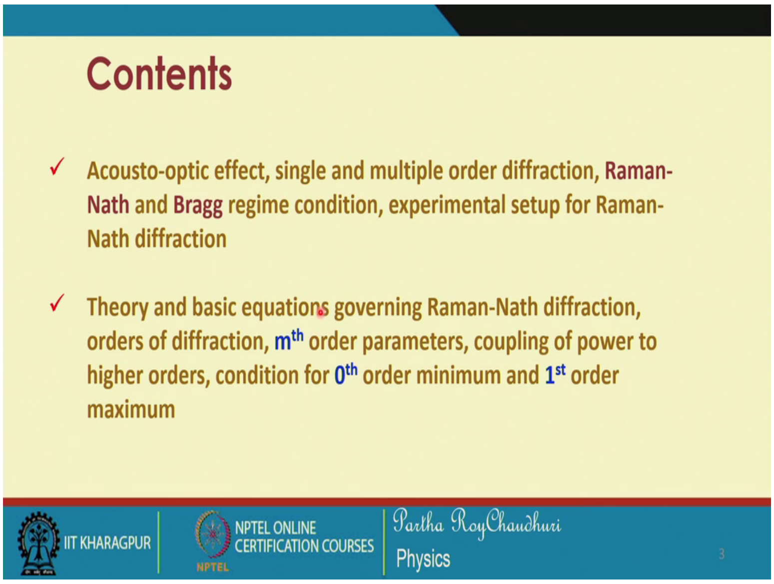
Modern Optics
Prof. Partha Roy Chaudhuri
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture – 47
Acousto-optic Effect (Contd.)

In the last few discussions we were looking at the various possibilities of the acoustic wave traveling in isotropic and anisotropic medium and we analyzed that when the acoustic wave is oriented in different directions for isotropic and anisotropic medium particularly this lithium niobate crystal, then the periodic refractive index variation can be also oriented at specific directions and this finding is very useful when we will be discussing then diffraction of light from such a periodic grating.

So, by now our grating moving grating in a medium is ready, we will now consider the incident light wave and we will see how this diffraction takes place from such a moving grating.

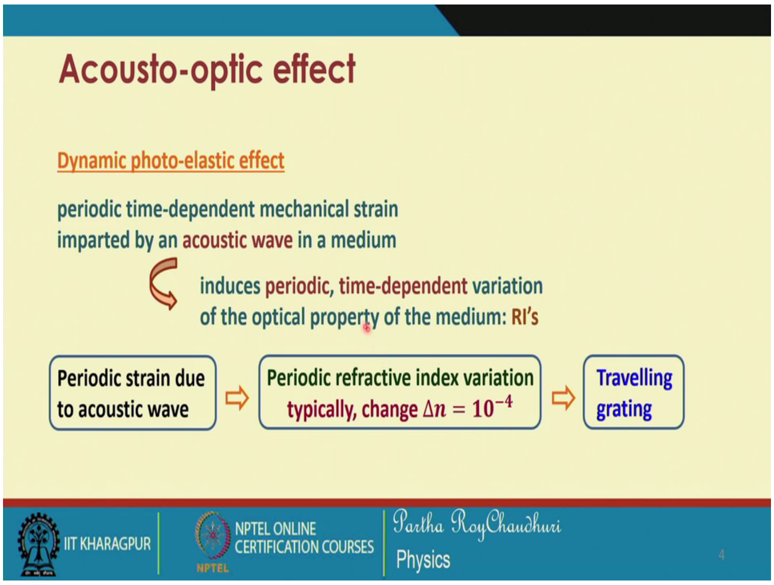
(Refer Slide Time: 01:27)



So, first we will consider this the case of diffraction, how it can be categorized looking at the width of the width and frequency aspects of the acoustic cell, acousto-optic cell and then we will divide that into Raman-Nath diffraction, regime and Bragg diffraction regime we will find out this condition for Bragg diffraction.

Then we will discuss the experimental setup for Raman-Nath diffraction, we look at how various orders of this Raman-Nath diffraction are coming out as a consequence of an incident light field. Then we will discuss the basic theory and basic governing equations of the Raman-Nath diffraction.

(Refer Slide Time: 02:23)



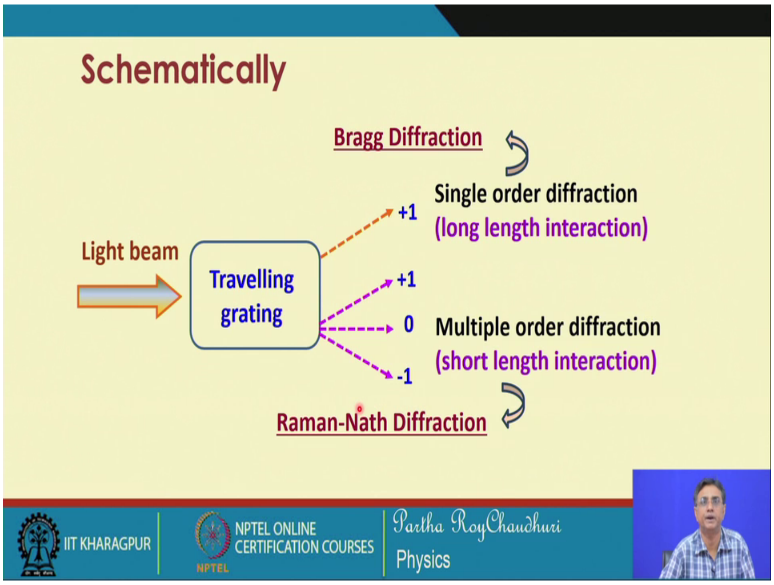
So, first thing is that this acoustic acousto-Optic effect is a dynamic photo elastic effect. We have seen that even when it may not be an acoustic wave, but if there is a strain in the medium the optical properties are modified.

But if the strain is of periodic nature time dependent mechanical strain, then this acoustic wave which comes from acoustic wave a time dependent mechanical wave, then the there will be a periodic time dependent variation of the refractive indices in the medium. And this property is nothing but the property of a grating, but this is a dynamic grating this is a grating, which is present as long as the acoustic wave is present in the medium and because the acoustic wave frequency acoustic wave amplitude all these things can be controlled externally, so, the grating can also be the grating properties can also be controlled.

So, we will see that that this periodic strain due to acoustic wave changes this periodic refractive index and there will be a travelling grating, but this traveling grating formation can cause the diffraction in two ways. If the if it corresponds to a thin phase grating with a as if a static grating that we use in the laboratory, then it gives you various orders of diffraction starting from the undiffracted wave to plus order minus order plus 1 minus 1 and all.

So, that corresponds to the Raman-Nath diffraction whereas, if it is a is a if the width of the if the if the light wave has to travel a longer length in the refractive index medium then it will under it will see a volume grating and that will correspond to Bragg diffraction.

