Modern Optics
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Lecture – 52
Acousto-optic Effect (Contd.)

We have considered the small Bragg angle diffraction and then after we discussed the large Bragg angle diffraction also, but still we have some more insight to discuss about the small Bragg angle diffraction in terms of the vector model.

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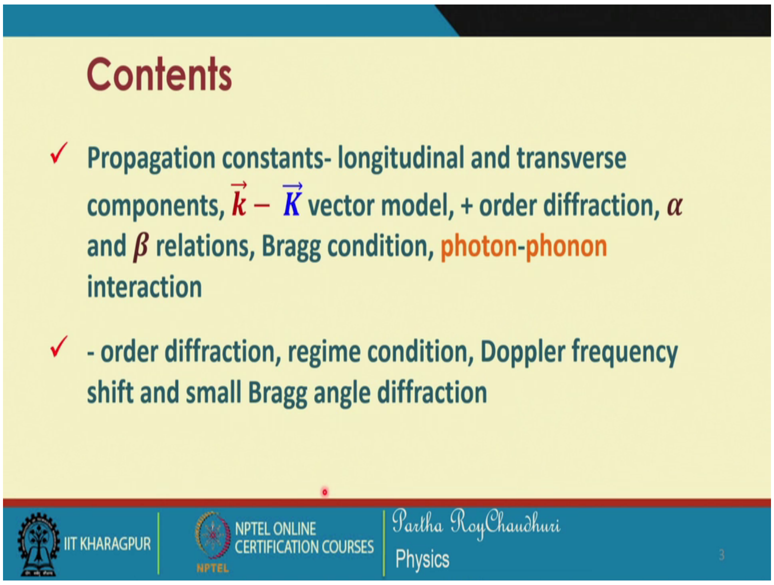

That is k K, small k which represents the propagation constant of the optical beam and capital K which represents the propagation vector of the acoustic beam. So, under that with that understanding we will look at the plus order diffraction, alpha beta relations, Bragg condition we will see that these are the consequences of photon phonon interaction.

And in a small discussion we will see that and then we will look at the minus order diffraction. From here we will try to do now this regime condition that is the width of the acoustic wave which is going to decide whether it is a Bragg type or a Raman Nath type diffraction. Then there is another interesting thing that is Doppler frequency shift we will see that because of this Doppler frequency we get the frequency of the refracted beam modified and exactly by the same amount which is the Doppler shift, under small Bragg condition.

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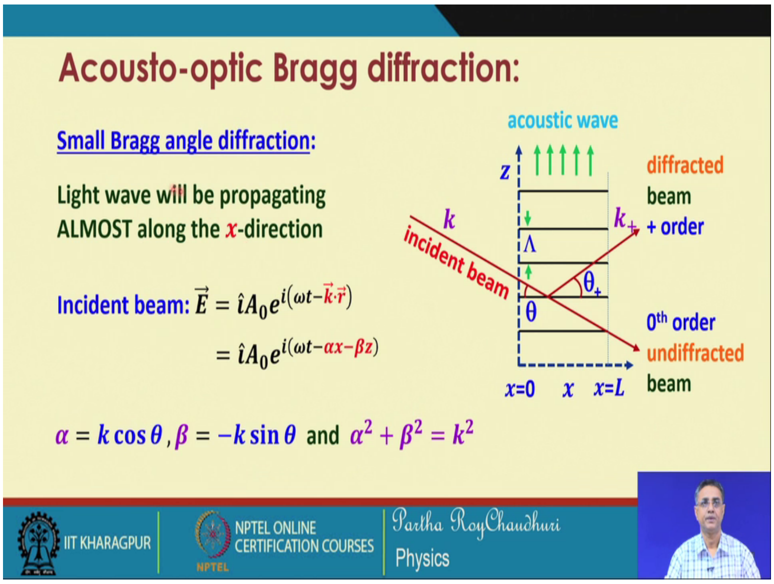

So, this small Bragg condition, Bragg diffraction we remember that the incident optical beam is traveling almost along the x axis, along very close to the x axis. This angle is very small way this is an exaggerated figure where this angle is very small that is we have this beam which is traveling almost along the x axis.

The diffracted wave plus order will be almost along the x axis and this was already along the x axis. So, under this condition we have seen that this the incident beam can be represented by e to the power of i omega t minus k dot r. This k dot r now contains alpha x minus beta z, where alpha is the x component of the propagation vector of the incident beam, and beta is the z component of the propagation vector of the incident beam.

So, therefore, this picture is complete and this alpha that is x component of the propagation vector will be alpha will be equal to k cosine theta. And z component of propagation vector beta will be minus k sin theta along this direction. And for incident beam this alpha square plus beta square must equal to k square. At this point x equal to 0 the beam is incident and at x equal to L the beam is out from the acousto-optic wave.

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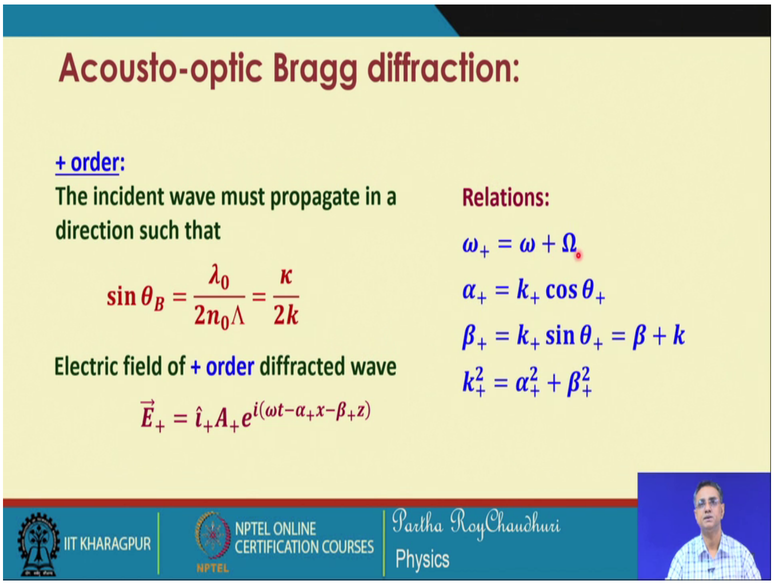

Therefore, for plus order diffraction the incident wave has to propagate, must propagate in a direction such that this condition is satisfied. We will see this how it condition comes once again and the electric field of the plus order of the diffracted wave. You see this is for the minus order this is for the incident beam omega t minus k dot r alpha and beta where the x and z components.

Similarly, for the plus order diffracted beam A plus is the amplitude alpha plus and beta plus will be the x and z component of the propagation vector of the diffracted way plus order diffracted wave. And they must hold this relation alpha plus, we have seen from the basic couple mod equation.

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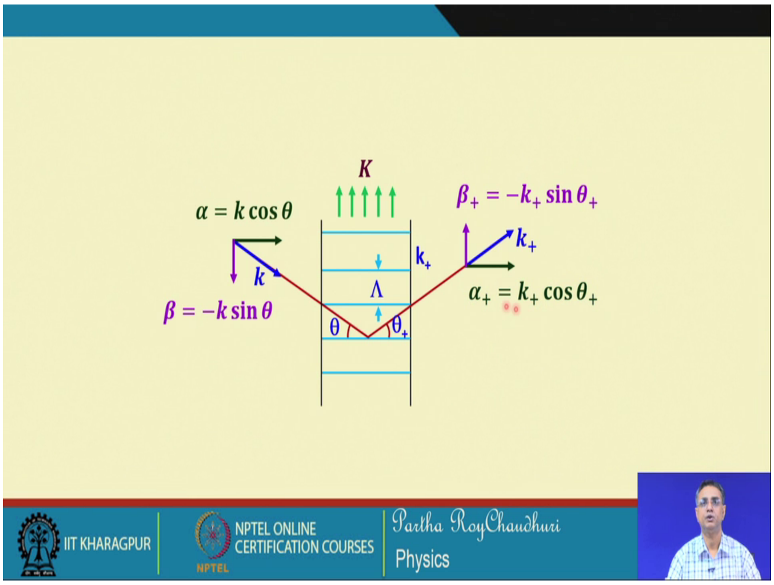

Alpha plus will be equal to omega, omega plus will be equal to omega plus capital omega alpha plus will be we can see from here, alpha this is the incident beam. So, alpha equal to k cosine theta this is the angle theta and beta will be minus k sin theta.

