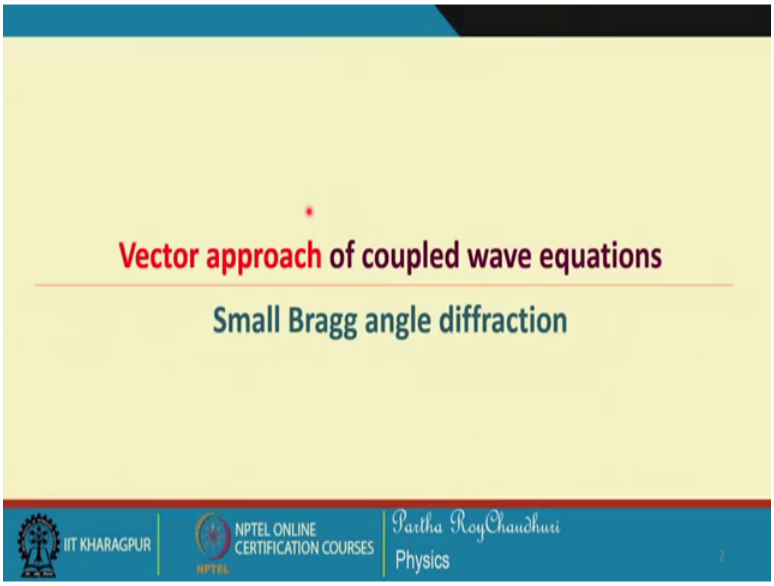
Modern Optics

Prof. Partha Roy Chaudhuri
Department of Physics
Indian Institute of Technology, Kharagpur

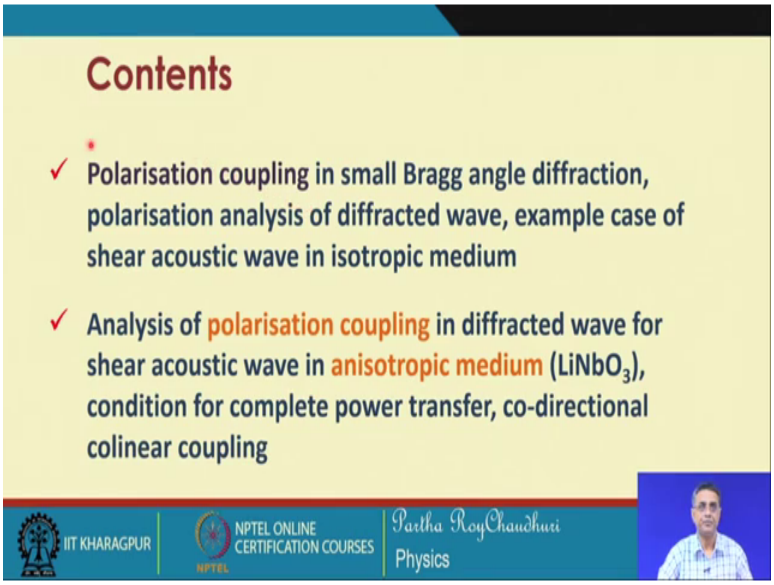
Lecture - 54
Acousto-optic Modulators and Devices

(Refer Slide Time: 00:25)



We were discussing this polarization coupling for small Bragg angle diffraction. And we used this vector approach of the coupled wave equation for this small Bragg angle diffraction.

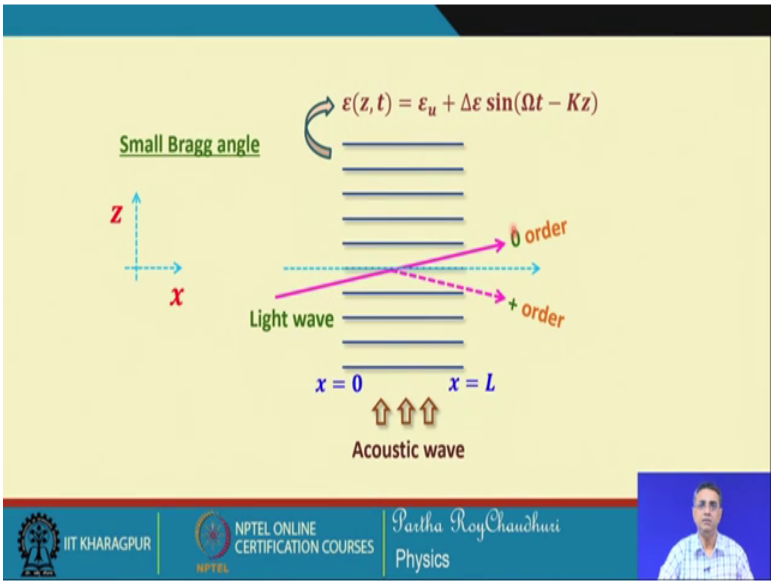
(Refer Slide Time: 00:31)



Now, we will see the other possible situations for this polarization coupling in the case of small Bragg angle diffraction using this vector wave formulation, we will analyze the diffracted wave in terms of its polarization with example cases like in now we will consider the shear acoustic wave propagation. In isotropic medium, we have already seen the longitudinal acoustic wave. In isotropic medium, so this part we will see and we will see that how the polarization coupling takes place. In the longitudinal case there was no coupling; there was coupling, but it remains the same.

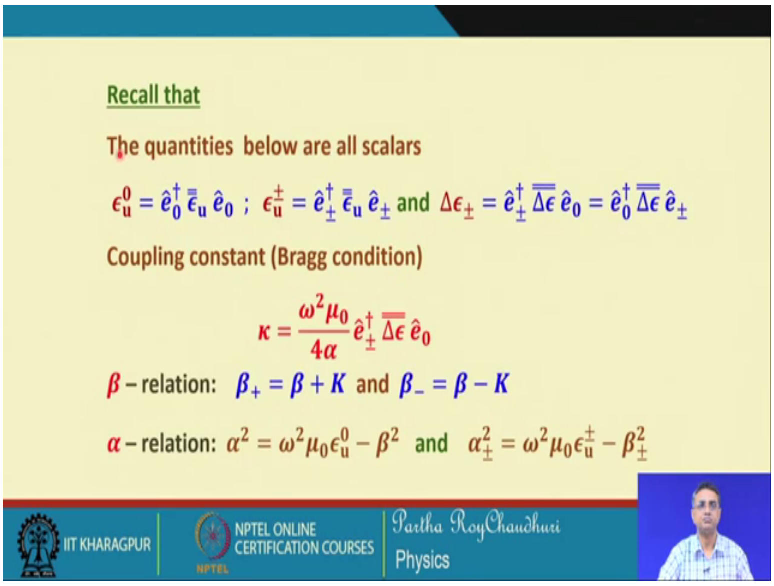
And then we will also consider the polarization coupling in the case of anisotropic medium; particularly this lithium niobate crystal, which is a very important crystal. And there we will consider the shear acoustic wave, and how this permittivity changes, and that gives rise to the coupling of the incident beam to the refracted beam looking at its polarization changes. And we will also consider the complete power transfer cases that: what is the condition for co-directional, and collinear coupling. We will look for the requirement of acoustic power for complete transfer of power from these considerations.

(Refer Slide Time: 01:56)



So, let us again continue with the same situation that for a small Bragg angle light wave is incident, which is very close to the x-axis. And you have x equal to 0, x equal to L that defines the length of the acoustic wave. And this permittivity as a function of the acoustic wave it has this form that we have seen.