(Refer Slide Time: 04:37)

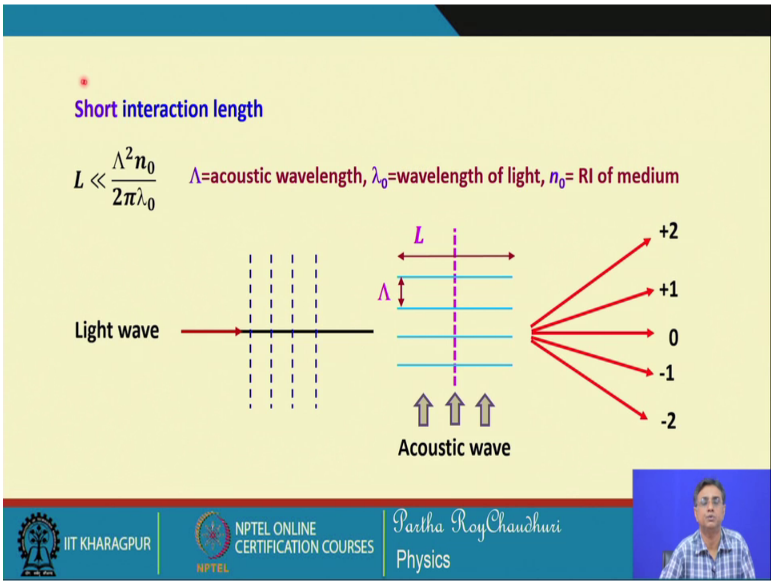


So, schematically that we have a travelling grating, but let us suppose the width of the grating is very small, that is it is a thin grating as good as a grating that we see in the laboratory for this undergraduate experiments, then the light beam which is trying to travel through that grating will be diffracted with various orders plus 1 minus 1 plus 2 minus 2 etcetera and you can see that a multiple order of diffraction will take place when the length of interaction through the grating will be short.

But if the length of interaction is large, that is if the light beam has to travel a longer length in the grating, then we will see instead of a thin phase grating. This is also a phase grating, but you will see a volume grating like the x ray see, a x ray diffraction in a through a crystal. So, there will be a periodic variation of the refractive indices, which will be looked upon as the lattice planes and this light rays will be diffracted. So, you will get Bragg diffraction, but in this case there will be a single order Bragg diffraction. It and light will be switching back and forth between the single order and the undiffracted beam depending on the parameters of the parameters of the travelling grating.

So and we will see how this light coupling from the undiffracted beam to the first ordered diffracted beam can take place, depending on under which conditions that we study in details ok.

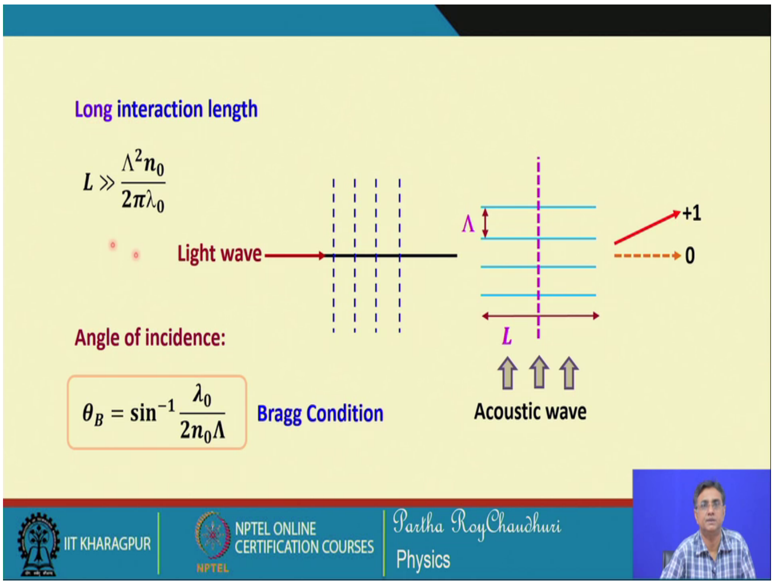
(Refer Slide Time: 06:23)



So, for short interaction length are this figure we define that one can see that if the length of interaction; this length of interaction of the light wave within the acoustic wave within the medium is very small, that is if it is a very thin phase grating which is a mathematically expressed as this lambda square n 0 by twice pi lambda 0. This capital lambda is the periodicity of the refractive index grating, which is generated by this acoustic wave; lambda 0 is the free space wavelength of the light wave and n 0 is the refractive index of the medium when there is no acoustic wave.

So, if we have a very thin phase grating, then we will get multiple orders of diffraction and we mentioned that this is like the normal diffraction that we encounter we work in the laboratory stationary grating. In this case also we will see that even though the grating is travelling, but it can be treated as a stationary grating with respect to the light because of a very big difference in the frequency of the light, with the frequency of the acoustic wave.

(Refer Slide Time: 07:55)

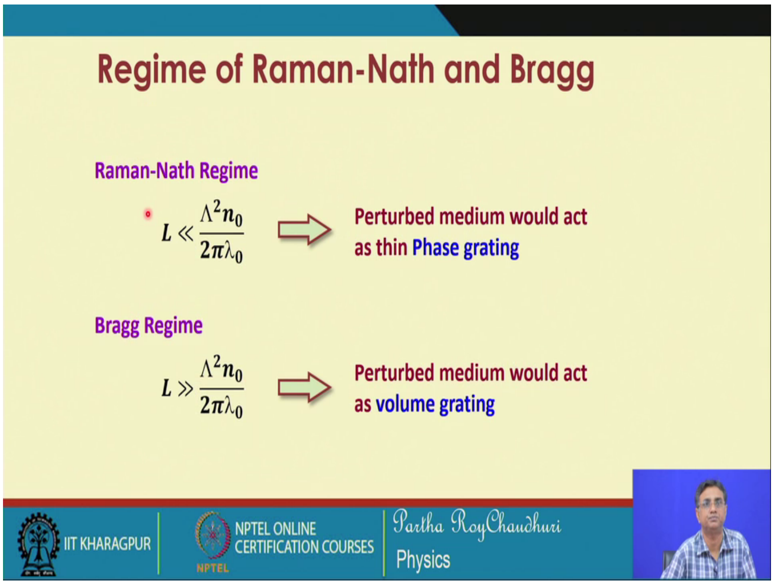


In the case that this length of interaction of the optical wave light wave with the acoustic wave is very large of the order of L much greater than we will see that this will be more than for normal conditions; if it is more than two centimeter or so, then this will be this will correspond to a diffraction by diffraction which is a Bragg type of diffraction.

And the angle of incidence angle of incidence that is required for this power to get transferred to the first order, that is called this theta B and we are very familiar with this Bragg angle condition because twice d sin theta equal to n lambda n equal to 1 in this case. So, first order diffraction which is very straightforward one can easily derive from here.

So, that is the Bragg condition you have an acoustic wave, which is traveling along this which has a width of L and the periodicity is lambda and we define this is the quantity that we will be always checking whether it is to correspond to a Raman-Nath diffraction or a Bragg diffraction.

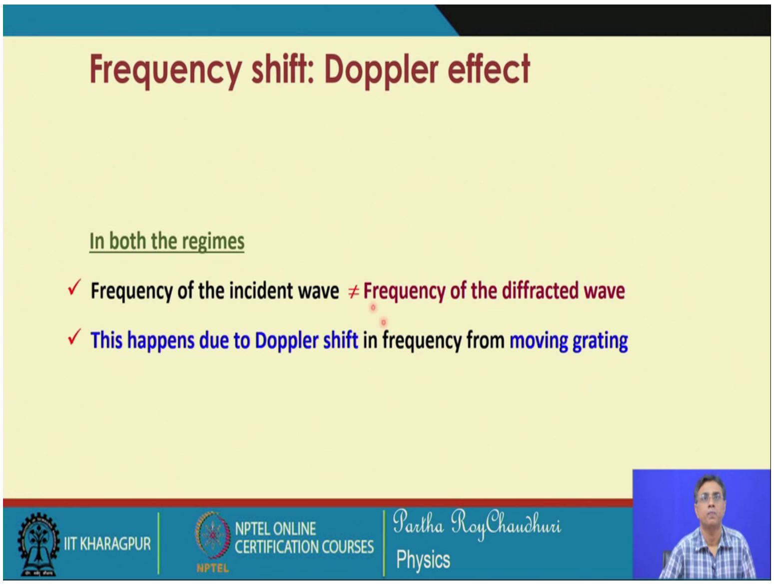
(Refer Slide Time: 09:20)



Now this is all that we summarized here that for Raman-Nath discuss diffraction L the length of interaction of the optical wave with the acoustic wave will be will be less than this quantity, which could be at considered as the width of the width of the acoustic wave cell acoustic acousto-optic cell.

