Modern Optics
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Lecture - 50
Acousto-optic Effect (Contd.)

So, we looked at this small Bragg angle diffraction in terms of the basic set of equations representing the power transfer, periodic exchange of power.

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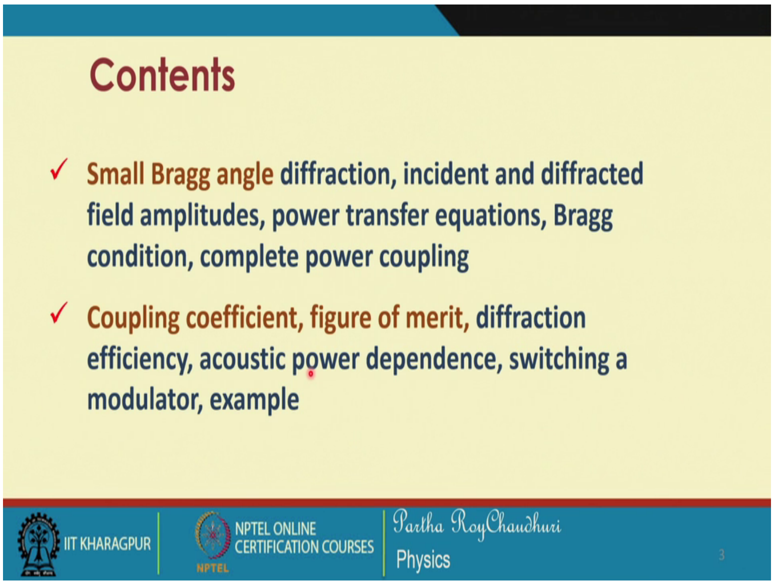

We will continue with that, and we will be discussing this small Bragg angle diffraction in terms of the power transfer equation for specific cases. Then we will look at the Bragg condition in this case and the condition for which there will be a complete transfer of power from the 0th order to the first order or vice versa from first order to the 0th order. We will see that this is a periodic exchange of power depending on the length of interaction and the factors those are going into the coupling constant.

And we will discuss this coupling coefficient, then the figure of merit for [vocalised-noise] very good coupling the high performance coupling, particularly which is very useful for modulation of the incident beam. Then the diffraction efficiency acoustic power dependence switching in the switching of a modulator and some example, numerical some numbers, to have a feel of what would be the parameters for a modulator.

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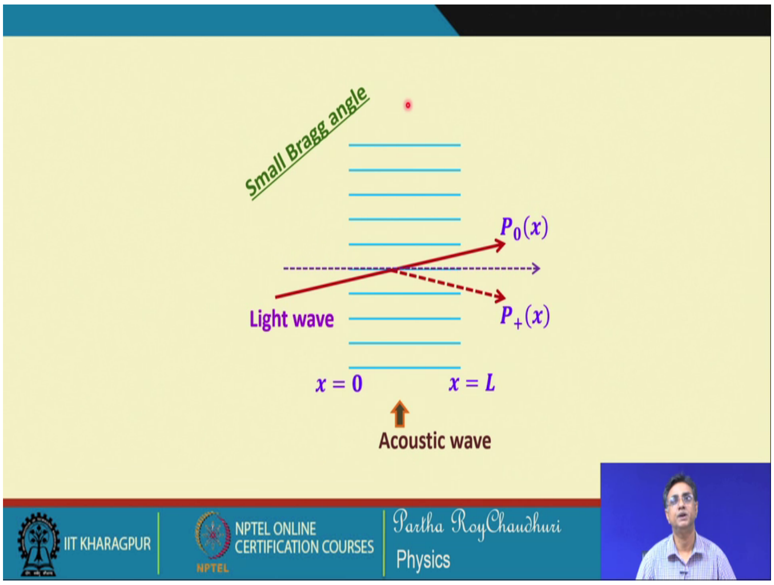

So, we will continue with this basic set of equations for small Bragg angle, let us recall that there was an incident light which was almost normal to the direction of the acoustic wave which is propagating along the z direction and the light wave is travelling along the close to the x axis, along the direction of x axis. And there will be a diffraction which is which we will call this plus order. So, this is your undiffracted beam so, there is a coupling between this undiffracted that is 0th order to the to the plus order diffracted beam.

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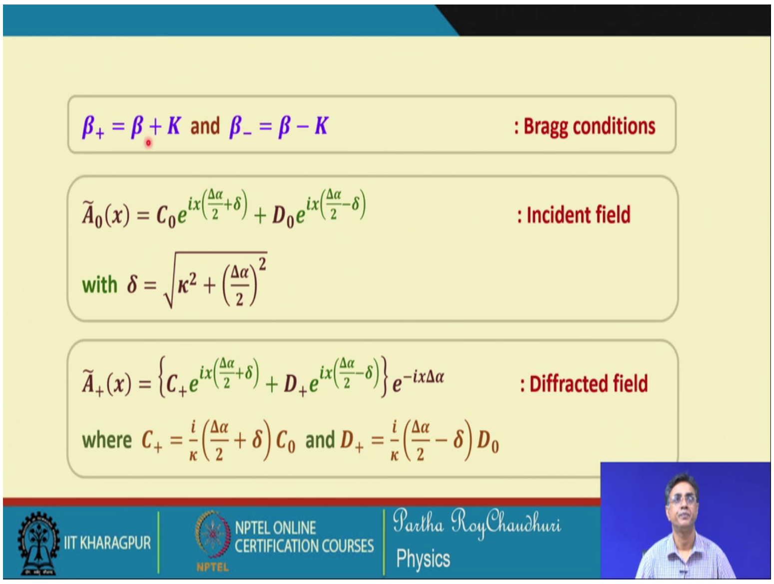

So, we also remember that this beta plus for the plus order diffraction was given by beta plus equal to beta plus K beta minus. We will see, we will try to look at this vectorially also we will try to understand that why it happens because, you have a propagation vector which is directed in this way, but finally, after diffraction this propagation vector of the light wave is in this direction.

So, there has been a change in the in the propagation vector z component as well which is the effect of this. So, we will look at the, conservation property then the small k beta and sorry this z component of the propagation vector for the incident wave that is beta beta plus and this k vector for the acoustic wave this phonon vector representing the phonon [vocalised-noise] that will be connected put together to get the equation. This we will we will try to understand.

