Modern Optics
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Lecture – 46
Acousto-optic Effect (Contd.)

We were discussing the propagation of acoustic wave in various media and, we started our discussion in the last time the propagation in isotropic medium. We considered longitudinal acoustic wave and transverse acoustic wave and analyze the induced birefringence. And now we will consider the propagation of acoustic wave in an anisotropic medium. We will consider the specific example of the lithium niobate crystal.

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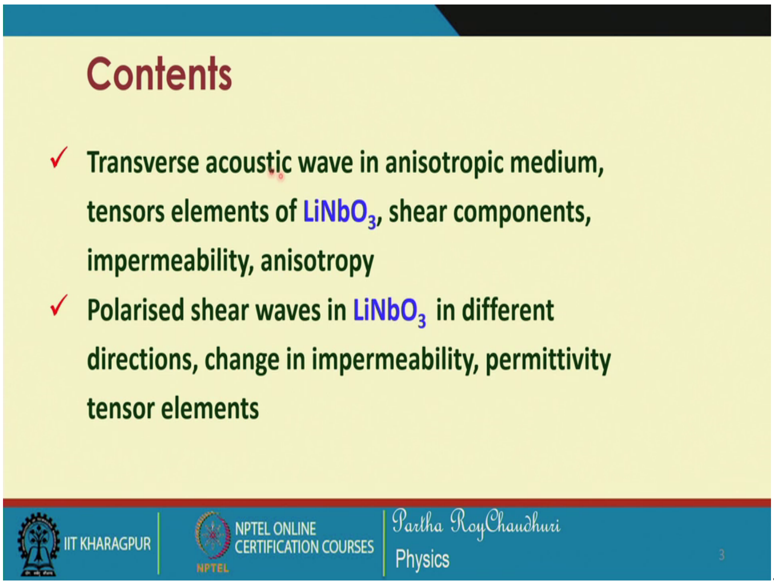

So, we will be discussing this transverse acoustic wave in anisotropic medium and we look at the resulting tensor elements for this lithium miobate crystal, which is a very important crystal and we look at the shear components permeability change in the in permeability tensor and resulting anisotropy.

We will also consider another example case of polarized shear waves in lithium niobate in different directions, and then in the same way we look at the change in the impermeability and permittivity tensor elements, the purpose of this to provide an exercise how we can configure the propagation direction with respect to the crystalline principal axis. So, as to get different effects in terms of the induced birefringence, which will form a three dimensional travelling volume rating, phase grating and that will cause this diffraction of optical beam and we will consider those cases in the later occasions how this modulation in the refractive index by the propagation of acoustic wave can be utilized for different cases of diffraction namely the Raman-Nath diffraction and Bragg type of diffraction.

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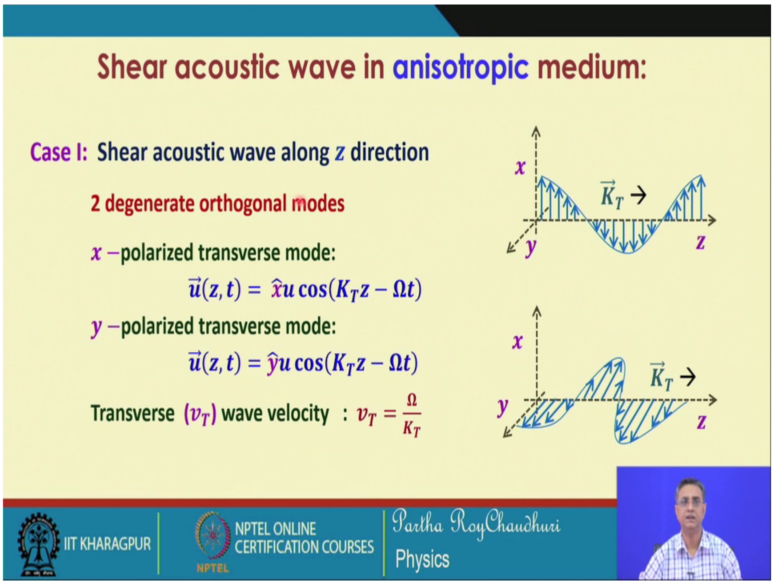

So, we first consider a shear acoustic wave in an anisotropic medium, where the wave is propagating along the z direction and the wave is polarized along y axis, but it could be as well the wave is propagating along z direction, but it is polarized along y direction. So, these are the two we have seen these are the two degenerate modes degenerate modes of these acoustic waves and they are represented by this equation wave equation, this one for the x polarized mode and this one for the y polarized mode, where this K T is the propagation constant of the acoustic wave and omega is the frequency of the acoustic wave and therefore, the wave velocity of the acoustic wave will come from this relation.

