Modern Optics
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Lecture – 51
Acousto-optic Effect (Contd.)

In the case of small Bragg angle diffraction by acoustic waves, we have seen that the incident optical beam travels almost along the direction which is normal to the direction of propagation of the acoustic wave. And we saw that there could be A plus order or minus order, and from there we obtained the conditions the Bragg condition. And now that we will consider the case when the incident beam optical beam is at large angle, large angle of incidence that is almost close to a 90-degree. In that case, we can approximate that the optical beam is almost parallel to the direction of propagation of the acoustic wave.

And we will bring out the Bragg condition, in this case we will also use the k vector model to understand how this diffraction. We have seen that the plus order and minus order cannot exist simultaneously.

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So, this we will be discussing starting from the basic coupled wave equations, and we will impose the condition of large Bragg angle diffraction.

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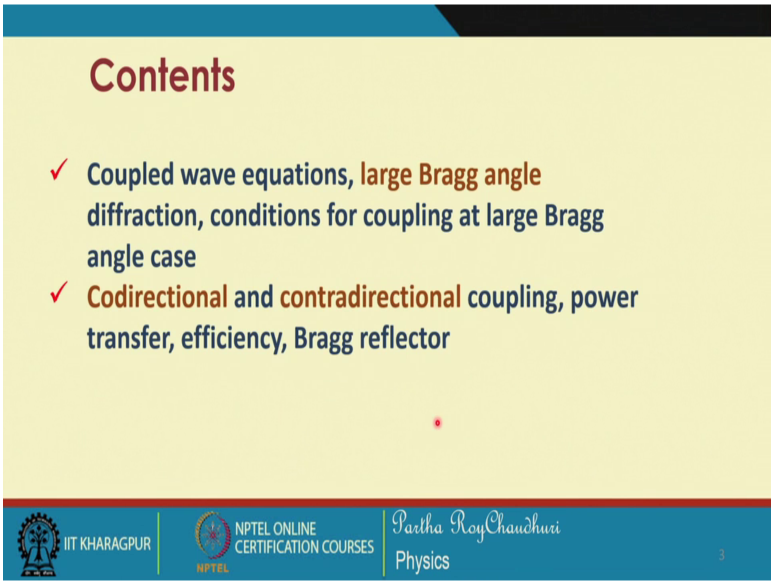

Then we will look at the condition for coupling for this case. Then in this case also, that is for a large Bragg angle diffraction, we will have codirectional and contradirectional coupling; that is the coupling may take place along the direction of the incident beam or counter to the direction opposite to the direction of the incident optical beam.

We look at the power transfer equation efficiency and the use of Bragg reflection. This will help us understand how these modulators will be working based on this Bragg diffraction, when we will take up the discussion of the acousto-optic modulators on the principle of based on the principle of Bragg diffraction.

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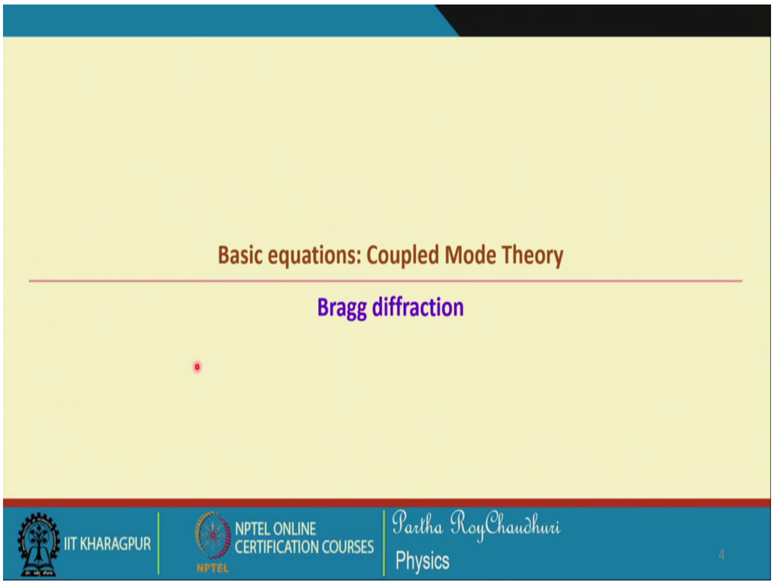

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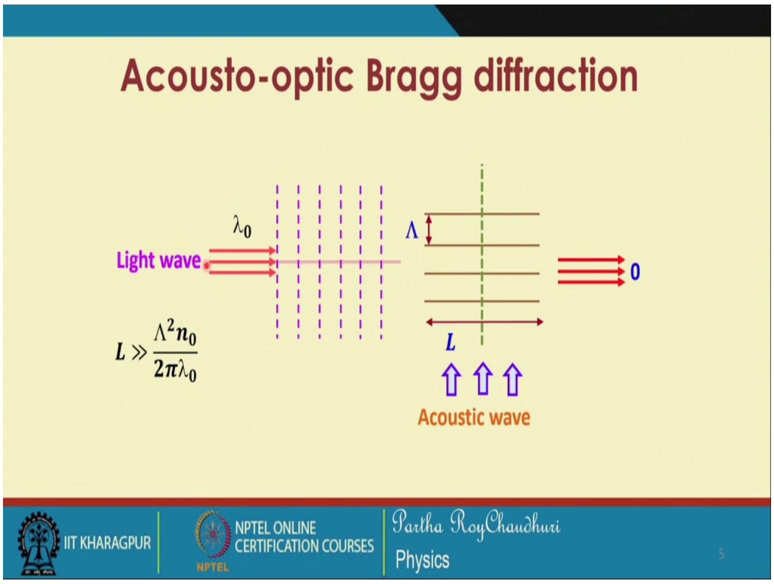

So, let us look at so, this is again the condition which will again see from this discussion also the light wave, earlier it was incident almost you know normal to the direction of the acoustic wave, and that case was small Bragg angle diffraction.

Now, if it travels very close to the direction of propagation like in this direction of propagation of the acoustic wave, then we will call this is a these are the 2 distinct cases of Bragg diffraction.

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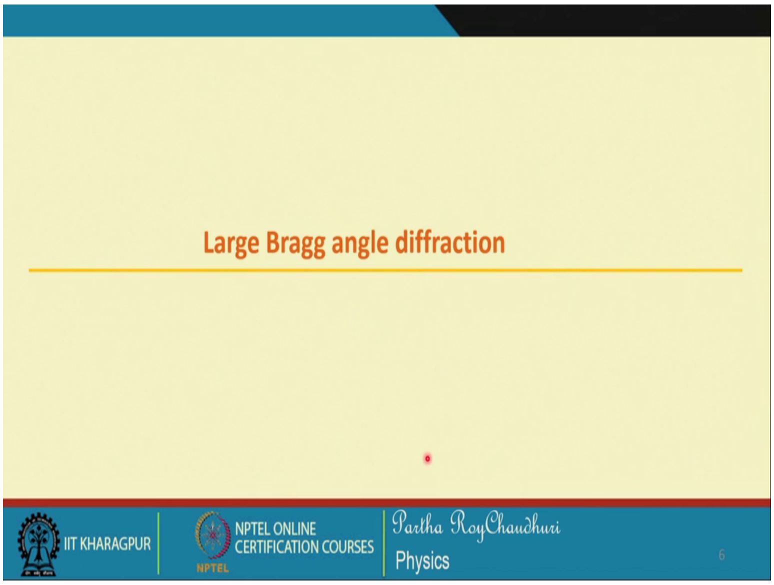

So, in the case of large Bragg diffraction, the light wave will be traveling almost along the direction of propagation of the acoustic wave.