Similarly, for the plus order diffracted beam beta plus will be. So, this will be equal to plus, this is mistake this will be plus k plus sin theta. And this will be your alpha plus will be equal to k plus cosine theta plus so, this should be plus.

Then we get this set of relations omega plus equal to omega plus capital omega alpha plus will be k plus this is all from here. This is all from here, you see alpha plus will be this beta plus will be k plus sin omega plus k plus sin omega plus and k plus square this is again true for the diffracted wave k plus square will be equal to alpha plus square and beta plus square.

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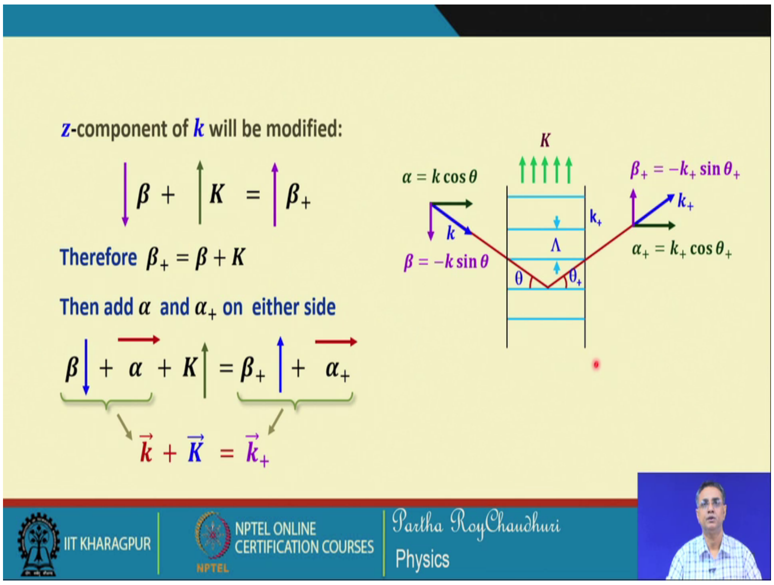

Therefore, we have this z component of k will be modified. This was the z component of k that is beta and this is the z component of k that is beta plus. Here again this mistake is there this will be plus. So, this beta will be modified to beta plus by the addition of this k vector which is along this direction. So, we put them in the order beta plus this k vector is equal to beta plus vector.

Therefore, beta plus is equal to beta plus beta k. And this is the relation; this is the relation, but we see it here using this vector model and then alpha and alpha plus, if we add on these sides that is if you put alpha plus alpha here and alpha plus on this side, then we get beta plus alpha. Because this as a vector this beta vector and alpha vector beta vector plus alpha vector is equal to k vector.

So, that is what we are writing here k vector plus this k which is the vector due to the acoustic wave will be this plus capital K is going to modify this into this k plus. So, you have added k plus alpha plus beta plus which represents this these are 2 vectors. So, this is the resultant k plus beta plus alpha plus will be equal to k plus. That is what is written here and beta plus beta plus alpha will give you k which is here the resultant of this then you add this K vector. So, this relation again comes out as a consequence of this simple vector diagram of the propagation constants in terms of their individual components, which is very bright to see.

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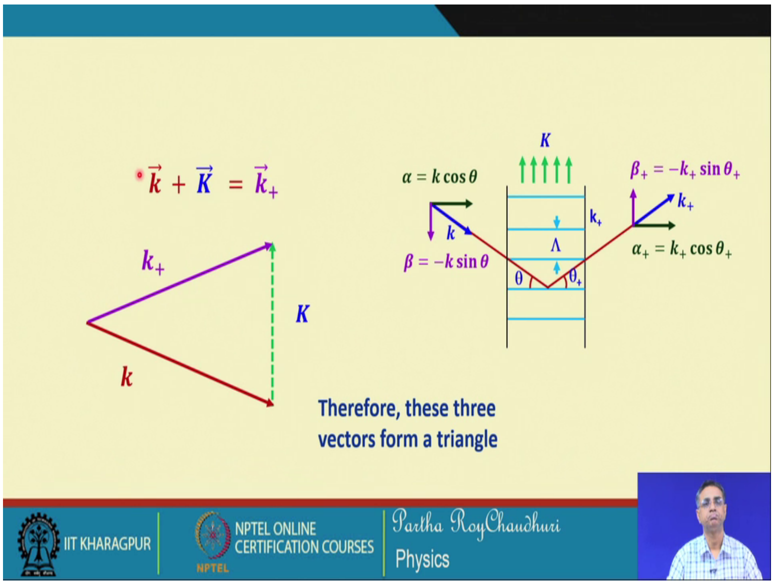

Then in the, in this form that is this was the incident sorry this is the incident wave direction of k, you can see from here incident wave there is a direction of k. And it is exactly in the same direction this is the diffracted wave k plus which is along this direction. And this is your capital K which is to represent the propagation vector of the acoustic wave. So, these 3 vectors will form a triangle. So, and you get the consequence as the condition where of diffraction.

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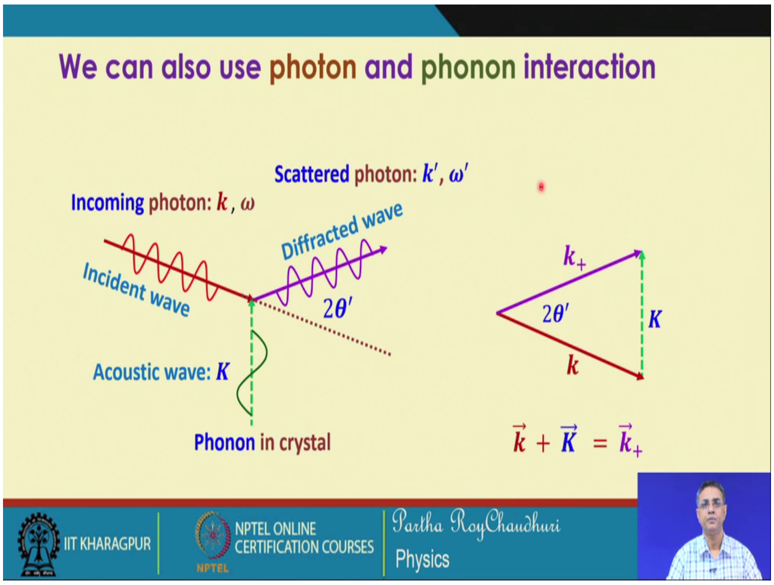

Now, the same thing we can have a look at from a different point of view, that is basically the same that is the photon and photon interaction you have an incoming photon which is hitting this acoustic wave and this acoustic wave which was a propagation vector k, incoming phonon has a propagation vector photon k, omega is the frequency the scattered photon will have k dash and omega dash. So, these 3 they are put together k plus this is k incident and this is K. So, and that is actually their conservation of momentum and energy will give you the same principle.