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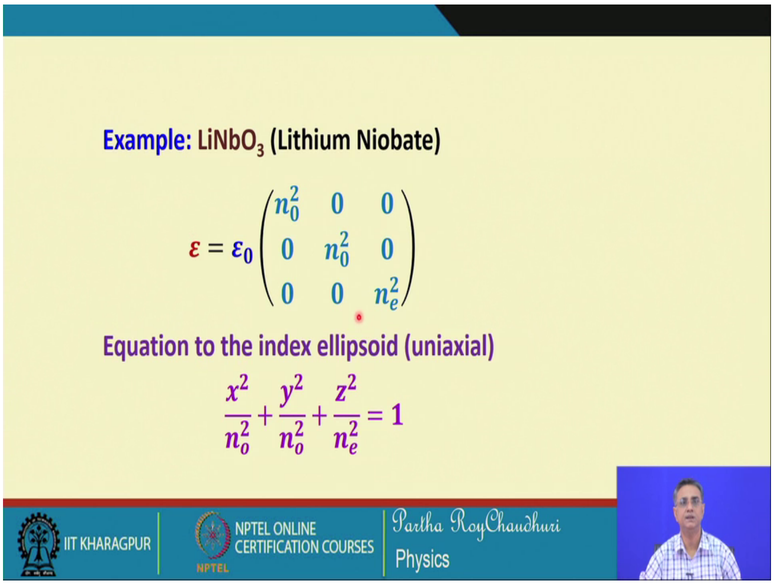

So, now we will consider the example of lithium niobate which turns out to be a very useful optical crystal, and it has many fold applications and for that lithium niobate which is naturally a uniaxial crystal, you have the permittivity tensor in the principle axis system like this, that is n 0 n 0 and n e these are to represent the ordinary refractive indices and this is to represent extraordinary refractive index.

So, the equation to the index ellipsoid of this uniaxial lithium niobate medium will be given by this equation where this is under the principal axis system the refractive indices.

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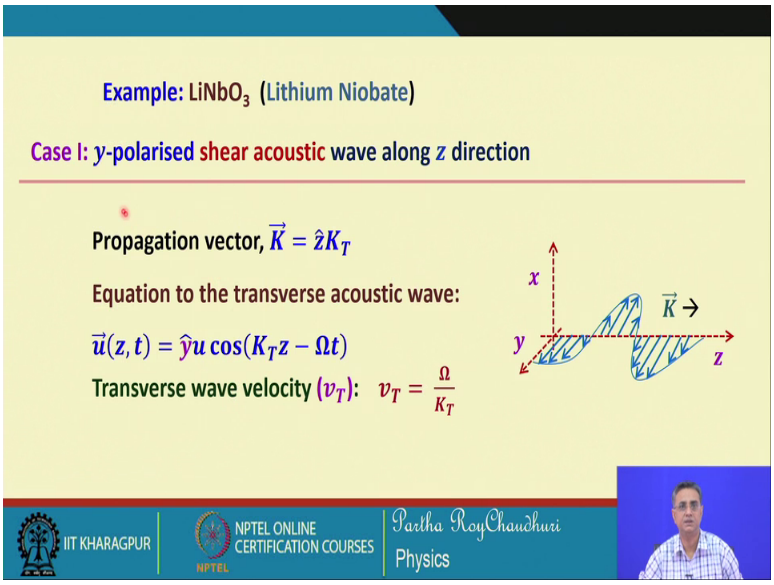

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So, this case that we are going to discuss is the y polarized acoustic wave that is travelling in the z direction, and you have this propagation vector for this acoustic wave will be represented by z unit vector and K transverse along z. So, this is the magnitude equation to the transverse acoustic wave is given by this unit vector y into u represents that it is y polarized and the transverse wave velocity is given by this. And these cases we have seen earlier also and by now it is very clear.

So, this is also to remind us that if the wave propagation involves z and y that is it is propagating along z direction and polarized along y. So, this y and z in the case of transverse wave in the case of shear wave only this y and z component y and z component y z or z y this component of the strain matrix will be non zero and rest all others will be. So, all the normal strain components are 0 because this is a shear wave and all other components they do not involve they will be equal to 0.