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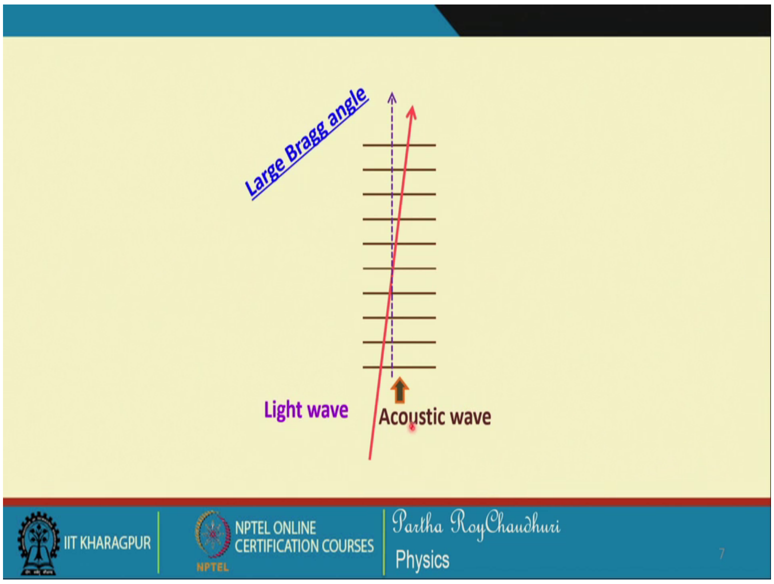

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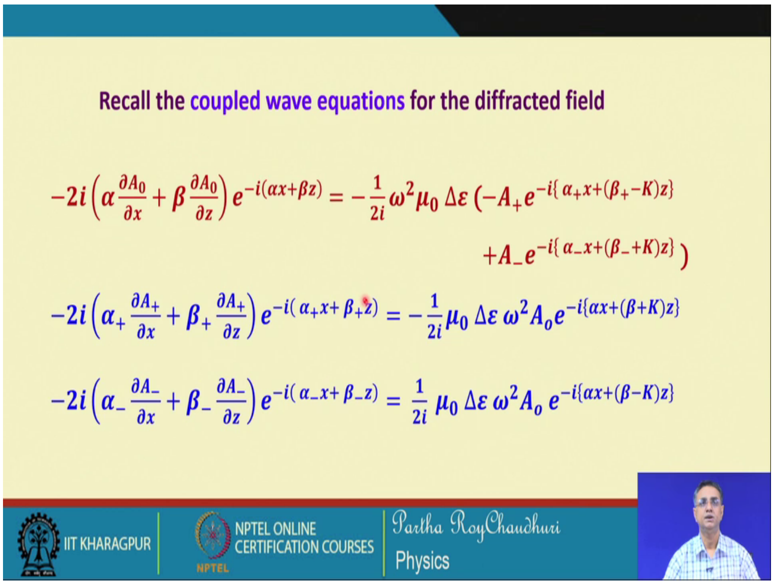

So, this is this picture is useful to understand the configuration, acoustic wave is travelling in this direction, and the light wave is almost along the same direction.

So, the basic set of 3 equations which we have seen starting from the couple mode equations. This A 0 and A 0 the amplitude of the incident wave is connected to the amplitude of the first order, plus order diffracted wave and minus order diffracted wave, through this here the phase of the incident wave is given by this alpha x plus beta z; alpha is the x component of the propagation vector and beta is the z component of the propagation vector. And likewise for the diffracted plus order diffracted beam, this alpha plus is the x component of the propagation vector, beta plus is the x comp beta z component of propagation vector. And see that the z component of propagation vector is modified.

And in the same way for the minus order diffraction, this alpha minus and beta minus will represent the same quantity respectively. This equation these 2 equations are to connect this plus order diffraction with the with the incident beam amplitude, and this is for minus order diffraction to connect the, and these are the straightforward consequences of the couple mode equations we have seen. So, this is the starting point, here we will put the approximation that the beam is traveling almost along z direction.

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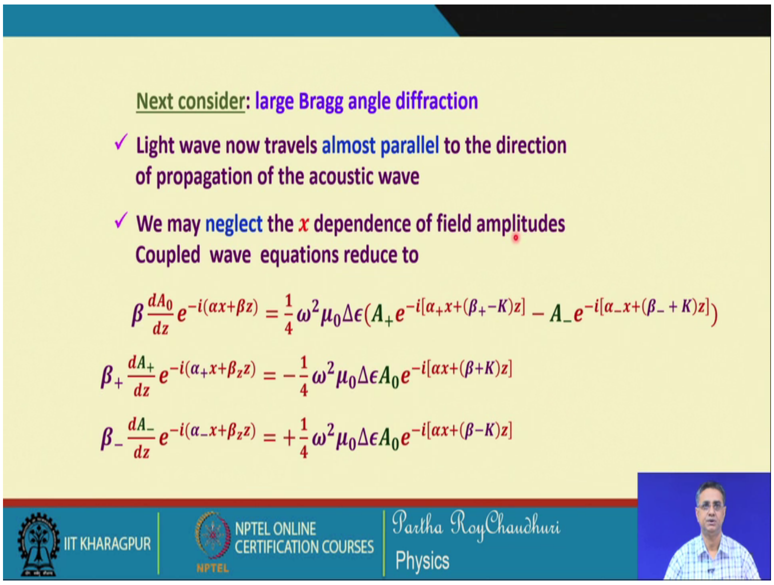

That is now the light wave travels almost parallel to the direction of the propagation of the acoustic wave.

So, we may neglect the x dependence of the electric field amplitudes, and then the coupled wave equation equations will reduce to this form. You see we have this del A 0 del x del A 0 del z. So now, that we have ignored the x dependence of this, because the wave is travelling almost along the z direction. So, this part the z depend x dependent part we can we assume that there is no variation along the x direction therefore, we can rewrite this equation in this form.

You can see del A 0 del z and that connects to this part which remains like this. And this is the other 2 equations that is for the plus order and minus order; where again we have we have neglected the x dependence of the field amplitude of the plus order and minus order diffracted wave. So, we arrived at a set of these 3 equations which are reduced where we have neglected the x dependence of the amplitude. Now these equations these equations will be valid.

So, again the x dependent factors should cancel out therefore, we must have see this e to the power of I by alpha plus x, must be equal to e to the power of I by alpha x.