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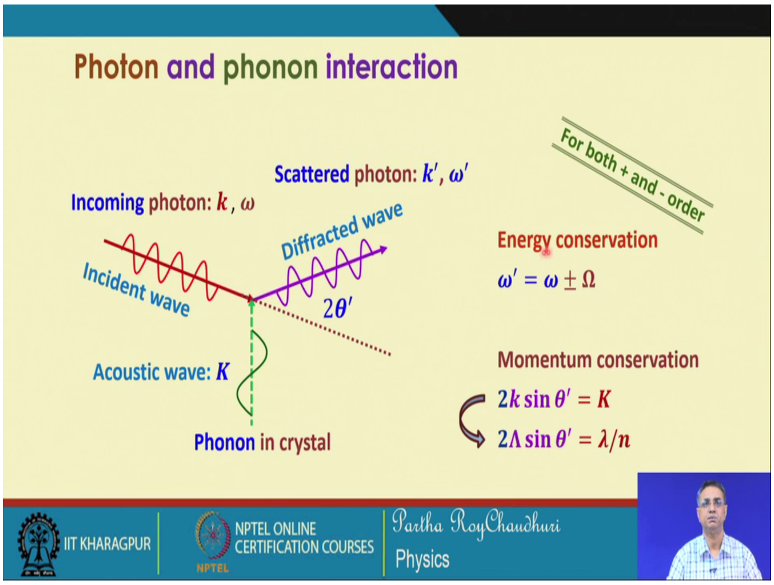

Let us see that.

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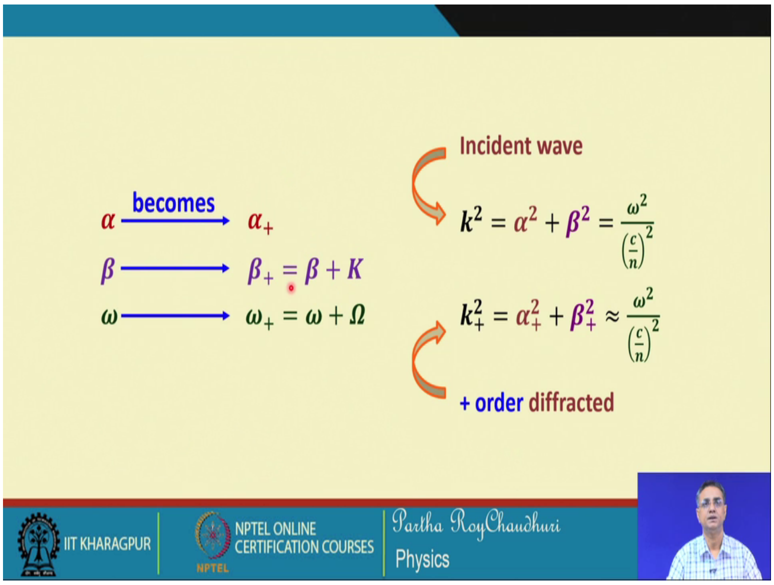

I think there is ha alpha becomes alpha plus beta becomes beta plus beta plus equal to beta, beta becomes beta plus. So, that is equal to beta plus k omega becomes omega plus which is equal to. So, this is the case for plus order diffraction, but this is true you come back to that this is true for both plus and minus this interaction is true for both plus order and minus order and only the sign will change.

So, omega dash which is the scattered wave, which is the diffracted wave will be equal to omega plus capital omega or omega minus capital omega. So, because this is the consequence of energy conservation, you write multiply h the Planck's constant on either side you get the conservation of energy.

Momentum conservation that gives you we will see this how this represents this momentum conservation. And from here the because h into k that gives you the conservation of momentum. So, this from this side the momentum the this component of momentum and this component of momentum, they will be conserved and we will see the sin theta component. So, this component and on either side for the incident and diffracted beam will remain constant and that gives you the Bragg condition.

So, from this simple picture that is photon-photon interaction we can bring out the Bragg relation that is twice lambda sin theta does equal to lambda by n, is k equal to you can put k equal to a 0 into n.

So, you get this same relation. So, again this is the modification the x component of the incident, x component of the propagation vector of the incident wave becomes alpha plus which is the x component of propagation vector of the plus order diffracted beam. Similarly, for the z component it, becomes beta plus which is equal to beta plus k we have seen this, omega becomes omega plus which is equal to omega plus capital omega which is the acoustic frequency this is the optical frequency, mind that this acoustic frequency is much smaller than this.

Therefore, we can write this for this because k square equal to alpha square plus beta square, which is equal to omega square by v square c by n square. This is true and there is no approximation, but k plus square that is the square of the propagation vector mod of square of the propagation vector of the diffracted beam plus order is equal to this x component square and z component square, but that is equal to omega square y v square which is the.

But c by n this because the it remains the same, but omega has become omega plus capital omega, in the numerator which is same because you can neglect this is of the order of 10 to the power of 14 15 whereas, this is only 10 to the power of 5 or 6 hertz. So, this is mega hertz this is 10 to the power of 14 or 15 hertz. So, you can easily neglect this. So, they approximately become the same. So, k plus square and k square they are actually equal. So, that is a very beautiful consequence.

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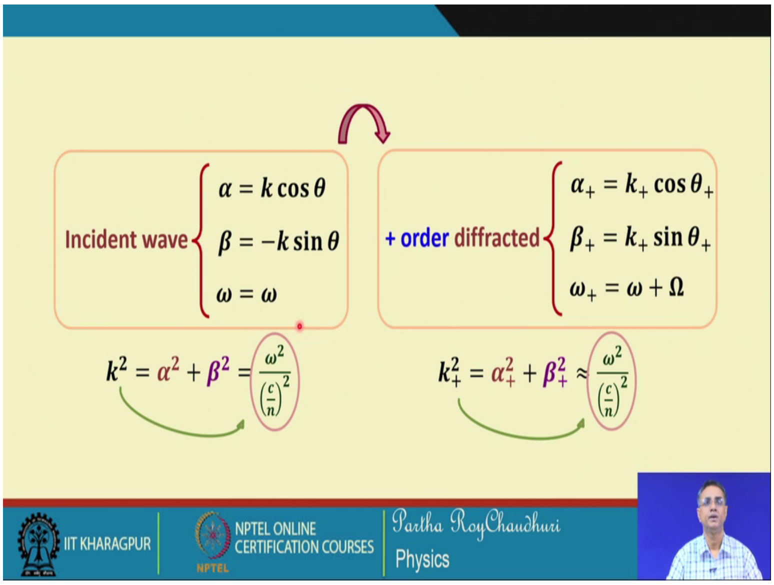

Therefore, alpha becomes k cosine theta beta is minus k sin theta. Here beta has become k plus this we have seen pictorially here from here also the same thing and then we have this relation that k square is also equal to k plus square. You can see these 2 are equal this is because this capital omega does not play any role in this becomes very small and negligible. Therefore, k and k plus we have shown that k plus is approximately equal to k.