And in the case of Bragg diffraction, this L will be much greater than this and we call this is the regime for Bragg diffraction and this is the regime for Raman-Nath diffraction; and this we have already mentioned that this is a thin phase grating whereas, this is a volume grating.

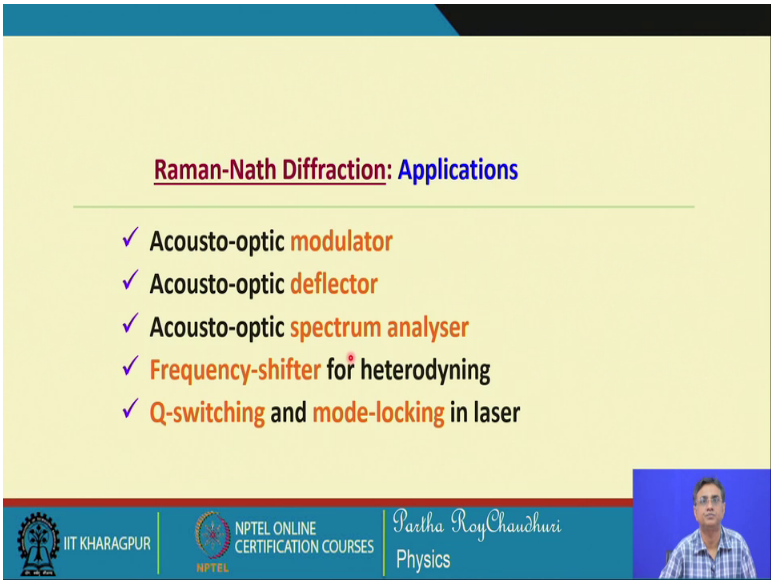
(Refer Slide Time: 10:07)



Now, in both the cases whether it is a Raman-Nath diffraction or a Bragg type of diffraction, there will be change in the frequencies with regard to the incident frequency of the optical light of the incident light wave and this is this happens due to the Doppler shift from the moving grating.

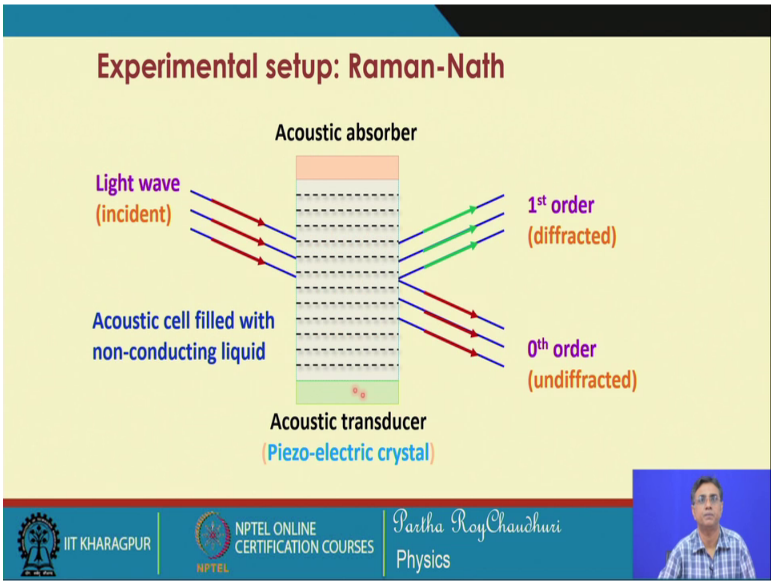
Because the grating itself is moving and we will see that really there is a change in the frequency, the grating is moving and because of the moving grating is a sort of moving reflector. So, there will be a change of change of frequency and this change in the frequency has to accommodate with the propagation vector and that is why there is a diffraction.

(Refer Slide Time: 11:05)



So, we will see that acousto-optic modulator in the case of Raman-Nath diffraction, there are large number of applications, but not limited to this then acousto-optic modulator we will we will discuss acousto-optic modulator and devices separately, then one of them is acousto-optic deflector, acousto-optic spectrum analyzer then frequency shifter for heterodyning and very commonly used in Q-switching and mode locking in for laser light for laser light this is these are also commercially available devices ok.

(Refer Slide Time: 11:46)

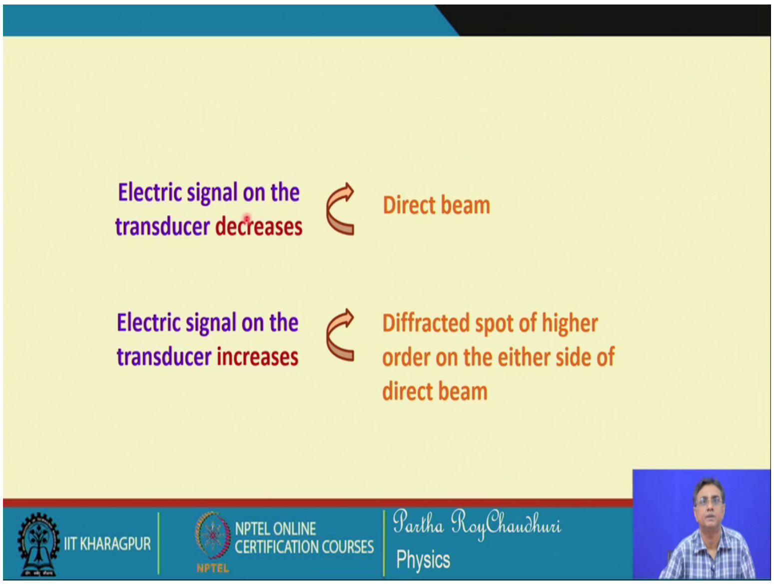


So, the basic experimental setup for this a Raman-Nath type of diffraction, you have a light which is incident here and this is the acousto-optic cell you have a transducer acousto-optic piezo crystal which is attach to this and we have some liquid which is non-conducting non polar liquid and this liquid fills and it behaves as the medium where the periodic refractive index will be generated.

There is an acousto-optic absorber; if this absorber is not present then there is a chance that acoustic wave will be reflected back and as a result there will be a formation of standing waves, along this and that case is also studied, but for the present discussion there is an absorber. So, there is no reflected wave, it is only a travelling wave travelling acoustic wave which moves to this medium in this directions.

So, the light is incident here and this will be the undiffracted beam direction, but the moment there is a diffraction first order let us suppose, then it will be diffracted in this direction so, as to satisfy the condition of this of this diffraction. There may be minus order plus order; so, we will consider with the basic diffraction as this and now we can.

(Refer Slide Time: 13:25)

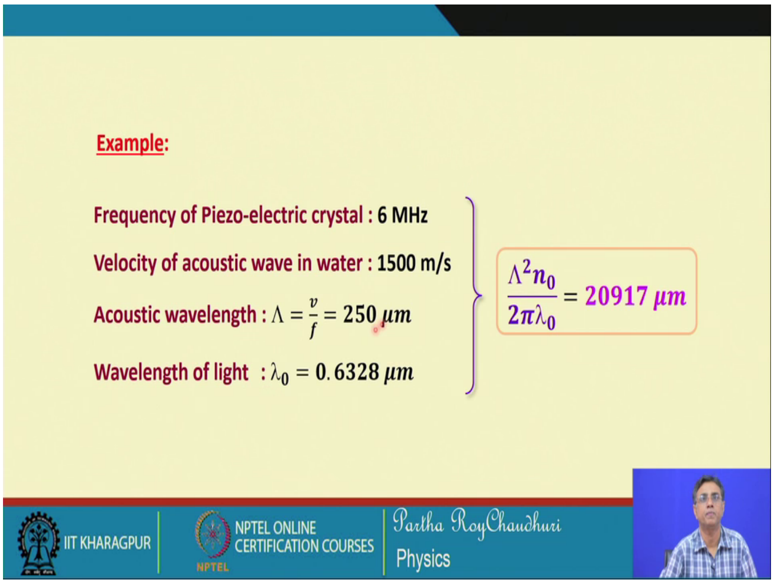


So, but the thing is that if we increase the electric signal in the transducer, then you will get direct beam, but if you decrease this, but if you keep on increasing the electric field, we will see that there is a dependence on the signal frequency and then you will get higher order diffractions on either side.