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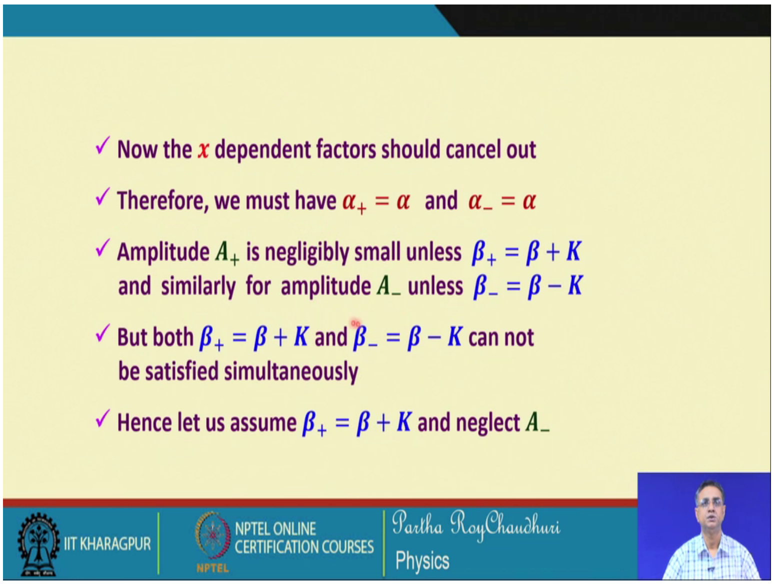

And from here also so, during that alpha plus is equal to alpha, and or alpha minus is equal to alpha. So, both the conditions have to be satisfied in this equation.

But that is not possible; it is not possible to satisfy both the condition simultaneously. The amplitude A plus will be negligibly small. This is unless this condition is valid beta plus equal to beta plus k, look at this equation you have beta, and this is your beta plus minus k.

So, beta equals to beta plus minus k this condition, because they are in the form of the exponential power. Here also so, they must be the coefficients must be equal so, this and this must be equal. And similarly this and this must be equal. So, that is what is here beta plus equal to beta plus k and beta minus equal to beta minus k. But both these conditions let beta plus equal to beta plus k and beta minus equal to beta minus k, cannot be satisfied simultaneously. They cannot coexist; we will see when we look at the vector configurations of this beta k, and or in terms of the components of the propagation vectors along the z direction.

You see that both the conditions are not possible to be satisfied simultaneously. Hence, let us first consider one case that is one of them will be satisfied that is beta plus equal to beta plus k. And in that case we will neglect A 0. As if this minus order diffraction is not happening only plus order diffraction is taking place. So, in that case how we organize the set of equations?

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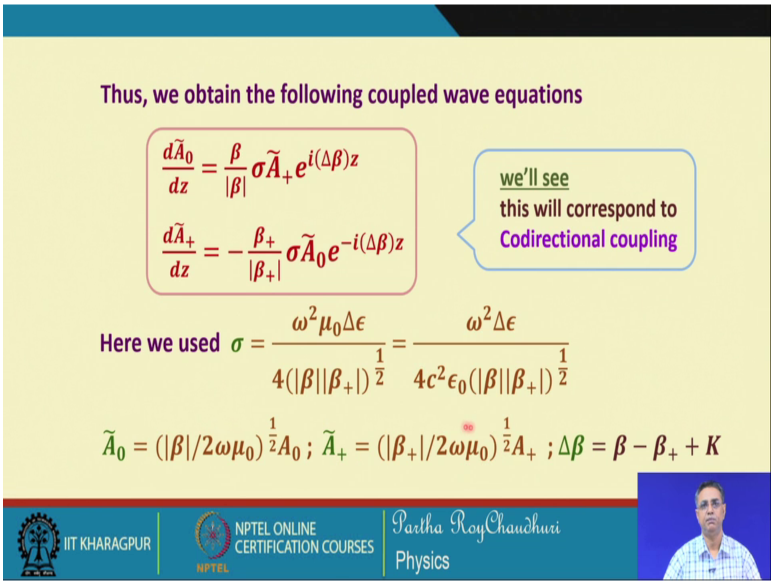

Now, from here from here we can rewrite this equation del A 0 del z equal to beta by beta z, this beta by mod of beta and sigma is actually if we rewrite that equation, we can write this will be appearing on the right hand side, and for that we use this symbol sigma; which is equal to because beta plus then omega 0 mu naught if you add one epsilon naught here, then this will give you c square at the denominator. And this epsilon naught will be appearing here. So, mu naught epsilon naught by epsilon naught; so, this will give you 1 by c square. So, that is the reduced form, and this is useful for the coupling coefficient calculation. And this A 0 we have written for this part that is beta mod by twice omega mu naught under root of that into A 0.

So, this is the amplitude in the modified form of the incident beam, optical beam. And similarly this is the amplitude of the first plus order diffracted optical beam. And this condition this delta beta has appeared, because if we simply take into account this beta minus beta plus minus k and this beta if you equate them. Then we can write that this difference of these 2 things; beta minus beta plus plus k. So, beta plus k equal to beta minus if it is not exactly 0, then in this case we will have delta beta.

So, this delta beta appears here, here also delta beta appear. So, this situation we will correspond to codirectional coupling; that is, the incident beam and the diffracted beam they are in the same direction, there will be in the same direction.

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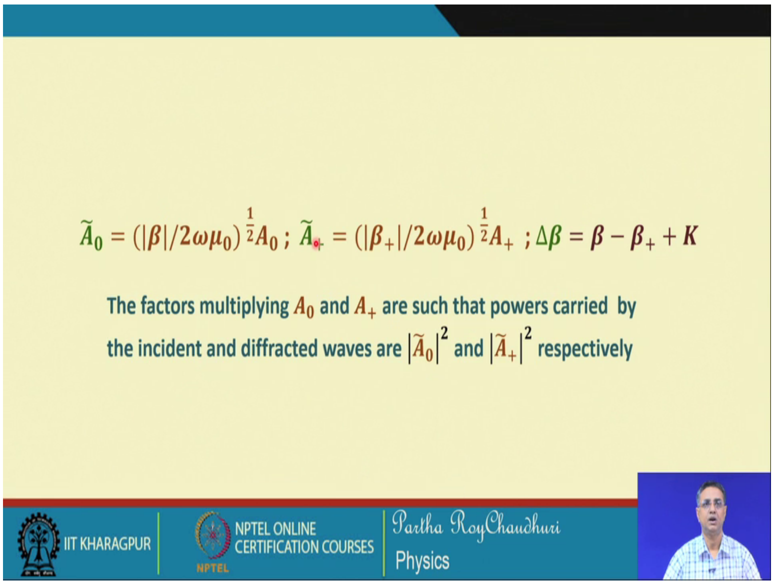

We will see how it happens and A 0, A plus delta beta they are all defined here in this in this equation. So, these are the coupled equations which couples A 0 to A plus; that is incident beam to the coupled first order coupled beam.

Now, these factors A 0 and A plus are such that the powers carried by the incident and diffracted waves this, and they will be like this. So, this mod of this square and mod of this square will represent the power which is carried by the incident beam and the diffracted beam. So, if we now use this condition for the plus order diffraction. This will represent the plus order diffraction. So, we will now we will consider the minus order diffraction, and we will bring out the same condition. In this case sigma will be represented by beta mod beta minus mod which will be appearing here in this.