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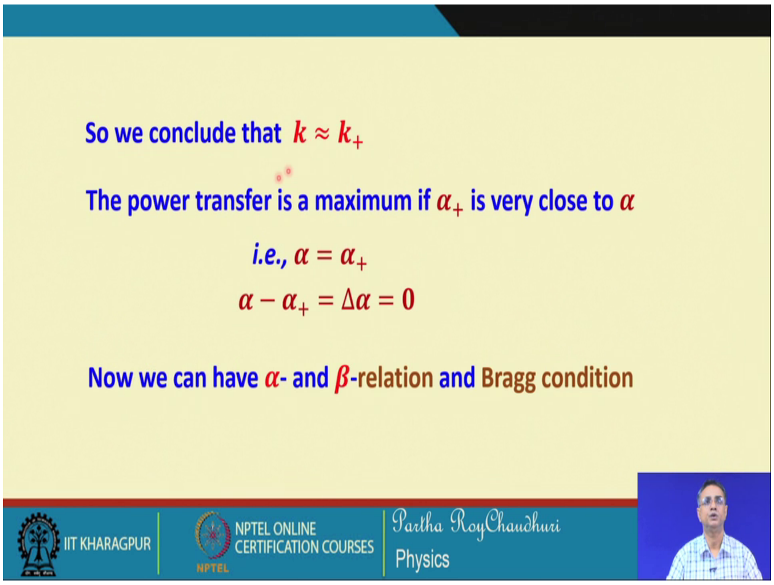

So, therefore, the power transfer is a maximum if alpha plus is very close to alpha. Then delta alpha is equal to 0. And now we can have this alpha and beta relations and the Bragg conditions from here.

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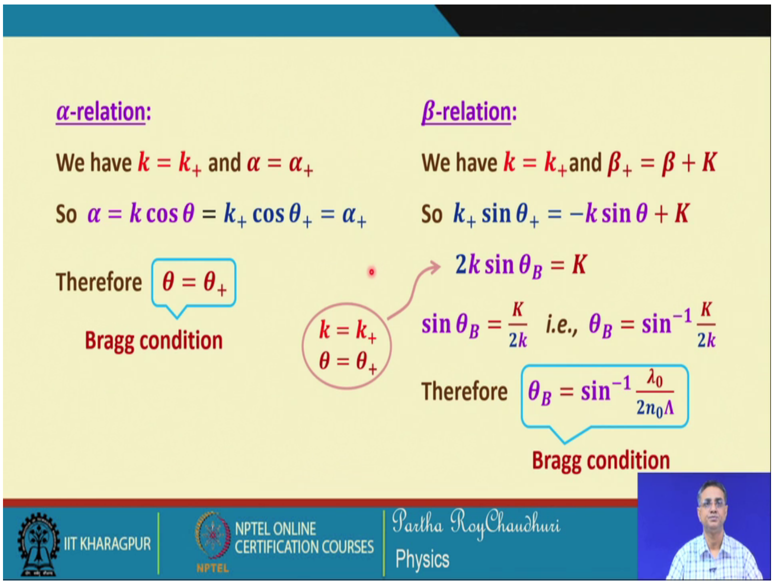

That is alpha relation is k has become k plus that gives you alpha is equal to alpha plus. Therefore, alpha equal to k cos theta we have seen which is equal to k plus cosine theta plus and that is equal to alpha plus from here; alpha equal to this and alpha plus equal to this.

So, therefore, these 2 are equal these 2 are equal. Therefore, theta is equal to theta plus from here theta is equal to theta plus. And that is the Bragg condition. That is the Bragg. So, this is the angle at which this theta and theta plus they are equal. For the beta relation we have k equal to k plus and beta plus is equal to this, just now we have seen.

Because this beta z component of the diffracted wave propagation vector of that will be modified by the propagation vector of the acoustic wave we have seen repeatedly. And therefore, k plus sin theta which will be for this beta plus is equal to beta which was minus k sin theta plus k and because k plus is equal to k, k plus is equal to k and theta plus is equal to theta.

So, it becomes just k sin theta and you have one more k sin theta here. So, if you bring them to the left hand side and this theta plus and theta will be same under Bragg condition and we will call this theta equal to theta plus equal to theta B which is the Bragg angle.

So, therefore, twice k sin theta B is equal to capital K from this relation, from the from the conservation of from the by equating the z component of the of the propagation vector of all the incident and diffracted beams and the acoustic wave. So, therefore, from here we get this relation twice k sin theta B is equal to capital K. And you know that this k equal to k 0 into n the refractive index and the free space propagation constant of the light beam, but otherwise sin theta B is equal to from this equation we can write k by k theta.

Which is again the Bragg condition theta B you can see is the same Bragg condition twice d, twice this is your capital lambda n 0, sin theta b is equal to lambda 0. So, that is the famous Bragg condition which comes just from the from by equating the propagation constants of the incident and the refracted beams along with that of the acoustic wave, which is a very beautiful outcome from this analysis.

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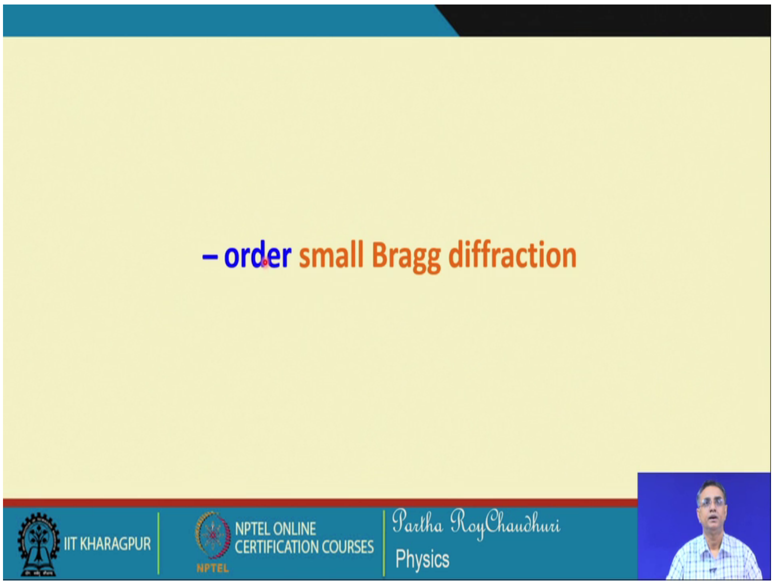

Now, we will see the same thing for the minus order small Bragg angle diffraction.

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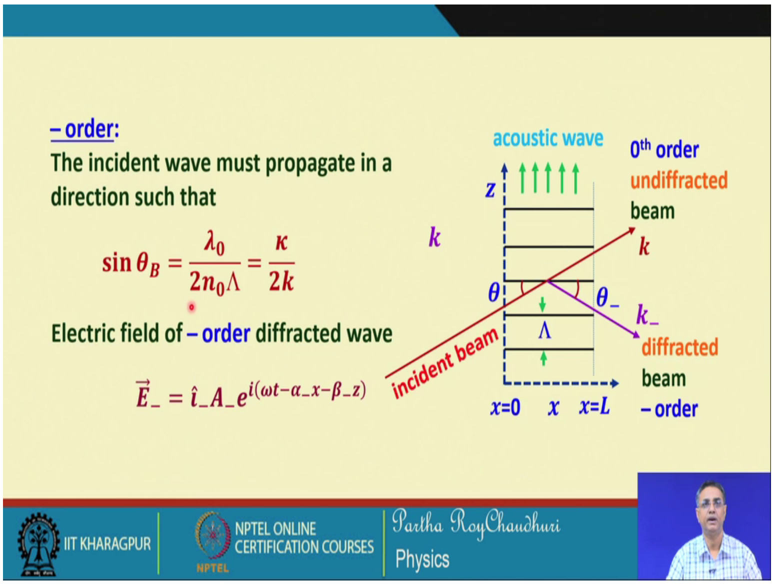

And it is almost the same only the signs will be different. And I will see that this is the condition for the minus order diffracted wave we have alpha minus k alpha minus x minus beta minus z. So, this alpha minus and beta minus are the x and z components of the propagation vectors of the incident and the minus order diffracted wave. So, there will be coupling from this incident beam to the diffracted wave and this coupling will be maximum when you satisfy the condition.