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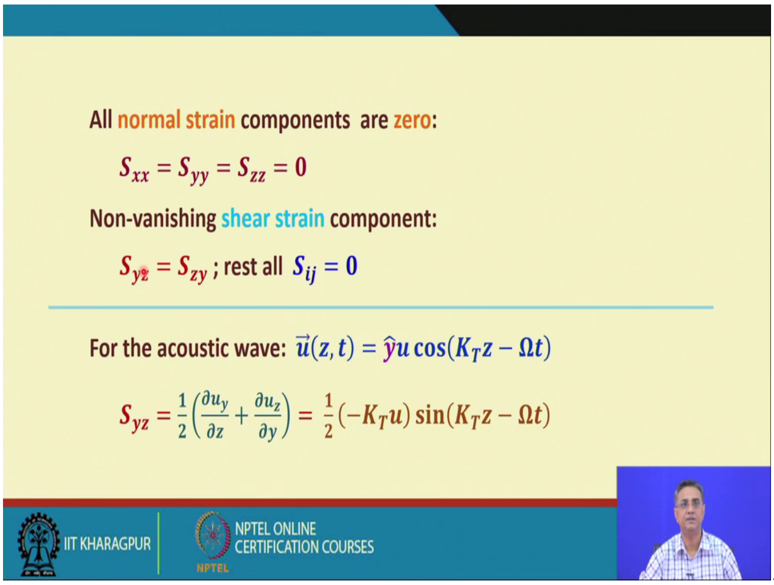

So, we have only one non vanishing strain component strain element that is S yz equal to S zy and for this acoustic wave very straightforward to obtain this S yz or S zy which is symmetric element symmetric tensor element. So, therefore, if we calculate this y variation with z and z variation with this one of them will be surviving. So, this will give you this quantity that is S yz and we call this is the amplitude of the strain amplitude.

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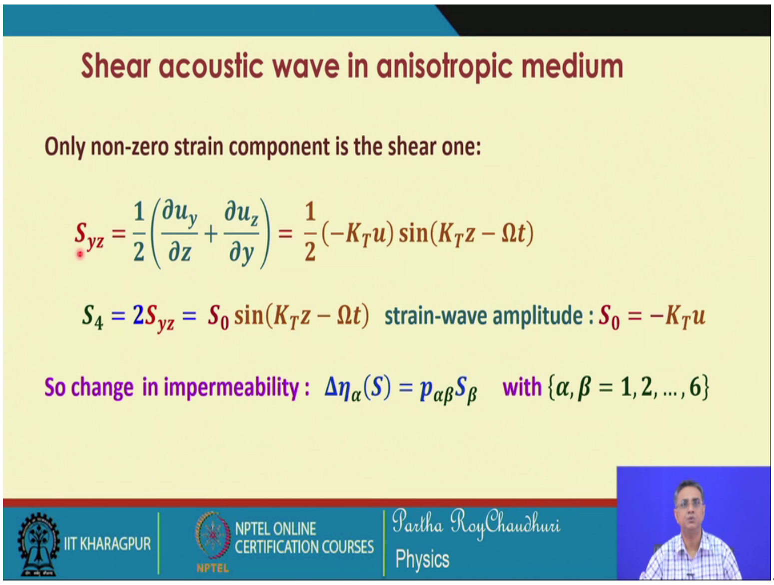

Therefore S yz equal to S yz is actually the fourth S 4 element S 4 element of the strain matrix. So, S 4 equal to twice y z is given by this, where S 0 is to represent the amplitude of the strain wave this minus side does not matter because where to start from whether it is it has a negative peak or the positive peak. So, it does not make any difference.

So, therefore, the change in the permeability in the compact notation we can write that delta eta alpha as a function of the strain can be written as the photoelastic coefficients and the strain matrix.

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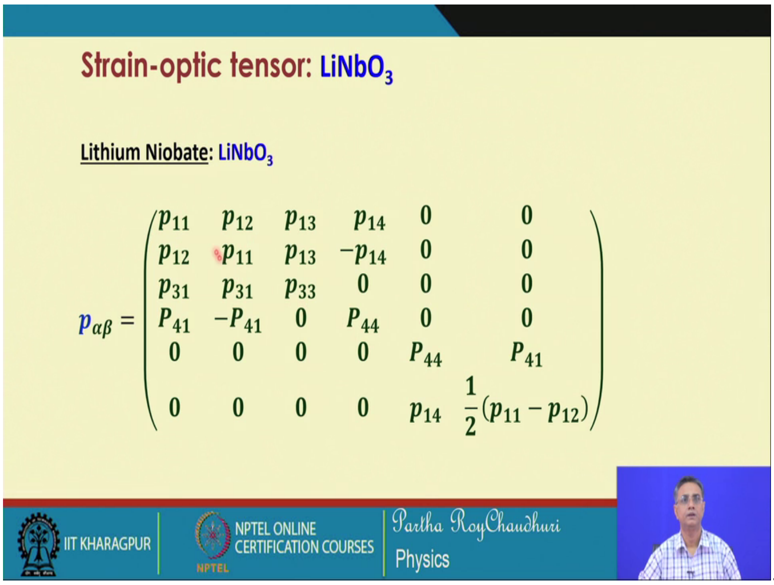