So, these are the very simple and direct consequence of the couple mode equations where we have used those conditions. Similarly, we get the sigma in this reduced form. And this amplitude of the modified amplitude of the incident beam is this, they are the same, but now it represents the minus order diffraction, and this is your delta beta the difference of the propagation constants, z component of propagation constants.

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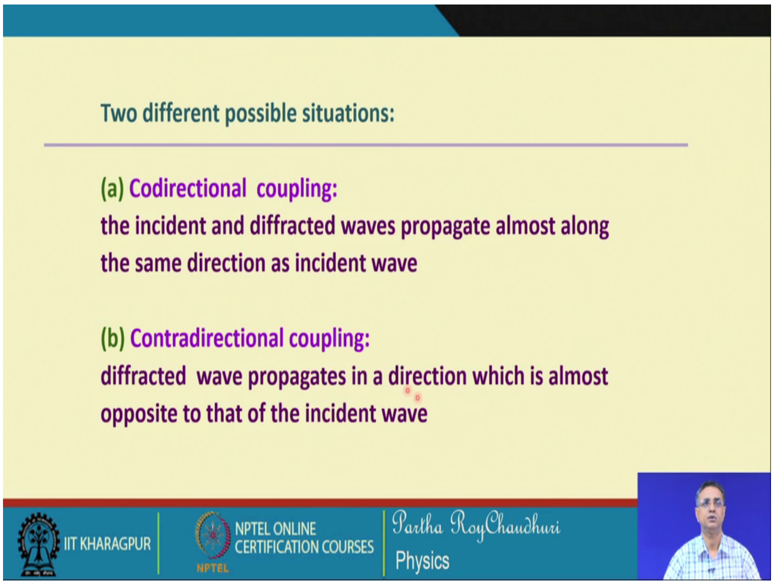

So, by doing so we could arrive at 2 possible situations, one is codirectional coupling that is the incident and diffracted waves propagate almost in the same direction. As the incident beam and the other situation is contradirectional. That is the incident beam and the diffracted beams will be traveling in the opposite direction; that is, diffraction diffracted beam will be talk traveling in the opposite to the incident wave.

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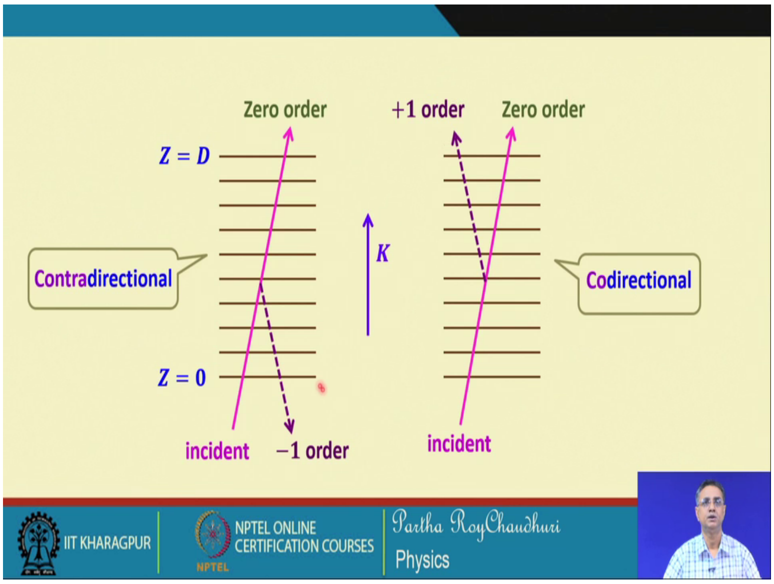

So, these are the 2 situations this is quite self-explanatory, you have an incident beam; which was trying to travel in this z direction. But the diffracted beam which we will call the minus order, will be diffracted in this way, k is the direction of the propagation of acoustic wave. And this is the case of the plus order diffraction that is the incident beam direction, and the diffracted beam they are almost in the same direction so that is codirectional.

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Now, we will look at this codirectional diffraction.

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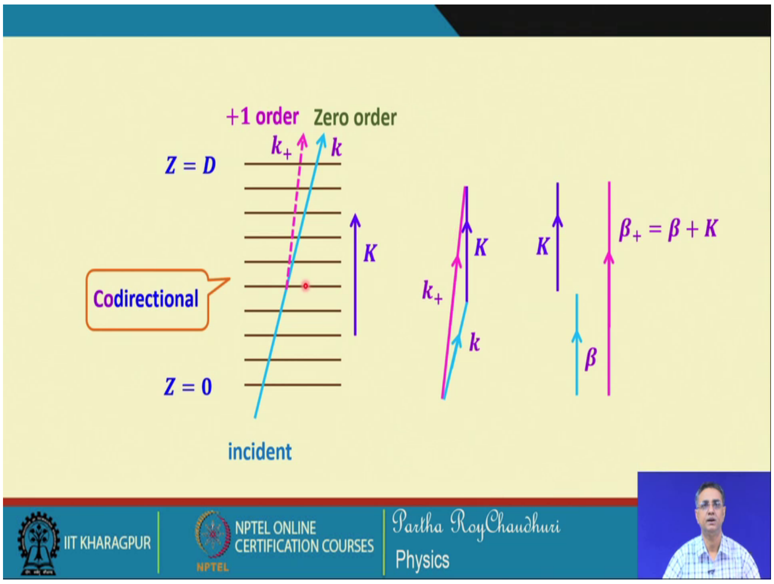

Look at this you have your incident beam whose propagation z component the propagation vector is k which is this. And this propagation vector will be modified by adding the propagation vector of the acoustic wave. So, this k plus capital K will be equal to k plus. So, as if this is the consequence of this is the consequence of the modification of the incident propagation vector of the optical beam by the propagation vector of the acoustic beam.

So, these 3 put together will form a triangle. And you directly get the consequence that is beta plus. So, this is the z component of propagation vector in this case; if I take the z component along this, then I take the z component of k along this. And k is already along the z direction so, you have beta plus equal to beta plus k. So, this is again the Bragg condition.

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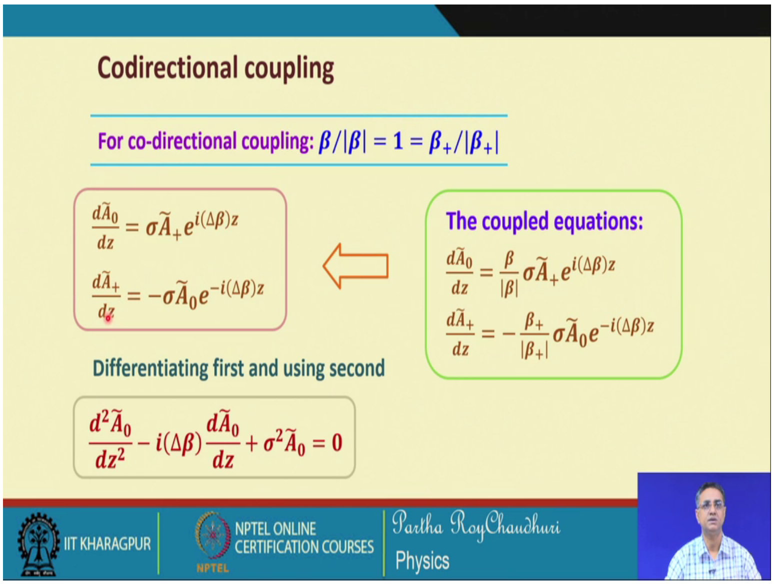

So, for codirectional coupling this quantity beta by beta mod has to be equal to 1; which will be again equal to beta plus 1, this is the consequence of this.