(Refer Slide Time: 02:27)

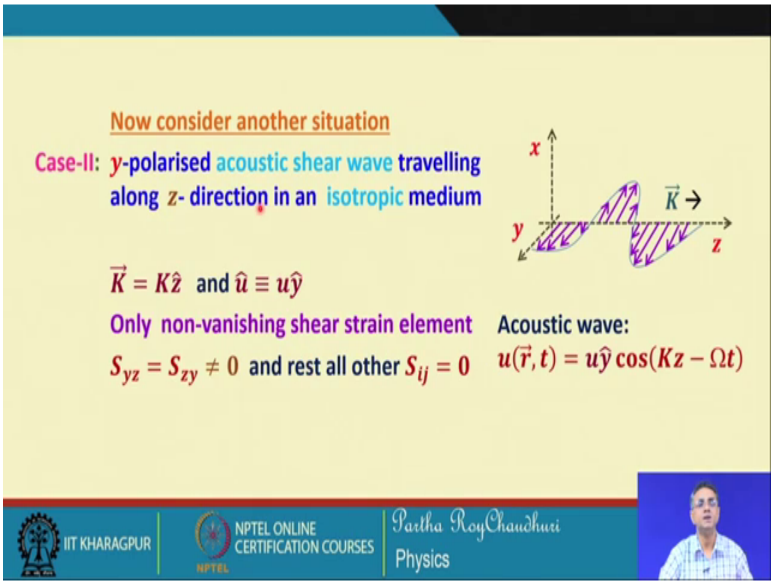


And now let us recall from the earlier discussion, these quantities that this quantity, which connects this the polarization of the incident wave incident optical beam, and the polarization of the plus or minus order optical beam, and the polarization of the incident beam with the polarization of the plus or minus order optical beam, they are all they have to be all scalar. And the coupling constant under the Bragg condition, because alpha alpha plus they are equal delta alpha equal to 0 this we have seen. So, kappa is equal to omega square mu by 4 epsilon.

And this quantity we when we will evaluate this quantity, because rest or all other quantities are known. When we will evaluate this quantity, we will be able to calculate the coupling coefficient also as a consequence of this we will see that how the polarization coupling takes place between the incident optical beam to the diffracted optical beam.

So, the beta relation this these are the things that we have seen even when we were considering the scalar wave equation it is the same that beta plus equal to beta plus K beta minus we have seen this these relations vectorially also, and with the consideration of the energy and momentum conservation also. So, they are very well known and familiar by now. And these relations are also the consequence of that those equations.

(Refer Slide Time: 04:06)

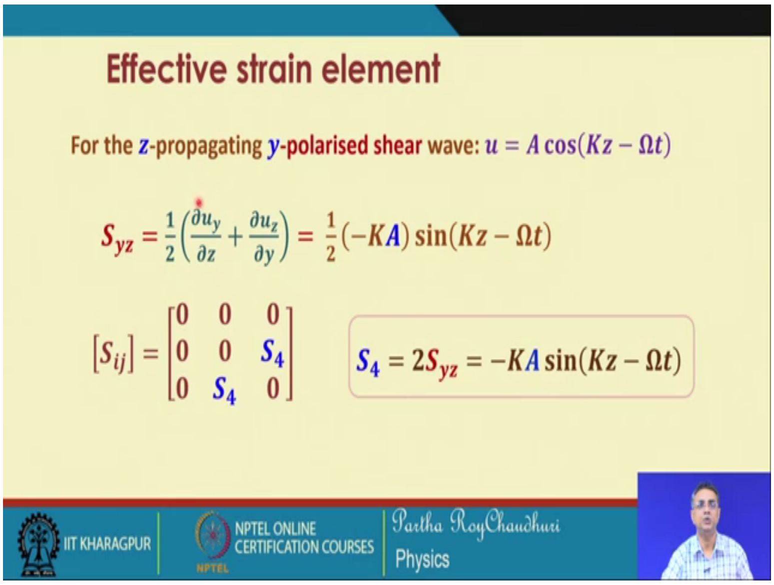


Now, let us consider the acoustic wave that is travelling along z direction, because in our consideration in our investigation, we have seen that we always consider the acoustic wave to be traveling along the z direction. So, this acoustic wave is traveling along z direction, but this time it is a transverse wave shear wave. Therefore, the shear wave and that is polarized along the y direction. This is the case that we will consider we can as well consider that it is polarized along x-axis also, but because it is isotropic.

So, there will not be a considerable there will not be any change, because it is the same for isotropic medium x polarization and y polarization. These are the two degenerate modes of the acoustic wave that also we have seen in the beginning of the acoustic wave discussion. For this case this propagation vector k is k and z and this u y changes as a function of z. So, this will be u suffix y with unit vector y. So, it appears in this form we can as well write that u y and then unit vector y that will make this understanding more simple, but that is this is also as well find S yz and S zy they are the only non vanishing components.

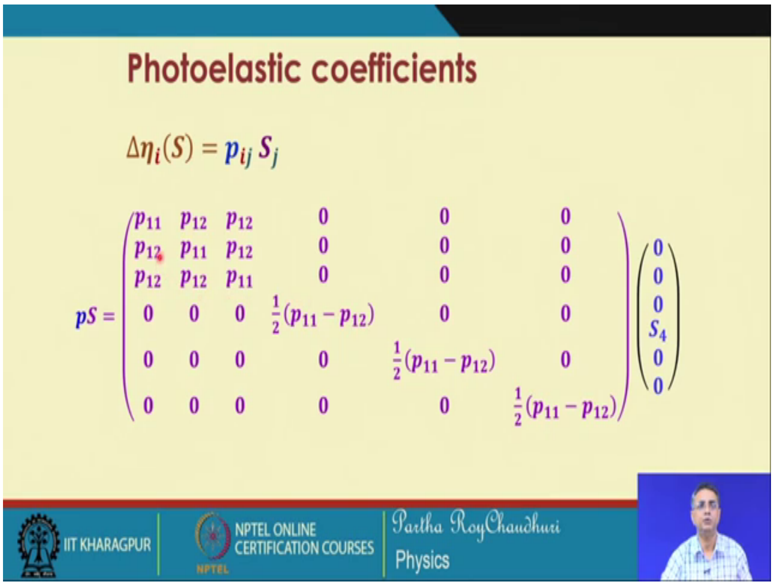
Let us recall that while discussing all these acoustic wave formulations that only the polarization direction and the propagation direction. They appear in the case of shear wave. Here it is y is the polarization direction and z is the propagation direction. So, this is interesting to note the shear strain we will contain only these come these two components y z and z y, because they are symmetric, so S yz equal to s z y which is equal to non 0, and rest all other components are 0. So, they will occupy two of diagonal positions. So, in this case, we are looking for the variation of u y as a function of z.

(Refer Slide Time: 06:31)



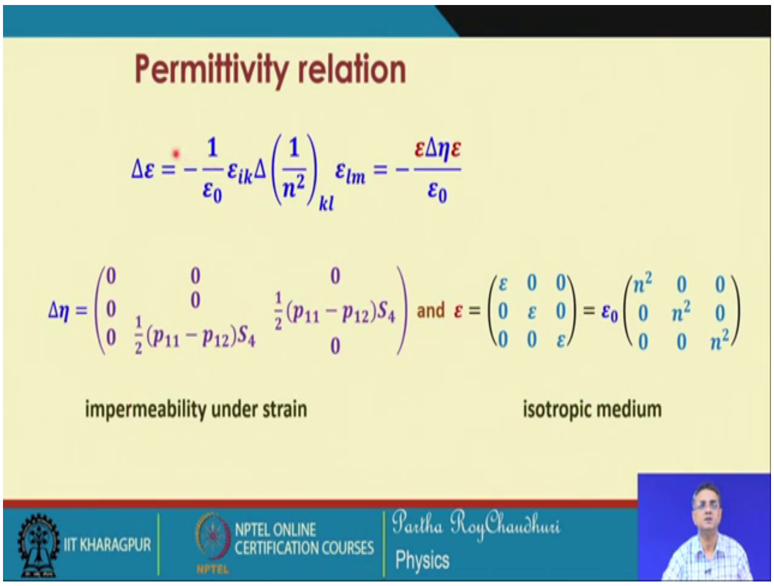
Therefore, del u y del z is the non vanishing non-zero quantity, so that gives you this quantity S yz so S 4, because it is y and z. So, this will occupy these two positions up diagonal positions of the strain matrix. And then S 4 is equal to because you have a half it goes here. So, it gives you this, we have seen this earlier also the same expression.