Therefore we can obtain the change in the impermeability from the photoelastic tensor, which is for the lithium niobate crystal the one which we are considering is given by this quantity, this tensor we have seen this previously also and this time we will be using this column only I will see though. So, this is your from here, delta eta alpha as a function of S we write here.

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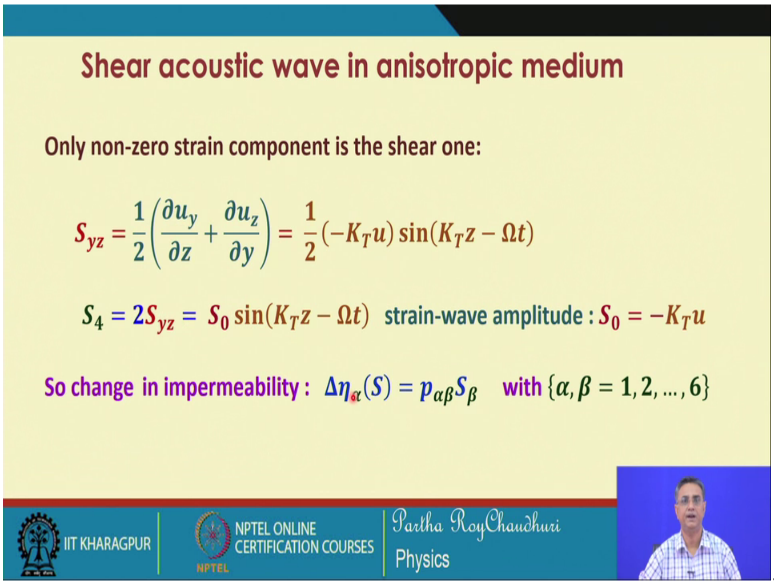

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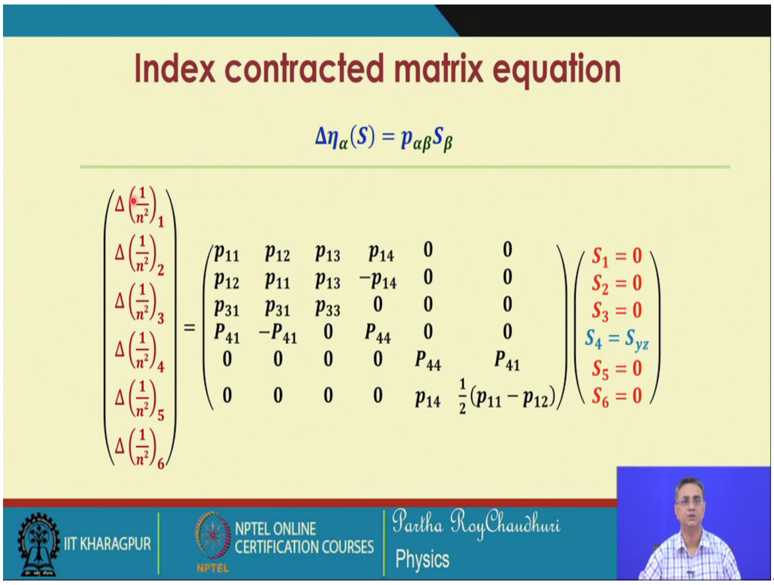

So, this is the change in the impermeability as a column vector and this is the photoelastic tensor, electro optic strain optic tensor for the lithium niobate crystal and this is the. So, you have seen that only this S 4 S yz is the non zero component. So, this will effectively provide this will effectively give you only these column as the non zero elements of this.