And then we have this modified form of this equation, because beta plus if you go back and see this here, beta by beta mod this is for minus order of course, the same thing is true for plus order as well. And condition that beta by mod of beta will be same, beta 1 mod of beta will be the same, and then we have this equation; which has come from here beta by mod of beta beta plus by mod of beta plus will be put equal to 1 so, you get this.

So, differentiating this first equation, if I differentiate this with respect to z, you will get D 2 A 0 dz square, and that will give you da plus dz in which I will substitute the value of da plus dz here. So, after differentiating this we get a term which will be da plus dz; in that place, we will substitute this and we will readily get this equation. Which is the second order differential equation, but it is uncoupled now. The cost that we pay is that it has become a second order differential equation.

The solution of this equation is similar to the small Bragg angle, and we have seen in that case it was delta alpha, because the wave was traveling along almost along the x axis. So, all the quantities will be modified, and we will get a set of equations to represent the amplitudes of the electric field of the incident wave and the diffracted wave respectively.

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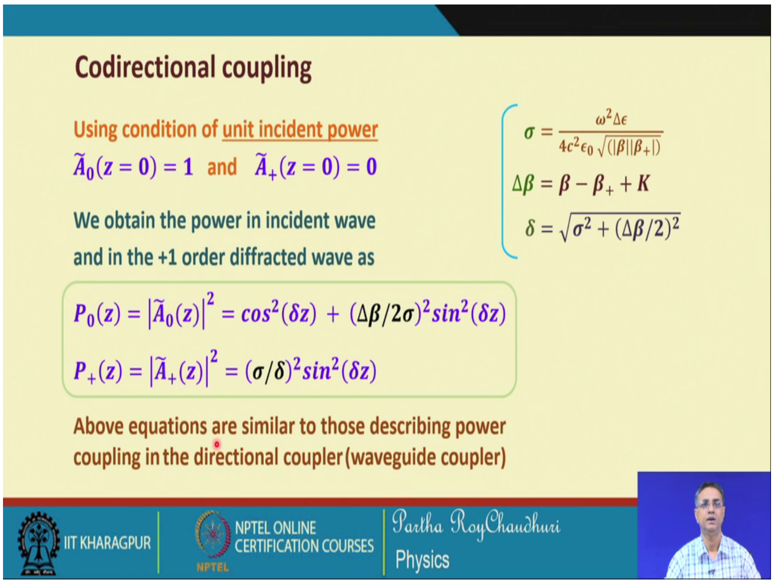

Now if we put the condition in that equation that A 0 at z equal to 0 which is the initial boundary condition that we assume the incident beam carries an amplitude of unity. And the diffracted beam at z equal to 0, because there is no diffraction at z equal to 0 that amplitude is equal to 0.

So, that is this condition if we impose we can bring out the relationship that is the, to get the condition the constants of those are associated with the field amplitudes. And from there this is exactly in the same way we did it for the small Bragg angle. So, it will be a good task to carry out this algebra. And from there we can calculate that P 0 the incident power will be A 0 of z square. This z is any intermediate proposition between z equal to 0 to z equal to D or z equal to A.

So, that appears in this form and P plus will be A z plus mod square which will be in this form. So, if delta beta equal to 0, but these 2 equations they remind us about the these are the similar equations that was used to describe the power coupling in directional coupler. And this was this set of equations are similar to the equations which we used for small Bragg angle. So, basically the principle is the same in all these cases, the coupling principle; that is why the equations they appear to be similar only parameters are different, because of the orientation and configuration.

So, this is again very useful equation from here if you put the condition delta beta equal to 0.

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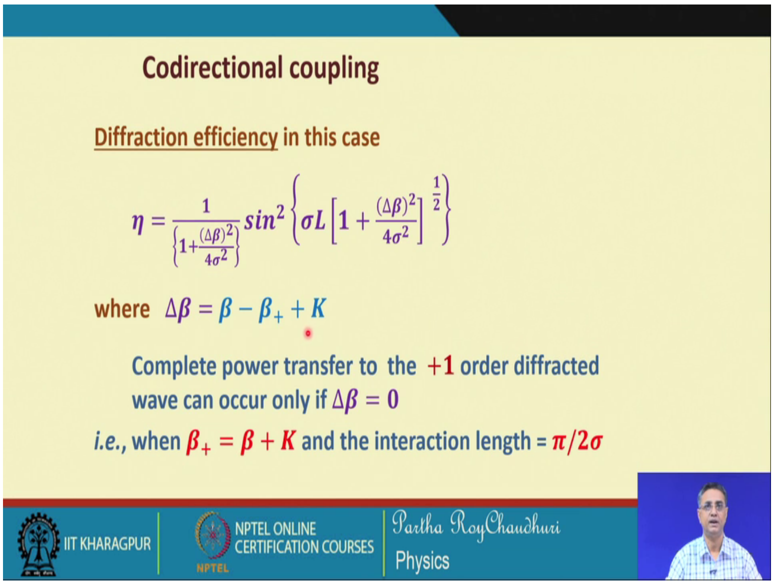

Then we will get the sinusoidal power transfer sin, cosine variation of the power transfer. So, initially the entire power will be represented by cosine square, and the coupled power will be represented by sin square of delta z; where delta will be the coupling coefficient, this we have seen. So, eta the diffraction efficiency in this case from here will come from here, because you see you can see delta beta delta. So, if you put this in this equation sigma by delta, you will readily get this equation. And this expression will be useful will be we will be using also for the efficiency of the modulator diffraction efficiency; delta beta equal to this. Complete power transfer will take to the plus order only if delta beta equal to 0.

That is beta plus equal to so; this will happen and the length of interaction will be pi by twice sigma sigma is the coupling coefficient. And these are all the same that we have seen in waveguide coupling, and also in the small Bragg or a Bragg angle diffraction case. So, these are all in the same line only the parameters are changed.