Now, in this equation we left here that A 0 of x amplitude of the undiffracted of the incident beam which is given by this where delta was equal to a Kappa square plus delta alpha by 2 square. Under root of that A plus was given by a similar equation C 0 was, here C plus, D 0 is here D plus and then you have a factor of e to the power of ix delta alpha. So, C plus this we have seen that can be represented knowing the constant C 0 and D 0 they are represented by this.

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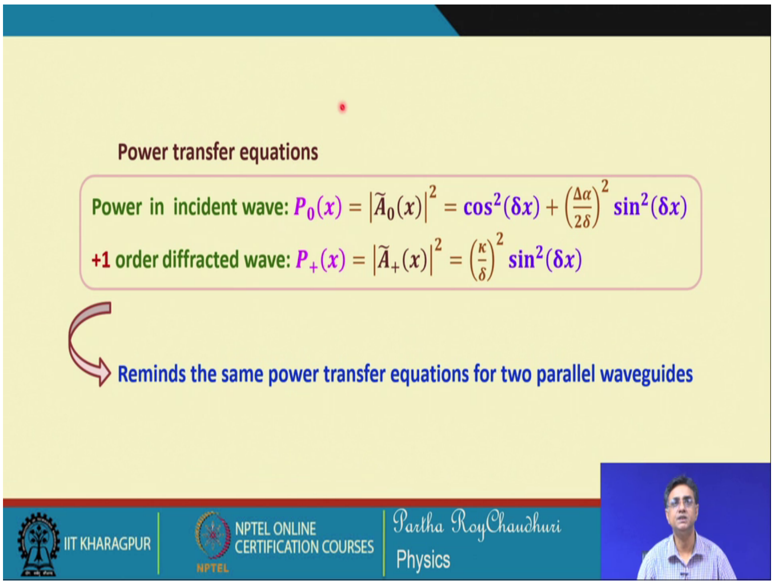

So, and using that boundary condition for the unit incident field we calculated the power that is a power in the 0th order beam, 0th order wave and power in the plus order wave. So, that was cosine square sine square variation

and this reminds we mentioned that it is the power transfer periodic exchange of a power various power between the two parallel wave guides and that also we understood from the from the consideration of the couple mode theory.

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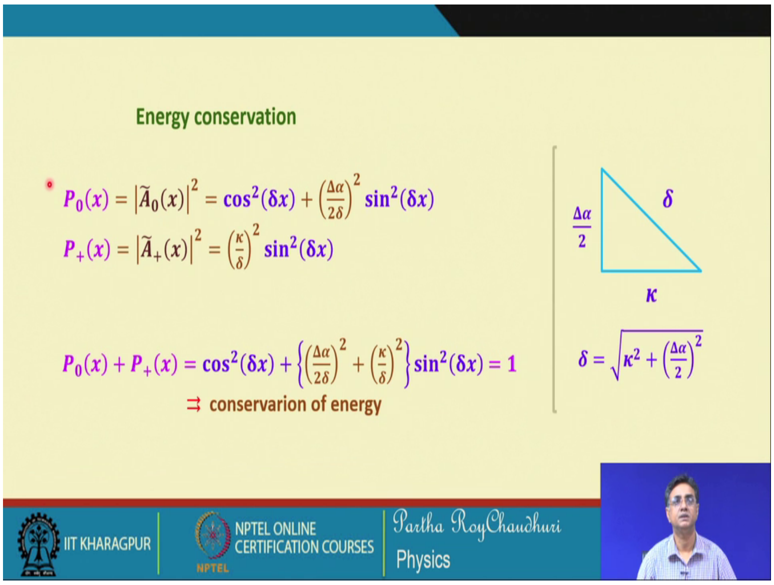

So, this is the power transfer equation and for energy conservation you see this plus this put together or it should be equal to 1, and that is what is happening here also and that tells you the conservation of energy. This can you can understand that delta alpha by 2 this quantity will be equal to you see delta alpha by 2, 2 delta so, this quantity will be equal. So, therefore, this and this put together is again equal to one this quantity is equal to 1 so, that is why you get this total power equal to 1 and we started off with the unit power in the incident, incident field.

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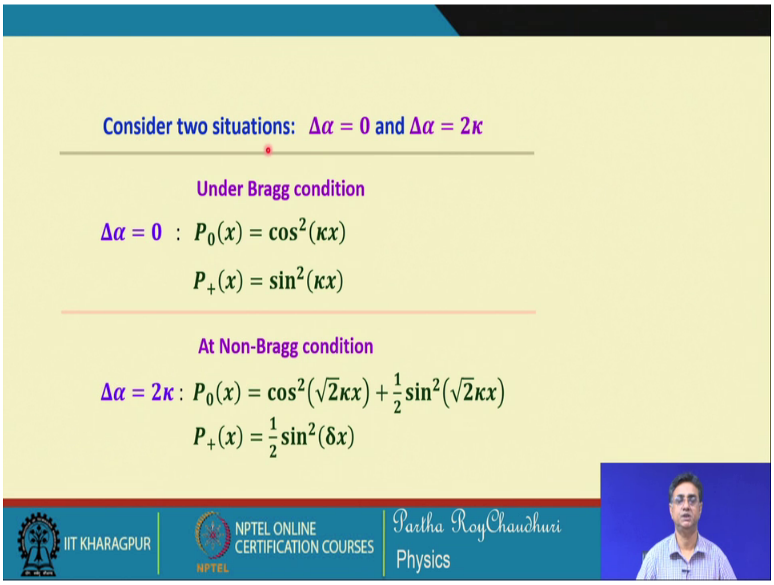

Now, let us consider the two situations that delta alpha equal to 0, we just want to see that this will correspond to the complete transfer of power, but if this delta alpha is not equal to 0 that is we take a very convenient value of that to see that this power transfer is not complete. You can see that this is the factor delta alpha, when alpha plus and alpha that is [vocalised-noise] the x component of the propagation vector of the incident beam that is alpha and x component of the propagation vector of the diffracted beam plus order that is alpha plus.

If these two they differ by a small amount that is they are almost close to each other then this is this delta alpha will correspond to a value which is equal to 0, in that case delta and Kappa will be equal. That is in that case the power transfer will be complete, the power transfer will be complete. So, you can see from here that when delta alpha equal to 0, if you put it into this equation this becomes 0 and this Kappa and delta they become equal. So, Kappa equal to delta, so this is equal to 1 as a result you get only cosine square Kappa x because, Kappa and delta has now become equal if delta alpha equal to 0. Therefore, you get this equation P 0 x equal to cosine square Kappa x P plus is equal to sine square Kappa x.