(Refer Slide Time: 06:58)



Therefore, for this isotropic medium we have we have seen this is the form of the strain optic tensor. And in this case, because it is S 4, so this S 4 will make all of them 0 except this column. So, it will appear with this quantity multiplied by S 4. So, this will be the form of the p s that is delta eta change in the impermeability tensor will have only these two up diagonal elements.

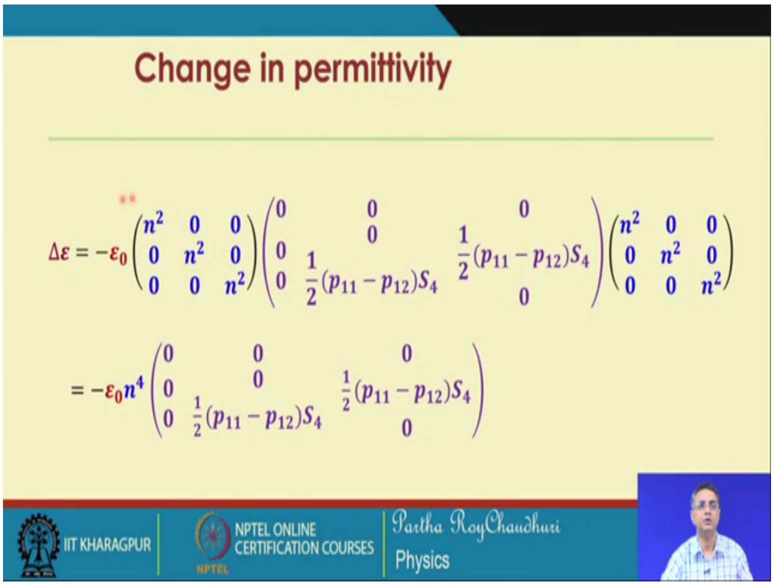
(Refer Slide Time: 07:36)



So, you can see from here. So, we are to calculate this delta E effectively finally delta E and for that we have to post and pre multiplied with the permittivity matrix, and its transpose. So, this delta eta is equal to from here you can see this will give you only the S 4 multiplied by this component. So, this will come here.

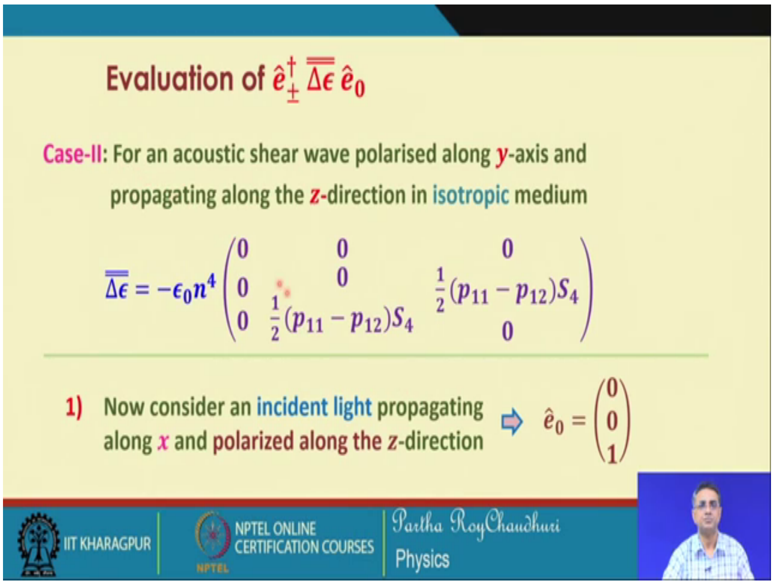
So, delta eta is equal to this. Only delta eta 4 will be there, because you have seen that this is this will give you delta eta 1, delta eta 2, delta eta 3, delta eta 4. So, this delta eta 4 will be only the non-zero, which will occupy these two positions in the impermeability change matrix and then because it is an isotropic medium. So, this is the form of the permittivity tensor, and this, so this is impermeability tensor under strain, this is the isotropic medium property.

(Refer Slide Time: 08:46)



Therefore, delta E again you multiply post multiplication, and pre multiplication by this permittivity matrix to this delta eta matrix. Then it gives you this equation which is in the same line that we have done earlier in the case of longitudinal waves. So, therefore we end up with this, but this time these are the two non-zero elements in the delta in the permittivity tensor change in the permittivity tensor.

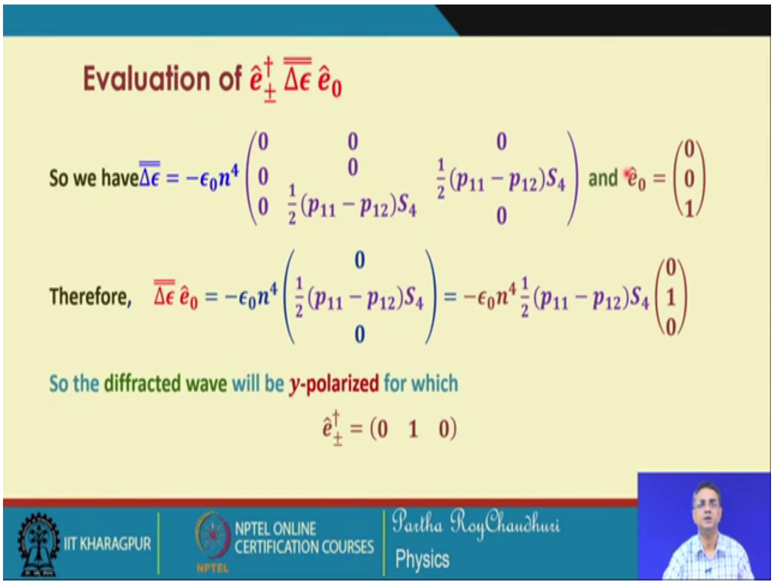
(Refer Slide Time: 09:22)



Therefore, now we consider the second case where the incident optical wave is propagating along x direction. This is the situation that we have been considering. If we go back to the first this case, if you go back to this we are throughout considering that for small Bragg angle, the incident wave is traveling very close to x-axis, and the acoustic wave is traveling along the z axis.

So, keeping that configuration same we now just change the polarization, and the nature of the acoustic wave. So, this time it is shear acoustic wave incident beam is propagating along x, but polarized along z direction. So, immediately we can write this polarization vector as 001. So, this is z polarized light which is the incident optical beam.

