Modern Optics
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Lecture – 53
Acousto-optic Effect (Contd.)

We have seen Acousto-optic diffraction coupling of small Bragg angle and large Bragg angle. We used the scalar wave formulation to understand the principle of coupling the relation between the capital K vector small k vectors k plus vectors and the Bragg condition.

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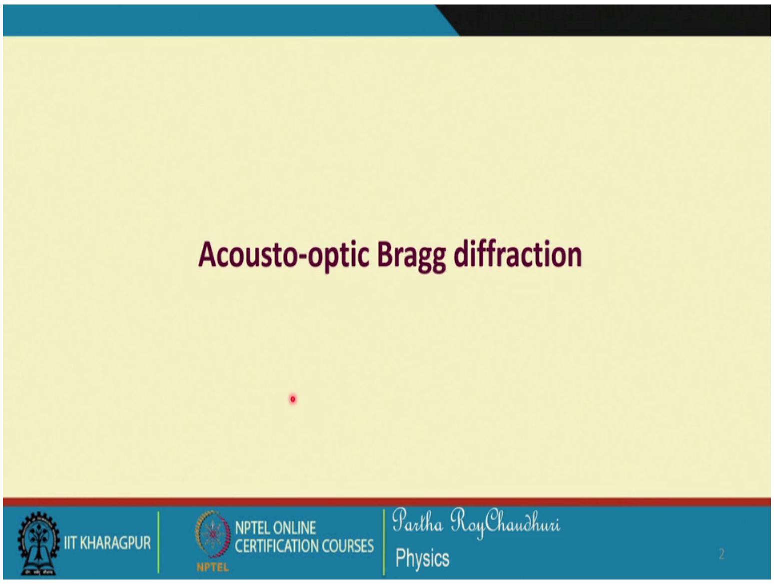

And now we will use altogether a different mechanism that is using the rigorous vector formulation for the coupling of the diffraction, from the incident wave to the diffracted wave this mechanism we will study.

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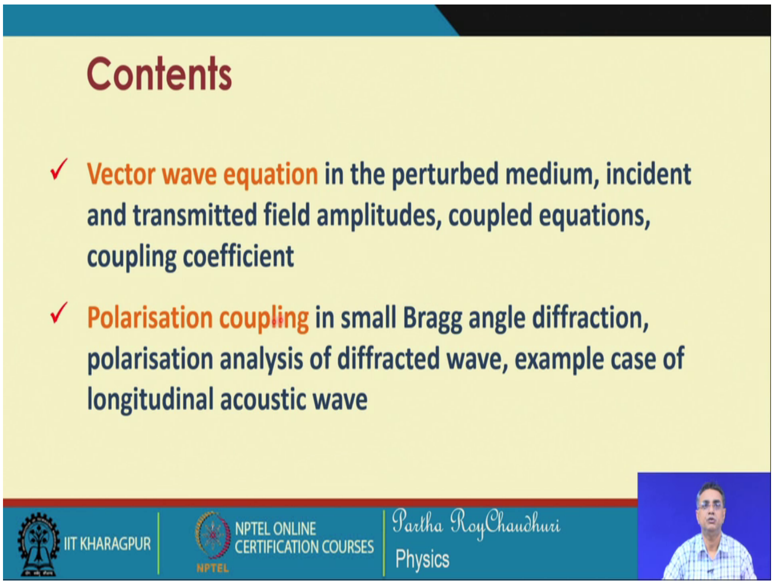

And we will be discussing under the following heads that the vector wave equation in perturbed medium, incident and transmitted field amplitude, will try to write. Those equations, then we will formulate that coupled mode equation coupling coefficient. Then a very interesting aspect that we will see is the polarization coupling. If we have some incident polarization, we will see that it can be coupled to a different Eigen polarization of the system.

So, this polarization analysis of the diffracted wave, and we will also study the example cases for longitudinal and shear acoustic waves in both isotropic and anisotropic media.

Therefore, we start with the permittivity relation.

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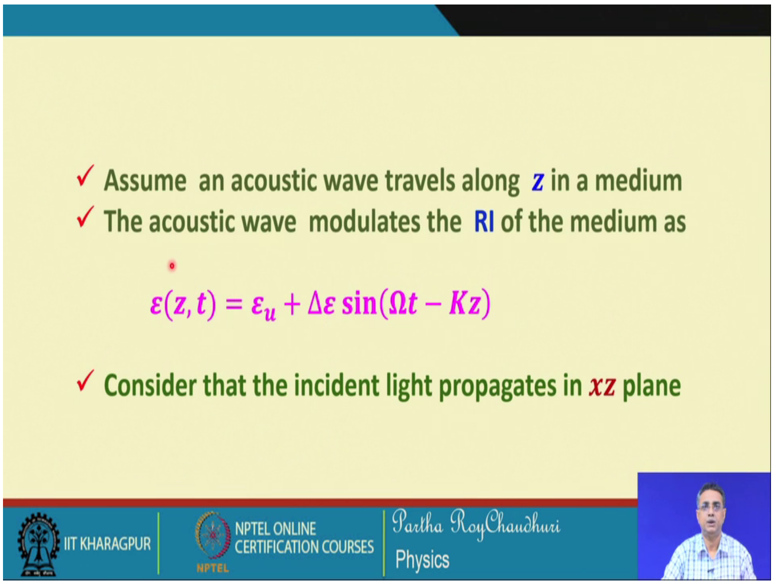

That is the acoustic wave modulates the refractive index of the medium. In this way this we have seen assuming that the acoustic wave is traveling along the z direction. This is the permittivity in absence of any acoustic wave; permittivity of the medium that is unperturbed permittivity. And this is the peak change in the permittivity due to the traveling acoustic wave. And we assume that the wave is traveling along the z direction.

And then we also consider keeping the same notation the same configuration that the incident light propagates in the xz plane.