And this form of the power variation is already known it gives you complete exchange of power between these two, these two beams are representing the 0th order and the plus order. But and this is what is called the Bragg condition that is delta alpha equal to 0, we will see that if delta alpha equal to 0, we will have other consequences also and that is your Bragg condition.

At Non-Bragg condition, let us suppose that delta alpha is equal to this value we choose this value just to see that what will be the power variation, how much fraction of power that will undergo the periodic exchange. So, if we put this delta alpha equal to twice Kappa, delta alpha equal to this is equal to twice Kappa. So, this is again Kappa square, this is also Kappa square so, this will be under root Kappa which is equal to delta that is what we have written under root Kappa is for delta.

And this is also the same this should be delta equal to under root Kappa into x. So, therefore, you can see this they look at this two equation this is cosine square, this is half sine square, and if you if you express this as 1 minus cosine square then this half and half they will go together and you will get a variation.

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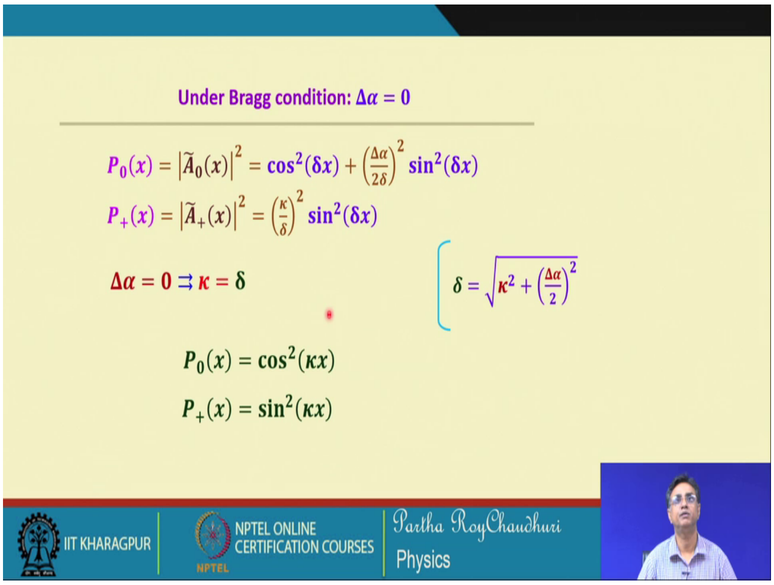

So, this is this is this is the kind of power variation, we will see we will look at the power variation curves to understand that how much fraction of power will be can be at the best couple to the diffracted way from the incident wave.

So, under this Bragg condition this delta alpha equal to 0, Kappa equal to delta it gives you this value.

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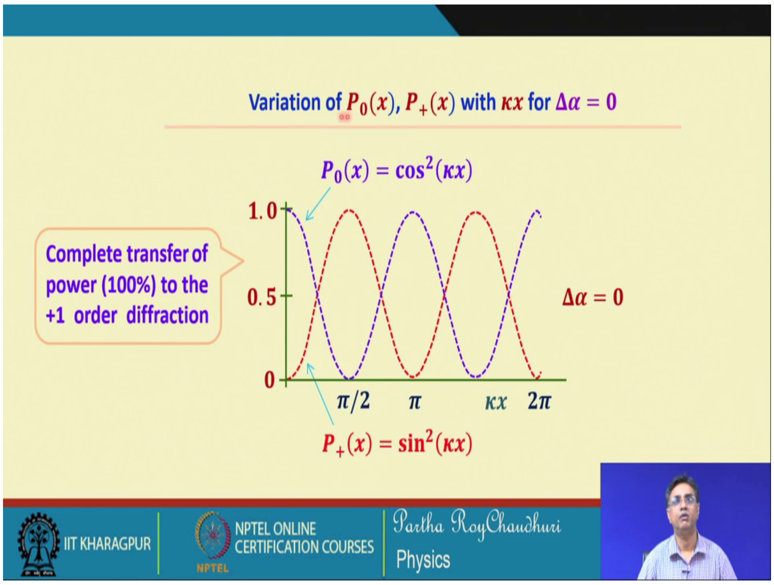

Power variation, so again we consider this Bragg condition under this Bragg condition delta alpha equal to 0, Kappa equal to delta we get this equation, and the power variation is represented by this between the diffracted and 0th order plus order and 0th order beam.

So, here we start with the when there is no perturbation, when there is an acoustic field the maximum power will be here that is P 0 x. And then with the perturbation as you increase the value of Kappa x then it falls and the power in the in the diffracted order it increases, then it goes to the maximum when it has become minimum. And this is how when the power in the diffracted order will be maximum there will be no power in the case of ideal situation, and no power in the 0th order and when there is the maximum power in the 0th order the diffracted order will have no power.

And this will happen at these values of Kappa x pi by 2, because you can see if you put pi by 2, here if you put pi twice pi so, these are the, so this is the purely Bragg condition and delta alpha equal to 0. And you can see that complete transfer of power 100 percent to the first order diffraction is possible if the diffraction is under the Bragg condition.

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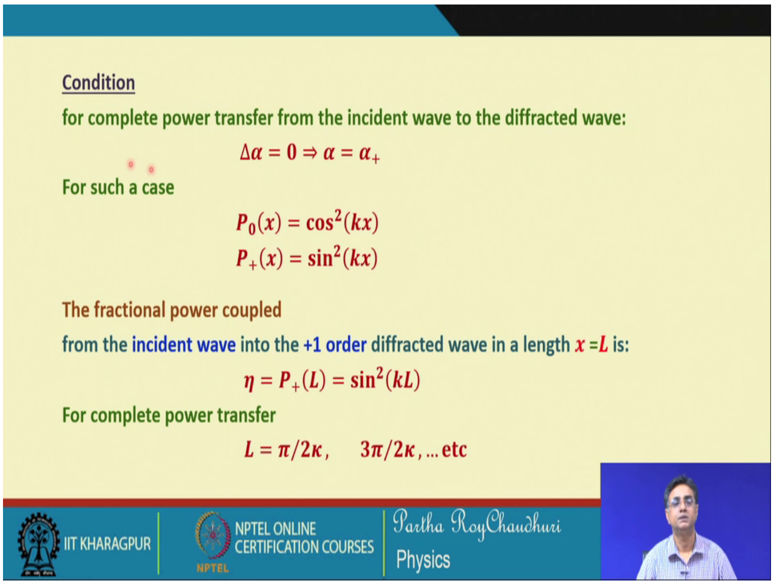

This condition to complete this condition is alpha will be approximately will be very close to alpha plus, but in the for the sake of this calculation this alpha is equal to alpha plus, if you consider you get this equation. The fractional power that is coupled to the plus order is given by this, this is the expression that we will be using for the modulator purpose and that is what we will call the diffraction efficiency. So, P plus will have sine square of kappa L this is the fractional power which will be coupled to this.

