ,  $f_0 \rightarrow 2'$ , and  $f' \rightarrow 1''$ . A small inset video of the speaker is visible in the bottom right corner.

**aim:**  
To find maximum  $f'$ , such that  $v + f'$  will be along  $2'$

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Now, we modify the situation. Finite angular divergence of delta theta, that is now this optical beam has a finite angular divergence. And the acoustic beam contains  $f'$  and  $f_0$ , these 2 frequencies; which is over a range and it has an angular width of delta theta a.

Now, if you consider this 1 optical beam which is corresponding to this direction, then this will be giving you a diffracted beam 1 dash along this P dash.

There will be another beam along this direction optical beam 2, which will give you 2 dash and these 2 things are due to  $f_0$ . You see look at this for a frequency acoustic frequency  $f_0$ , 1 undergoes a diffraction 1 dash; for the acoustic frequency  $f_0$ , 2 undergoes a diffraction 2 dash. But  $f$  dash is also present.

Therefore, as a result this 1 may undergo diffraction because of this  $f$  dash which will occupy some position between these 2 dash and 1 dash. So, 1 may undergo 1 double dash by this acoustic frequency  $f$  dash. So, this is here, but our task is to maximize this so that it goes along this direction. So, that will give you the maximum value of the acoustic frequency; that means, to say precisely that the optical beam 1 will be maximum, will be diffracted along the direction at the most it can go along the direction 2 dash.

So, that is the maximum possible value of  $f$  dash compared to  $f_0$ . And that gives you the range  $f_0$  minus I mean  $f$  dash minus  $f_0$ . So, to find the maximum  $f$  dash such that this  $\delta \nu$  plus  $\delta f$  this frequency will be along 2 dash; so, that is the task.

(Refer Slide Time: 25:25)

To get angular overlap:  $2\theta'_B < 2\theta_B + \delta\theta_0$

Largest  $f'$ :  $\theta'_B = \sin^{-1}\left(\frac{\lambda_0 f'}{2n_0 v_a}\right) = \theta_B + \frac{1}{2}\delta\theta_0$

$f' = \frac{2n_0 v_a}{\lambda_0} \sin\left(\theta_B + \frac{1}{2}\delta\theta_0\right)$

$\approx f_0 + \left(\frac{n_0 v_a}{\lambda_0}\right) \cos \theta_B \delta\theta_0$

assuming  $\delta\theta_0 \ll 1$  and using relation  $f_0 = \frac{n_0 v_a}{\lambda_0} \sin \theta_B$

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So, that you get an overlap of all the beams, you get an overlap of all the beams, I think. So, to get an angular overlap this must be the condition, because you will not allow this incident beam to be overlapped with the diffracted beam. So, this much of offset must be

there, that is what this condition says that, this Bragg diffraction twice will be this delta angular.

Therefore, it gives you the condition that delta theta B plus half the angular width, that must be equal to the for the largest frequency of this, right. For the largest frequency of this, it must be half the angular width of this.

(Refer Slide Time: 26:31)

**Modulation bandwidth**

**Modulation bandwidth:**

$$\Delta f_{max} = f' - f_0 = \left( \frac{n_0 v_a}{\lambda_0} \right) \delta \theta_0 \cos \theta_B$$

If the minimum width of light beam is  $w_0$ :  $\delta \theta_0 \approx \frac{\lambda_0}{n_0 w_0}$

$$\Delta f_{max} = \left( \frac{v_a}{\omega_0 / \cos \theta_B} \right) = \frac{1}{\tau} \quad \tau: \text{time of transit for acoustic wave to cross the light beam}$$

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Therefore, f dashed through simple algebraic manipulation we can show that it is equal to f 0 plus this. So, this is your the delta f, the excess of the frequency which gives rise to this. So, f 0 equal to this, we have again used that for small Bragg angle diffraction. Therefore, the maximum value of the frequency delta f max will be difference of this; which is equal to this quantity.

If the minimum width of the light beam is w 0, then delta theta 0 is approximately proportional to because from here only, so, you get lambda 0, this is the from the fundamental condition that lambda 0 by n 0 into with the diffraction of this. So, if I use this then you get delta f max, which is equal to acoustic velocity by this one; which is a direct consequence of this expression. And then this is the inverse of the transit time, the reciprocal of time, the time of transit for the acoustic wave to cross the light beam.

So, this we have seen earlier that the delta f and we will make use of this parameter to understand how it happens.

(Refer Slide Time: 27:28)

**Condition for no overlap**

In order for the undiffracted beam lying in angular region between OC and OD not to overlap with diffracted beam lying within OR and OS, it is necessary that  $DOR > 0$

$2\theta_B - 2\delta\theta_0 > 0$

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So, let us now look at this. You have a beam along this optical beam, you have another beam along this and this is the angular width of all the beams. And therefore, corresponding to A there will be a deflected beam; which will be given by this and corresponding to this there will be a diffracted beam which will be up shifted. This is frequency down shifted. And these are the 2 frequencies acoustic frequencies, one is  $f_0$  plus  $\Delta f$ , another is  $f_0$  minus  $\Delta f$ ; which correspond to the diffracted beam S and R respectively.