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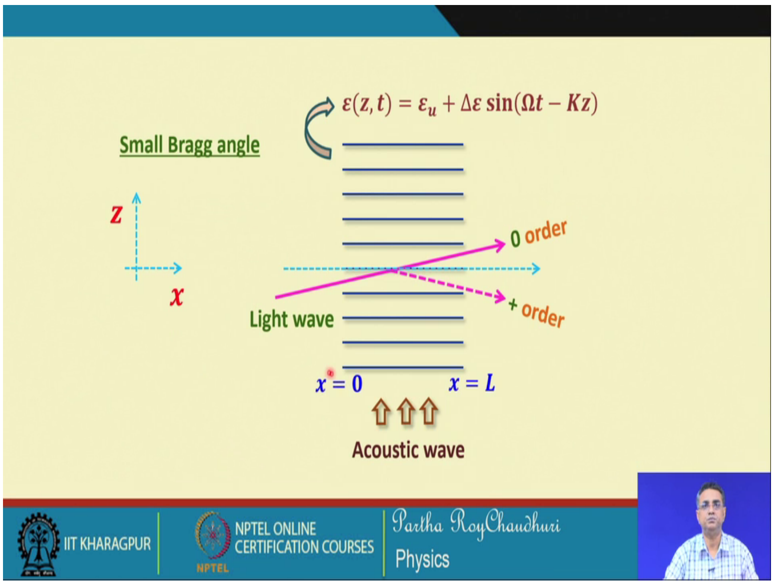

Therefore, so this is the schematic representation we have seen that the acoustic wave is traveling along the z direction. And the incident optical beam is traveling almost along the x direction, very close to the x axis with a very small angle; that is why we call this a Small Bragg angle diffraction. We have seen and this is the undiffracted that is the 0th order beam and this will be the coupling the of the first order beam this is the direction. And this point is from the x axis it is x equal to 0 and it is x equal to L. This length L will be used for the interaction for defining the interaction length and hence the coupling coefficient. And this is the periodic variation of per part of the permittivity because of the traveling acoustic wave.

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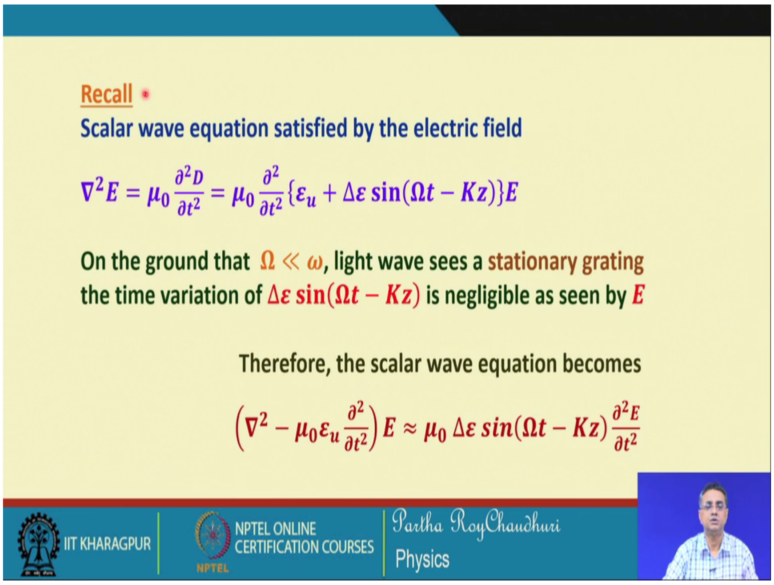

By now we have seen that using the scalar wave equation. We have seen that we can write the wave equation for the electric field in this form del square e equal to mu naught del 2 del d which is equal to this. So, this is on the basis that is on the ground that, because we assume that omega is much much less than the small omega. The acoustic frequency is of the order of 10 to the power of 6, and whereas, this is of the order of 10 to the power of 14 to 15.

So, this is quite negligible and the light wave sees a stationary grating. Therefore, even though this is within this del square del t square within this, but still this t variation will be treated as almost a constant. So, that is the time variation of this quantity this delta e sin of omega t minus K z is negligible as seen by the electric field, because electric field time variation is at a much higher frequency.

Therefore, that helps in simplifying the wave equation. So if we take this outside this time variation then we can write that mu naught delta e sin of omega t minus K z and then del 2 e del t square. So, this is the reduced form of the wave equation which we used in the scalar wave equation formulation.

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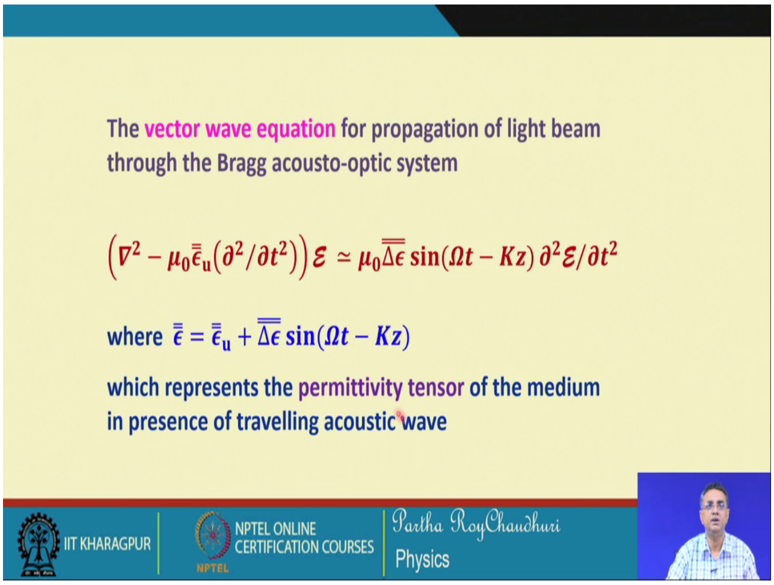

So, this is the same equation, but now we will consider this equation as a rigorous vector wave equation. So, the vector wave equation for the propagation of the light beam through this Bragg Acousto-optic system will be represented by this which is similar to this equation. You can see this del square and mu naught epsilon un part of the epsilon and this relation which appears here, but here it is the tensor and delta e that is again the permittivity change tensor.

So, this is the form of the vector wave equation where e is the electric field vector now. And this total permittivity is equal to this unperturbed permittivity tenser plus this change permittivity, so which represents the permittivity tensor of the medium in presence of the traveling acoustic wave. So, this is the total change.