For complete power transfer these are the length of interaction the values of x, that is values of x because, x is the length across the acoustic wave. So, the interaction length L that that comes here, so twice pi a pi by twice Kappa is equal to L and this condition this condition and similarly those points we will get the maximum transfer of power.

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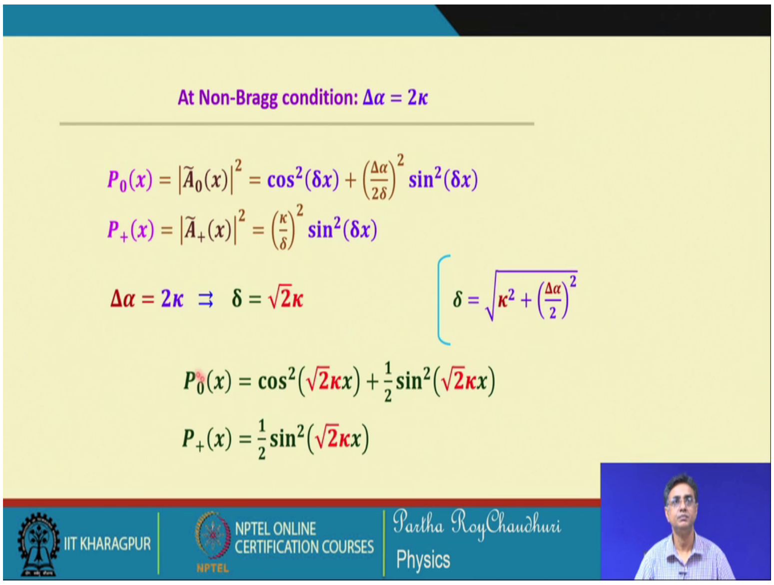

At Non-Bragg condition when we have substituted delta alpha equal to twice Kappa then you can see that the power variation will be given by this equation we have seen and now we look at the various power variation here the maximum power.

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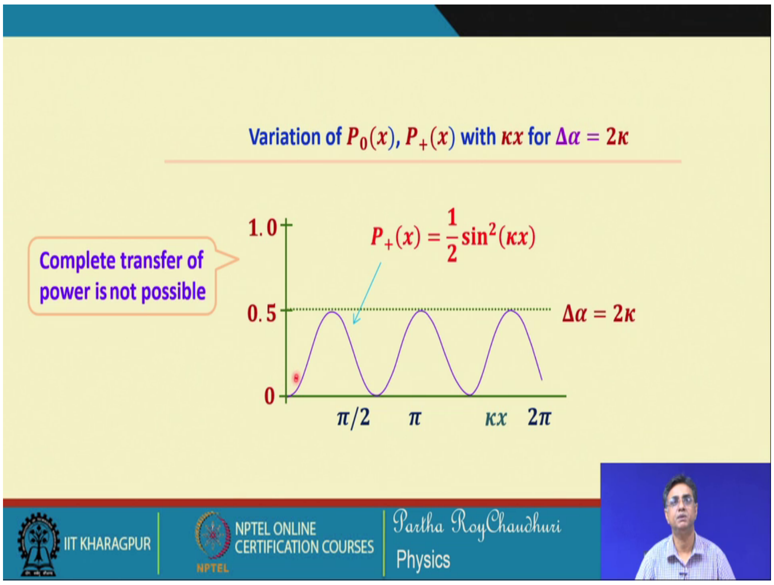

So, this corresponds to plus order power it cannot go beyond 50 percent, you can see plus off, so this is half sine square kx which is given by this equation.

So, the maximum value of this can assume one therefore, this is sine square so, maximum value is one. So, you can get maximum 50 percent power here that that is that complete transfer of power is not possible for this situation whereas, delta lambda is equal to twice k.

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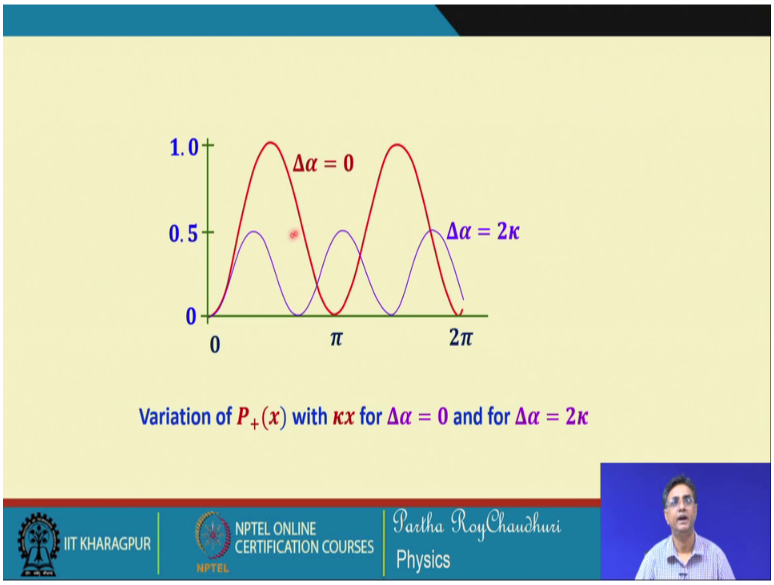

And in that case the P you have the power variation in the 0th order that can go to this is the power variation for delta lambda delta alpha equal to 0, and this is the power variation for delta alpha equal to twice k. So, you can see the difference the maximum power transfer is possible only at the Bragg condition, but at any non-Bragg condition other than delta alpha equal to 0 the power variation is incomplete.