But otherwise for only  $f_0$ , this frequency A and B will be deflected along P and Q. These 2 will be diffracted along P and Q. This S and R are the diffracted beams as a consequence of  $f_0$  plus  $\Delta f_{max}$  and  $f_0$  minus  $\Delta f_{max}$ . In order for the diffracted beam lying within the angular region, now that this within this region ODR, DOR, within this region, you cannot allow this diffracted beam to come down. So, OC and OD not to overlap with the diffracted beam lying within the within this OR and OS, OR and OS within this range.

So, this is the necessary condition that this DOR must be greater than 0. So, that means, this should never overlap, this zero-th order and the diffracted order beam should never overlap. So, that gives you this condition  $2\theta_B - 2\delta\theta_0 > 0$ , because anyway this is already known to  $\delta\theta_0$ . So, that is the condition for no overlap of the zero-th order beam with the diffracted beam.

(Refer Slide Time: 29:33)

That is, the condition to remove any overlap of diffracted beam with undiffracted beam,  $\delta\theta_0$  must satisfy relation

$$\delta\theta_0 < \theta_B$$

$$\Delta f_{max} = \left(\frac{n_0 v_a}{\lambda_0}\right) \delta\theta_0 \cos \theta_B$$

Assuming,  $\sin \theta_B \approx \theta_B$   
 $\cos \theta_B \approx 1$

$$\Delta f_{max} < \frac{1}{2} f_0$$

Largest modulation bandwidth is half the center frequency

Using that we can bring out and assume in that sine theta B is very small, is approximately equal to theta B, cosine theta equal to 1, you get this delta f max which is equal to half f 0.

So, largest modulation bandwidth is half the center frequency of the acoustic wave; is a very beautiful finding from this analysis, and I hope it has become very clear now.

(Refer Slide Time: 30:01)

An intensity modulated acoustic wave crossing a light wave of beam width  $w_0$

For light beam width  $w_0 \ll \frac{v_a}{\Delta f}$   
 i.e., the modulation frequency of the acoustic wave  $\Delta f \ll \frac{v_a}{w_0}$

At time  $t = 0$ :  
 when the acoustic wave is at peak the light beam crosses and acoustic wave amplitude is fully utilised for diffraction at Bragg angle

$w_0 \ll \frac{v_a}{\Delta f}$

$\frac{v_a}{\Delta f}$

$t = 0$

We will make it even more understandable by assuming this fact that let us consider an acoustic wave crossing a light beam width  $w_0$ . This is the width of the light beam, and



we consider the situation that this light beam width  $w_0$  is much less compared to this width; that is from the minimum to the minimum of the acoustic wave under modulation condition which is given by  $v_a$  acoustic wave by  $\Delta f$  because this is the separation of this gap, minimum to minimum  $v_a$  acoustic,  $v_a$  by  $\Delta f$ .

So, this optical width must be less than these value; that is the modulation frequency of the acoustic wave  $\Delta f$  must be less than  $v_a$  by  $w_0$ ; which is a consequence of this expression, this relation, you can see that. Now, let us consider this time when the acoustic your optical beam is always here, but the acoustic beam is traveling through it. This acoustic beam when it is at this state; that is, when the optical beam is just crossing through the middle of the acoustic wave with the peak positions here at this position, let us call this timing as time  $t$  equal to 0.

Then the acoustic beam is at it is at the peak, the light beam crosses and acoustic wave amplitude is fully utilized, you can see that this acoustic beam amplitude is the maximum amplitude here; which is fully utilized by the optical beam. And therefore, it is well diffracted. So, this Bragg diffraction is complete here, because the optical beam is completely enclosed by the maximum acoustic wave amplitude. So, you get the maximum complete Bragg diffraction.

(Refer Slide Time: 32:28)

Under the SAME CONDITION as before

For light beam width  $w_0 \ll \frac{v_a}{\Delta f}$   
i.e., the modulation frequency of the acoustic wave  $\Delta f \ll \frac{v_a}{w_0}$

At time  $t = \frac{1}{2\Delta f}$ :  
acoustic wave is at minimum amplitude  
the light beam crosses and acoustic wave amplitude is **not available** for diffraction

$w_0 \ll \frac{v_a}{\Delta f}$

$t = \frac{1}{2\Delta f}$

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Next consider the other situation under the same condition, you have a this optical beam which is of the same width, passing through the same position, but this time the acoustic

beam is traveling in such a way that it is now here. The beam is just at the position where the acoustic wave is minimum, amplitude is acoustic wave is at minimum amplitude. And this timing we know from the property of the acoustic wave must be  $t$  equal to  $1$  by twice  $\Delta f$ .