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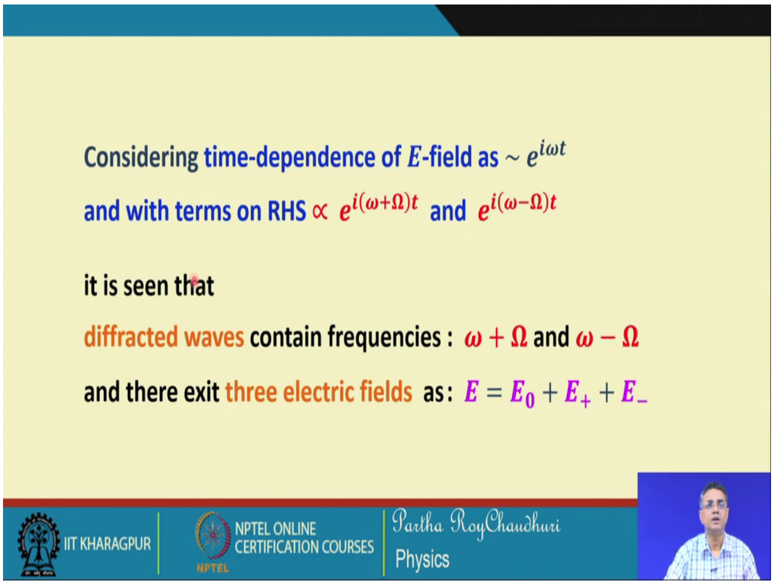

Now considering the time dependence of the electric field this e to the power of i omega tau. And we have seen that the right hand side of this wave equation will contain terms which are proportional to e to the power of i omega plus capital omega t and i omega minus capital omega t this we have seen. So, they are proportional and as a result the diffracted wave will contain frequencies which are proportional to which will be omega plus omega and omega minus omega.

And there exist three electric fields in the system. This also we have seen one will be the incident wave, and the two others are the plus order diffracted wave electric field, and electric field corresponding to the minus order diffracted field.

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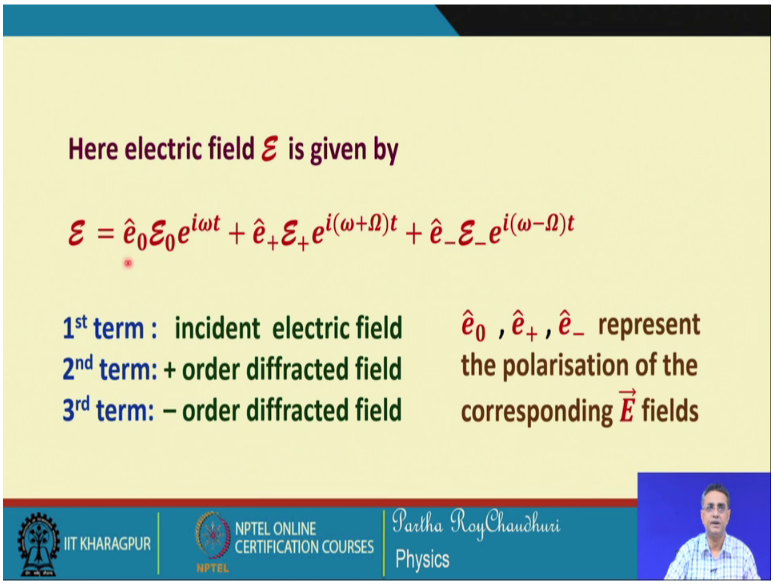

So, this total electric field will be will consist of this incident electric field or the plus order electric field and minus order. Even though both plus and minus order cannot coexist one of them will be existing, but for completeness we write this e total field equal to sum of all these three. And the first term it represent the incident, second term the plus order, and the third term the minus order.

Here, this e 0 e plus and e minus they are the polarization of the electric field of the corresponding incident diffracted plus order and minus order fields. So, this e 0 is the incident polarization of the incident wave, polarization of the plus order and polarization of the minus order.

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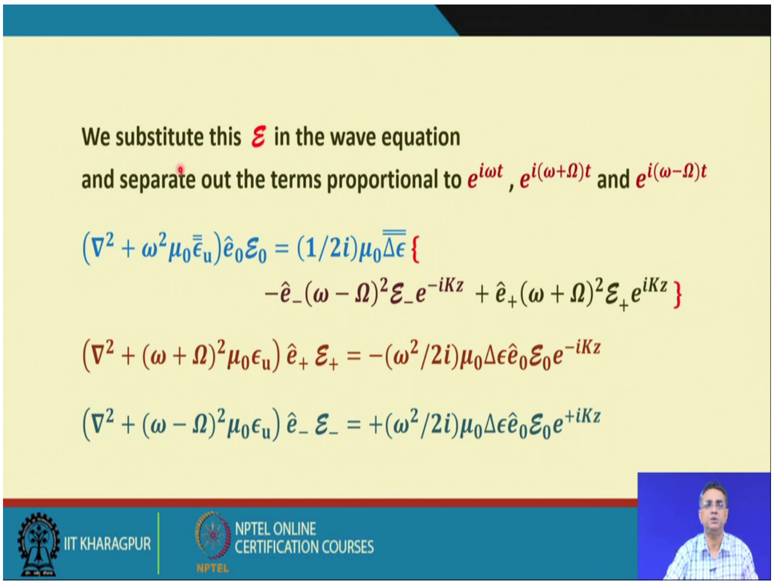

So, we substitute this e in the wave equation; in this equation. We substitute this sum of the three quantities. And then in the same way we can separate out the terms which are proportional to e to the power of i omega t, omega plus omega t, omega minus omega t.

In the same way as we did for the scalar wave and then we end up with they set of these three equations, but this time there vector wave equations. So, this can these are the polarization of the plus order polarization of the minus order. So, it is very straightforward and simple from there and we can write the three equations. Now we will use this the same approach as we adopted in the case of small Bragg angle that the light wave is traveling almost along the direction along the x direction and then we can approximate we can use the same formulation.