So, if we start with a very low frequency transducer and the thin under width of the acousto-optic cell is critically between this Raman-Nath and Bragg regime, then we will see that initially there will be only a zeroth order then there will be higher order higher order diffraction plus order and minus order. So, for the case of a thin the width is such that it corresponds to only Raman-Nath diffraction.

In that case starting from low acoustic frequency, we will see only undiffracted and as you keep on increasing the signal then we will see that more number of orders are appearing.

(Refer Slide Time: 14:37)

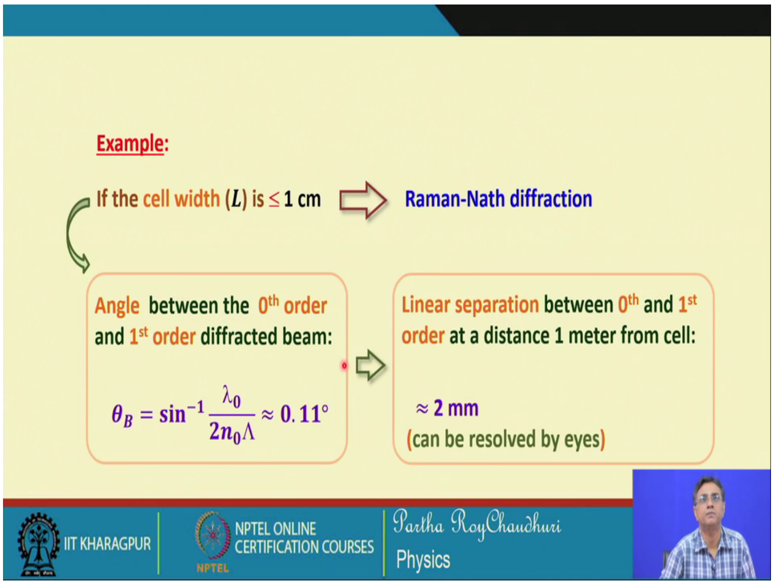


And we will account for how this we can see one example that, if the crystal frequency is 6 megahertz; we know that the acoustic velocity in water is of the order of 1500 meter 1450 meter per second.

And this acoustic wave length knowing this velocity and frequency one can calculate the acoustic wave length, which is of the order of one fourth of a millimeter 250 micrometer, then wavelength of light that is used is let us suppose helium neon laser light which has this wavelength. If we plug in these numbers into this expression, we can find we can define the L that is the width of the acoustic wave which will be of the order of 2 centimeter 20917 close to 2 centimeter.

So, if the acoustic cell width is less than 2 centimeter, it will very well correspond to a situation for Raman-Nath diffraction.

(Refer Slide Time: 15:34)

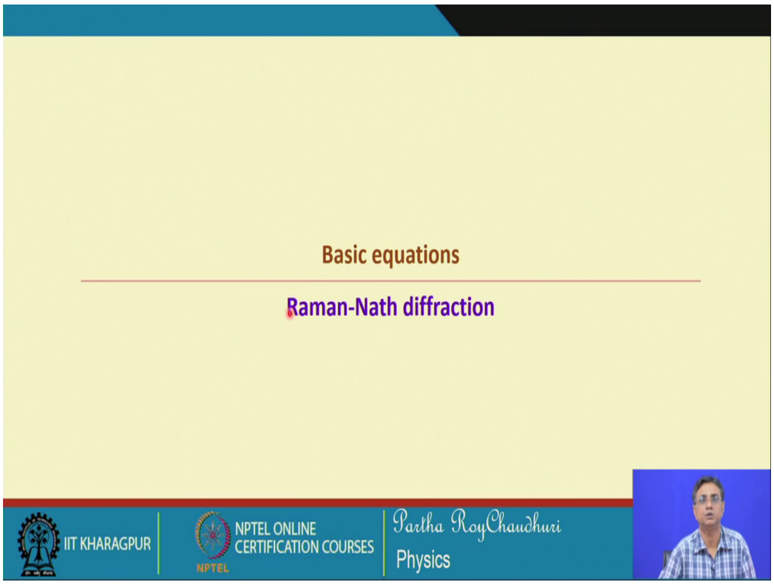


That is if it is less than 1 centimeter, then it is very much we are very much safe into the Raman-Nath regime and the angle between the first order and second order in that case will be this, which is which can be clearly seen let us suppose that the linear separation between the first order and the zeroth order this separation is 1meter.

If it is 1 meter then we can use this theta B to get the separation in terms of in terms of distance, angular separation can be translated into linear separation which will give you 2 millimeter and it means that this two millimeter can be resolved with the two spots can be easily seen with their eyes.

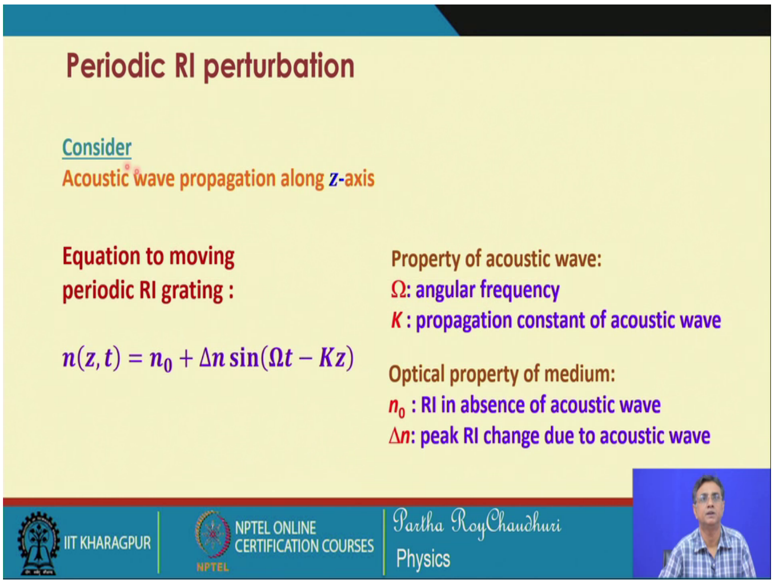
So, just by using this frequency and using this much of length less than 1 centimeter of length of the acoustic cell, the random that is the zeroth order and the first order diffracted spots can be visible very well visible.

(Refer Slide Time: 16:47)



So, let us look at the look at the basic equations that, how the formulation how we get various orders try to understand from here.