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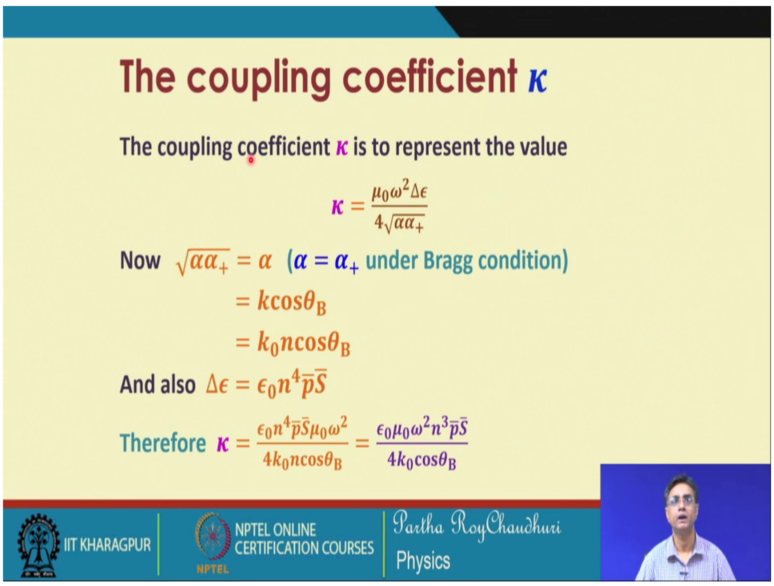

So, this coupling coefficient Kappa we have to look at the basic equation from where we actually started, let us see that the basic equation where we define this coupling coefficient Kappa the value of Kappa. So, the value of Kappa that will be given in terms of in terms of mu naught omega square delta lambda actually that they need to write down that basic set of the equations where Kappa is will be given by this equation this expression mu naught omega square delta e by 4 alpha plus.

This under Bragg condition alpha is equal to alpha plus and then we can write this is equal to alpha, coupling coefficient under Bragg condition it will be given by this, but alpha equal to k cosine beta. We will see that if under Bragg condition this is the incident wave the angle made by this incident wave is theta B.

So, this is the Bragg angle at which the power transfer is maximum at which the alpha is equal to alpha plus then we call that the x component of the propagation vector will be given by cosine k cosine beta this is equal to alpha. And k 0 n cosine beta because k is equal to k 0 into n, n of this medium is this and also delta lambda this we have seen is equal to epsilon 0 n to the power of 4 p and S therefore, Kappa is equal to is given by this expression.

And you can see that here this mu naught and epsilon naught these two put together will give you 1 by c square, there is one omega square. So, that will correspond to k 0 square and there is another k 0 here.

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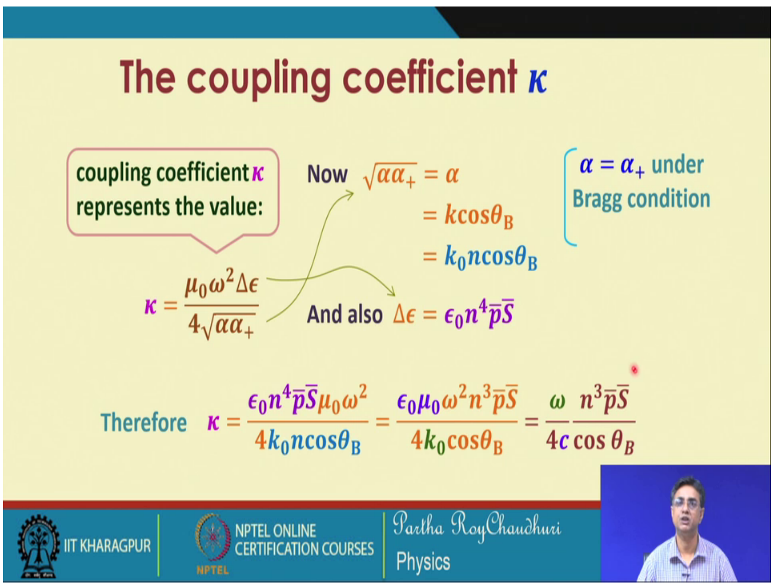

So, we will see that utilizing this we can write down this Kappa coupling coefficient where the exactly mu naught, epsilon naught and omega square that is also equal to correspond to k 0, but this k 0 k 0 cancels you can write omega by c.

So, this is the form of the coupling coefficient which is used where cosine beta cosine theta B, theta B is the Bragg angle. The x component of if this is the angle of incidence then the cosine theta of this angle small Bragg angle that gives you the that is the propagation x component of the propagation constant and that appears in this.

So, in terms of the Bragg angle, in terms of the refractive index of the medium and part of refractive index and effective strain element and the strain optic element in terms of that one can represent the coupling coefficient for the coupling of wave from the 0th order to the first order or from the first order to the 0th order and vice versa.

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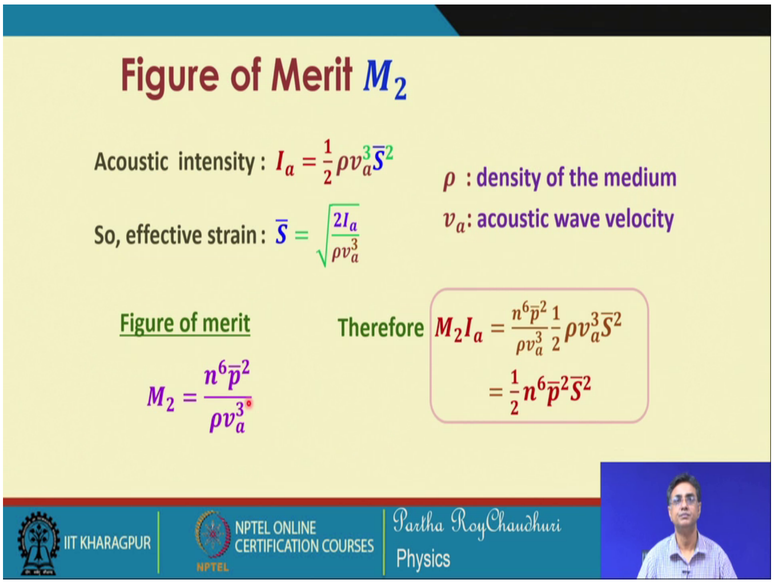

The figure of merit which is usually defined in this way that the intensity of the acoustic wave is given by I a half rho and this is again known where rho is the density of the medium and this is the acoustic wave velocity which gets into this and this is the effective strain a square of that so, that is the acoustic intensity.

In terms of the effective strain we can revert back we can write this equation this expression in this form twice I a rho v a cube, that is again very useful because, we will have the coupling coefficient in terms of this, you can remember this in terms of this effective strain so, that is really useful. Now, we define this is a very useful quantity the figure of merit M 2 that is defined in this way that n to the power of 6, p bar square by rho v a cube.