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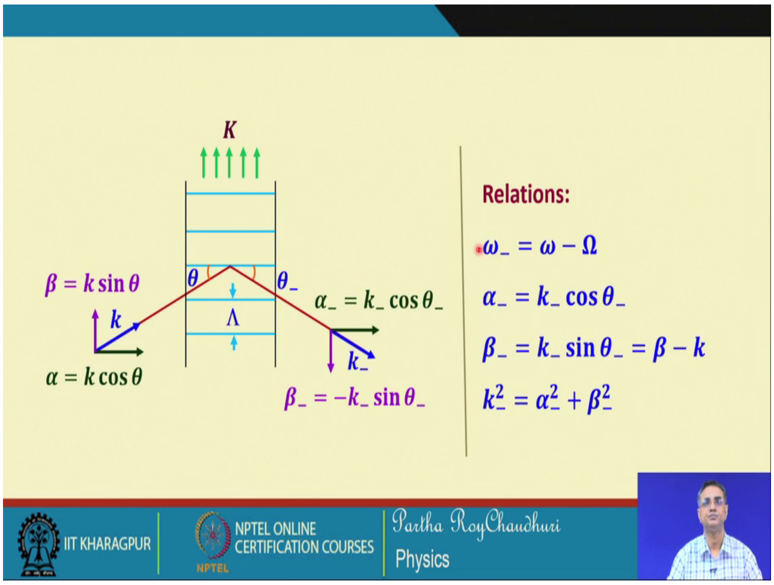

Therefore, once again pictorially this is your z component of the propagation. You see now this time it is plus because the wave is incident making an angle in this direction. So, you have beta equal to k sin theta plus, but this beta equal to minus k minus sin theta. And alpha is k cosine theta this is equal to minus k cosine theta. And rest all of them are similar and identical. So, omega minus equal to omega minus capital omega alpha and this is very clear, alpha minus equal to k minus all that we have written from here in this form.

So, beta minus equal to this which is equal to beta minus k. And this is again true for the diffracted wave that is minus order diffracted wave k minus square will be equal to you see this is your alpha minus, this is your beta minus if you add vectorially vector alpha minus plus vector beta minus must be equal to vector beta k and in terms of the so, therefore, if you take the mod you will get this condition because they are at 90 degree to each other, is that the sin and cosine component. So, you get this relation which is very clear.

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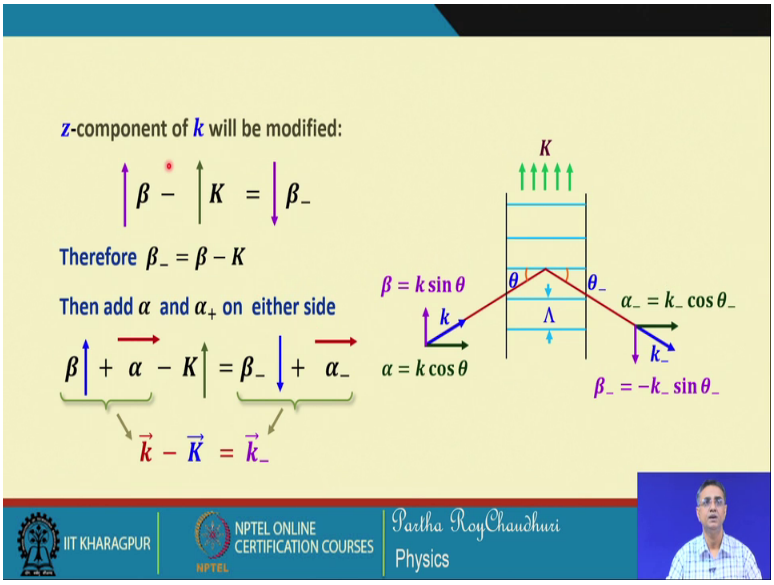

Therefore, in the same way you have beta incident beam will be modified by this k vector, which will give you this. So, as if you bring it to this side then sum of these 2 will be equal to this. So, beta is this beta is getting modified because of the this acoustic wave vector K. So, if you add this then you will get this equation.

Now you have beta minus equal to beta plus K. Therefore, if you add this alpha and alpha plus on either side in the same way beta plus alpha which will complete the propagation constant of the incident beam. This is z component vector this is x component vector form. So, the resultant vector is this. This is itself is the is only one component that is the z component acoustic wave and you have beta minus z component you have alpha minus x component.

So, these are the vectors if you add them you get this vector minus k so, which is very elementary. And therefore, this equation comes directly from the vector addition of this now, alpha plus and alpha.

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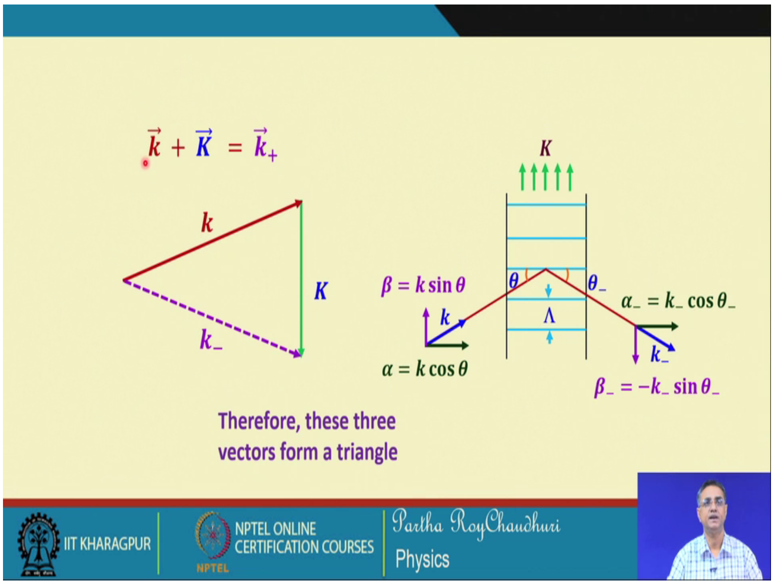

We have added on either side. So, k plus K equal to k incident wave propagation vector plus this acoustic wave gives you this so, again the form a triangle.

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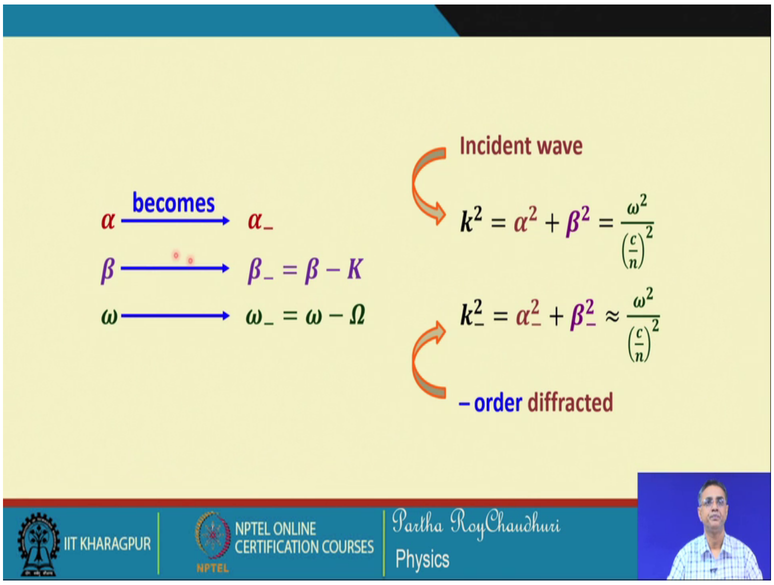

And this comes pictorially from here the same figure, here in this case alpha becomes alpha minus beta becomes beta minus equal to beta minus K. And omega minus becomes omega minus capital omega. So, these are the modification we will see these are this is equal to the Doppler shift. And it is once again same because this omega minus and omega they are approximately same, these 2 frequencies are widely different in magnitude.