(Refer Slide Time: 16:57)



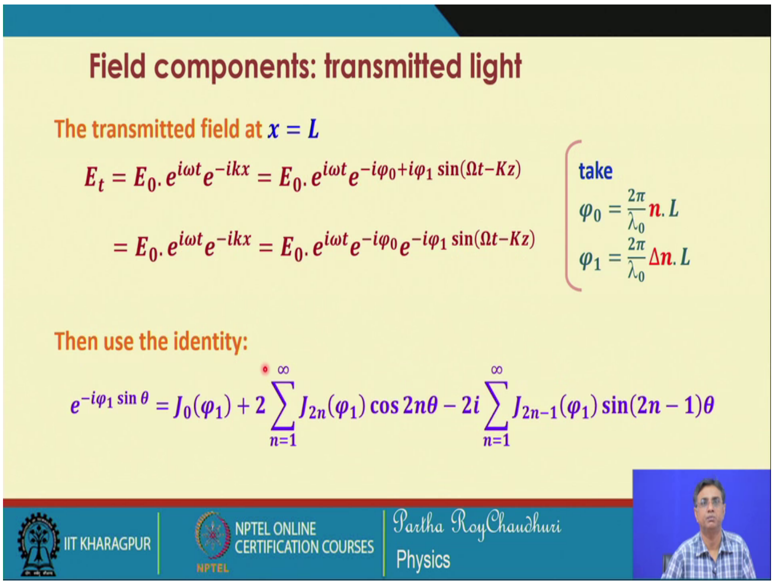
Well we have acoustic wave we consider that is propagating along z direction acoustic wave and the equation to the moving periodic grating.

Therefore because the wave is traveling along z direction, we can write this equation as this where delta n we have seen that this corresponds to this s matrix, multiplied s 0 and also this is the variation in the refractive index. So, this is the peak refractive index change due to acoustic wave, so, because sin theta can take plus minus 1 value. So, once it could be n 0 plus delta n, which is the negative minimum value and the maximum value will be n 0 plus delta n.

So, it modulates between n 0 minus n to n 0 plus n at different positions along the length of the acoustic wave del omega capital omega is the angular frequency, K is the propagation constant of the acoustic wave and n 0 is the medium refractive index refractive index of that liquid acoustic that fills the acoustic cell and that is in absence of the acoustic wave.

So, these are all known parameters when we have learned, how this equation comes as a consequence of propagation of acoustic wave in the medium.

(Refer Slide Time: 18:30)



So, now this periodic moving grating how it diffracts the light let us see; let us suppose that the transmitted field at x equal to l that is at the end of the at the end of the acoustic cell this length is l. So, at x equal to 0 corresponds to this position, x equal to l corresponds to this position.

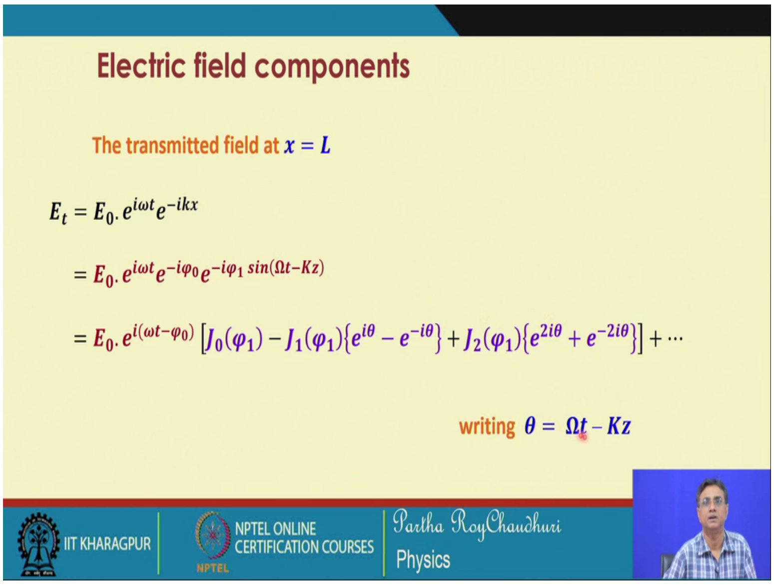
. So, at x equal to l we will write this optical field, which is the transmitted field you can write this equal to this is the form of time variation of the electric field of electromagnetic wave and this is e to the power of ikz that is the propagation constant k. And because the electromagnetic wave is traveling along the x direction, we write this equation in this form and we can write this k contains k 0 n into x. So, k 0 this n now is represented by delta n and delta n is delta n is nothing, but you see twice pi by lambda 0 n into L and this is twice pi by lambda 0 delta n into L. So, both of them are sitting here.

So, now it is this k will be equal to k 0 n plus delta n and this attached delta n is attached with this coefficient into x. So, therefore, we can write this equation in this form. So, you have the time dependent part in the electric field, you have the phase change which is which is which will happen in absence of the acoustic wave and this is the effect of the acoustic wave this is due to delta n and this is due to only n 0 that n this effect this will be n 0.

And then use the identity, because if we have an equation of this form expression of this form e to the power of i p theta; e to the power of i p theta here it is e to the power of i phi 1 sin theta and if we call this omega t minus kz equal to theta. Then one can write this identity these are the Bessel functions J 0 and J 1 J 2 all in pairs with plus and minuses.

So, we will use this identity to look at the diffraction of this electric field in the transmitted wave.

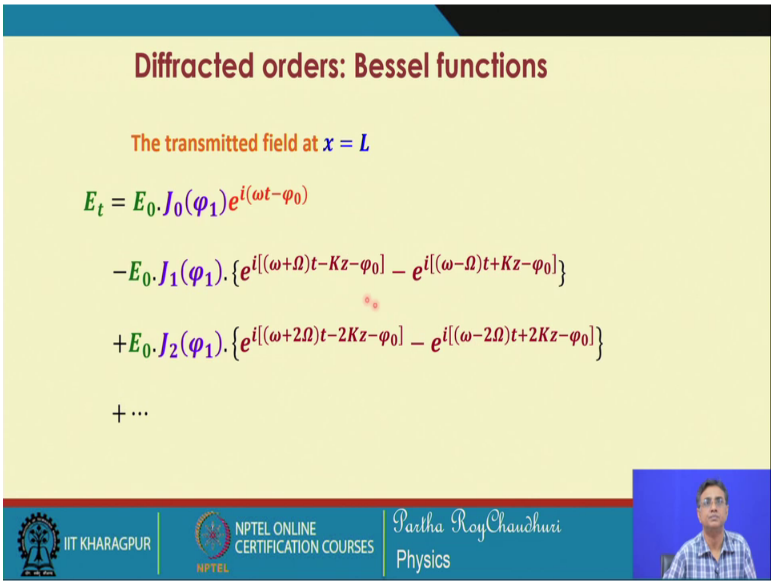
(Refer Slide Time: 21:09)



So, we can write this equation in terms of this Bessel functions. So, the transmitted electric field at x equal to L using the Bessel functions because e to the power of i phi 1 sin theta omega t minus K z which we have called theta we can write in this form.

And we have taken omega t minus phi 0 into one bracket and rest all the terms putting here. You can see that you have e to the power of i theta e to the power of minus i theta e to the power of 2 i theta e to the power of minus 2 theta all in pairs with the complex conjugate. And then this is this we have written theta equal to omega t minus K z.

(Refer Slide Time: 21:51)



And therefore, the transmitted field at x equal to L can be we are writing this equation in a more lucid way visible way. So, that each of the terms each of the Bessel functions are now attached to the incident amplitude of the electric field. So, E 0 J 0 this quantity, E 0 J 1 of phi this quantity and you can see that this J 1 phi J 1 phi this has a frequency of the light, which is because this is e 0 this corresponds to the undiffracted or the incident frequency, but this is the diffracted light frequency omega plus capital omega and similarly omega minus capital omega.

