Modern Optics
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Lecture - 48
Acousto-optic Effect (Contd.)

We were studying Raman Nath diffraction and we classified the diffraction acousto optic diffraction into 2 parts the where the Raman Nath resign and the Bragg resign. So, in the we studied the Raman Nath resign with the diffraction orders.

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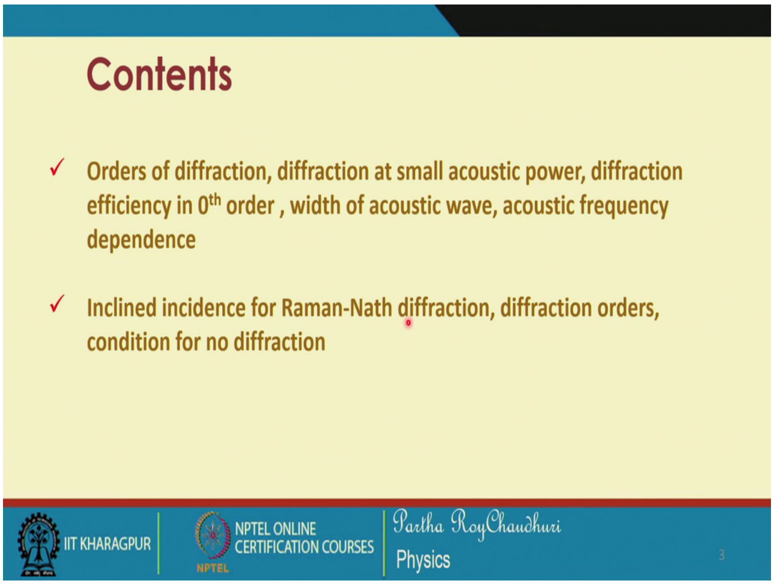

The various orders of diffraction and we will continue with that that for small acoustic power how this efficiency in the 0th order diffraction we have seen. In the case of Raman Nath, it is a multiple order and how it is the 0th order diffraction; that efficiency is very yeah very important because, this entire power has to go to the other orders like plus 1 minus 1 etcetera and how there is a dependence of this acoustic wave on the diffraction and efficiency of diffraction we look at this.

And then also, the frequency dependence of the acoustic wave we will we would like to study. Now, apart from that, there is one more thing, that is a still very important because in all practical situations. It is not that the light wave is incident exactly normally on to the acoustic wave, but let us suppose it is it could be deliberate also.

Let us suppose that, the light wave is incident obliquely with some inclination. Then, how this set of equations they modify and how they this inclination factor goes into the diffraction efficiency. The orders of diffraction they are amplitude, their intensity and the dependence that we would like to study. With this example, when you have a oblique incidence for this acousto optic cell.

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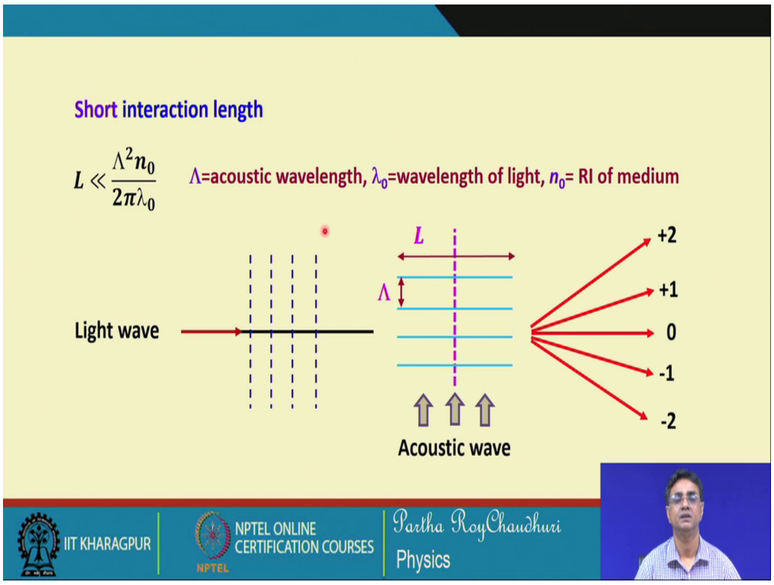

So, we will recall the basic equations that we brought out for this Raman Nath diffraction for because, if this the acoustic wave has a very small width, it is a thin phase grating. In that case, the condition we have seen that, the width of the acoustic wave should be much much less than we will study with a numerical example that with the known values of the wavelength of the acoustic wave, the refractive index of the of the medium and the wavelength of the light wave that will be used.

In terms of this quantity we will also look at we will also check this expression at other occasions also, that it really this quantity if it is greater than if the length of the acoustic the width of the acoustic wave is much much less than this quantity, then it forms a thin phase grating with respect to the light wave that is propagating through it.

And, we will have multiple orders of diffraction. This is the 0th order diffraction which is along the same line and then you have symmetrically located on either side of the 0th order. The first order the plus 1 plus 2 orders like the way the grating forms you have symmetric orders formed about the 0th order.

So, you have this acoustic wave which is travelling along this direction. It forms a periodic grating we have seen this with the peak value of delta n and it is a sinusoidal variation. So, there is a variation in the refractive index along this direction and it moves with a velocity of the acoustic wave, but the light wave the frequency of the light wave is much more than the frequency of the acoustic wave.

So, effectively this light wave we will see almost a stationary grating as if the moment the instant at which it hits the acoustic wave induced grating, it will see a grating which is fixed and then it will move away. By the time the grating moves in this direction the light wave has already interacted and moved out.

So, because the frequency of this is of the order of 10 to the power of 6, 10 to the power of 5, 7 whereas, the frequency of the light wave of the order of 10 to the power of 14 is very high. So, it will see a fixed or standing grating, that is why, this even though the grating is traveling. But, it will see a and we will have this order of diffraction, which will be we will see that will be represented by the various orders of the Bessel function. We have seen in the last discussion and we will continue with that.