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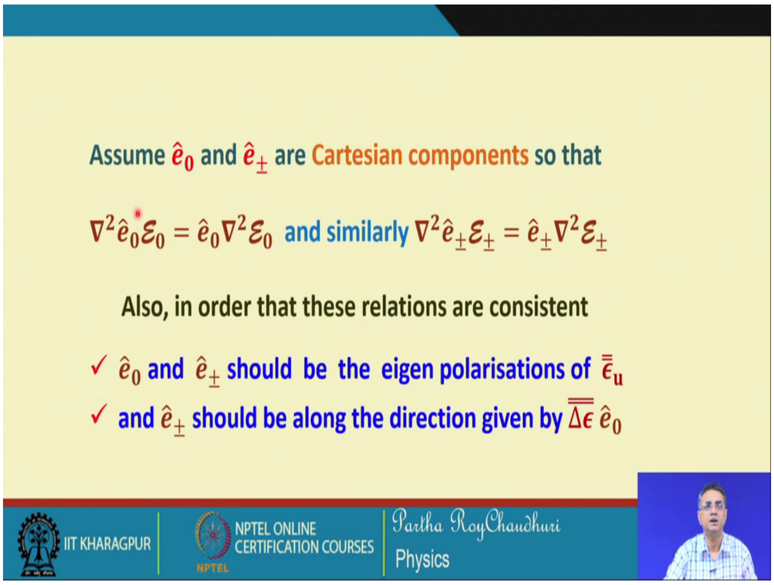

But in between that del square epsilon this e 0 e we can write this equal to this quantity. And similarly for the plus and minus order polarization we can write this you can take out this e plus minus outside and this is the polarization vector.

So, also in order that these relations are consistent this e 0 and e plus minus they must be the eigen polarization of the system that is in Cartesian system it must be x y or z. Similarly this will be also xy or z if the wave is travelling along the z direction as in the present case then it can have two polarization that is x or y, similarly this can have two polarization that those are x and y. So, in Cartesian system you can in accelerate this relation. And this e plus minus should be along the direction given by this, because they are associated with delta you will see this, we will see this.

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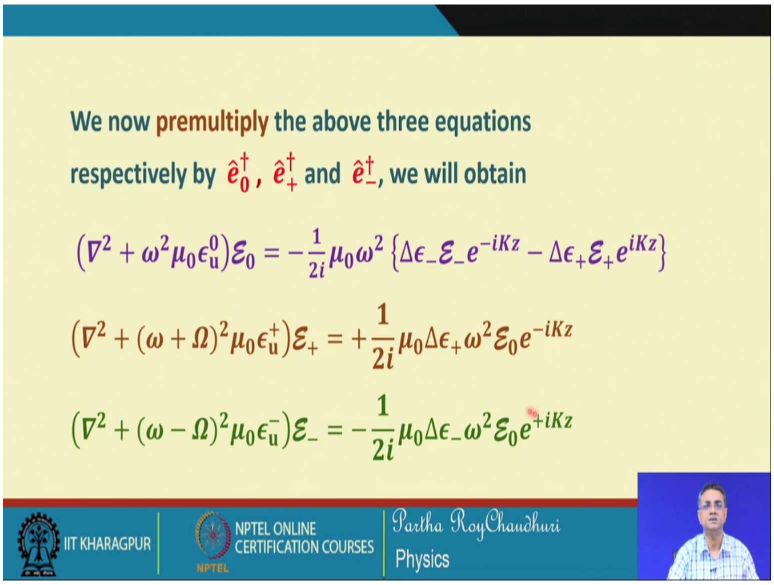

Now so this if these three equations will again lead to the in the same way that if you pre multiply this equation with this V 0 dragger e plus and e minus from the left side. These are the row vectors, these are row vectors and you have a column vector on the right hand side. You can see you have this e 0 this is the column vector and here also you have a column vector. If you multiply with this e 0 dragger then you can get rid of this polarization here and you can write this equation.

But on the right hand side because it was the column vector was e 0, but you are multiply e e plus or e minus then you are pre multiplying with e 0. So, they will give rise to this new quantity which is a scalar delta e because they are Eigen polarization. So, it will give you only a scalar quantity, and similarly for this equation and similarly for this equation.

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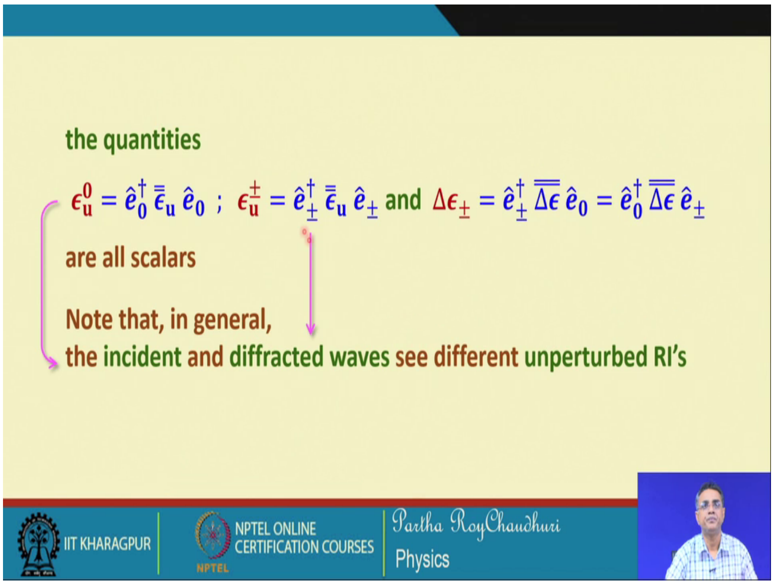

Now, this because you are so this quantity is e 0 and you are pre multiplying with this. So, they will give you a scalar quantity, here also this will give you a scholar quantity, and here for these quantities for these terms delta plus e delta plus minus e. So, you get this quantity, because you have already this and your pre multiplying with this quantity so these are all scalars.

So, the incident and diffracted waves see unperturbed refractive in different in unperturbed refractive indices so they are different in general. Now proceeding in the similar way for solving these equations that we approximate that the x dependent of the electric field of the incident beam is negligible.