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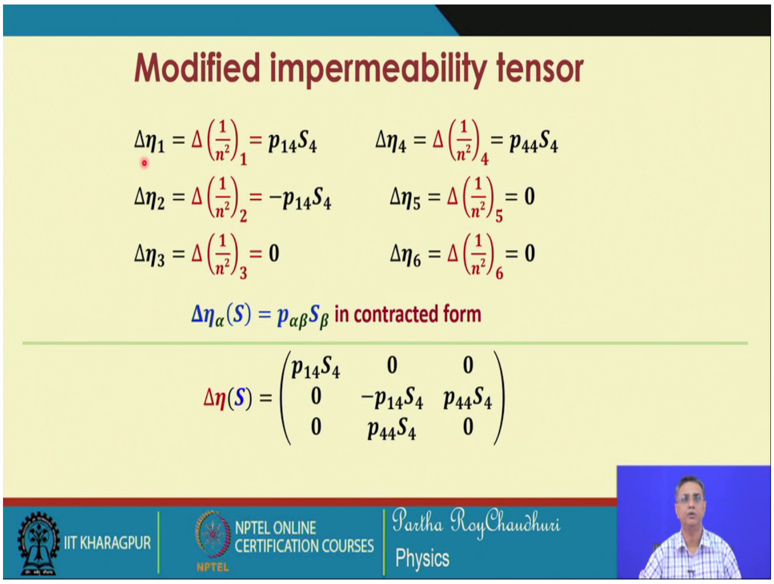

So, we can write from here delta eta 1 is equal to P 14 S 4 delta eta 2 this will come from here P 14 minus P 14 and this will be P 44, you can see from here P 14 S 4 minus P 14 and this eta 4 will be P 44 S 4 which is very clear from here P 14 S 4 minus P 14 S 4 P 44 S 4. So, these are the only non zero elements of this delta eta tensor therefore, delta eta of S we can write in this form in the contracted form otherwise; it will represent a 6 represent this column vector.

So, this is now 3 by 3 matrix element, where you have P 14 S 4 minus P 1 and 4 and this is the only cross term because these are the two quantities, which will be the diagonal elements first two y x and y z diagonal element is 0, but this fourth quantity that cross of diagonal element this will be non zero rest all of them are 0.

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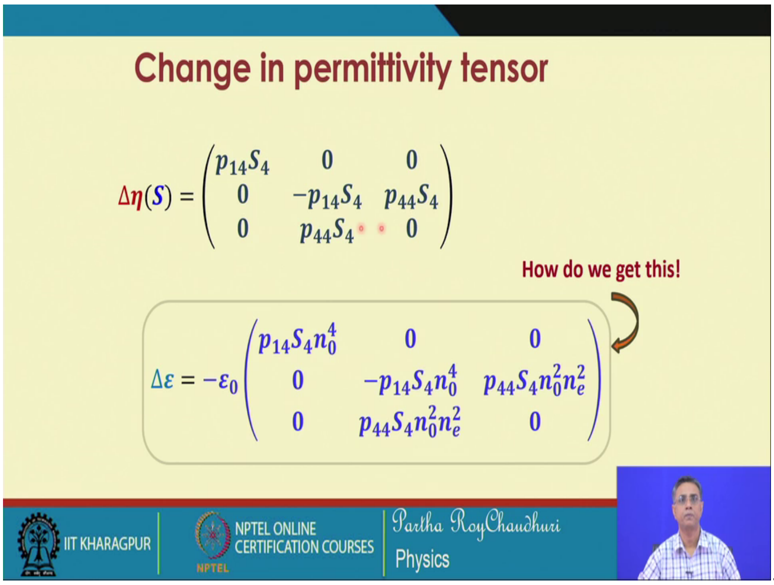

Therefore delta eta S delta eta S of this change in the impermeability tensor, will lead to this change in the permittivity tensor, we will get this equation.

So, how do we get this form because once we have this hand in hand then we can calculate the refractive change in the refractive indices and the birefringence.

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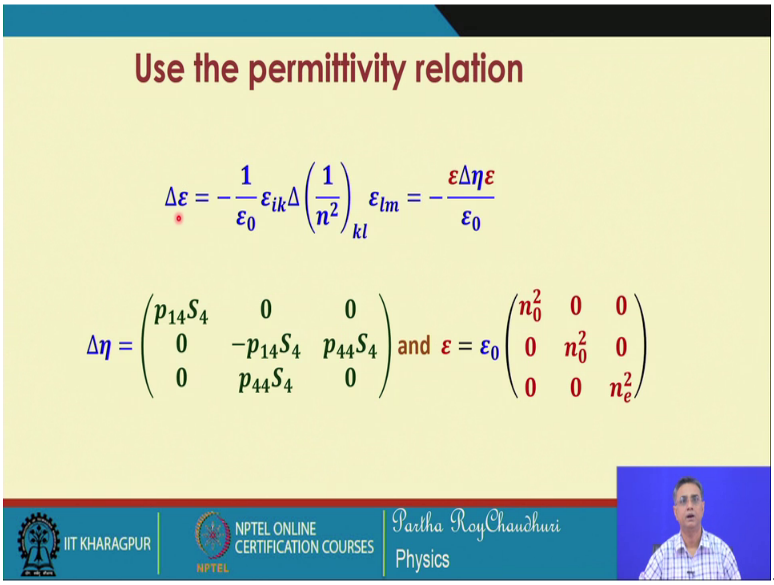