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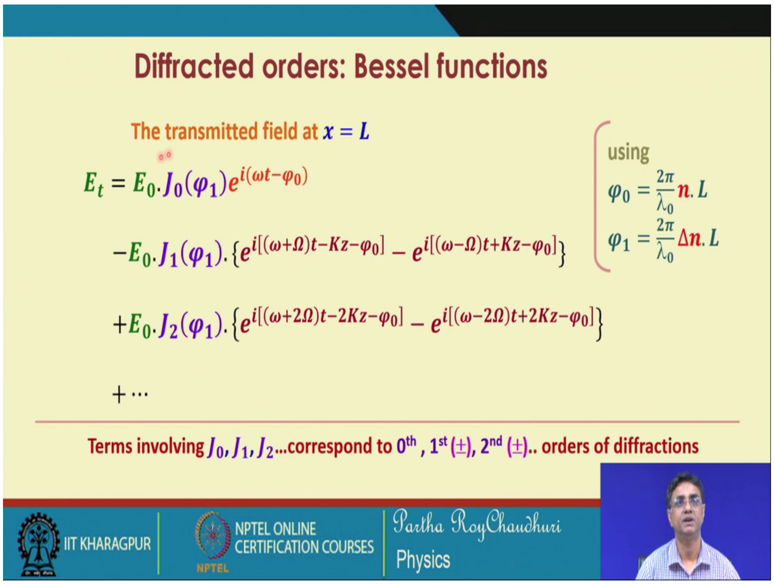

So, the transmitted light field at this end, that is the exit end of the acousto optic cell that at x equal to L. The transmitted light field will be represented by this we have seen this how we get this equation and J 0. This J 0 is to is corresponding to the 0th order J 1 corresponding to the first order, but it will give you the plus 1 order, this will give you the minus 1 order. So, they are very symmetric and very easy to understand. Then, this is the second order, you have twice omega here you have twice omega here this is minus, this is plus.

So, it tells you that this is the second order plus 2 order, this will correspond to the minus 2 order and you see that phi 0 will be the base refractive index. This twice pi by lambda. So, the phase due to phase due to if there is no acoustic wave, this will be the phase of the light, which you will travel through the medium. But, because of the acoustic wave there will be a formation of the grating. So, there will be a local change in the value of the delta n depending on the phase of the sine or cosine function.

So, this phi 1 is actually modulated over this length. So, this phase phi 1 is now modulated. Therefore, we have these various orders of diffraction 0th order, 1st order, 2nd order plus and minus etcetera.

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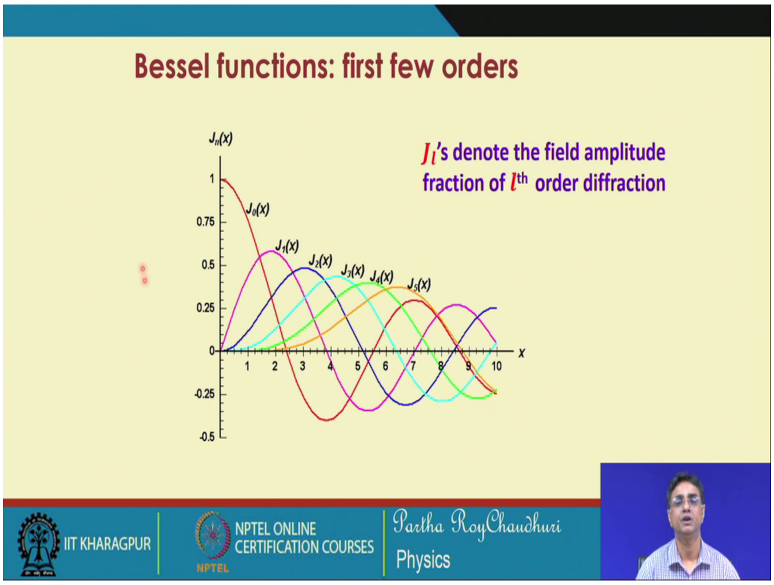

And, if we recall this Bessel function, which are to represent the various orders of diffraction, this is J l's denote the field amplitude fraction of the l th order difference is very interesting. If it is J 0, this will be 0th order. If it is first order plus or minus this will be given by the order one Bessel function, then J 2 Bessel function will be given all of them starts from 0, but the 0th order it starts from the maximum.

So, 0th order it starts from the maximum and as you increase X. X is the is phi 1 you see you can see this X is phi 1 here. So, as you increase X, that is when phi 1 is 0, when phi 1 is 0, if there is no delta n you can see that when phi 1 is 0. Then only, this quantity J 0 X is present and all of them are absent. Because, at phi 1 equal to 0. All the orders of Bessel function is equal to 0. Only phi J 0 is J 0 of phi 1 is non zero and that will carry the maximum amplitude of the electric field.

So, in absence of the acoustic field acoustic wave in the medium, you have the entire amplitude of light concentrated contained in the 0th order of diffraction. And as you move away from this 0 position in terms of increasing value of, the phase phi 1 because delta n changes; as delta n changes, then the value of phi 1 change and you see that you have other orders are appearing other orders are coming into play. When you are here, then you can see that all the orders are here. This order, this order, this order.

So, you have to just draw a vertical line along this direction, a vertical line and the intersection with the various Bessel curves will give you the fraction of the amplitude of the electric field corresponding to that particular order of diffraction in the Raman Nath diffraction case.

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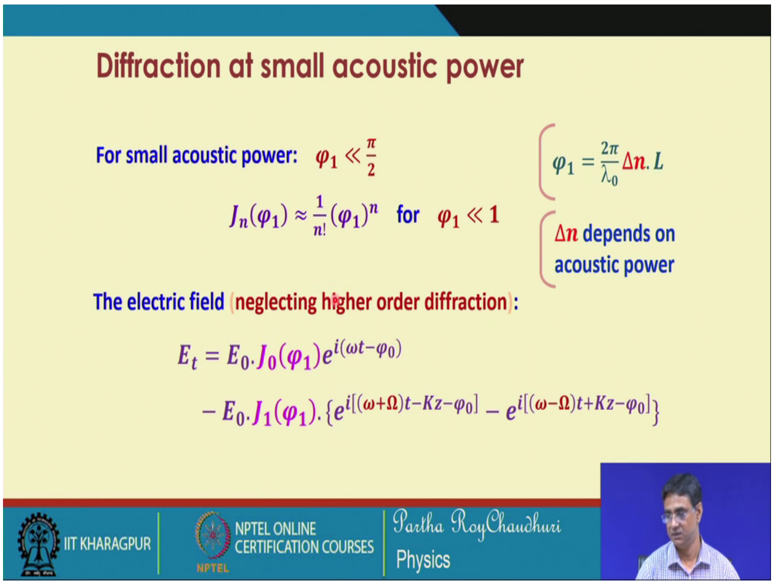

So, for small acoustic power as you have you have seen that this is phi 1. So, phi 1 equal to 1233.14. So, 3.14 corresponds to pi. At this point, it will be somewhere here this is pi and pi by 2 will be half of that 1 point 1.57 or so. It will be somewhere here. So, at this value, when phi is equal to pi by 2 or even less.