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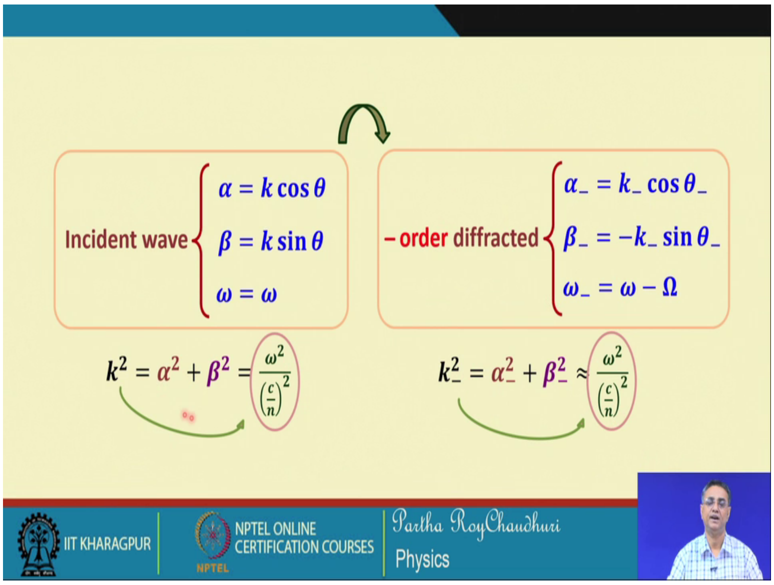

So, we have this is for the incident wave and this is for the minus order diffracted wave. And this shows that k square equal to omega square plus by the velocity square. Here also omega square by this velocity square they are approximately the same. They are the same that tells you that k and k minus they are equal.

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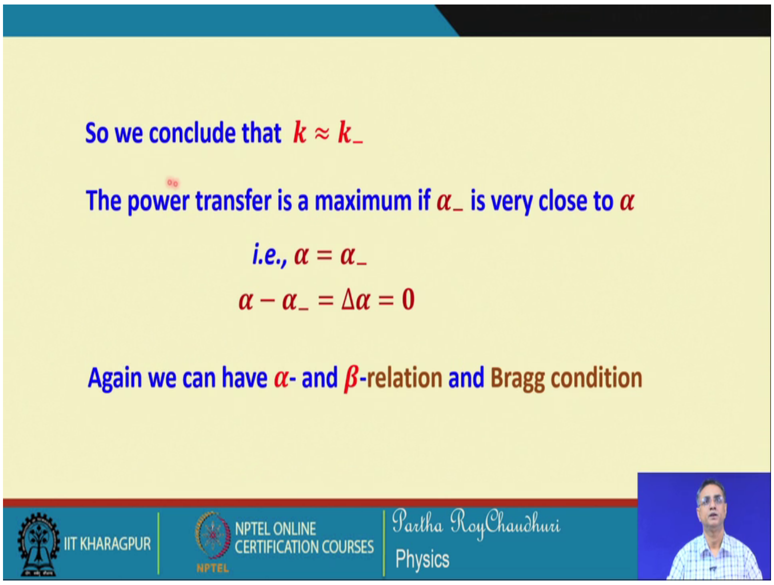

So, we conclude that they are approximately equal k is approximately equal to minus k. And this power transfer will be maximum is alpha minus this time is very close to alpha. So, that gives you the delta alpha equal to 0 and we again get the beta relationship in the same way.

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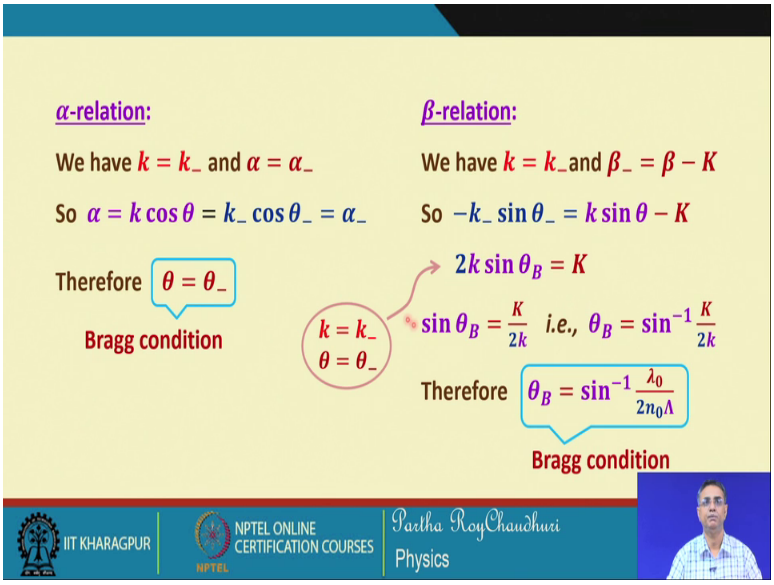

So, you have theta equal to theta minus as a consequence of this we have seen the same thing in the case of plus order and here also minus k, minus sin theta minus equal to this, but this is again effectively equal to k sin theta. Under this condition that theta equal to theta minus equal to theta b. Therefore, this equation this gives you twice k sin theta b equal to capital K and which gives you readily the Bragg condition this we have seen. So, very interesting to see that from here we can directly arrive at the Bragg condition for both plus order and minus order diffraction.

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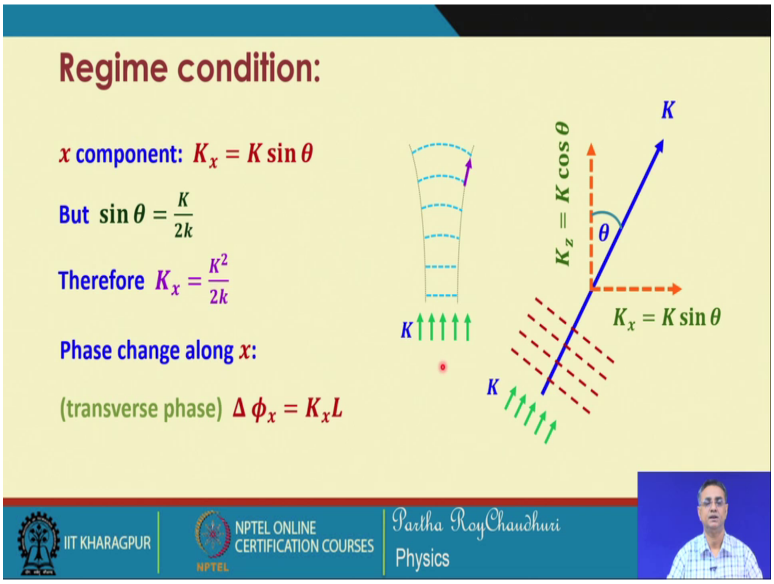

Now from here we can also bring out the regime condition with this you see you have an acoustic wave. So, there is a small divergence of the beam and; that means, initially it was along the z direction, but effectively there is a small divergence.

So, there is a component along this x, along the x. So, this x component will be k sin theta, you can see that. If it is this is little exaggerated form of this exaggerated form of this. So, you have a z component which is K cosine theta and K is x equal to K sin theta. So, that is what we have written.

But, sin theta equal to k by twice k; therefore, K x equal to I just substitute in place of sin theta in this equation. So, it becomes k square by twice k. Now the phase change. So, this is the condition now the phase change along this direction will be this propagation constant along x multiplied by this length L. So, this gives you delta phi x which is equal to K x L this is the transverse phase change that is happening in this direction phase change along this.