So, this now here in this place I will use this expression rho v a cube is equal to twice a by s square, if I use this here you can see from you can see here that M 2 into I a if I multiply then you can get this equation. Once again in place of rho v a cube acoustic velocity cube if I can use this as equal to twice a by s bar square. So, that if I substitute here if into this equation and this I, I take it to this side then we can get this equation or otherwise if we just multiply I a into M 2 we can write down this equation. And that is very useful because this n cube p S p bar and S bar they appear in the n cube p bar S bar they appear in the coupling coefficient this quantity.

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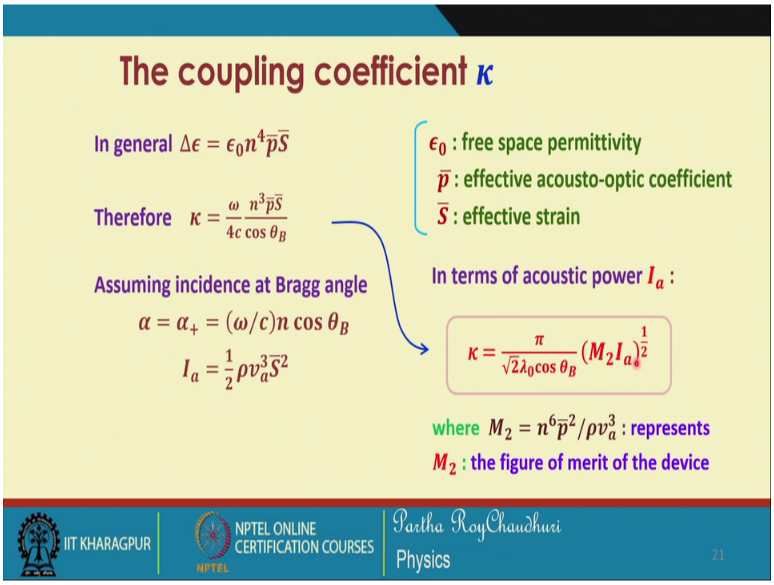

So, twice of M 2 I a under root of that will represent the a part of the quantities appearing in the coupling coefficient so, that is the intention. So, in general we have delta epsilon equal to this we are trying to express the coupling coefficient with the known quantities therefore, we can write this in this equation we have seen. These are the quantities which we have already defined, we have talked about this then assuming the incidence at Bragg angle we have seen that omega by c n equal to alpha equal to alpha plus we can write this acoustic power in this form.

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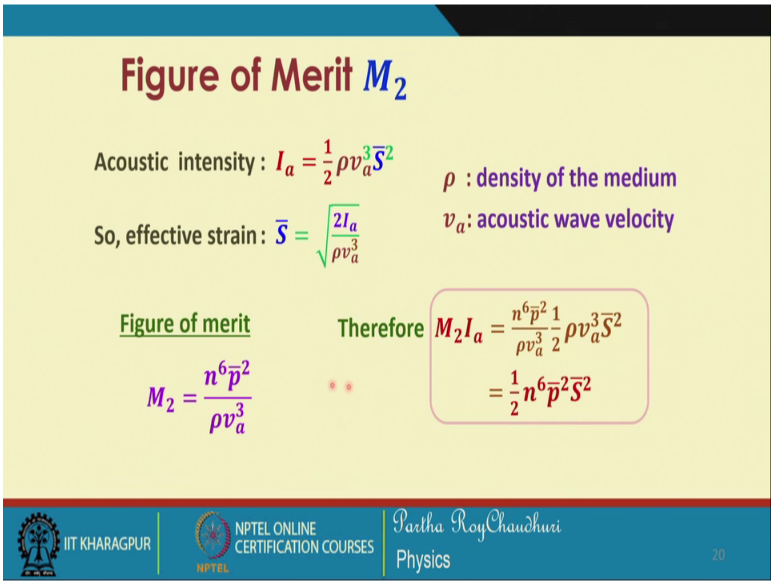

Now, from here we can write this equation, you see M 2 into I under root of that M 2 into I under root of that is equal to you know twice M 2 into I under root of that will be equal to n cube p and S which will go into this coupling coefficient. Therefore, we can write this coupling coefficient in this form which will be twice pi under root lambda 0 cosine beta and this figure of merit multiplied with the acoustic wave.

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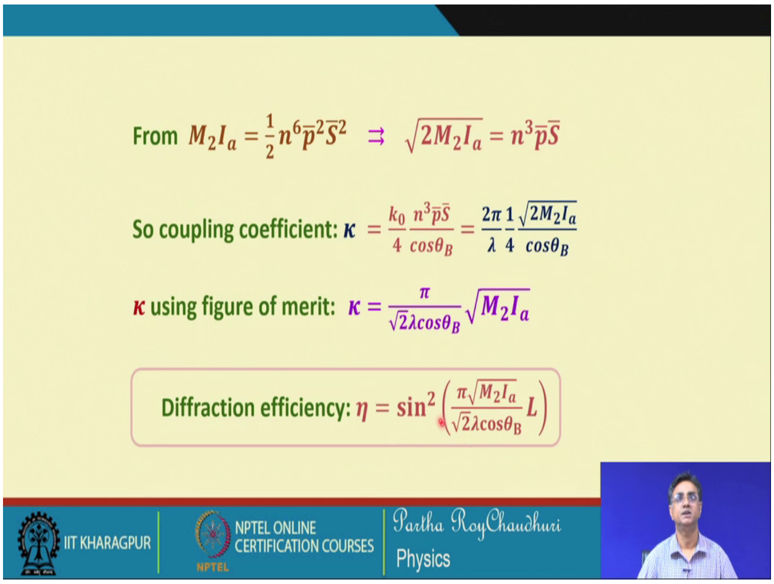

This figure of merit in terms of that we represent this coupling coefficient which is very useful that is why I am talking about this the these numbers for from the design point of view will be useful for. So, using this Kappa we can have this diffraction efficiency in the first order we have defined earlier will be sine square of Kappa into x, L is fixed width of the acoustic wave and for Kappa we write this expression this quantity.