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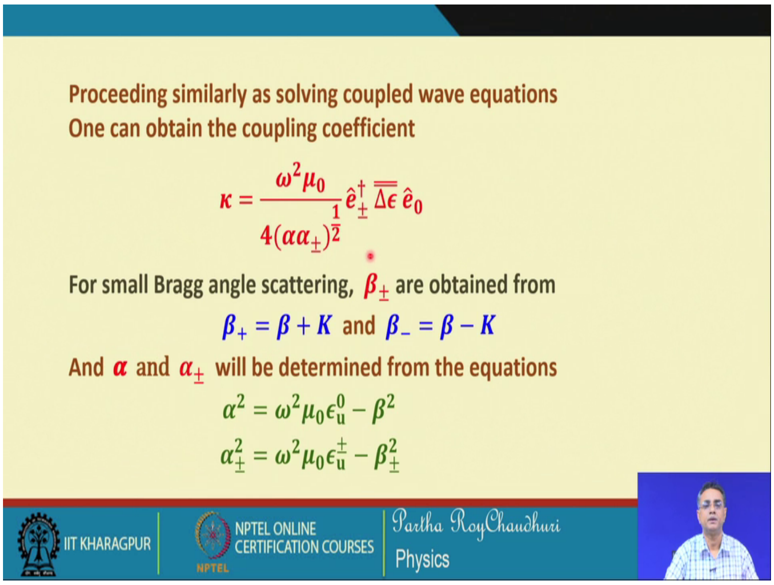

Then we arrived at this in the proceeding in the same way we arrived at this coupling coefficient which is given by this you have seen it is the same thing that we have already derived in the earlier equation for the case of scalar wave formulation. And for small Bragg angle scattering you also have seen that beta plus minus they are obtained from this equation. So, all these things are valid except that they are now vectors so, but effectively this is a scalar.

So, the coupling coefficient will be a scalar multiplied by the quantity this. So, the task lies in to calculate this delta epsilon this tensor. And if we know the incident polarization we can calculate the diffracted polarization, because this has to be a scalar. So, for small for alpha and alpha plus minus will be determined from these conditions this also we have seen repeatedly that this alpha square must be equal to this minus this. And alpha plus minus will be the same quantity, but for the plus or minus order diffracted wave.

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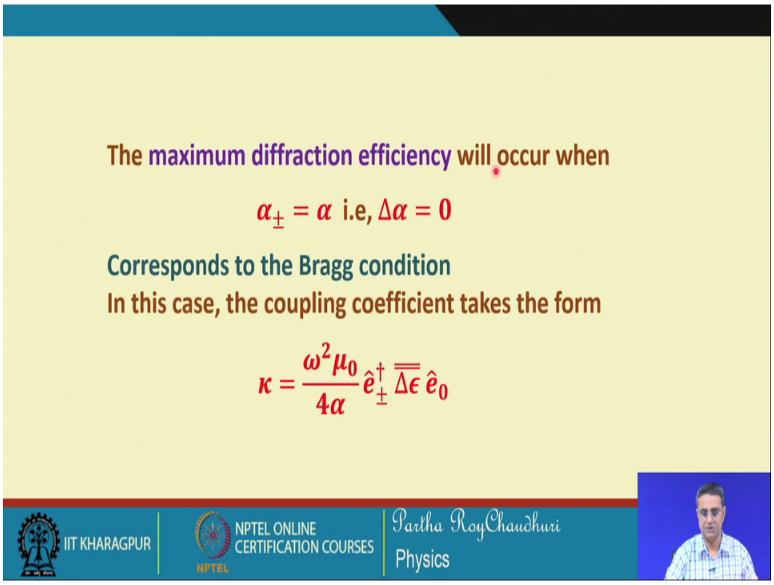

Now the maximum diffraction efficiency will occur when alpha plus minus is equal to plus and delta alpha equal to 0 this condition we have seen earlier also. And in that case this alpha plus minus this will become only alpha. So, you get the coupling coefficient under the Bragg condition which will be just omega square mu by 4 alpha, and this quantity this quantity which will be a scalar again.

And we will now study because we have seen this coupling coefficient. We will now study the specific cases example gases to understand in a better way that how this coupling from the incident wave coupling of the polarization takes place to the diffracted beams whether it is plus order diffracted or minus order diffracted.

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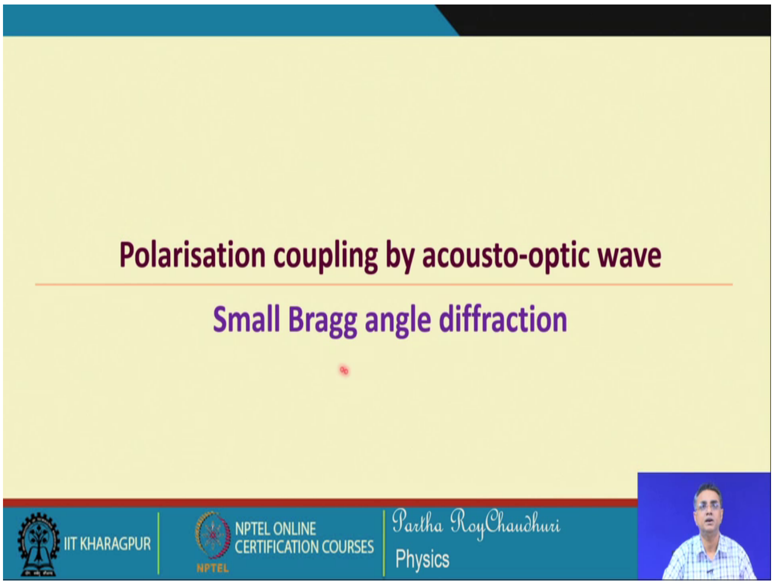

So, for that we take an example of a longitudinal acoustic wave that is traveling along the z direction.