This is the time starting from  $t$  equal to  $0$ , now this is the time at which you have this optical beam which is traveling the acoustic wave, where the acoustic wave amplitude is minimum. So, it is the optical wave is not used for Bragg diffraction because there is no acoustic wave amplitude here. So, at this time the diffracted beam will have minimum amplitude, the acoustic wave amplitude is not available for diffraction. So, this diffracted beam will have the minimum amplitude, because there is not enough amplitude of the acoustic wave.

(Refer Slide Time: 33:34)

**Amplitude modulation of light beam**

At time  $t = 0$  :  
acoustic wave at peak amplitude fully diffracts the optical beam

At time  $t = \frac{1}{2\Delta f}$  :  
acoustic wave amplitude is not available for the light diffraction

Thus, the amplitude of acoustic wave varies with time giving rise to amplitude modulation of light beam through Bragg diffraction

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So, at time  $t$  equal to  $0$  you get the fully diffracted optical beam, at time  $t$  equal to  $1$  by twice of  $\Delta f$ , the acoustic beam is not available for diffraction with minimum  $0$  diffraction, minimum diffraction. Thus, the amplitude of the acoustic wave varies with time giving rise to amplitude modulation of the light beam. So, because the amplitude of the acoustic wave is varying with time, the light wave will be modulated in the same direction; sometimes maximum, sometime minimum. So, that is the mechanism of amplitude modulation of light.

(Refer Slide Time: 34:11)

Under a **DIFFERENT CONDITION** of the waves

For light beam width  $w_0 \approx \frac{v_a}{\Delta f}$   
i.e., the modulation frequency  
of acoustic wave  $\Delta f \approx \frac{v_a}{w_0} \approx \frac{1}{\tau}$

On the whole, the light beam sees the same  
amplitude distribution of the acoustic wave  
So **NO AMPLITUDE MODULATION** takes place

$w_0 \approx \frac{v_a}{\Delta f}$

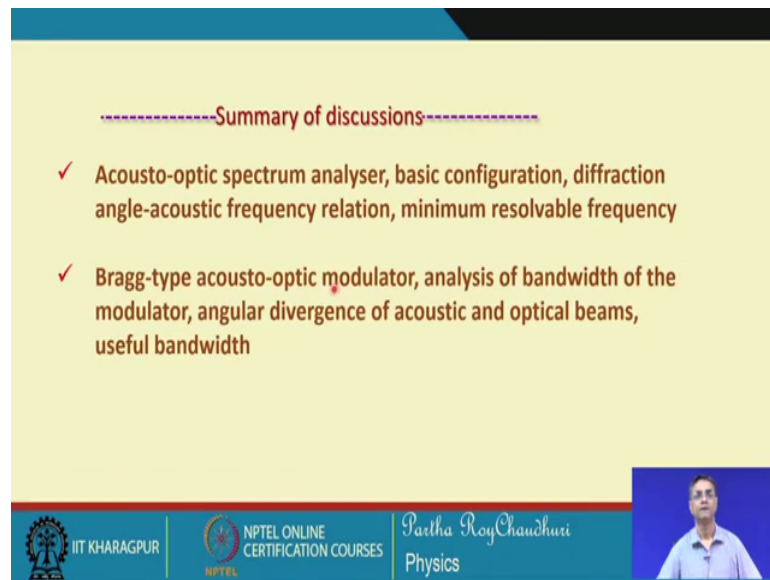
The diagram shows a vertical acoustic wave with a sinusoidal amplitude distribution. Two parallel light rays, represented by purple lines, pass through the wave. The distance between the rays is labeled  $w_0$ . The acoustic wave's amplitude is labeled  $w_0 \approx \frac{v_a}{\Delta f}$ . The rays are shown to be unaffected by the wave's amplitude modulation.

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Now, under a different condition, when this condition is not satisfied or it could be more the optical beam width is more or at least critically equal to this distance, that is from  $v_a$  this is the distance;  $v_a$  by  $\Delta f$  and it is completely engrossed by this. In that case, whether this beam changes passes through this optical beam, on the average on the whole the light beam sees the same amplitude distribution of the acoustic wave. And therefore, there will be no amplitude modulation.

Therefore, this amplitude modulation condition is completely decided by this condition, that  $t$  will be equal to  $\Delta f$ .

(Refer Slide Time: 34:58)



-----Summary of discussions-----

- ✓ Acousto-optic spectrum analyser, basic configuration, diffraction angle-acoustic frequency relation, minimum resolvable frequency
- ✓ Bragg-type acousto-optic modulator, analysis of bandwidth of the modulator, angular divergence of acoustic and optical beams, useful bandwidth

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So, that is what is how this amplitude modulation of the optical beam can be accomplished by acousto-optic modulator. So, in summary, we discuss this acousto-optic spectrum analyser, basic configuration, diffraction angle and acousto-optic acoustic frequency relation, minimum resolvable frequency.

Then at length we try to understand the several parameters associated with acousto-optic modulators in terms of the bandwidth of the modulator; which are due to the angular divergence of the acoustic beam and optical beam. And we could locate that what will be the useful bandwidth, minimum value of that useful bandwidth for a acousto-optic modulation.

Thank you very much.