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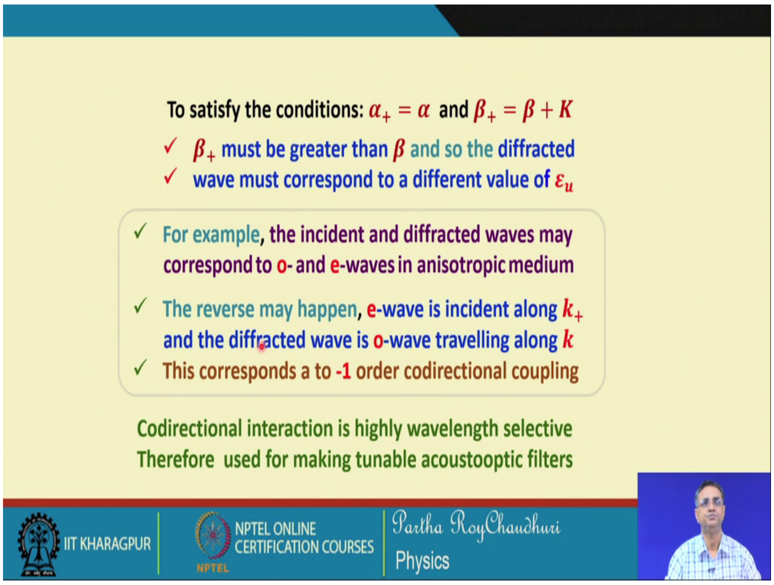

So, these to satisfy the condition, this alpha plus equal to alpha and beta plus equal to beta plus must be greater than beta. So, the diffracted wave must correspond to a way different value of this permittivity of the medium.

For example, the incident wave, and the diffracted wave may correspond to o ray and e ray. So, because of this we will see that it can modulate the polarization; we have a different discussion about how this polarization modulation in by this acoustic wave takes place how it can modify the incident polarization to a different polarization in the coupled wave that we will discuss. So, in codirectional interaction is highly wavelength selective and therefore, it is used for making tunable acousto-optic filters. We will see some examples later.

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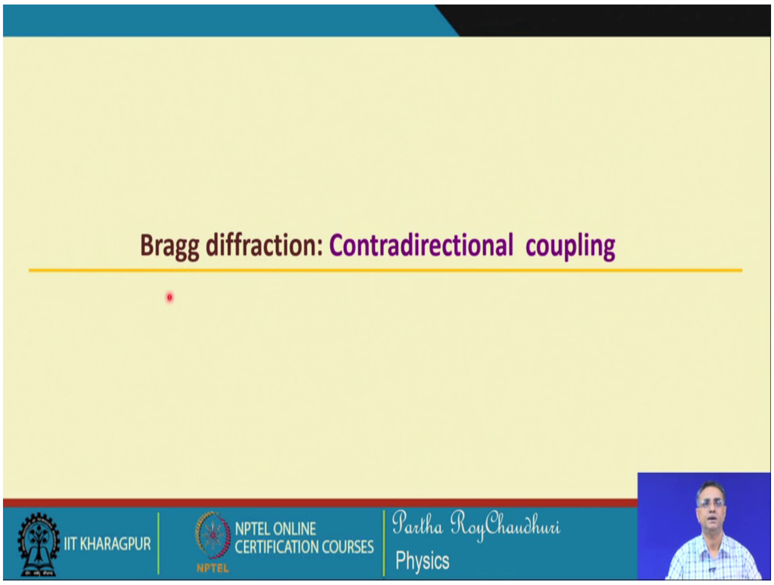

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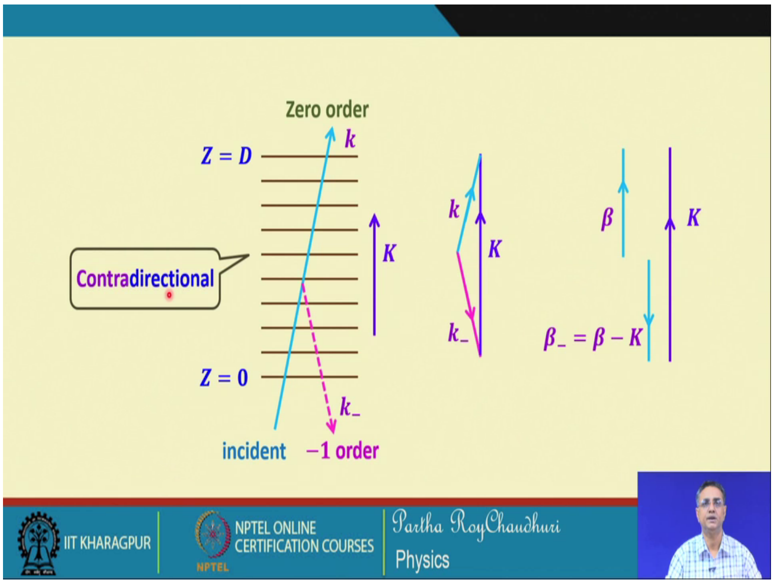

Now, contradirectional in the same line this is the case where we consider the contradirectional coupling; incident way wave is traveling in this optical beam is traveling in this direction, and the minus order is diffracted in this way. And if you take this k vector which is represented by hey k vector, and this is the k minus which is modified form of this k vector because of the addition of this vector this k plus. So, this and this put together this and this put together will be equal to this.

So, you have this vector, you have this vector, and you have this vector. So, this is very, very nice way of looking at this the components of the z component of the propagation vectors which will be put together b beta is the z component of propagation for the incident wave, this will be the minus order diffracted wave, which will be modified by the presence of this k vector due to the acoustic wave.

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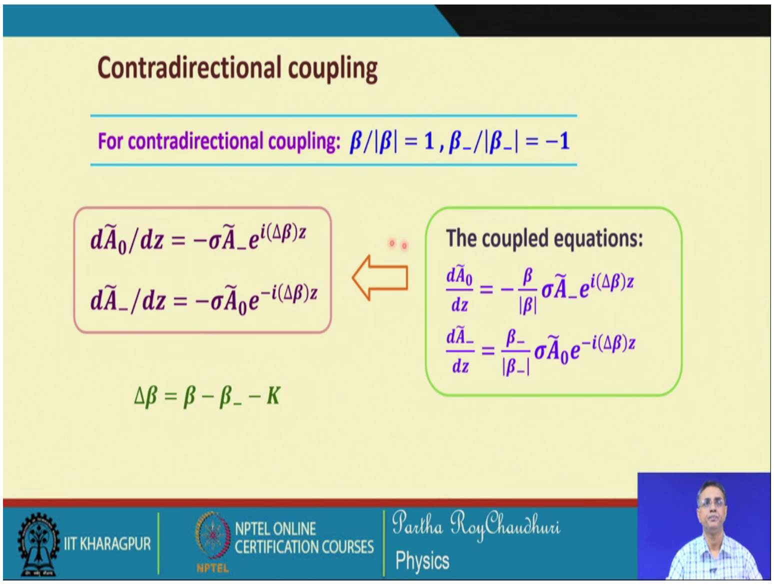

And for contradirectional coupling, we will have the similar set of equation in this case beta by beta mod will be equal to 1, beta minus by beta mod is equal to minus 1, it indicates the directionality also. Therefore, from this equation we get this, and delta beta in this case is given this is along the same line as we discussed in the case of plus order diffraction.