So, we want to refer to a situation. When this phi is much phi is less than pi by 2, in that case this is the property of the Bessel function that this Bessel J n the order n. All the n is to represent the order of the Bessel function can be approximately can be approximately given by this 1 by factorial n phi 1 to the power of n. So, for if it is less than 1. So, that is the situation, that is when it is close to 1. So, actually it is 1.57 somewhere here, but if it is much less than that, in terms of the phase that is phi by 2. So, we can represent this Bessel function using this relation.

So, in that case and you can achieve this value phi 1 because you have a control in your hand that is the delta n. So, because delta n it comes through the power, acoustic power. Because, it is the amplitude of the acoustic wave or the intensity of the acoustic energy density per unit length per unit volume the intensity of the acoustic wave that decides this or amplitude of the acoustic wave that decides this delta n value. So, because by changing this acoustic power, we can actually change this phi. So, they are proportional. So, if you bring down the acoustic power for us with a small value, then we can achieve this condition.

And, in that case, we can if we neglect the higher order diffraction terms. So, J 0 will be given this is the transmitted field component we know. So, if we consider only the 0th order and the first order plus 1 and minus 1 order we can write this equation.

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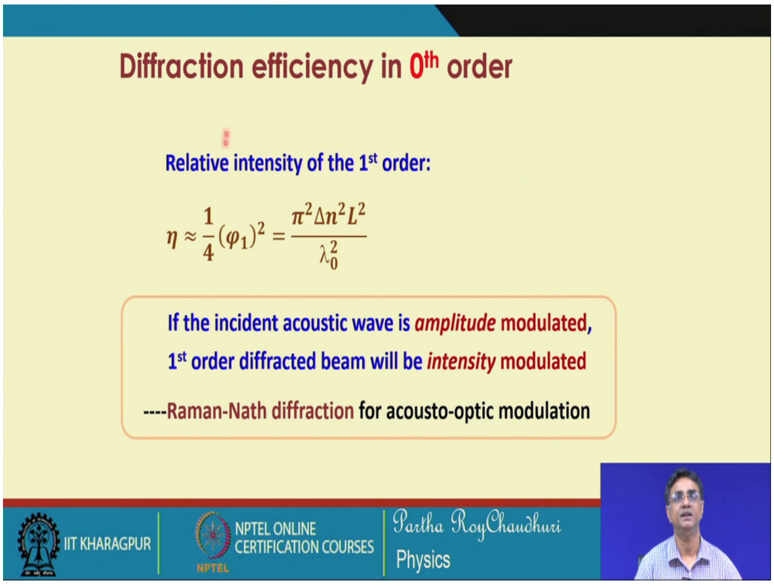

And, in that case, eta the diffraction efficiency for this will be represented by this n equal to 1, n equal to 2. So, we can write in this form. So, and phi is equal to 2 pi lambda. So, phi square will be 4 pi square lambda 0 square n square l. So, that is delta n square l square lambda 0 square and 4, 4 cancels. So, we get an expression for this which will be the relative intensity in the first order diffracted wave in the Raman Nath diffraction.

So, for low acoustic power. So, this is a very important, very useful result that, if you have a low acoustic power of the acoustic wave in the transducer, then you can get a diffraction efficiency which will be given by this knowing the length, knowing the delta n which is a consequence of the acoustic power and this lambda 0 the wavelength of the light wave. We can calculate the efficiency of the this.

So, if the incident acoustic wave you can see that this is amplitude modulated. If it is; if you modulate the amplitude of the acoustic wave, this delta n square this delta n square this modulation will come from this acoustic wave. Because, if you change the value of delta n, the efficiency will also be modulated that is for this.

So, the first order diffracted beam will be intensity modulated because, this phase will be modified and as a result of that, the amplitude the field amplitude will also be modified and square as a square function. So, so, that is how this amplitude modulation of the first order diffracted beam can be implemented by using this result, ok.

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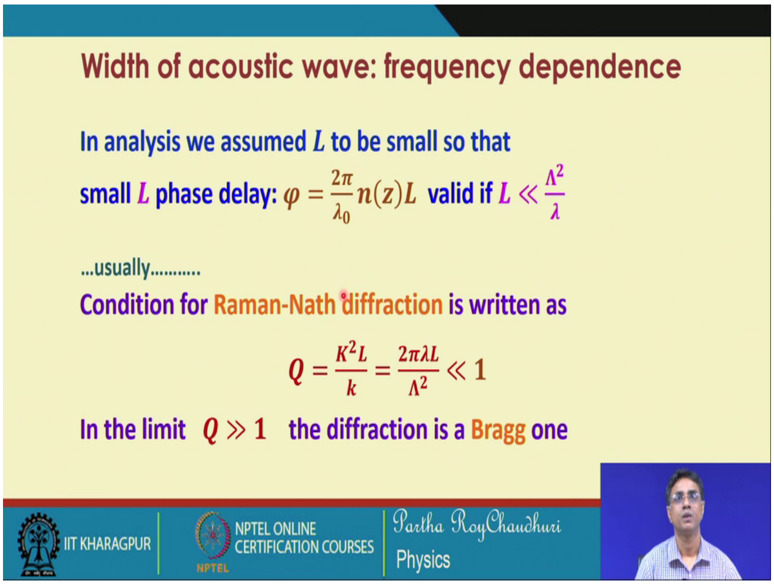

Now, in the analysis we assume that L to be small so that, the small L phase delay. In this case, is equal to this because delta n as a function of z and the length of the width of the acoustic wave is this. So, this much is the accumulated phase by the light which is travelling a length of L, that is along the along the across the width of the acoustic transducer.