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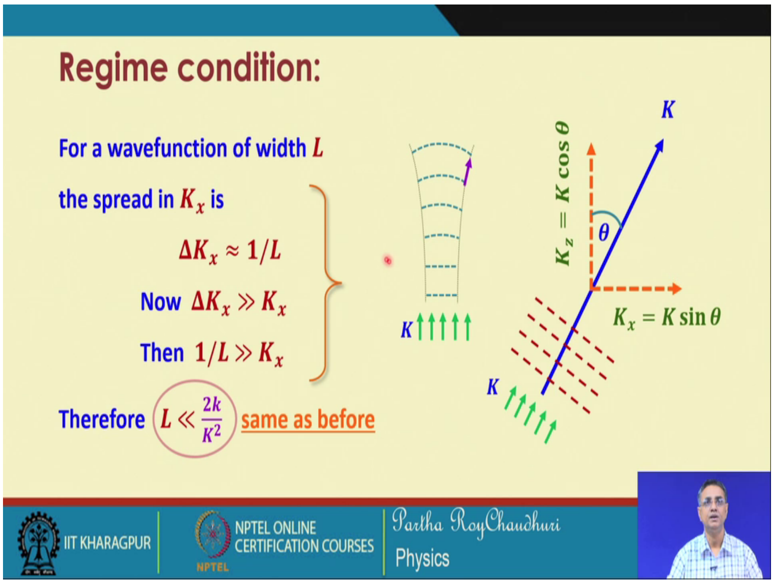

So, for a wave function of width L we know that the spread in K x will be delta K x is 1 upon L and therefore, therefore, delta K x is much greater than K x. This delta K x is much much greater than Kx the spread then 1 by L equal to because in place of delta Kx if I write which is approximately equal to 1 by L. So, 1 upon L is much much greater than Kx.

And therefore, this L much must be much much smaller than twice k by capital K square. This is the resign condition that we have seen earlier for to decide the condition for Bragg diffraction and Raman Nath diffraction. So, this L must be much much less than this and if it is greater than this then that will correspond to the Bragg diffraction.

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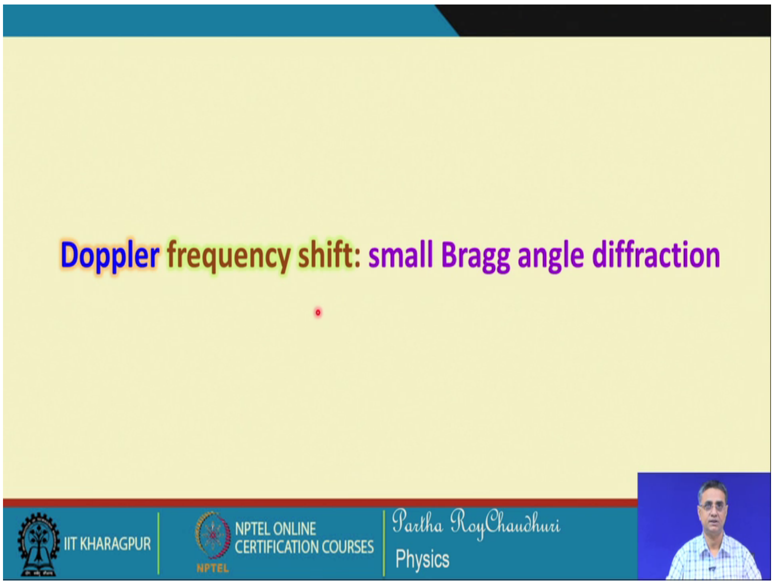

Then there is another very interesting part associated with this Bragg angle diffraction.

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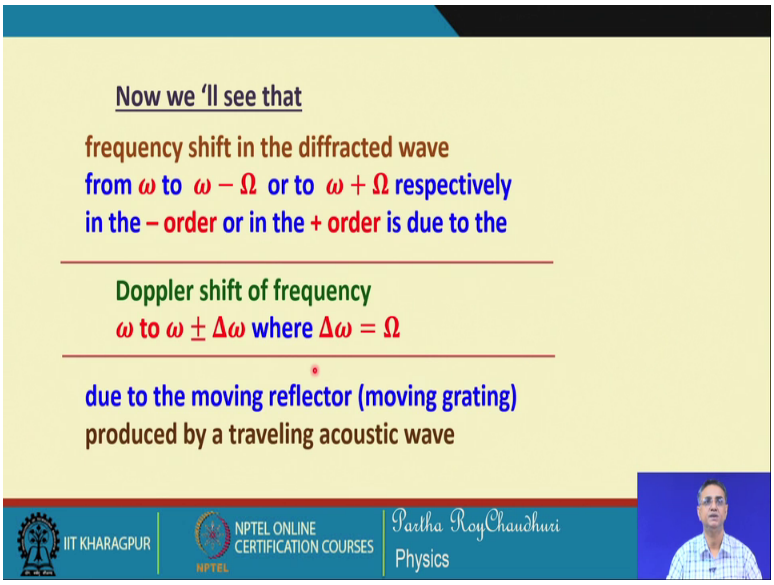

You can see that their frequency of the diffracted wave undergoes a change by an amount that is equal to the acoustic frequency minus omega or plus omega for minus order and plus order. This is very clear and very distinct and the Doppler shift we will see that this delta omega is exactly equal to this capital omega delta omega. And that is (Refer Time: 25:19) treated as a moving reflector.

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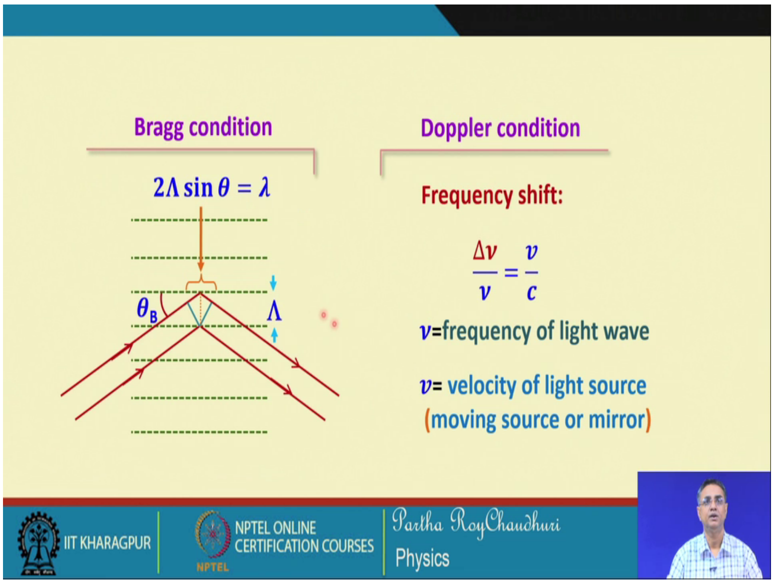

We can see from here this is the Bragg diffraction condition you have an incident beam. And this is reflected from this stack of this periodic variation of the refractive indices. And the Doppler condition is delta nu by nu which is equal to nu by c. This is the Doppler frequency set condition and where, v is the velocity of the light source or in this case it could be a moving source or moving mirror. So, these 2 conditions we put together then we will see what happens.