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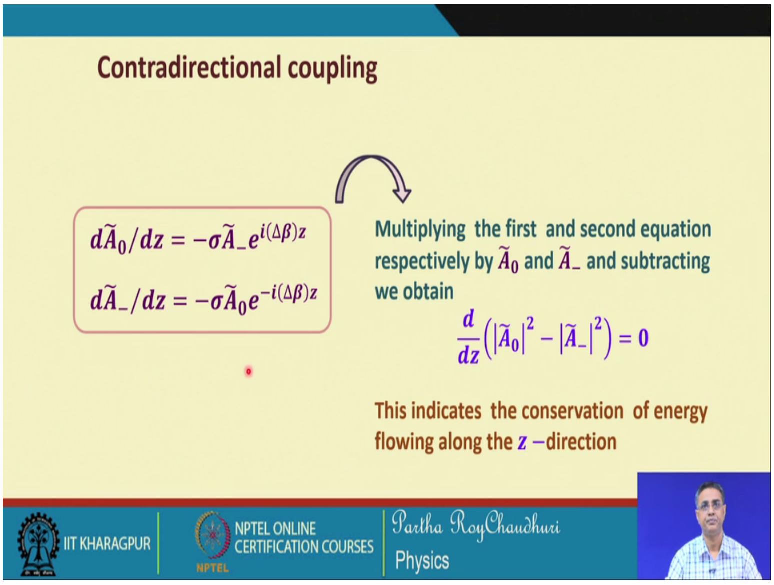

And from here if you the multiply the first and second terms respectively by A 0, and A minus on either side then you can show that from here, we can show and then if you subtract we can show that A 0 mod square, this is the power contained in the 0th order in the in the incident beam optical beam and this is the minus order diffracted beam.

So, the difference will be equal to 0. So, that really indicates the conservation of energy that is flowing along the z direction so, that must be equal to 0. At any point the energy that is flowing in the positive z direction will be equal to the minus so that it, maintains the conservation of energy.

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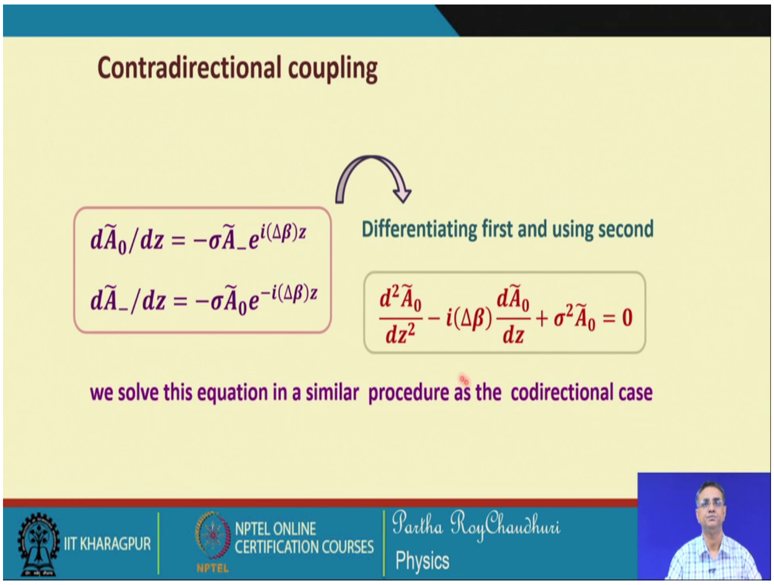

And similarly differentiating this equation with respect to z and using this equation del A z the way we have done it. So, we get this coupled this second order differential equation.

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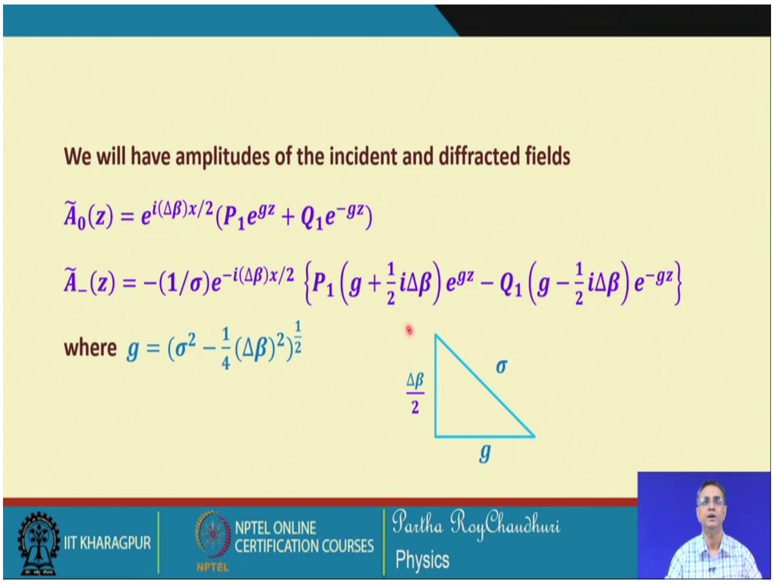

And we can solve this equation in the similar way which will yield that the amplitude of the 0th order, amplitude of the incident beam incident optical beam in this form P 1 and Q 1 are constants.

And in the diffracted beam this will be the a field amplitude. And this g which has appeared here is this detuning factor, sigma square minus delta beta by 2 whole square under root of that. So, g square equal to square of this; so, this when z equal to 0, then it becomes the maximum.

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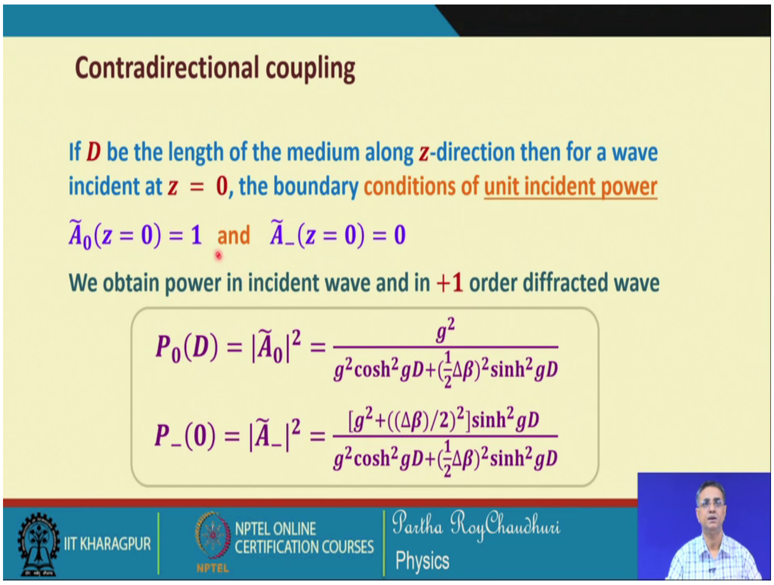

If D be the length of the medium that is the length of the width of the acoustic know the length of the acoustic wave propagation through which this optical beam, travels that is z equal to; and we apply the unit amplitude condition here as well, then we can see that this power in the initial incident beam will be represented by this as a function of function of D and power in the diffracted beam as a function of this should also be is there.