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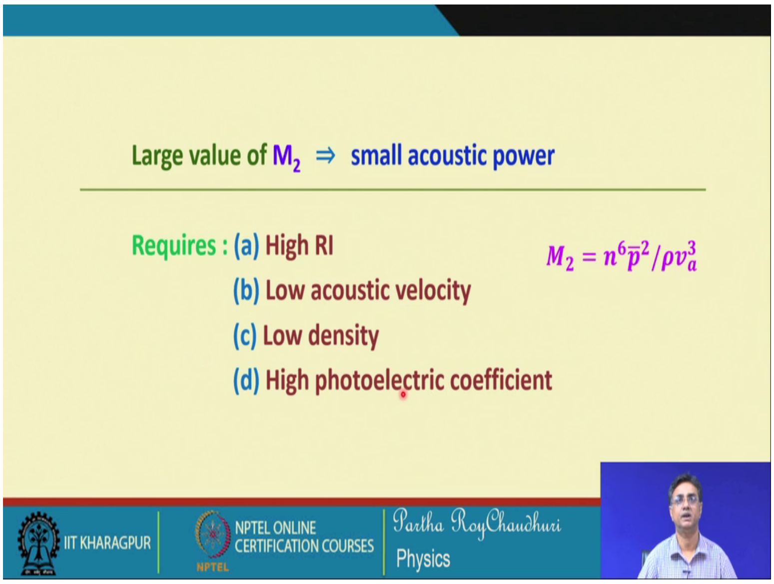

So, that is a very useful relation which will be used for the design of modulators. Now, it tells us from this expression that this figure of merit to be very high large value of it requires a small acoustic power and then high refractive index; refractive index should be fairly large. Low acoustic velocity it appears in the denominator and cube of that, low density, and high photoelectric this should be photoacoustic coefficient, photoacoustic coefficient, photoelastic coefficient this, sorry this is a mistake.

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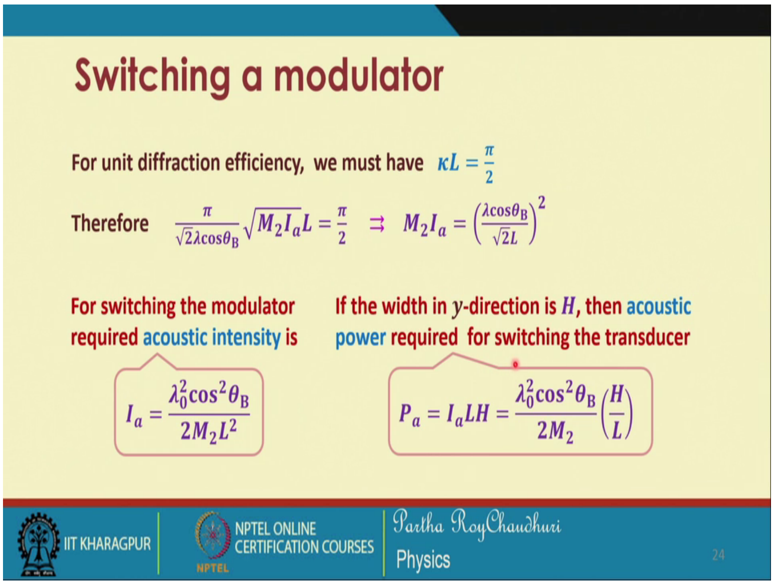

So, for unit diffraction efficiency we must have Kappa into L equal to, we have seen that the diffraction if you go back and just see this power variation offered you for diffraction efficiency to be one. This happens is the minimum value is pi by 2; it could be this also 2 pi by 2, so, these are the position where the diffraction will be maximum to the plus order, and there is no intensity available in the 0th order.

Therefore, using that condition for this diffraction efficiency here, but this quantity we can we can calculate the we can obtain the condition for unit diffraction efficiency we have Kappa in to L equal to pi by 2 and therefore, this must be equal to pi by 2 because this is your Kappa we have defined Kappa into L. So, that tells you M 2 into I by squaring both sides we get this is equal to this.

So, for switching a modulator because this is your acoustic [vocalised-noise] this will tell you like it is an analogous to the situation of the electro optic effect where you where looking for a half voltage, where the modulator will be switched the phase difference will be pi. In this case, the power from the incident wave that is the 0th order wave to be completely transferred to the plus order wave, is defined that is what is the switching of the modulator and that corresponds to pi by 2 in this case because the variation is a sine square variation. So, at this define is the condition for the acoustic power requirement. So, this is the requirement for acoustic intensity and from here we can calculate the acoustic power.

So, there is acoustic intensity comes here because you can bring it back this M 2 figure of merit here that gives you I a LH equal to this so, lambda 0 square cosine square theta B, Bragg angle twice M 2. This is the geometry of the accousto optic cell, L is the width of the acoustic wave and this is the height of this. So, put together the width and the length that put together that gives you that this acoustic power requirement comes from here, acoustic power. So, we can use some numbers to see what is the power requirement for switching the modulator like the case we discussed in the case of electro optic switching where we define this half voltage.

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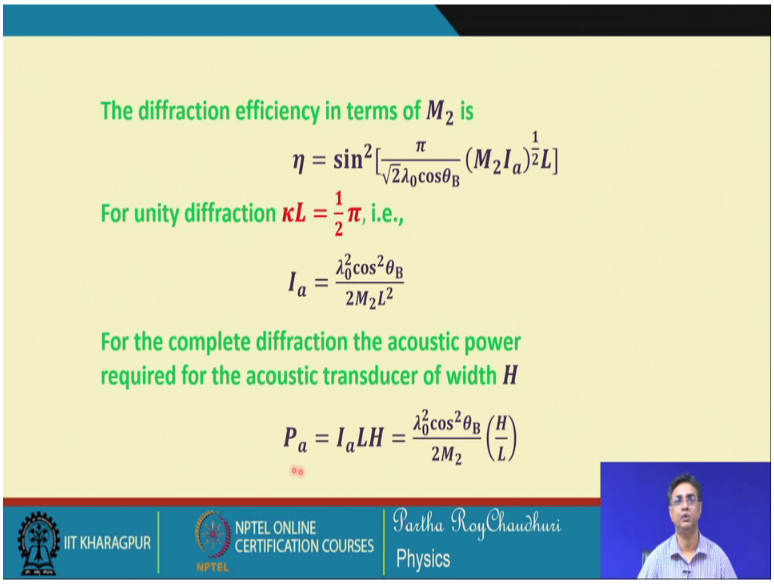

So, the diffraction efficiency for this putting this Kappa into L equal to pi by 2, the intensity requirement is this, for complete diffraction of the acoustic power, required is given by this equation.