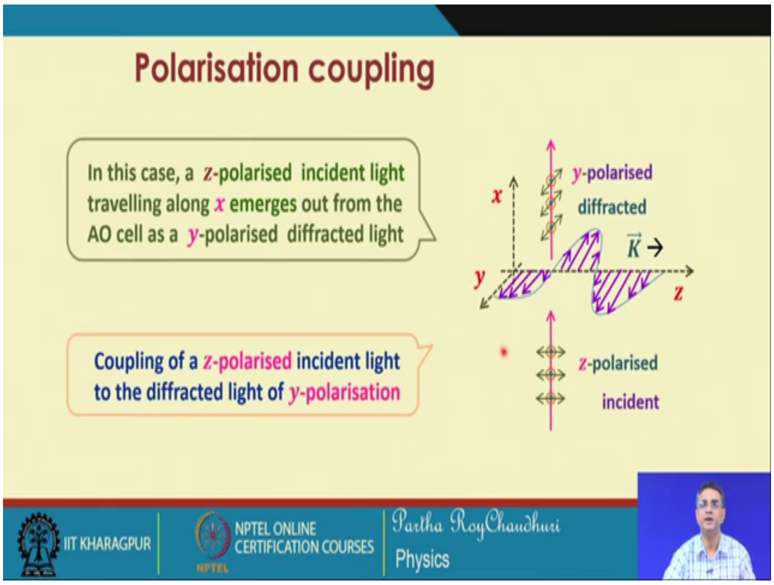
(Refer Slide Time: 10:22)



Now, e 0, if you multiply this with this quantity very straight forward, we can see that we can see that epsilon 0 n to the power of 4, and this quantity will be the non-zero quantity. If you multiply this, this, this, and this so all this thing only this quantity will be surviving. Therefore, this expression to become to be a scalar quantity this quantity epsilon 0 n to the power of 4, and this change this p s quantity will be a scalar only if your diffracted polarization is this, because if you multiply pre multiply with this vector, then only you get this quantity a scalar.

So, therefore, it suggests that the polarization of the diffracted wave should be a y polarized. So, you started with you see that we started with a z polarized optical beam which was the incident beam, but as a consequence of coupling by diffraction the polarization coupling we get a refracted wave which is polarized in the in the y direction. So, we started with x polar z polarized, but we get a y polarized diffracted beam.

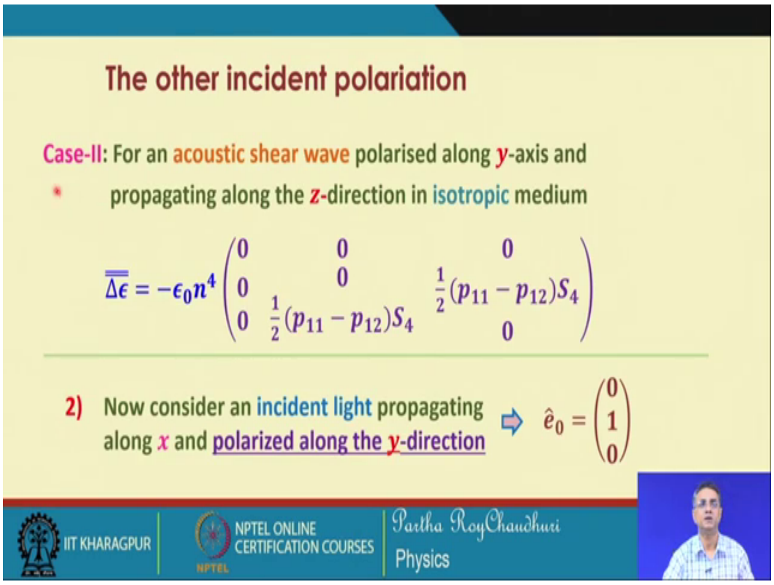
(Refer Slide Time: 11:53)



So, this is pictorially shown here you have the acoustic wave which is traveling along z direction. And the incident optical beam is z polarized, this is your z direction, so this is z polarized, but when it is diffracted out as a plus order or minus order optical beam, then we see that the polarization has become hyper.

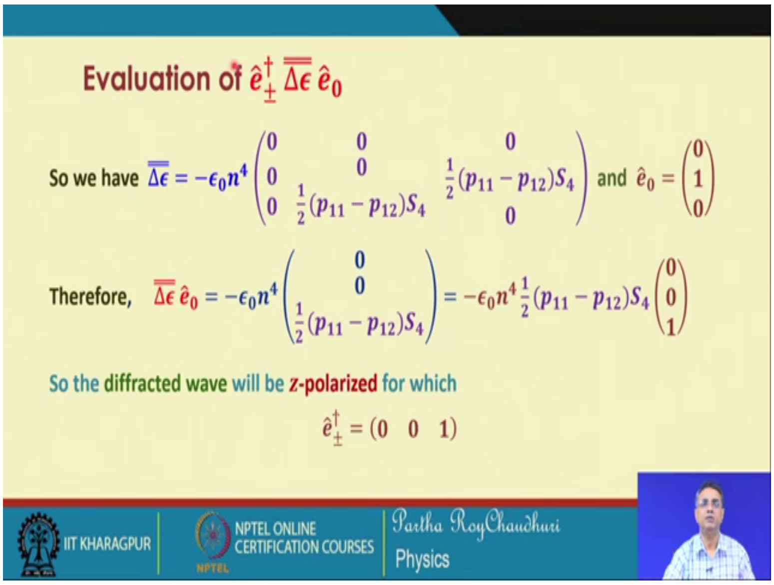
So, there has been a coupling of the z polarization to y polarization of the optical beam through this acoustic wave. Through this perturbed medium per changed permittivity, because of the change in the periodic perturbation of the medium we get a polarization coupling from z to y polarization. So, coupling of z polarized incident light to the diffracted light of y polarization that happens, if you consider the shear acoustic wave propagating in the medium.

(Refer Slide Time: 12:51)



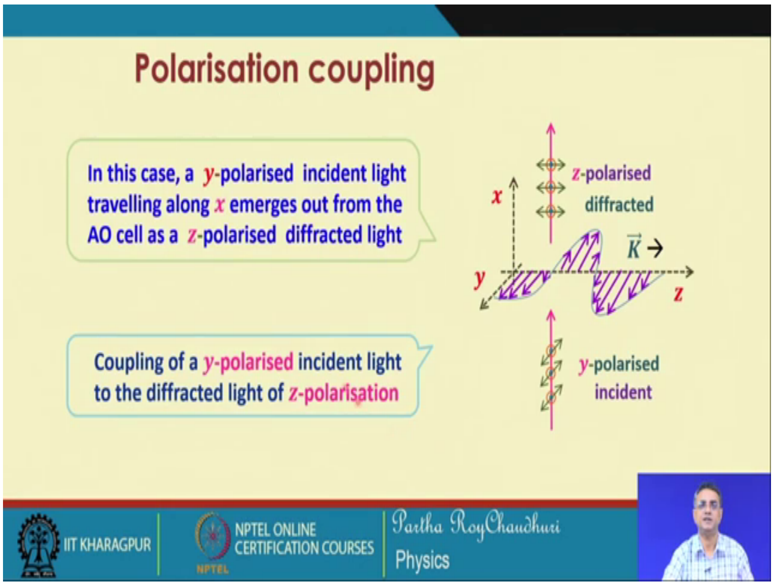
The immediate second case for this case too is that just to check the completeness. If we have now y polarized incident beam y polarized incident beam this beam is instead of being z polarized, if it is y polarized beam, then what happens to the optical beam which is diffracted out just to check that because this is the form this is independent of the incident polarization of the optical beam. This is the consequence of the acoustic wave that is traveling in the medium and which yields this change in the permittivity, now that if we consider that the incident wave is propagating along x, but it is polarized along the y direction, then we will write the incident wave polarization in this form 010.