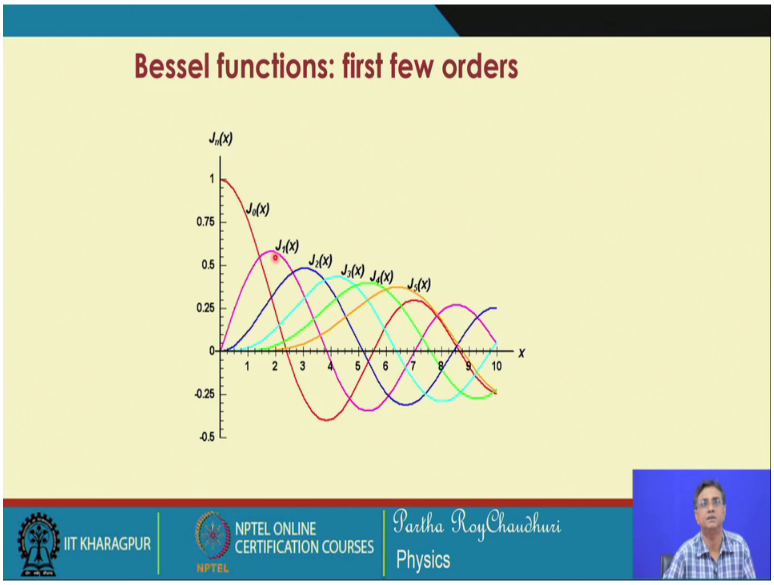
Here also you have omega plus 2 capital omega minus 2 capital omega; that means, the diffracted light contains frequencies which is which are omega plus capital omega, omega minus capital omega and we see that this corresponds to the first order diffraction that is plus 1 order and minus 1 order.

And this term will correspond to the second order diffraction that is plus 2 and minus 2 order of diffraction. You can see that plus 1 order diffraction amplitude is this E 0 times J 1. So, it is a factor which decreases the original amplitude J 1 and E 0 J 2 this factor J 2 function of phi 1; phi 1 is known to us, so, this phi 1 J 2 phi 1 that reduces this factor of the initial amplitude.

So, this E 0 total amplitude will be shared by J 0 this undiffracted beam; the first order diffracted beams on either side of the undiffracted beam and the second order diffracted beams and so, on and so, forth. So, all the orders will now contain the incident electric field in some fractions J 1, J 2, J 3. So, very clearly that if we have an incident amplitude E 0 in the diffracted zeroth order beam the amplitude will be E 0 J 0. In the first order diffracted beam it will be E 0 J 1 and in the second order and so on and so forth and the frequencies of the light will be for the plus 1 order it is omega plus omega minus 1 order will be omega minus omega and so on.

So, we will see this that because this J 1, you see that each order is now attached to the different order of the Bessel functions. In fact, the same order of the Bessel function.

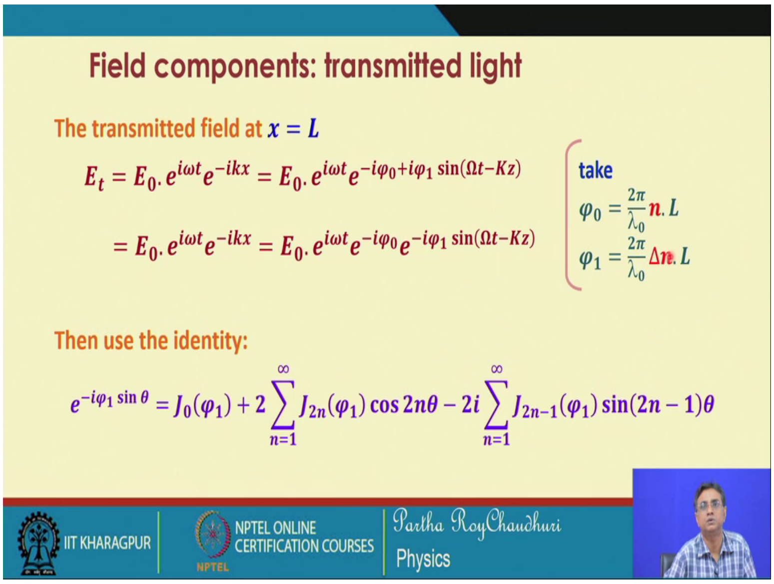
(Refer Slide Time: 25:03)



That is first order J 1 is attached to this, J 2 will be attached to this and so, this is the variation of this. One thing very interesting to note here that if phi 1 is 0 then phi 1 that is the argument of this and you know what is phi 1? Phi 1 is equal to this quantity.

If there is no periodic grating formation if there is no periodic grating formation delta n; that means, there is no acoustic wave. So, this delta n equal to 0 and as a result we can see that when as a result this phi 1 is also 0.

(Refer Slide Time: 25:25)

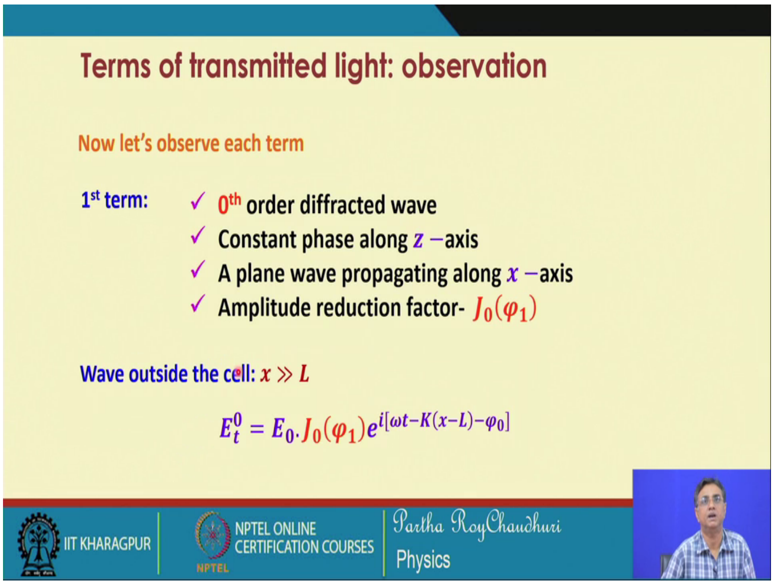


If phi 1 is 0 then at phi 1 equal to 0 all the other higher orders that is J 1 J 2 etcetera everyone is 0 at phi 1 equal to 0.

But J 0 is present so; that means, this will be present there and; that means, that incident light is now completely in the zeroth order that is in the undiffracted beams. So, if the incident lies just passes through the medium. But once you switch on the acoustic field phi 1 increases and there is a sharing of the intense of the amplitude of the electric field as per this J 1 J 2.

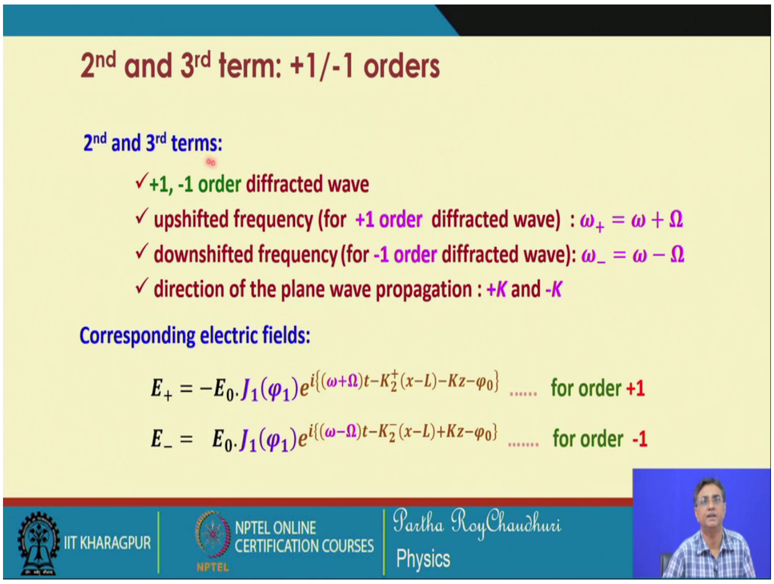
So, we have to draw a line across this which will correspond to the value of phi 1 and then we can calculate the values of J 1, J 2, J 3 and that will give me the amplitude of diffraction.