So, and this expression will be valid. If L is much much less than lambda square by capital lambda square by lambda, this is the wavelength of the acoustic wave and this is the wavelength of the light and this L the width of the acoustic wave. So, this relation gives you the phase delay. What will be the phase delay?

So, usually the Raman Nath diffraction condition in terms of this is also written as a Q; Q number which is given by K square L by small k and this K square equal to twice pi by lambda square and lambda twice pi will also come from here which will be twice pi by small lambda and this will give you 4 pi square by capital lambda. So, effectively you get this and this quantity will be much much less than 1. And, so that will correspond to Q is less than 1 will correspond to that Raman Nath diffraction and contrary to this fact, if Q is much much greater than 1, then you have a volume grating because width of the acoustic wave is now much more.

So, that the traveling the wave light wave which is going through this medium will see a volume grating. We will see a large interaction length and that will result to the Bragg diffraction. We will study this case in details how this Bragg diffraction takes place with the possibilities of various coupling from the incident wave or the 0th order wave to the coupled wave in the form of diffracted wave. We will consider that situation.

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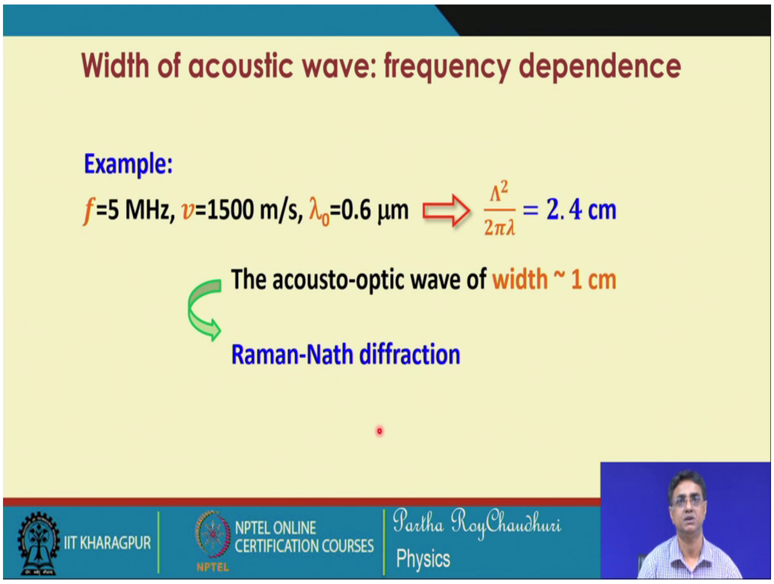

Now, let us consider one example that, if you have a the frequency of the acoustic wave generator or the oscillator, that is the Piezo crystal which is used to generate this acoustic wave which has a frequency of 5 megahertz, the usually the acoustic wave velocity in water or so to a non-polar liquid. The velocities of the order of 1500 meter per second and if we use a light wave of wavelength 0.6, which is close to this wavelength of this helium neon laser light which is 0.6328 micrometer.

So, a wavelength close to 0.6. We have taken a round figure number. So, using this, one can show that this lambda square by twice pi lambda is close to 2.4 centimeter. It is not exactly equal, but it will be close to 2.4 centimeter.

So, that defines the width of the acoustic wave that is the l. So, for this case, I have a frequency acoustic wave frequency which is 5 megahertz. The velocity of sound wave in the acoustic cell in the medium in the liquid which is contained in the acoustic cell is this or light wave which is going to travel through this medium is this. In that case, I define this acoustic wave width equal to 2.4 centimeter.

Now, this quantity is going to divide the 2 classes, the 2 regimes; that is whether it is Raman Nath or it is Bragg diffraction. So, if you are fairly below this, that is if the width is 1 centimeter or so; so, certainly that will correspond to Raman Nath diffraction. So, this is how by taking this example, we will learn how to decide how to calculate what will be the width of the acoustic transducer, that is the crystal and the medium such that, it is very much confined within the Raman Nath resign and if it is more than that, then there will be a divergence of the acoustic beam within the medium. We will take care of that and we will see that how it changes.

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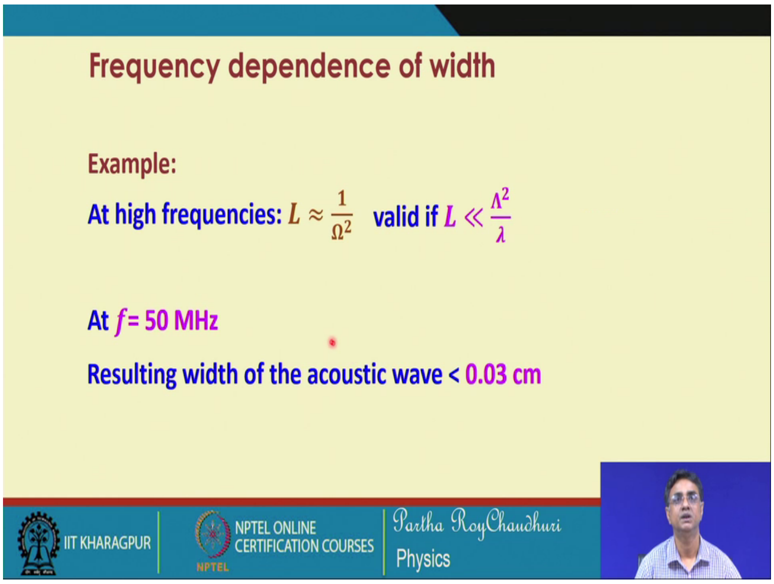

Now, the other. So, this example another example that we also know that at high frequencies, the width of the acoustic wave is inversely proportional to the approximately proportional to the square of the acoustic frequency and this is valid only if this L is much less than this. So, under this condition, at a frequency f equal to 50 megahertz.