(Refer Slide Time: 13:46)



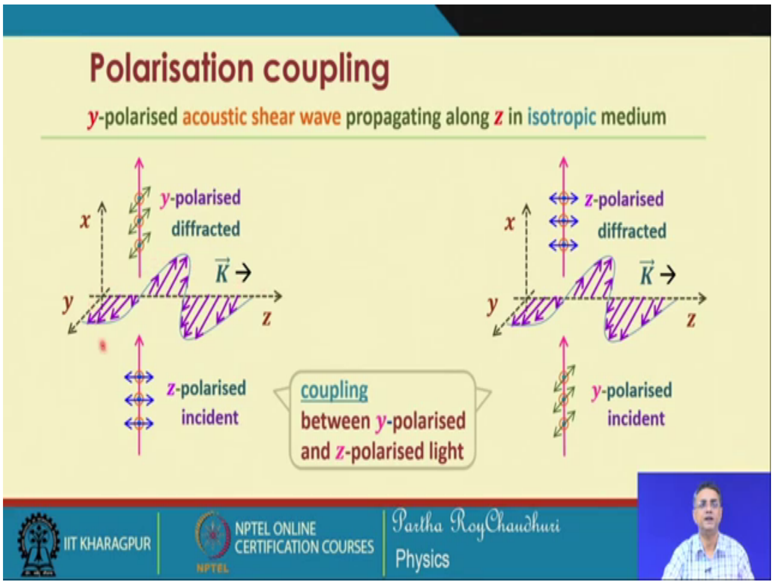
And again we have to evaluate this quantity. So, if we post multiply with this 010, a then you get that this quantity. You see you can if you multiply only this column, this will be the non-zero which appears here. And therefore, this is the quantity which immediately tells you that the diffracted wave should be, because this will be pre multiplied with the row vector which is 001. So, this pre multiplied row vector must be 0 0 1, and then you will get a scalar quantity for this which will in turn represent the coupling coefficient. Therefore, we find that if the incident wave optical wave is y polarized than the diffraction refracted wave will be z polarized.

(Refer Slide Time: 14:40)



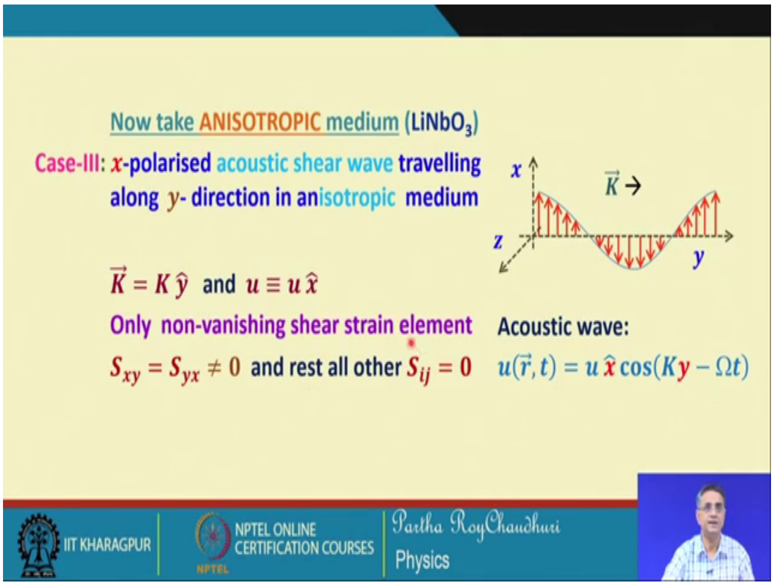
So, this is very interesting that in this case you have y polarized beam and you get a z polarized diffracted wave. So, it is very straightforward to understand that.

(Refer Slide Time: 14:52)



If you have y polarized z polarized beam, you get y polarization. If you have y polarized beam, you get z polarization so that means through this shear acoustic wave you get a coupling between y polarized and z polarized optical beam light. So, you have incident light which is y polarized you get a z polarization. If it is z polarized, you get y polarized. And these are the two possible polarization, these are two orthogonal eigen polarizations as far as this optical wave is traveling close to the x-axis close to the direction x direction.

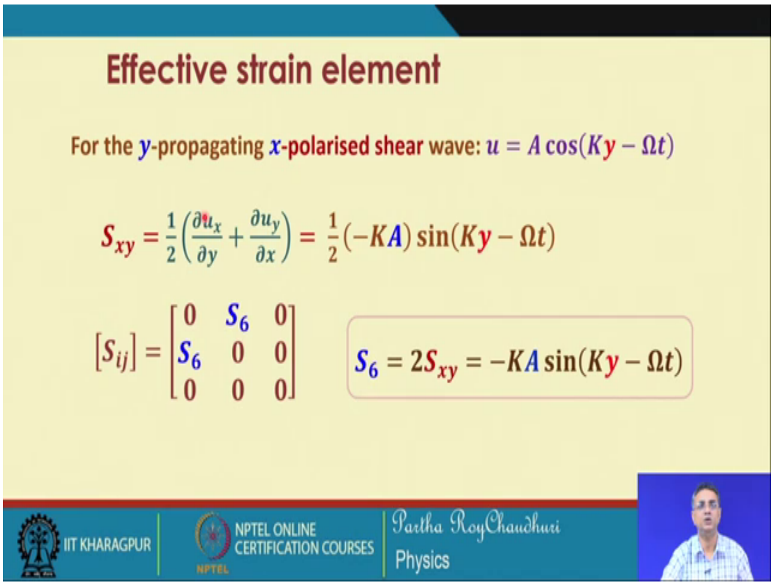
(Refer Slide Time: 15:33)



Now, we will consider the other case that is when the medium is anisotropic, and the acoustic wave is a shear wave. In this case x polarized acoustic shear wave traveling along y direction in anisotropic medium and the medium that will consider, because for anisotropic medium we have to specific in order to take understand we have to take an example of any specific strain optic tensors. So, we considered that lithium niobate which is a very important anisotropic crystal with many other properties.

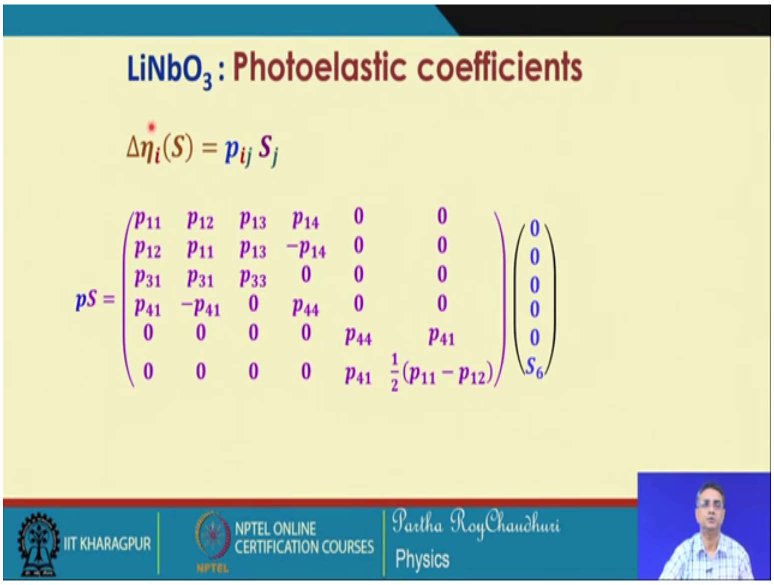
So, in this case the wave is acoustic wave is traveling along the y direction. And it is polarized in the x direction, it is polarized in the x direction, in that case because as I mentioned x and y the direction of propagation of the acoustic wave, and the direction of polarization these are the two indices which will appear as the as the strain components shear strain components x y and y x. So, this is the only non-zero component rest all of them are 0. Therefore, and you can see that there is a there has been a strain variation along x as a function of y.