Therefore from that this is the relation that, we have used earlier also delta eta 1 upon eta eta ik and delta eta eta lm. So, this expression if we evaluate, so, delta eta is already known to us from here this is delta eta and then we will post and pre multiply with this permittivity matrix this one and this one.

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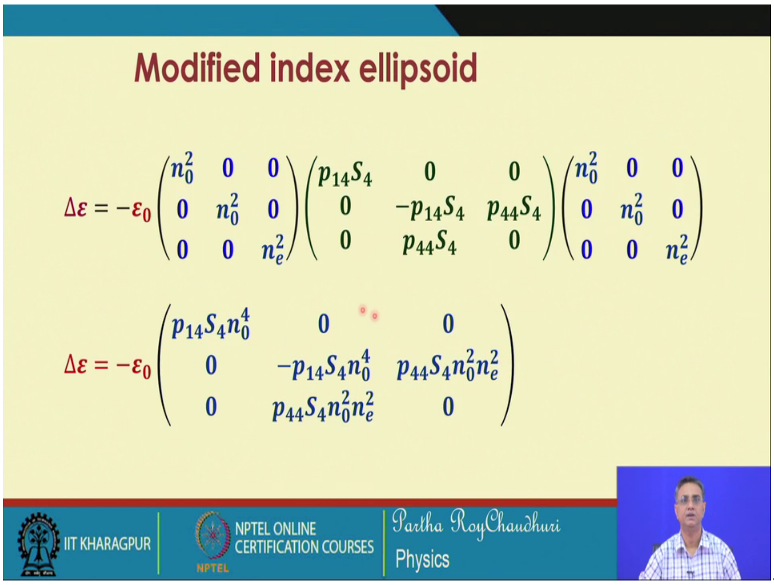

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So, which will result in this multiplied by this 1 by epsilon naught. So, this epsilon naught will appear twice, so it will be square and there is 1 in the denominator as a result we will get the change in the permittivity is of this form.

Now, this permittivity change in the permittivity tensor it contains two diagonal elements and two off diagonal elements these are because of the cross term.

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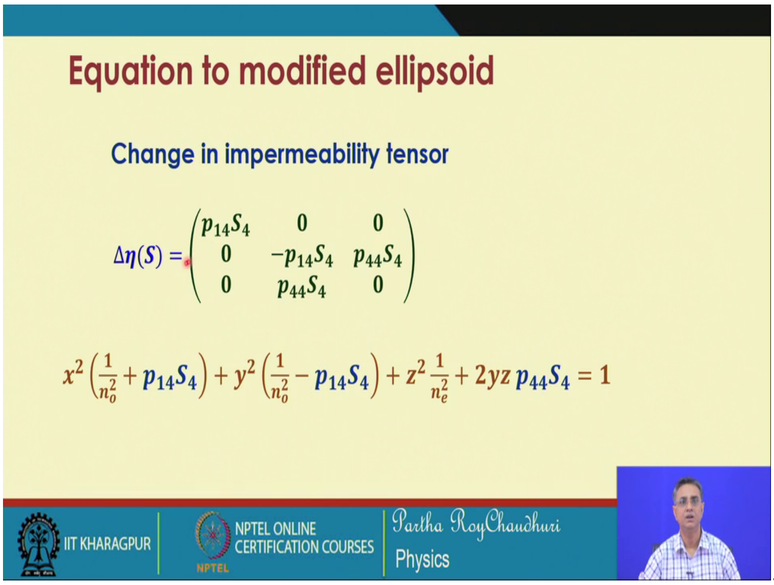

So, we can write from here the index ellipsoid in this form this x square by 1 upon n 0 square which is n o square which is the original, refractive index plus this and 1 by n 0 n o square minus this quantity and this two will go to the to the off diagonal inner to form this cross term that is y into z multiplied by this, which is also very simple and straightforward we have seen earlier also.