Earlier, it was 5 megahertz. Now, you take 50 megahertz, then with if you repeat this calculation, you will see that the resulting width of the acoustic wave is 0.03 center (Refer Time: 19:14) got a drastically brought down. This width that is 2.4 centimeter is now 0.03 centimetre. You have a very thin acoustic wave width which is which is much much less than the width that is normally used for this Raman Nath diffraction 1 centimeter.

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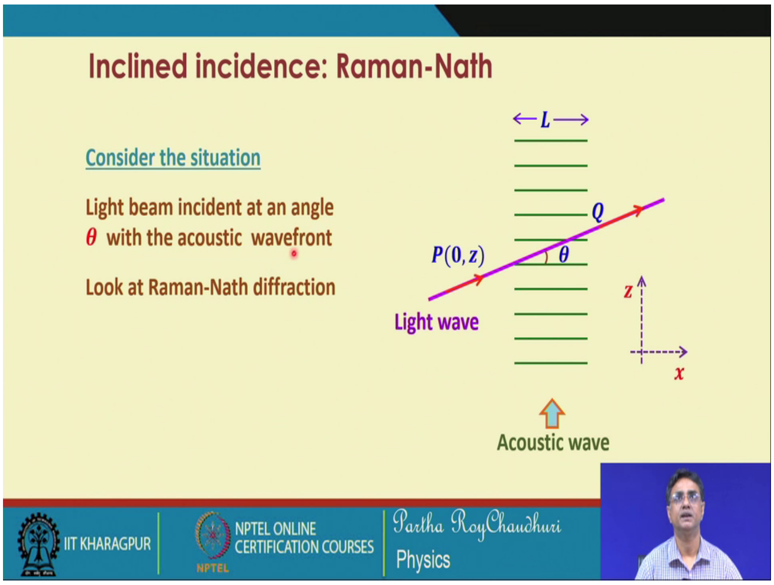

Now, there is another case which we would like to quickly browse through that. This you have a light wave which is inclined which is oblique which has an oblique incidence here. With this acoustic cell, it makes an angle theta.

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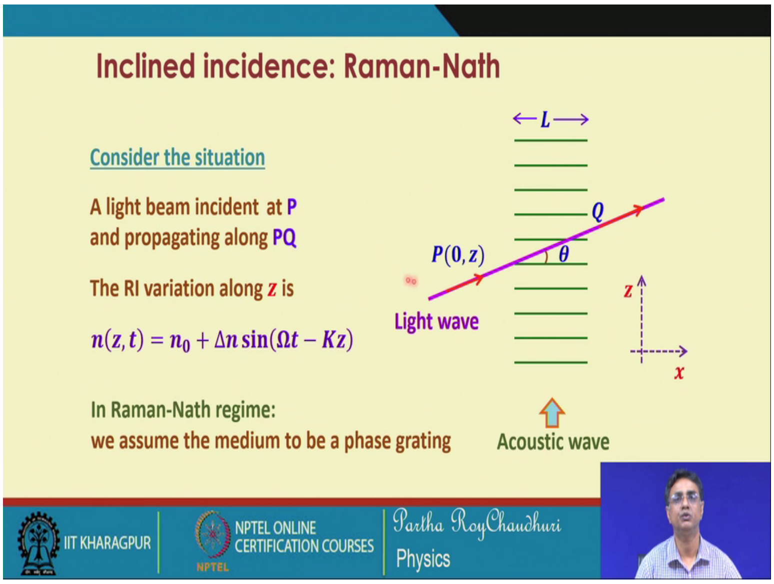

And then, we will look for the Raman Nath diffraction. You see, in this case, you can see that this equation is still valid, but this delta n will now be modified. It is not that delta n remains constant all along this length, but it changes because, you are changing the position in these direction also as well.

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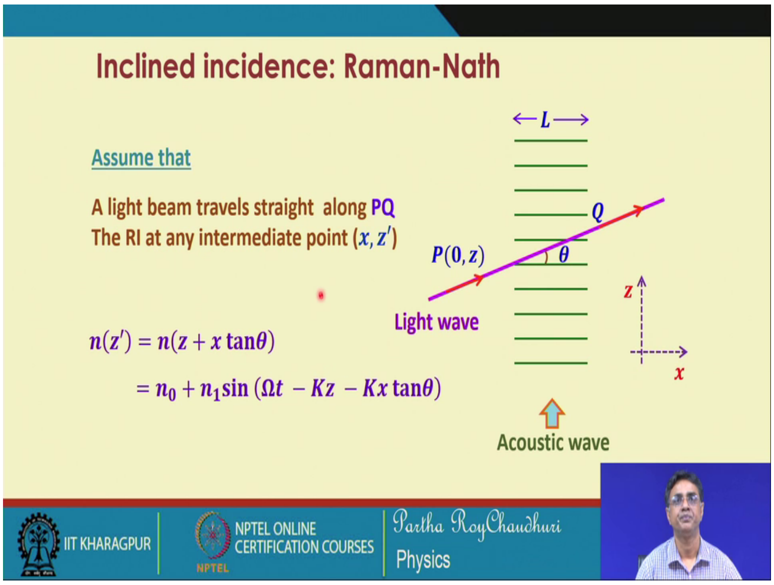

So, and analytically, we can calculate this because, this n is now x, this is delta n will be x tan theta and this x tan theta z n is a function of x as well as delta z. So, this delta z is your x tan theta this quantity. And so, it goes into this and you can rewrite this equation k z minus k z tan theta. So, that is very straightforward. I just replace this z by z plus x tan theta in the basic equation of the refractive index modulation.