(Refer Slide Time: 17:06)



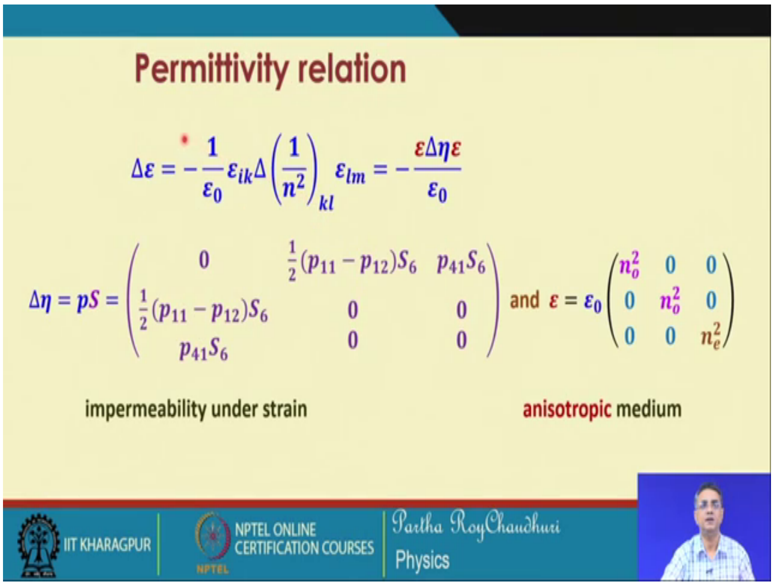
So, it is u x with y that change is non 0 del u x del y this quantity is non-zero, so that gives you this S x y is equal to this. And in the same similar way we can write that S 6 which will occupy these two up diagonal positions, and that gives you this S 6 equal to twice S x y. And this quantity this we have seen earlier also in the same way.

(Refer Slide Time: 17:37)



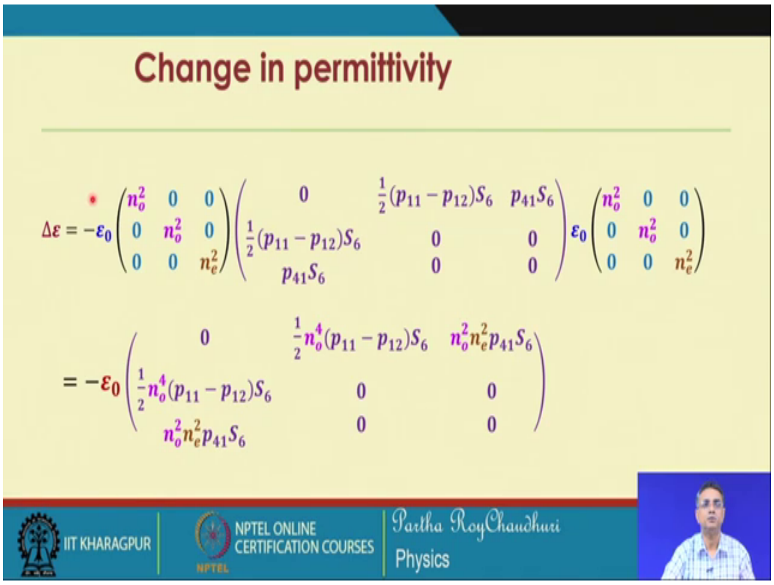
Therefore, for this lithium niobate a crystal we have this strain optic tensor, this photo elastic coefficients will appear in this form. And the change in the impermeability, this tensor p in to S we have seen that this will be will have this form. Here, S x S 6 is this quantity which is the shear strain that is non menacing. So, it will occupy this position this is again in the compressed notation. Therefore, this 3 by 3 matrix has become a column vector. And S 6 this S 6 will be multiplied and will give only this column to be non-zero. So, these two quantities will appear to be non-zero.

(Refer Slide Time: 18:35)



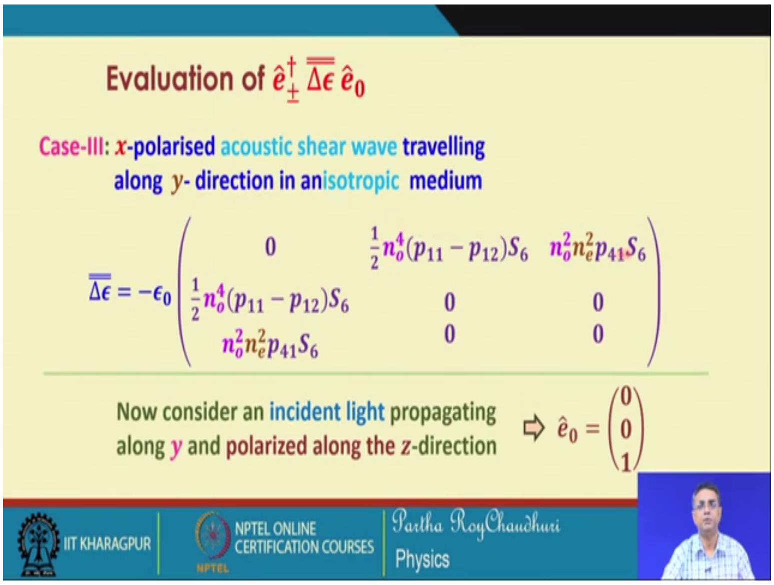
Therefore, from here because finally, we will have to calculate this delta epsilon, and that gives you. This for it comes from here, because we have this delta eta which is equal to this, because from here if we write in this 3 by 3 matrix notation, you can write that this quantity p 4 1 s 6 which is due to this p is 4 1 s 6. And this quantity half p 1 1 minus p 1 2 into s s6 will be appearing here in the off diagonal positions. And for an anisotropic medium these are the ordinary refractive indices this is the extraordinary. So, this is the form of the permittivity tensor.

(Refer Slide Time: 19:24)



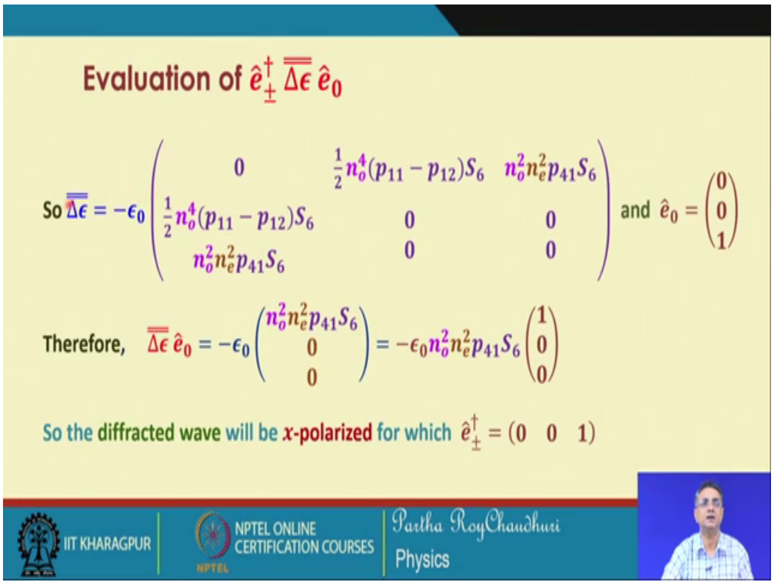
Therefore, post multiplying and pre multiplying by this permittivity matrix to this ps matrix, we find delta epsilon is equal to this, which is again very straightforward, but this time n 0 n o to the power 4 we cannot take out because they are appearing in the different positions. It is not that all of them are n o s square. So, there is n e square is also there. Therefore, these two positions they are appearing as n o to the power of 4, whereas here both of them are coupled. So, this is the form of delta epsilon e.