But this Z is not affected Z square term is not affected because the change contribution is 0 in this case.

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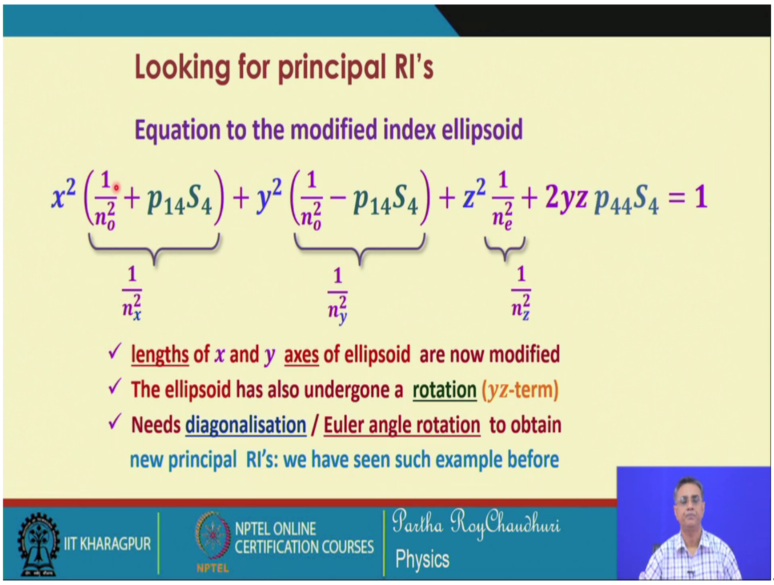

Therefore, now we can look for the principal axis principal axis refractive indices so, for that from here from this equation either we can use. So, this is the form x square into this we can called this entire quantity as n x square this quantity as n y square and this quantity if we change just for the completeness we can write this equal to n z square and you have this.

So, you can observe from here that the lengths of the; because these are the quantities which are to represent the length of the semi axis of the ellipsoid along x y and z direction. So, you can see that the lengths of x and y axis of the ellipsoid are now modified only these two are modified, but there has not been any change in the z axis along the z axis it remains the same.

So, once and also there is a cross term the ellipsoid has also undergone a rotation because it involves a cross term with y and z therefore, to bring it to the principal axis system either one can provide a Euler angle rotation to get the new principal refractive indices or one can do this diagonalisation of the relevant matrix to get the eigenvalues and the eigenvectors.

So, we will look at as an exercise we look at both the situations for this and how we can solve for extracting the change in the refractive index that is delta n x delta n y and delta n z.

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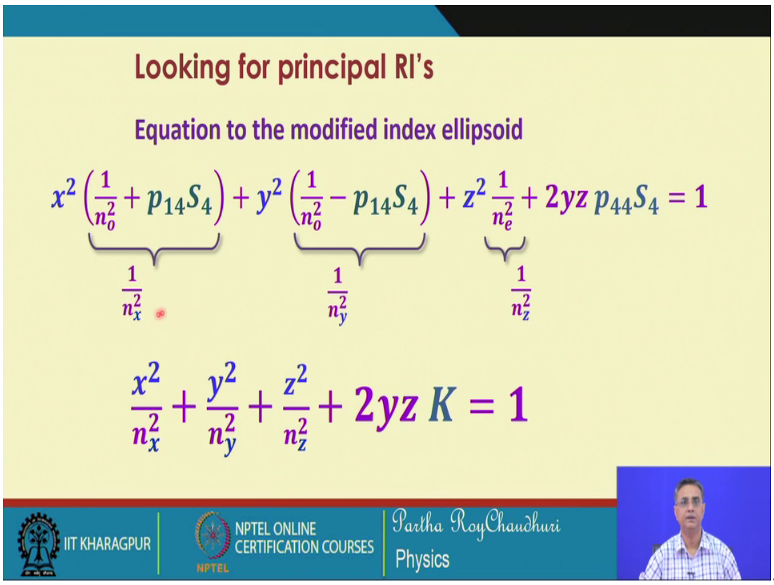
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So, to do that let us first rewrite this ellipsoid equation using this n x n y and n z. So, it takes the very simple simplified form of this n x square n y square n z square and twice y z k. So, this is the change index ellipsoid you will have to bring it to the principal axis system.