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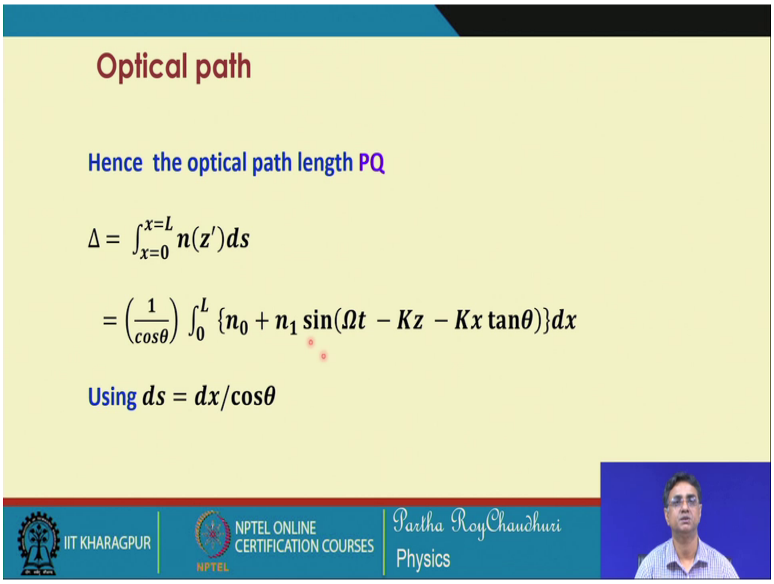

So, the hence, the optical path x equal to L from x equal to 0 to x equal to L the integrated change in the refractive index. And hence, the cumulative phase change you can write in this form n 0 n 1 and then, if you do this integration whereas, d s equal to d x. So, d s d x by d s equal to cosine theta, that is the distance. So, if I use this, I can write in this form x is the variable cosine theta has come out in place of d s. I have used this d s equal to right.

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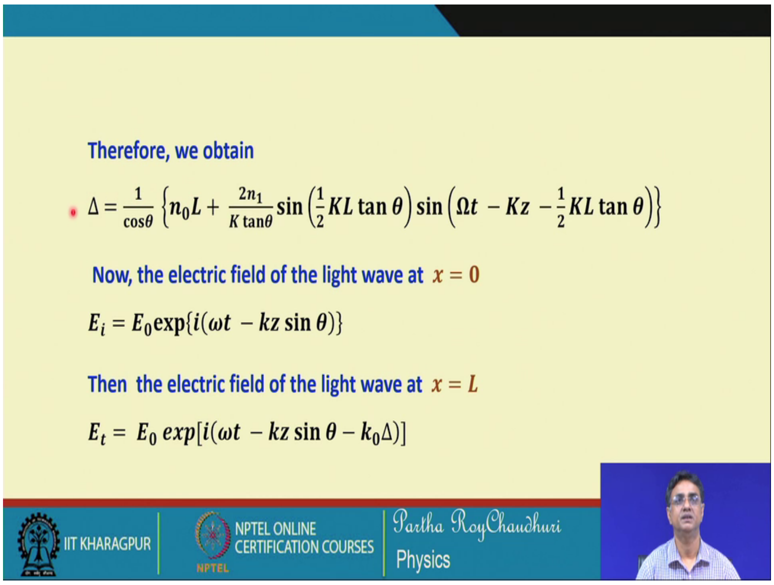

So, I get this delta value that is the change in the phase n L not exactly phase this change in this delta n into this you have to multiply by k 0 also to get the phase change. So, delta n will give you this quantity straight forward from this integration. And now, the electric field of the light wave at x equal to 0 is given by this where this k z sine theta and at x equal to L, the transmitted field will have k 0 multiplied by this k 0 multiplied by this quantity which has come from here.

So, this k contains k 0 into delta k k 0 into n 0 and then, z sine theta and this contains this refractive index delta n as well and the path length. So, this delta k 0, this is the x s of the phase which is coming into play and that is not just straightforward, but it is the cumulative effect because the wave the light wave has moved from here to here.

So, it has seen different refractive indices change in the refractive indices delta n values all along it is length, which is now integrated over when it appears here at the at x equal to L, this wave will have a cumulative change in the x s phase that is given by this is the x s phase.

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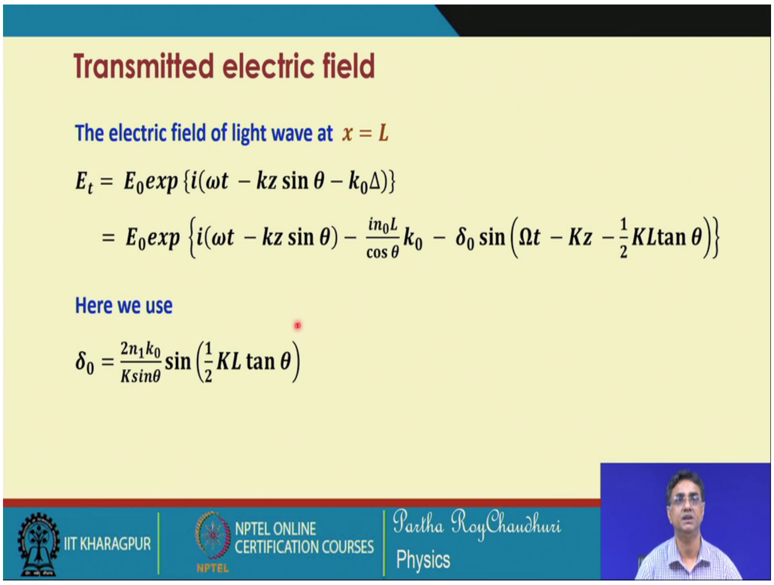

So, we get this equation and you have this delta 0 in place of this we have written in place of this. So, therefore, this delta 0 equal to this quantity because, I just simply put the value of delta into k 0 and then, sine theta. So, put together I get this value of delta 0 equal to this quantity.

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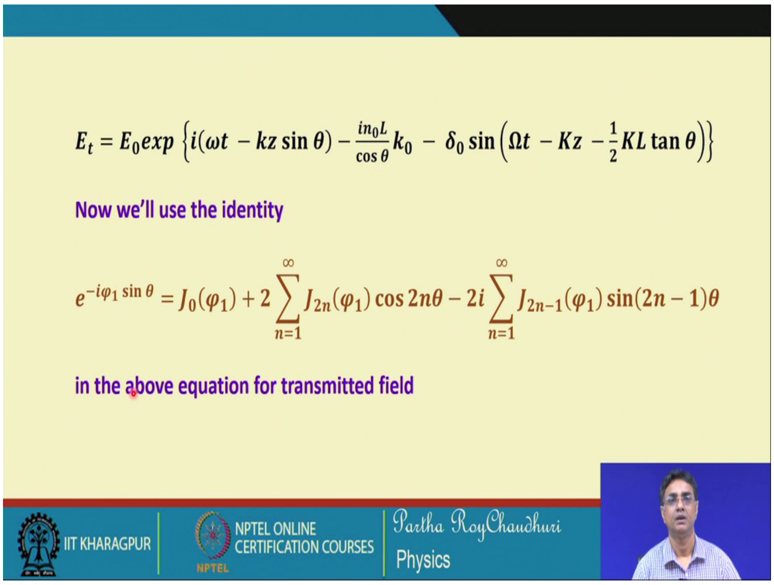

Now, this transmitted field once again if you look at it, if we look at this equation, you have i omega t, then, i of this quantity and this quantity. So, once again, you have i phi sine theta. It has this form i phi sine theta.