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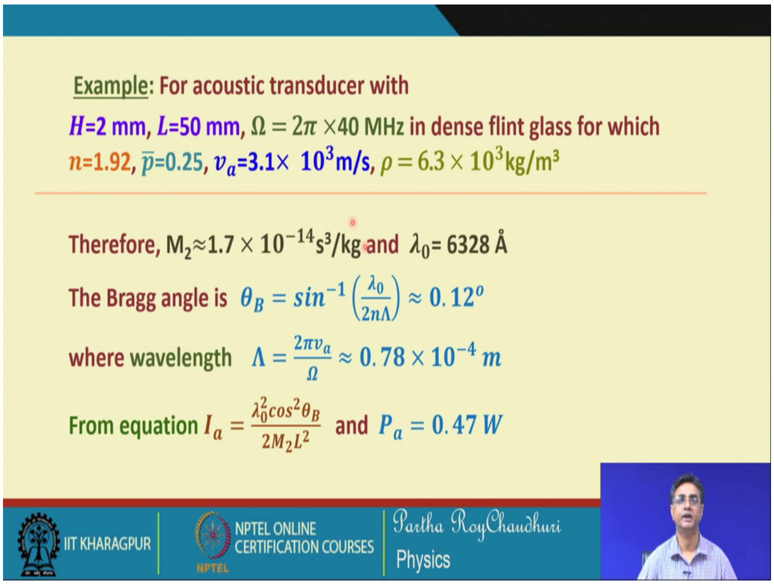

And we can look at the example, one just practical numbers as to have a feel about how this they for switching of this power from the 0th order to the first order what is the requirement. Let us suppose that these are all the numbers H equal to 2 millimeter, length of the that is the width of the acoustic wave, that is length of the accousto optic cell width of the cell is 50 millimeter. And your frequency of the acoustic wave is some 40 megahertz, and the dense flint glass for which this n that is the refractive index of the medium natural normal refractive index is this. And the strain optic tensor element effective value is this velocity of the acoustic wave in such a medium which is given by this and rho is the density of the flint glass medium.

So, with these parameters we can calculate and for a typical value of the figure of merit of let us say 1.7, this is again a practical number 10 to the power of minus 14, and for a wavelength of laser helium neon laser light which is 6328 angstrom, the Bragg angle comes out to be comes out to be approximately 0.12 degree which is very small, so almost grazing angle.

So, this is your very small Bragg angle almost close to the close to x axis and using this value of this Bragg angle pack into this equation where the wavelength of the acoustic wave with this data one can calculate is equal to 0.78 into 10 power minus 4 meter. That is the wavelength, which will be defining the periodicity of the of the grating in the formed in the medium because of the acoustic wave. And from this equation we can from that the equation we have shown here we can calculate this acoustic [vocalised-noise] intensity and from there we can calculate the acoustic power which using these numbers will turn out to be point almost half a watt.

So, by having these numbers we can see that to switch a modulator accousto optic modulator a power requirement of the acoustic power requirement of half a watt is required. And by doing this we can we will see later also that how this configuration can be used for switching for modulating intensity between the 0th order and diffracted order that is the plus order diffraction of the beam we will continue with this.

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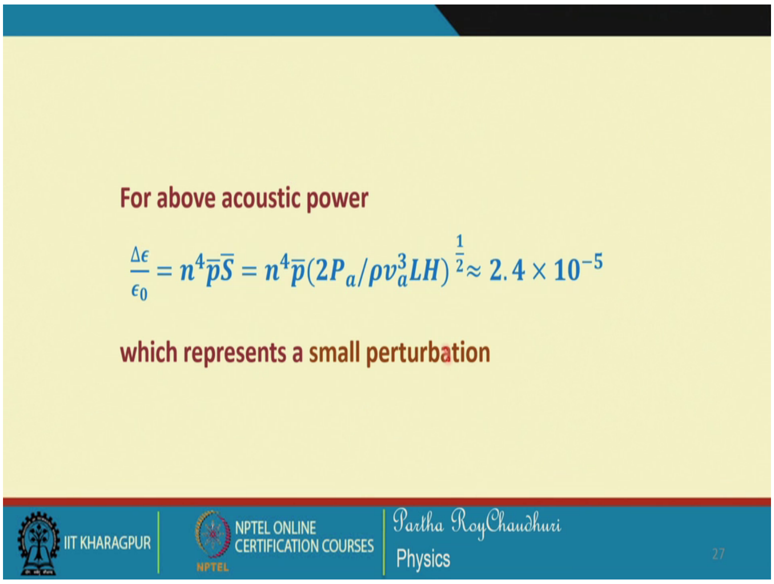

So, today and for this above acoustic power, this is the this is also a part of the of this calculation the delta epsilon by epsilon 0 will turn out to be this. And it requires a and then because to show that this perturbation fractional change in the perturbation is a really very small 2.4 into 10 power minus 5, this is a big change in the refractive index.

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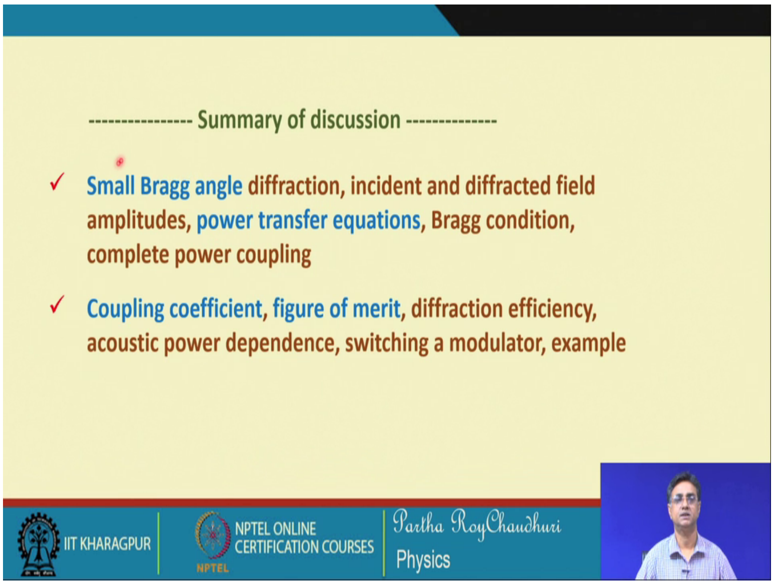

So, we discussed this Small Bragg angle diffraction and looked at the power transfer equations, the condition for complete transfer of power and that is the Bragg condition. We also discussed this non Bragg condition power transfer, then the coupling coefficient including that the figure of merit definition of that. And then we looked at the diffraction efficiency, acoustic power requirement for switching a modulator and we also discussed with a numerical example.

Thank you very much.