(Refer Slide Time: 26:41)



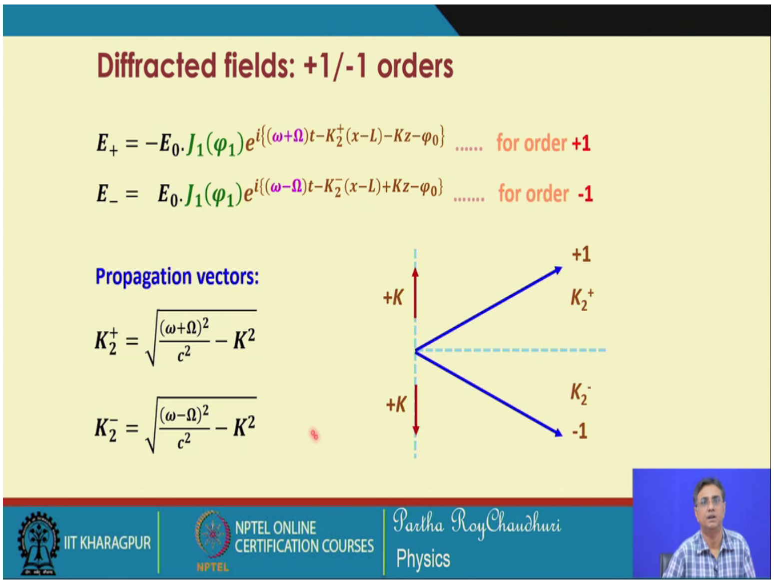
Now, let us absorb each of the terms, zeroth ordered diffracted wave constant phase along z axis we have seen that constant phase along z axis and a plane wave along x axis amplitude reduction factor for the zeroth order is this. So, in the zeroth order there is a reduction of the amplitude and that is entirely due to the presence of the phi 0. The wave outside the cell can be represented by this for this zeroth order.

(Refer Slide Time: 27:19)



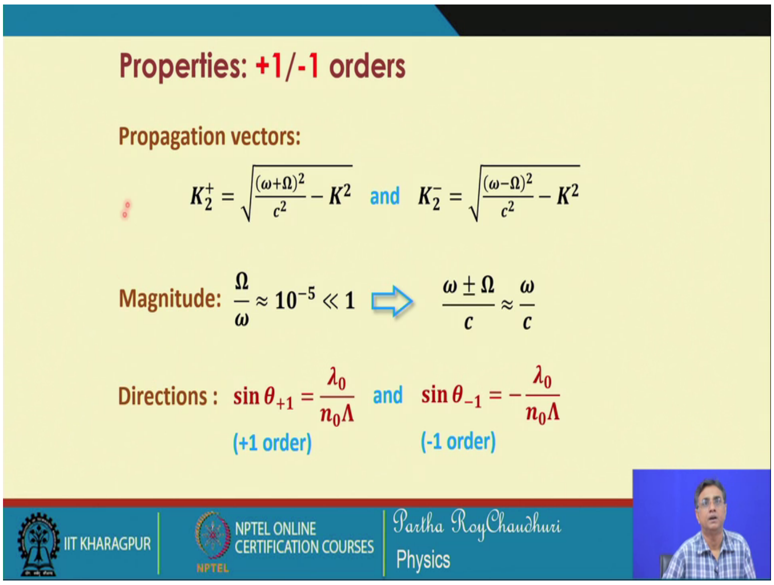
And for the second and third term; you have first order and second order diffracted wave we have seen this, that it is omega plus this is the upshifted frequency and this is the downshifted frequency, the responding electric fields are for the first order plus 1 and minus 1 order you can see that is upshifted frequency, this is downshifted frequency right.

(Refer Slide Time: 27:41)



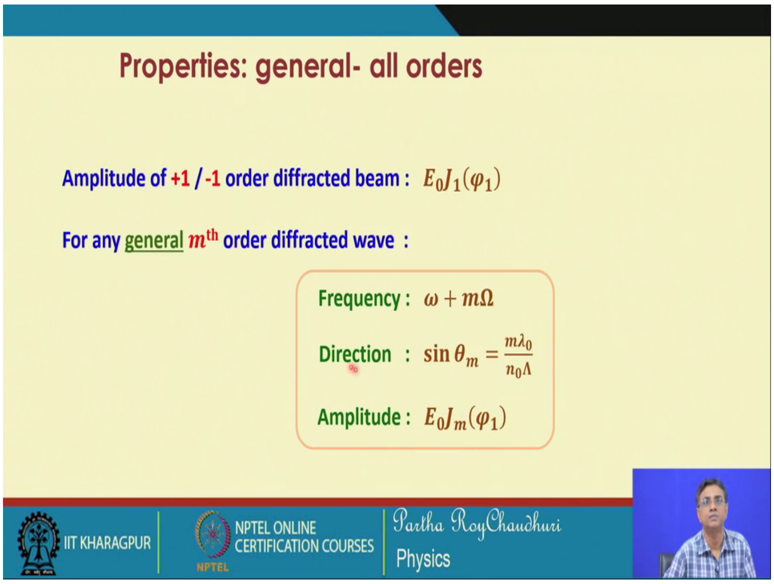
So, schematically we can show that the acoustic wave vector is K plus in this direction, this would be K minus actually, should be K minus; and for this you have this K 2 plus. So, this K 3 plus is nothing, but a modification of K vector; this K vector with this omega plus this. So, this is the conservation of this K and for K 2 minus also, we can write this k 2 minus as a difference of this.

(Refer Slide Time: 28:20)



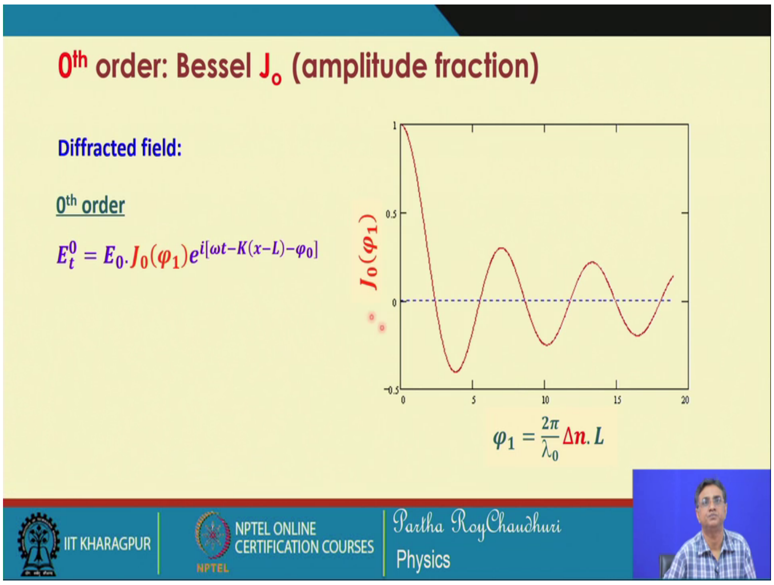
Propagation vectors k one and k two we have seen so, this is the summary for the second and third terms, omega by omega is of the order of 10 to the power of 5. So, it is approximately the free light frequency, we will not be able to see any change because approximately it is same and the directions of the first order and minus one order will be given by this sin theta plus and sin theta 1.