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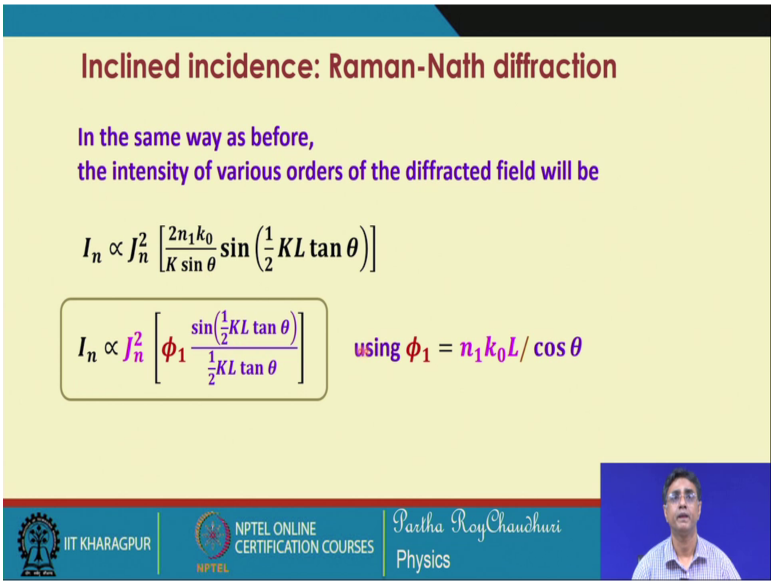

So, this phi 0 phi 1 sorry this phi 1 is given by this equation n 1 k 0 L by cosine theta, you can see k 0 L by cosine theta.

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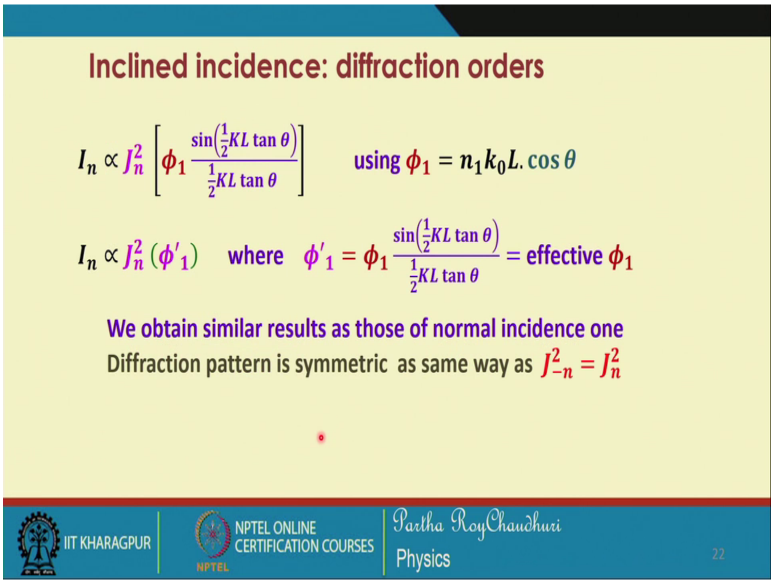

So, this is your phi 1. So, this is your phi 1 and from here we can write this equation I n is proportional to this. And therefore, I n is again proportional to J 1 square of phi. This quantity the oblique incidence that is theta is constant for a given incidence L is constant K is constant. So, this quantity is absorbed here in the form of phi dash, whereas, this effective phi is this.

So, after doing all these things, we could actually represent the transmitted that is the diffracted beam amplitudes in terms of the same form that is phi and we obtain the similar results as those. But, this phi 1 is now phi dashed effective phi because, all other quantities are going to change the modified the value of phi 1, originally this was phi 1 and now that this quantity is multiplied with this.

So, the diffraction pattern is symmetric as we obtained in the earlier cases because of this identity of the Bessel functions. So, for oblique incidence, we find that it is the same, but the effective value of phi is now modified.

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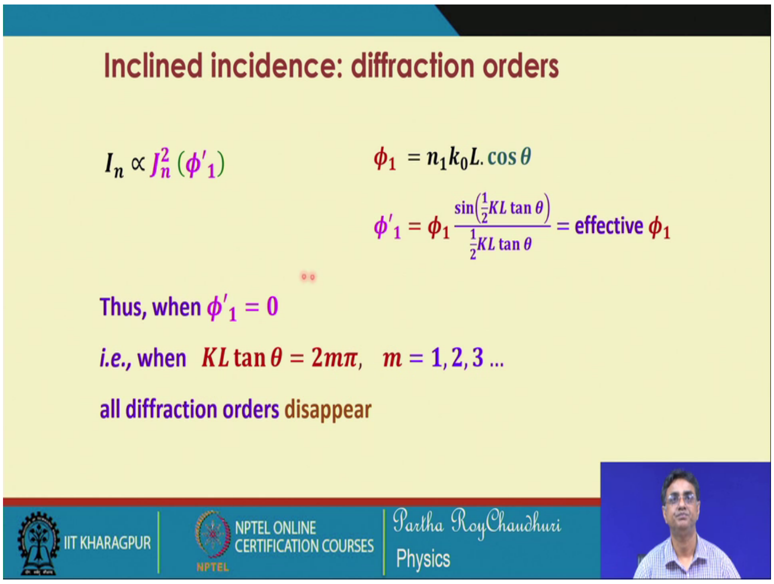

Now, there is. So, this is an interesting situation that it will give rise to that when you have this phi dash equal to 0. So, to how to get this phi dash equal to 0? At x equal to 0.as I have mentioned in the beginning, you have all other orders, all the diffraction orders are 0. The entire field amplitude entire intensity is now contained with the 0th order that is the undiffracted wave.