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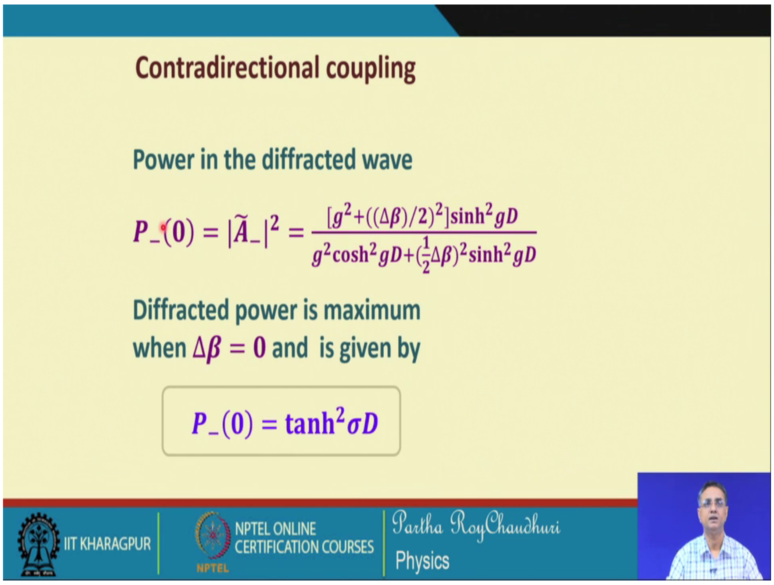

So, therefore, these 2 differences of these 2 will be equal to 0. This can be very nicely represented by this. And from here P 0, we can diffracted delta beta equal to if you put equal to 0, then you can show that this P 0, the diffracted beam in the 0 means z equal to 0, that is in the reverse direction at the point where the incident beam is the beam is a optical beam is incident at the acoustic wave. So, that gives you this very well-known equation tan hyperbolic square kappa in sigma into D.

So, this is the variation of power in the minus order diffracted beam.

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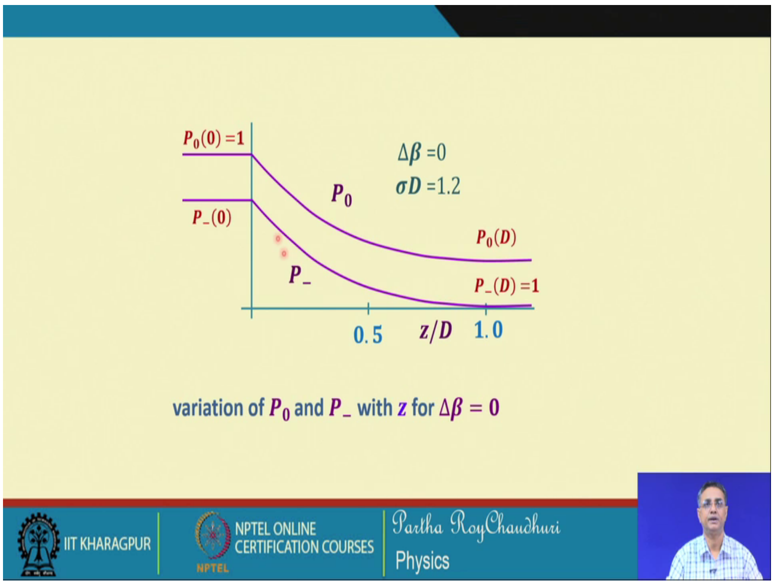

And this is a quite well known, you see you have P 0 of 0, that is that is at D equal to 0, that is at z equal to 0 is equal to unit power and P 0 p minus order power at z equal to 0 is somewhere here; which is initially it is 0, but because of the because of the diffraction coupling we get this has this value. So, it reduces the power in the incident beam that is reducing, and the power in the diffracted beam is increasing, and this is the consequence of this equation.

So, therefore, the power variation of P 0and so, at any point at any point you will see that the total power the difference of this power that maintains the conservation of energy; which will be coming from this equation. So, A 0 square this will this is now P 0 and this is your P minus; so, the difference of these as a function of z that will be in constant which will give you the conservation of energy.

So, these equations are very well-known familiar. These are also used in various waveguide coupling distributed coupling. For example, so, let us complete this for example, D tends to infinity, then minus order it becomes 1. That is what I want to mean that if you allow this scrub coupling for a long length, then there will be 100 percent Bragg reflection. So, we can treat this as if the wave was trying to travel across the acoustic wave, and entire power will be reflected back if you allow the interaction length to be very large.

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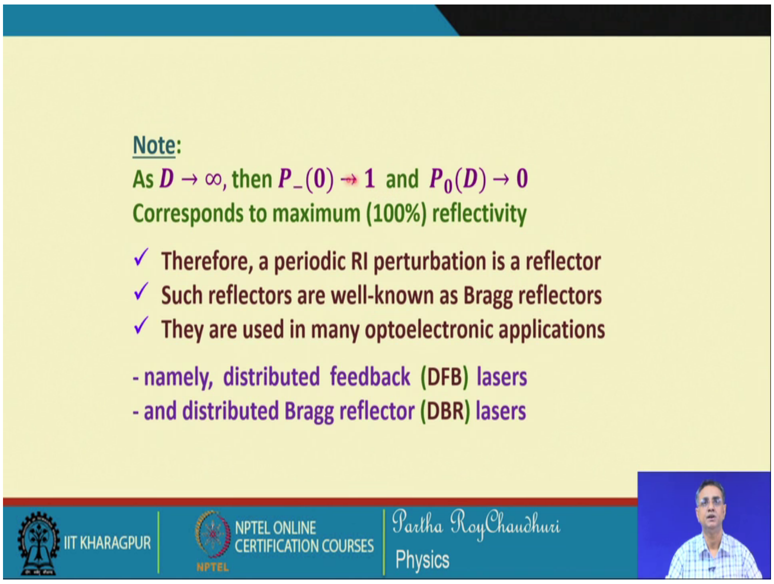

So, D tends to infinity, P minus will become 1, and P 0 will become 0; because entire power from the incident beam will be transferred to the minus order diffracted beam. This corresponds to the maximum reflectivity. And this property is used in various applications in up to electronic devices. So, we conclude that a periodic refractive index perturbation is a reflector and is a is a reflector; where we do not use any mirror, but it is the periodic variation of the periodic variation of the refractive indices in the medium. And such reflectors are very well known as Bragg reflectors, they are found in many optoelectronic applications; particularly, this distributed feedback lasers which are called DFB lasers and distributed Bragg reflector lasers DBR lasers.

But more interestingly this power variation in the contradirectional coupling, and this equation they are even more popular and familiar for this fiber Bragg grating analysis. And there also it happens in the same way you have a piece of fiber; where you create a periodic perturbation along the length of the fiber, and if the light which is trying to travel within the fiber as a guided mode will be reflected back entirely under this condition, but that will happen at the Bragg condition which will correspond to a particular width of particular wavelength with a small width with a very high reflectivity.

So, this Bragg fiber is also used as a reflector. So, these are there are various applications in optoelectronic devices and we will continue.

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So, today we discuss this coupled wave equation with large Bragg angle diffraction, condition for coupling. We also analyze this codirectional and contradirectional coupling, power transfer, efficiency and application as Bragg reflector.

Thank you very much.