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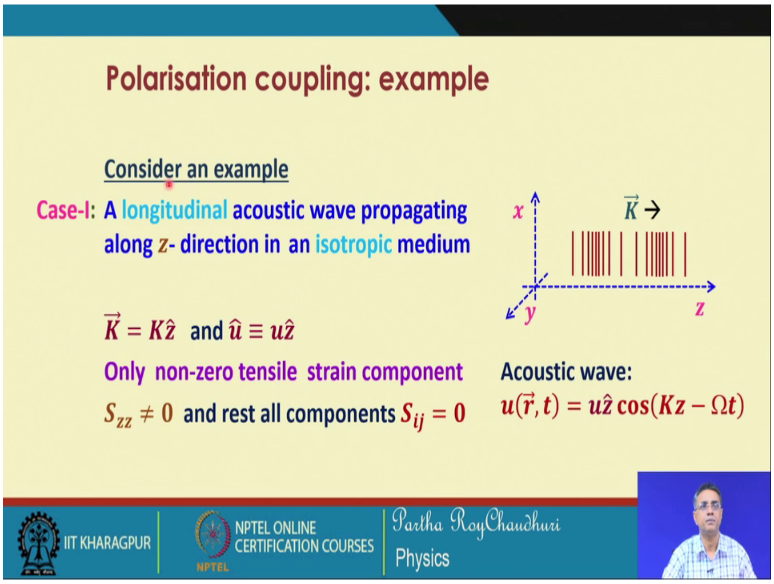

We remember that the longitudinal this compression and rarefaction that happens along the z direction and then the propagation constant is k into z and this is actually the ux uz equal to u of z. So, u suffix z is equal to this and in this case, because this is a longitudinal wave that is propagating along z direction.

So, it will give rise only tensile strain components and that is only one component. So, they are only non zero tensile strain component in this case will be S zz which is not equal to zero rest all other components will be 0. So, we obtained that the strain matrix which is by now we are very familiar.

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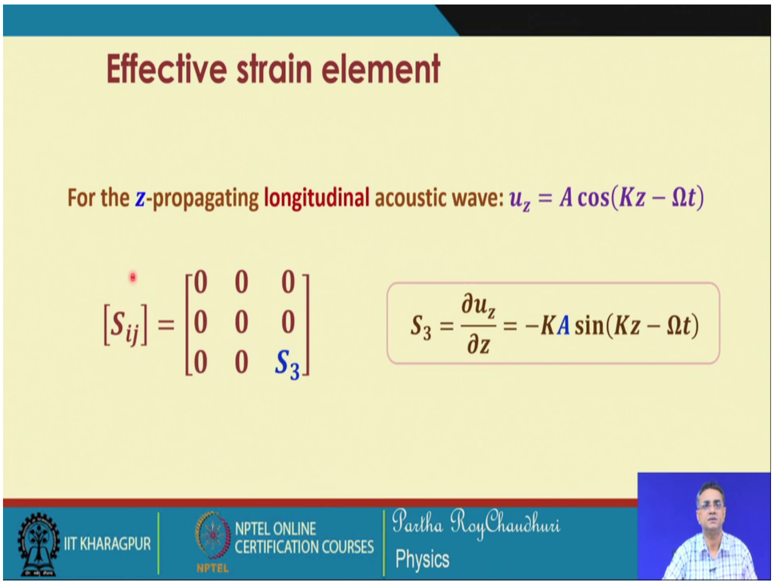

So, it is only the S 3 component z z Szz component will be non zero and which is equal to del u z del z, because it is the variation of z uz along the z direction that is the non zero quantity. So, we get that S 3 equal to this.

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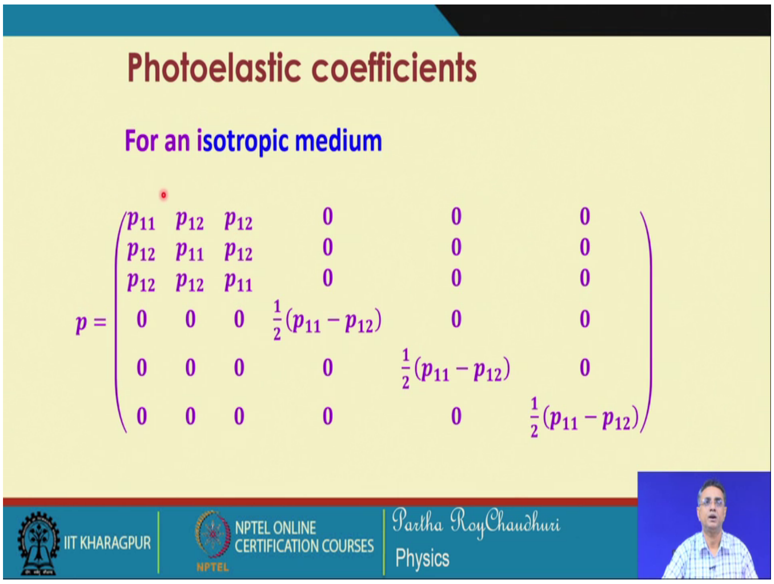

Now, we can use this to calculate the change in the impermeability and hence the change in the permittivity tensor. So, for an isotropic medium this is the form of the photo elastic coefficients the strain optic tensor this we have seen at various occasions.

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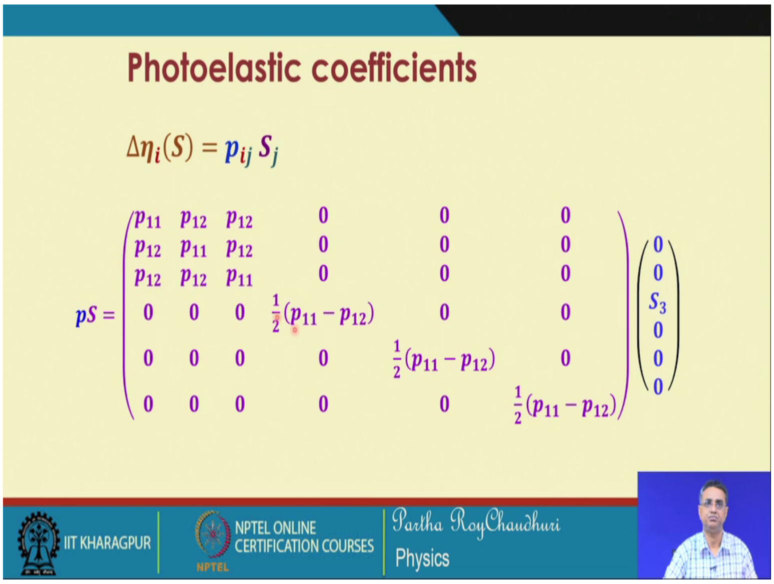

Now, in this case we can calculate the change in the impermeability in presence of the strain by taking the strain into account.