(Refer Slide Time: 20:06)



So, in this case, now up to here it is to represent we find that this is the kind of change in the permittivity change in the permittivity, because of this configuration of the acoustic wave that is it is x polarized, and traveling along y in an anisotropic medium. So, this will be the form of the delta epsilon e the permittivity change tenser. Now under this situation we consider a case that the optical beam is incident which is propagating along y and polarized along z direction. A different situation this optical beam is y polarized and polarized along the z direction. The acoustic wave the direction of the acoustic wave is x. So, the optical wave will be perpendicular to this. So, consider the propagating along y direction and polarized along the z direction.

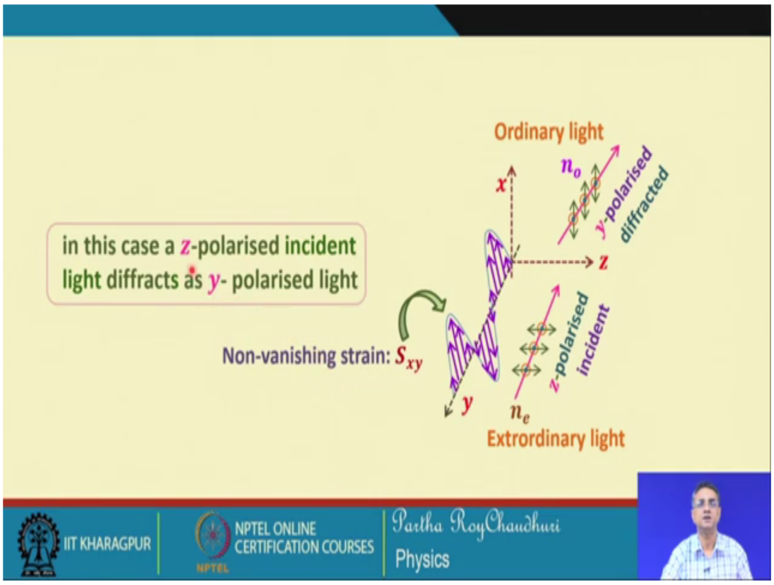
(Refer Slide Time: 21:14)



Under this situation what we can see that delta epsilon e will be given by this quantity 001. And therefore, delta epsilon e into this will give you 100. This is again very interesting you have started with a z polarized light as an incident beam and z polarized light. And then so the diffracted wave this is your 1 00. This is wrong it should be 1 0 0, this quantity is wrong this would be 1 0 0. So, if you pre multiply with 1 0 0, then only this quantity will be representing the coupling coefficient multiplied by this mu naught omega square and other parameters.

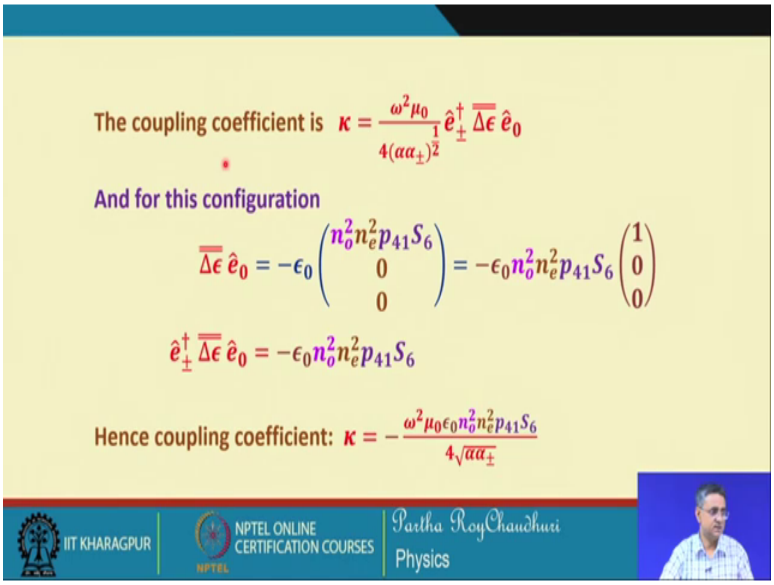
So, this quantity to be scalar it tells you that the diffracted wave will be x polarized the diffracted wave will be x polarized so that is very interesting. When you have an incident beam which is y polarized propagating along y and z polarized incident beam, and that gives you diffracted beam which is x polarized. So, there is a coupling from z polarized to x polarized.

(Refer Slide Time: 22:40)



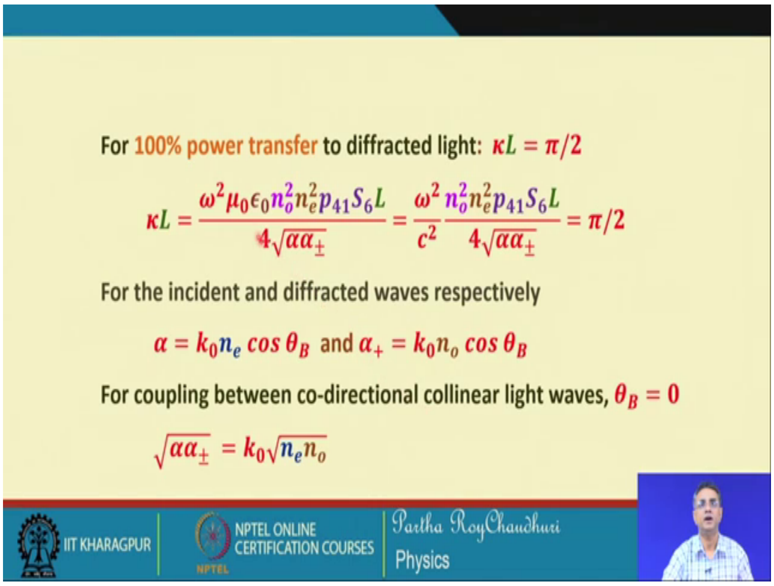
So, look at this in this case z polarized incident light diffracts as y polarized light. So, you have the acoustic wave which is travelling in this y direction and the extraordinary light which is incident here which has this polarization that is e wave which is polarized along this z direction. When it is diffracted out it will be a o wave that is ordinary light, and its polarization will be along the y direction so that is a very interesting consequence that from an extraordinary light we get an ordinary light in terms of the polarization.

(Refer Slide Time: 23:27)



In this case, the coupling coefficient k is given by this which is the same expression, but now that we have been able to evaluate this quantity. So, therefore for this delta e and e polarization of the incident beam, we get this quantity which is x polarized. And therefore this quantity gives you this scalar epsilon 0 n o square ne square then p 4 1 and s 6. So, the coupling coefficient is now available in terms of the in terms of all the known quantities s 6 will be the will come from the acoustic power. And this of the acoustic wave this is the strain element strain optic tensor element of the crystal that is lithium niobate here. These are the o and e polarization in lithium niobate. And these are all known quantities for Bragg angle diffraction this alpha and alpha plus minus there will be same.