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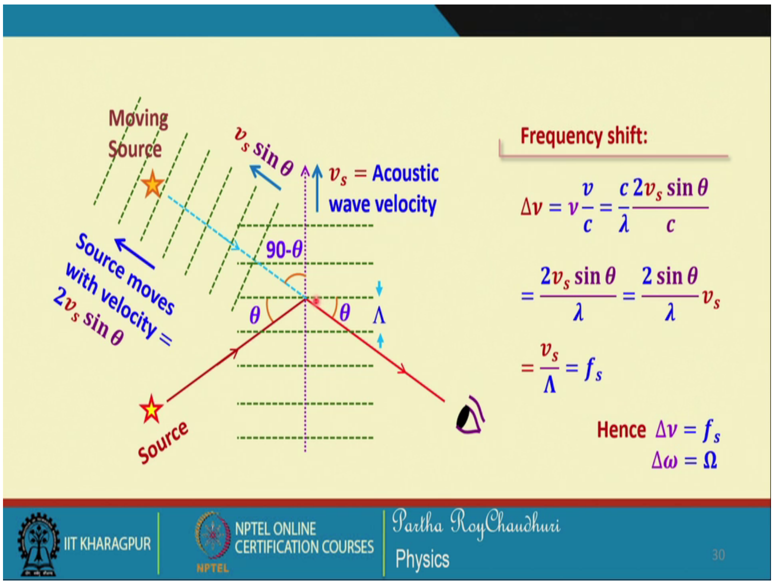

Let us suppose you have a source and which is actually the incident optical beam through the acoustic wave, through this acoustic wave. And this acoustic wave it undergoes a periodic you know it is a traveling wave.

So, therefore, this will be this can be thought of looked upon as a moving mirror. So, there is a velocity of the acoustic wave which is v s, velocity sound velocity, sound wave or v s. And there is a component of the sound wave v s sin theta along this direction because you can always decompose this velocity of the acoustic wave into 2 perpendicular mutually perpendicular directions.

So, this is the velocity of the acoustic wave in this direction. Therefore, when the wave the optical beam is reflected from here, it will see a mirror which is moving in this direction, which will see a mirror which is moving in this direction, but the velocity of this mirror will be v sin theta, you can see from here which is v sin theta. But this moving mirror the source will be traveling with twice the velocity of the mirror that is source moves with the velocity twice v s sin theta.

So, this is the effective velocity of the source or the moving mirror. And that is what we will use here to v a sin theta in the condition of Doppler frequency shift. So, delta nu by nu was equal to v by c. So, delta nu equal to nu times v by c nu the frequency of the optical wave can be written as c by lambda. And for v that is the velocity of the moving source twice v s sin theta by c the cc cancels, you get twice v s sin theta by lambda which is equal to 2 sin theta by lambda into v s.

2 sin theta so, sin theta by lambda sin theta 2 sin theta by lambda. This is equal to what? This is if you look at the Bragg condition to capital lambda sin theta equal to lambda. So, we can write this equal to lambda. We can write this equal to lambda. So, this twice capital lambda sin theta is equal to small lambda. So, 2 sin theta by small lambda is equal to capital lambda. So, v s by capital lambda, but this is equal to the frequency f s, which is the frequency of the this is a shift. This is the delta nu shift in the optical frequency and f s you see this is nothing but this the frequency of the acoustic wave. Therefore, f s that is the frequency of the acoustic wave is equal to delta, which is a very beautiful consequence.

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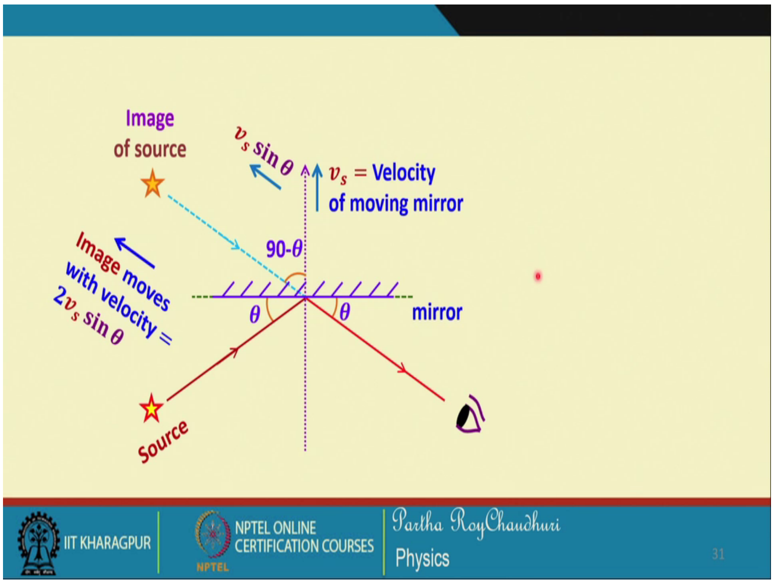

You have this moving mirror in this direction this is the source, we can use an equivalent figure for this. You have a mirror and the source is here, it is getting reflected from here and the observer here will see that the source is here the light is appearing to come from this side.

And because this mirror is moving with a velocity v s as if in this direction the mirror is moving with a direction with a velocity of twice v s sin theta. And this effect of this moving mirror in terms of the velocity that is twice v s sin theta moving source we get this condition that is twice v s sin theta by c.

So, very interestingly we see that this Doppler shift and the frequency of the diffracted beam is exactly the frequency of the acoustic wave. So, it can undergo a change increment by plus omega the acoustic frequency in the case of plus order. And in the case of minus order diffracted beam this optical frequency will be modified by a frequency which is equal to acoustic wave, but it will be a decrement, it will be subtracted from the original frequency omega, small omega minus capital omega and that is very interesting.

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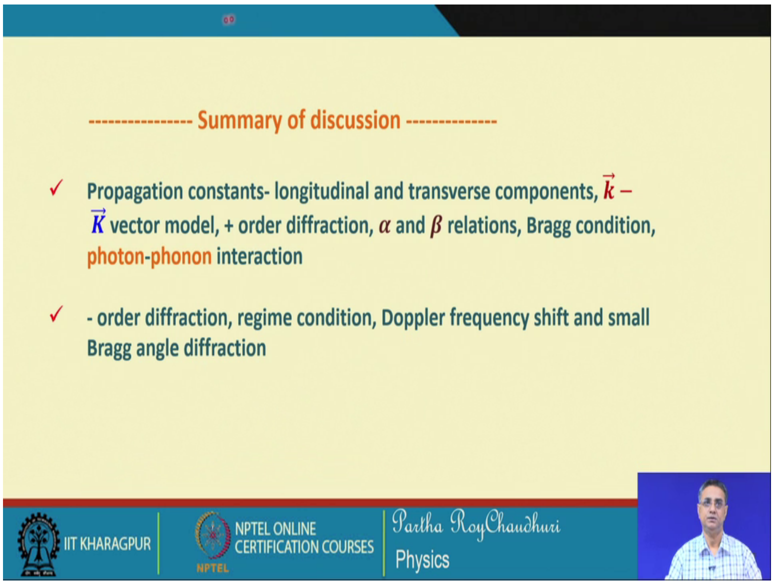

So, I conclude by saying that this propagation constants longitudinal and transverse components from this k K model we arrive at for the plus order diffraction and also for the minus order diffraction, the alpha beta relations the Bragg conditions.

And we also seen it from the point of view of photon phonon interaction, phonon is was attributed to the acoustic wave and photon is to represent the optical wave. Therefore, this interaction also brings out this Bragg condition. Then for minus order diffraction we looked at the regime condition that is the line below and above the Raman Nath and Bragg diffraction should occur.

Then finally, we looked at a very interesting aspect of this acousto-optic diffraction in terms of the Doppler frequency shift. Incidentally it is very interesting that this Doppler frequency shift which is the change in the frequency of the incident beam to the diffracted beam is exactly equal to the frequency of the acoustic wave.

Thank you very much.