So, this is again the compressed notation. So, this all three by three matrix has appeared as a column vector here and it is only the S 3 that is nonzero which will make only these columns nonzero rest all of them will be 0. So, if you multiply S 3 and P 12 S 3 and P 13 and S3 and P 11 they will occupy the diagonal positions and that you can see from here.

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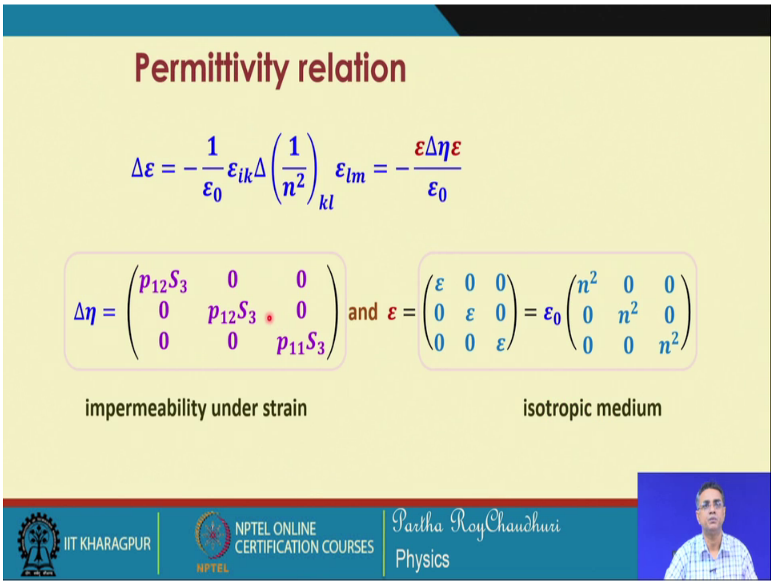

So, delta e these three quantities will be occupying the diagonal positions. So, that is your delta eta that is the impermeability change in the impermeability tensor and for an isotropic medium. This is the form of the permittivity tensor in the principal axis system all of them are n square n square n square. So, we have to post multiply and pre multiply with the row with these two matrices from this side and this side.

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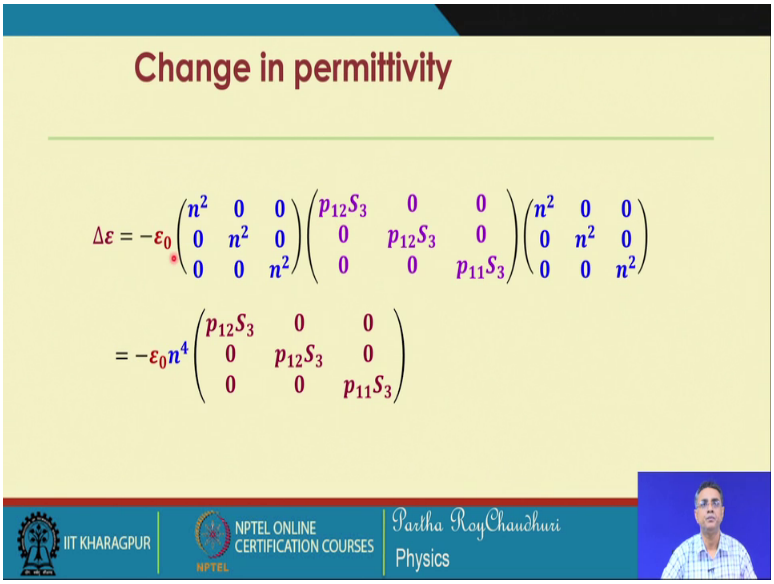

Therefore, the transpose and this matrix from the as a post and pre multiplier will give you this form that is a minus epsilon 0 and this permittivity tensor; and here it is the permittivity tensor which is then on multiplication with this and then on multiplication. With this we will get n to the power of four out and the side you will be able to retain the same form, because of this the specific nature of the form of this. So they are all diagonal elements.

So, you will retain only the diagonal elements n to the power of four will be out. So, this will be the change in the permittivity tensor. So, that is what we were looking for in order to calculate the coupling coefficient as well as the polarization of the diffracted wave when we know the polarization of the incident wave.

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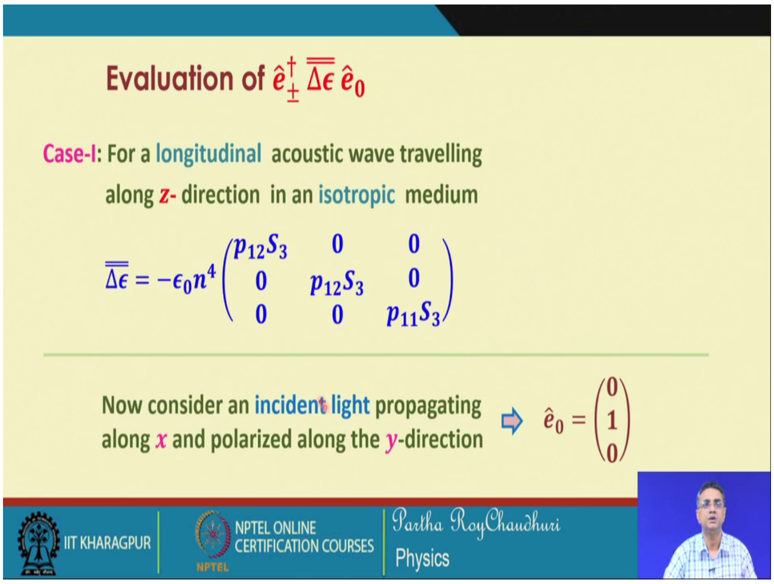

So, let see that now the case is the one which we are studying is a longitudinal acoustic wave, which is traveling along the z direction. And we find that the change in the permittivity tensor as this form this for this particular case. So, they have only the diagonal elements. And so now, the case is that the example is that let us consider an incident light wave which is propagating along x and polarized along y direction. So, it is x propagating wave and the polarization is along y. So, we have we can write that incident beam polarization as 010. And then we will have to multiply post multiplied with this you can see this.