(Refer Slide Time: 24:33)

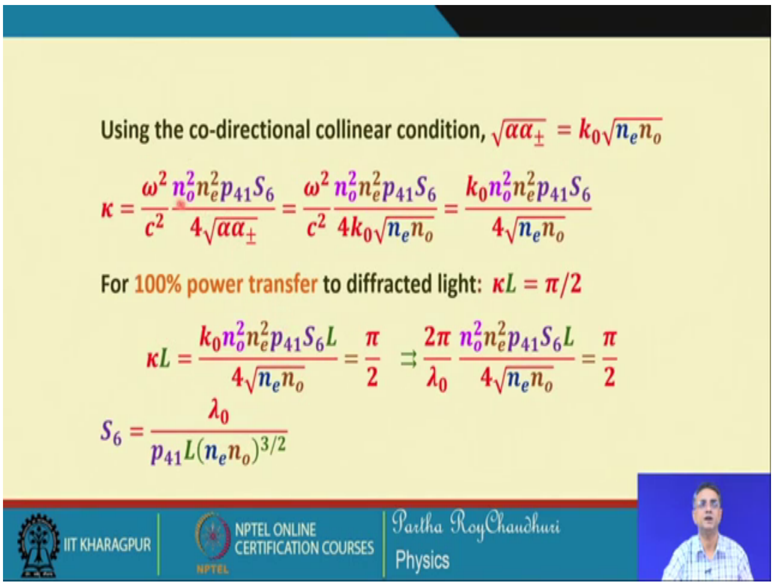


And we can reduce this to only alpha in the case of, but otherwise the incident and diffracted waves they are related by this we have seen for this k 0 n e and cosine of b which is the Bragg angle alpha plus will be k 0 n 0 cosine of b which is the Bragg angle. So, these are now well defined and for coupling between two co-directional collinear waves delta theta b. If it is co-directional and collinear wave that is large Bragg angle case, if this is the case where, you get the large Bragg angle diffraction, because your x polarized acoustic wave traveling along y direction. The wave is traveling along y direction and your light is also propagating along y direction.

So, this is a case which corresponds to a large Bragg angle diffraction. Both acoustic wave and the optical wave they are traveling in the same direction incident beam particularly. The diffracted beam may be in the minus y direction if it is minus order contra directional coupling, but it will be plus y direction traveling, if it is co-directional coupling.

So, you can see that this is a clear case of large Bragg annual diffraction where the optical beam is under goes a coupling from incident to the diffracted beam from e polarization to o polarization that is any extraordinary light becomes an ordinary light through this coupling. And also the coupling coefficient is given by this these are the quantities which are known for co-directional coupling theta b equal to 0 otherwise theta b will be equal to 180. So, these two quantities are also well known. So, k 0 n e n o under root of this will now represent we will replace this alpha alpha plus minus under root of that by this quantity n e and n o.

(Refer Slide Time: 26:47)

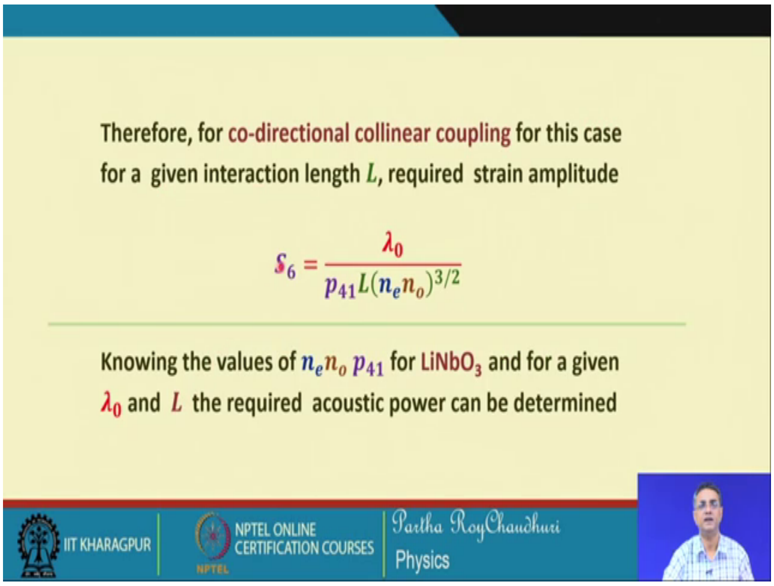


Therefore, it is straight forward we can now calculate the coupling coefficient from here. And this is simple algebra I just have to plug in. For 100 percent coupling this quantity k into l kappa into l must be equal to pi by 2 we have seen this in the waveguide coupling small Bragg angle coupling for periodic exchange of power from the incident wave to the optical wave we have seen this. So, if we use this condition then, because this is equal to pi by 2 which will give you this equal to pi by 2 pi pi cancels, and then you end up with a with this expression for S 6 which is equal to lambda 0 by this quantity.

And this quantity you see all the quantities are known l is the length width of the acoustic wave. In this case it will be the length of interaction of the acoustic wave, because this is a large Bragg angle rest where the light wave is propagating in the same direction as the acoustic wave. So, it is the length of interaction of the acoustic wave with the optical wave. And this is the non-zero strain element which is appearing here.

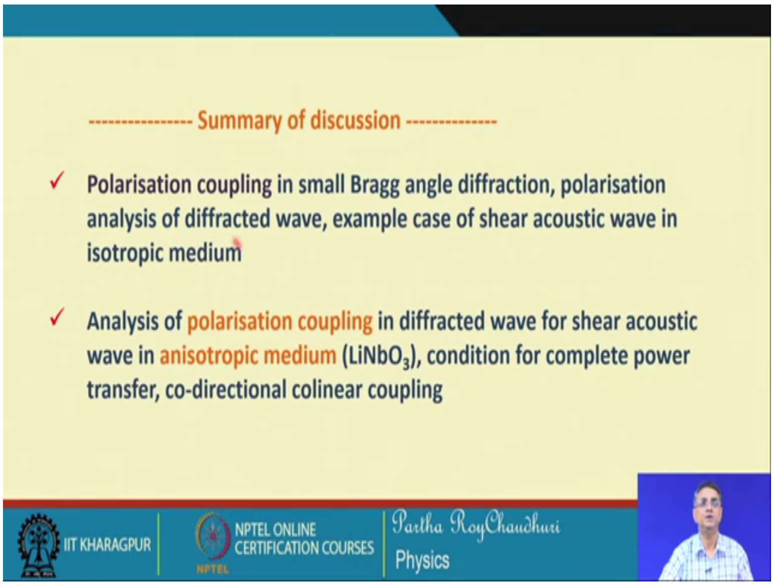
And these are the ordinary extraordinary refractive indices of the crystal here it is lithium niobate. So, all the quantities are known here. So, from here we can estimate this S 6 or otherwise we can estimate the complete transfer of power from the optical beam to the co-directional coupling diffracted beam width from extraordinary light to ordinary light coupling for that.

(Refer Slide Time: 28:28)



This is the amount of this should be the value of S 6 and that tells you that what should be the acoustic power. So, therefore for a co-directional collinear coupling as for this case for a given interaction length L, and for all other quantity is known, because this is also known for a given wave length the required strain is this s 6 this much of strain is required that is the peak strain. This will define the peak strain, and that will give you a complete transfer of power. Knowing all these values then we can calculate the acoustic power that is required for this complete exchange of power, so which is very interesting, and very useful from the design point of view.

(Refer Slide Time: 29:24)



So, we conclude this part by saying that we discuss this from the vector approach of the couple mode formulation for the acoustic acousta-optic diffraction for the case of small Bragg angle. We consider the polarization coupling analysis for the diffracted wave. Took up the example of the in this discussion example of the shear acoustic wave in isotropic medium.

Then we consider the example of anisotropic medium which shear acoustic wave propagating in lithium niobate is an anisotropic medium. Then we consider the condition for complete transfer of power this was the case of a large Bragg angle diffraction with co-directional and collinear coupling. And it was we show that from this calculation, from this expression we can determine how much acoustic power is required for complete transfer of power from the extraordinary incident beam to the ordinary diffracted beam in the collinear direction.

Thank you very much.