So, that is when there is no effect of the strain, so, at phi equal to 0. But even when there is an effect of the strain, then this phi dashed equal to 0 is possible. Because, if this quantity this quantity itself is equal to 0 half K L tan theta equal 0. If K L tan theta equal to 0 or if tan theta is equal to 0.

So, that means, this is the condition when you will have this numerator equal to 0, if this numerator is 0, then it will make everything the entire quantity equal to 0. Even when there is an angle of incidence, even when there is a delta n value and even where everybody is present, still because of this condition that tan theta is equal to 0. You will see that phi dash equal to 0 and if phi dash equal to 0, then if then the entire light is now contained in the 0th order.

So, the acoustic wave is there the strain the is also there the traveling grating is also formed everything is present. But, if the condition is such that the inclination is such that it satisfies this condition, then we will still get no diffraction that the all the diffracted diffraction orders will disappear just because of this.

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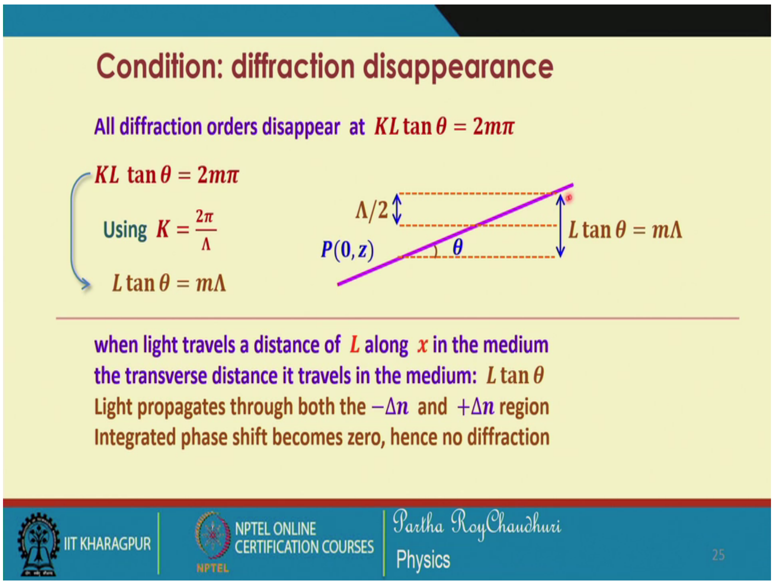

To understand physically we note that this K L tan theta equal to 0 that is L tan theta equal to m into lambda K equal to twice pi by lambda. So, lambda goes to this side and it simply gives you L tan theta equal to lambda and this will clear the understanding that when it has travelled through a length L. So, this much of length L tan theta which corresponds to a phase which corresponds to an effective length of m times capital lambda m times if you m equal to L.

So, it is only one lambda when light travels a distance of L along x in the medium. The transverse distance it travels in the medium is L tan theta. So, this much of distance it travels.

Light propagates both minus delta n and plus delta n you see, if it corresponds to 1 period, then it has started travelling from the peak position which is the negative peak; that is minus delta n. Let us suppose that and when it appears, then at this position, the peak is plus delta n. Therefore, if you average it over throughout this length because, it is a linear you know you see it is moves straight and you have an equal weight of delta n on either side. So, this minus delta n to plus delta n integrated over the entire change in the phase is equal to 0.

So, that is why, you have the total effect cumulative effect of phase change is equal to 0 and you even though there is an acoustic wave, even though you have a travelling grating, you have this periodicity everything remaining here. Because of the inclination of the of the incoming light wave, it sees no effect; it sees the cumulative effect of and for a parallel beam of light. It happens all throughout for every individual wave.

So, it happens the same. So, the total cumulative effect of the phase equal to 0. And as a result, it sees no delta n related change in the phase in the medium and therefore, there is no diffraction. So, the integrated phase shift becomes 0. Hence, no diffraction.

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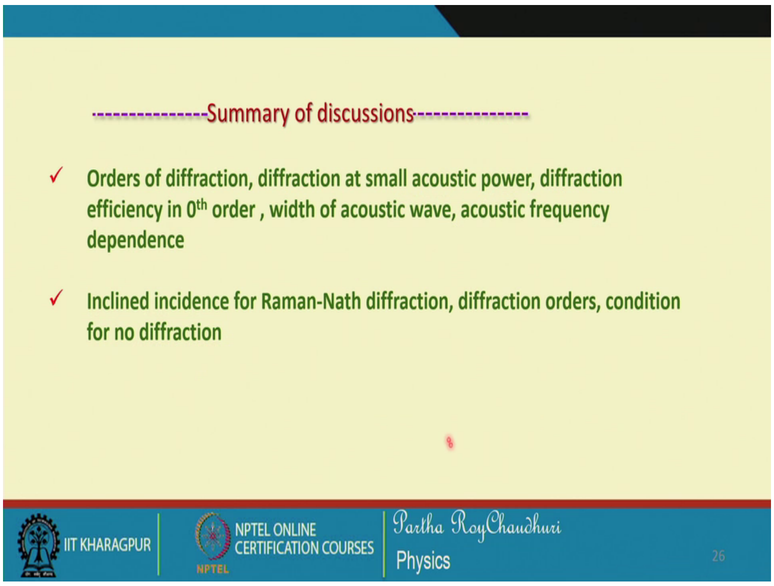

So, we studied this Raman Nath diffraction case individually for the 0th order diffraction and for small acoustic wave diffraction efficiency in the 0th order. When we assume the acoustic power is very small, then acoustic power with dependence of that and then frequency dependence.

We also studied a very beautiful case of this oblique incidence of this Raman Nath diffraction, where even though there is an acoustic wave, but still there is a situation when the light wave will see no grating and it will be passed to the media without any diffraction.

Thank you very much.