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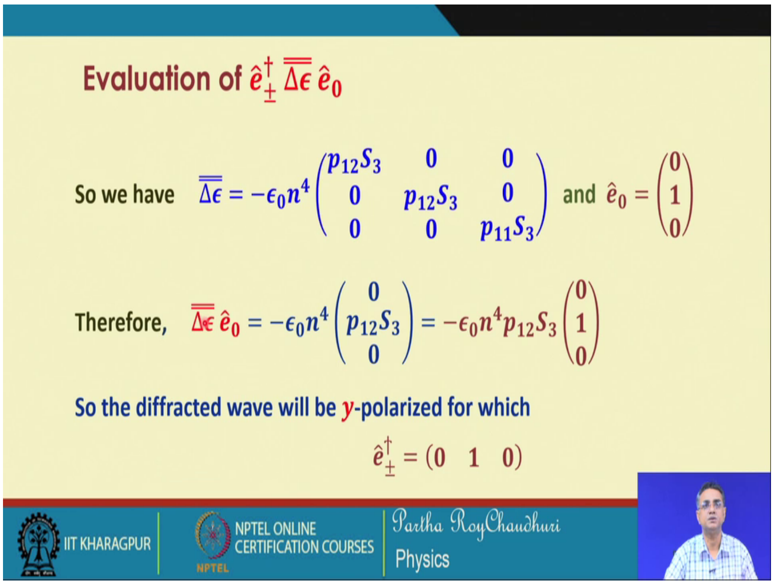

We multiply this with this polarization 0 1 because it is y polarized wave and propagating along x direction. The other possibilities it could be x polarized wave propagating along no sorry this is your x propagating wave. So, it can have why polarization or z polarization. So, the other possibilities it can have z polarization as well in the incident wave if the incident beam is propagating along x direction. That is the case that we are studying, because your acoustic wave is along the z direction and the optical wave is propagating along x direction.

So, an x propagating optical wave can have two polarizations; these are y polarization or z polarization. So, in this case we consider the incident wave is polarized in the along the y direction. So, on multiplying this e 0 with this permittivity matrix; if we multiply we will get this form which. And finally, you get this form which suggests that if it has to be a scalar. Then you have to multiply pre multiply with these vector pre multiply with this vector look at this.

So; that means, the diffracted beam the polarization of the diffracted beam will also be y polarized the diffracted beam is also y polarized. So, what we get from here is that if the incident beam optical beam is x polari y polarized the diffracted beam is also y polarized because of the nature of the change in the permittivity tensor. And we can repeat the same procedure for z polarized wave in the same way.

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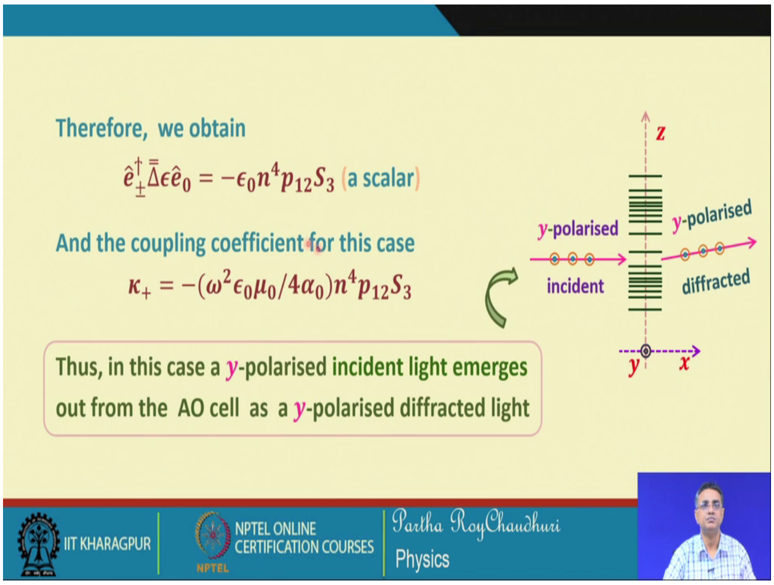

And in this case so this to get a more clear picture about what is happening you have an acoustic wave which is traveling along the z direction, which is traveling along the z direction. And the incident wave optical wave light beam is traveling along x direction very close to x direction. And it has y polarization this wave has y polarization. When it is diffracted out whether it is plus order or minus order the case we studied.

Then it is again y polarized; that means, in this case for the case of longitudinal acoustic wave. The polarization of the incident optical beam is not changed it remains the same. Thus in case of y polarized incident light that emerges out from the acoustic cell as a y polarize diffracted light. So, there is no change in the polarization because of the coupling due to diffraction in this case.

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So, what we have discussed in this case is that you can use this vector wave equation model to identify the polarization content of the using knowing the incident beam polarization. What will be the polarization of the diffracted beam, when the wave is a longitudinal wave acoustic wave that is traveling in the medium?

We will study the shear wave and then also we will study the caustic wave in an anisotropic medium. So, in summary we discussed in this case that the vector wave equation and then incident and transmitted field amplitude how we can write. Then we rewrite the coupled wave equations coupling coefficients in a wave which is similar to the scalar wave formulation. And then we looked for the coupling of the polarization from the incident wave to the diffracted wave.

And in the present case we studied when the acoustic wave is a longitudinal wave. And for small diffraction Bragg angle diffraction we saw that the polarization coupling it remains the same the incident polarization and the diffracted waves polarization they are the same.

And in the next case we will consider an isotropic medium, but for Shear wave and also an isotropic medium with shear wave propagation in the Acousto-optic cell.

Thank you very much.