(Refer Slide Time: 28:48)



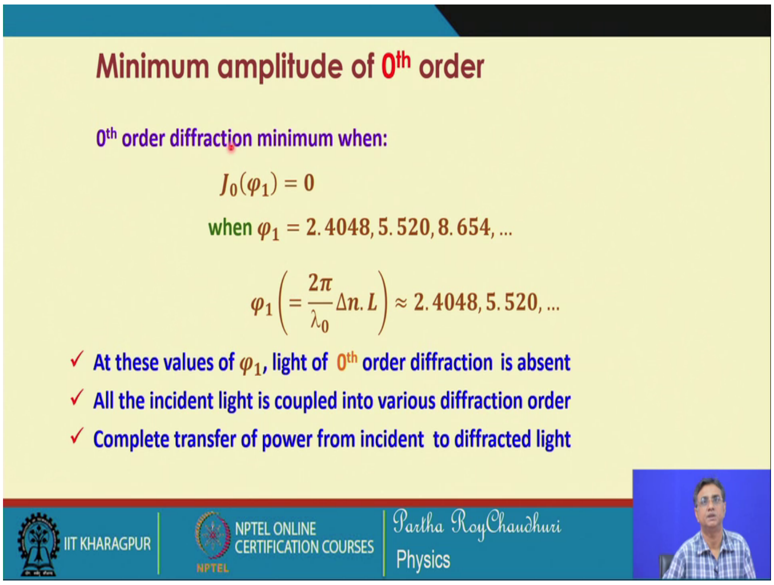
Frequency any general m-th order diffracted wave we will have this frequency the direction of. So, this is by induction for and this order will be J m.

(Refer Slide Time: 29:01)



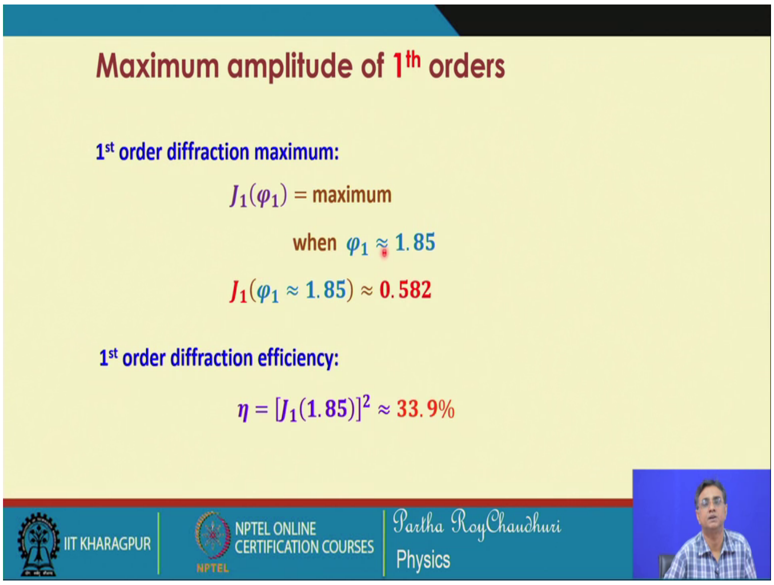
Now, for the diffracted wave zeroth order you can see.

(Refer Slide Time: 29:05).



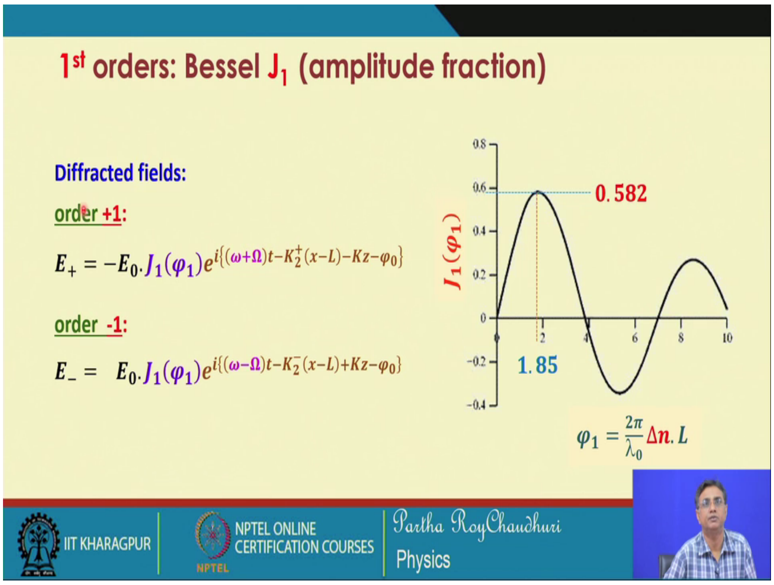
And J 1 will be 0 i just mentioned that when J 1 equal to this, for this is the first Bessel order, here the power will be power will be at this point there will be no intensity in the zeroth order at this point at these points there will be no intensity and the phi 1 is. So, 0 this zeroth order undiffracted beam will be absent and all the electric all the light power will be now will be now distributed into the various orders, but complete transfer of power from the incident to the diffracted beam will take place when phi corresponds to these values these values, where the Bessel J 0 has 0.

(Refer Slide Time: 29:56).



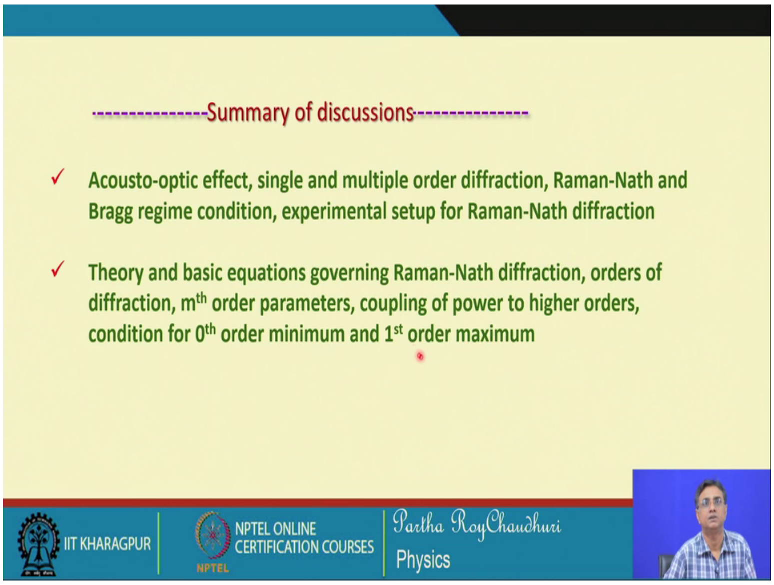
And in the same way for the first order you see this is J for the this will be maximum when phi 1 equal to 1.5.

(Refer Slide Time: 30:05)



You can see at 1.5 you have the Bessel J 1 maximum and at that point the value is 0.52 therefore, for this value of phi 1 the 50 percent of the close to 60 percent of the initial amplitude, initial electric field amplitude will be available in the first order beam and the efficiency will be close to 34 percent.

(Refer Slide Time: 30:33)



So, we have seen that how this diffracted entire power can be transferred to from the undiffracted from the zeroth order to the side bands that is the first orders plus order and minus order, when the value of phi 1 it corresponds to this 1.85, the efficiency is 50-60 percent. So, that we can also see observe the other aspects of this Raman-Nath diffraction and we will continue that discussion in the subsequent discussion.

So, in this case today we now discuss the acousto-optic wave and the resulting single and multiple order diffraction. We looked at the Raman-Nath and Bragg regime condition in terms of the width of the acoustic cell. Then we looked at the typical experimental setup for Raman-Nath diffraction, we discussed various orders of the diffracted wave in terms of the Bessel functions analytically.

And we have seen the condition when the entire power will be transferred from the zeroth order to the diffracted beams and we also looked at the other aspect that is when there is no acoustic wave that is phi 1 corresponds to 0, then the entire power is with the undiffracted beam that is Bessel J 0 of phi 1 equal to 0 multiplied by e 0. So, that is very interesting and we will continue the other aspects of this Raman-Nath diffraction in the subsequent lectures.

Thank you very much.