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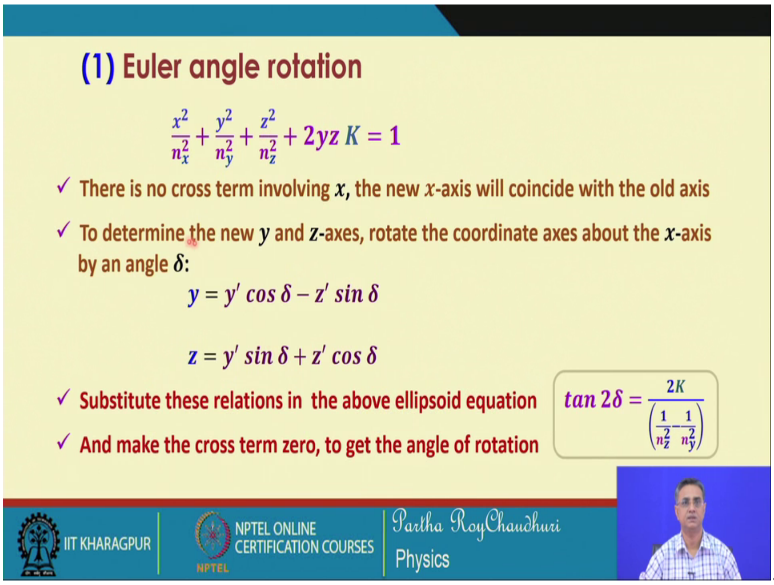

So, as an observation you can see that there is no cross term involving.

So, first we will consider this Euler angle rotation and will provide rotation to the coordinate access to make it coincide with the principal axis system. So, we can see that this cross term does not involve in x and as a result the new x axis and the old x axis they will coincide and to determine the new y and z axis a rotation will be required and this rotation by an angle delta will come from this relation. So, y equal to y dash cosine delta minus z dash sine delta and so on for the z also.

Now, if we substitute this for y and z back into this equation and then if we make the cross term y dash, z dash because after you substitute for y and z there is no y z it will be y dash and z dash. So, it will be x y dash z dash and y z y dash and z dash. So, if I make this cross term equal to 0 that will give me a condition that will lead to this rotation angle rotation angle tan of twice delta is equal to this. We will see this expression from a tutorial discussion that for any general index ellipsoid a x square b y square plus c z square plus d into x y equal to 1 of this form how we can get this relation you will see that.

So, anyway here the after substituting this relation in the above equation and if we make the cross term equal to 0, we will get the required angle of rotation of the coordinate axis about the x axis will be provided by this. So, this is 1 by n z square minus 1 by n y square.

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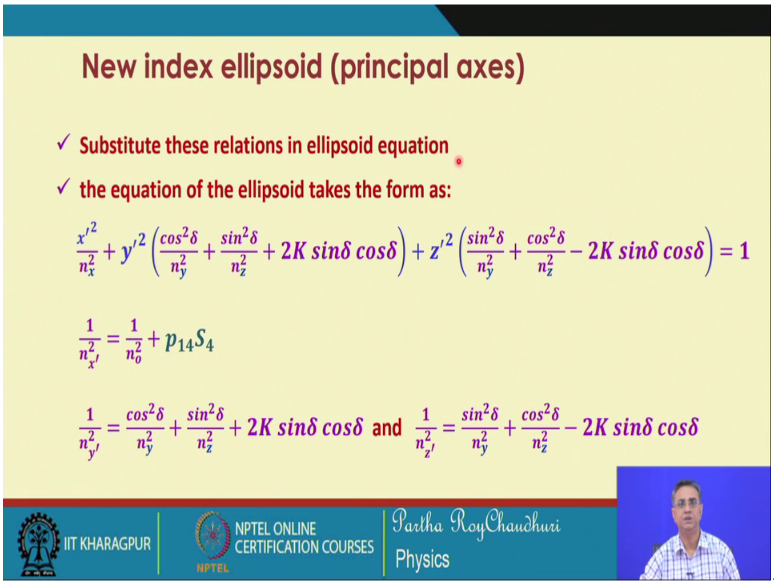

So, after substituting this in the equation the new equation will be of this form because you have replaced y by this and z by this therefore, you get a new equation and from here you can see that n x square or n x dash square the new because it does not undergo any change apart from the one which was because of the change in the access length along x the semi access length of the ellipsoid. So, this quantity will be representing the new 1 by n x dash square and this quantity will be representing this new 1 by n y dash square and similarly this will be for 1 by n z dash square. So, these are very straightforward and a direct outcome of substitution of this y and z by y dash and z dash equations.

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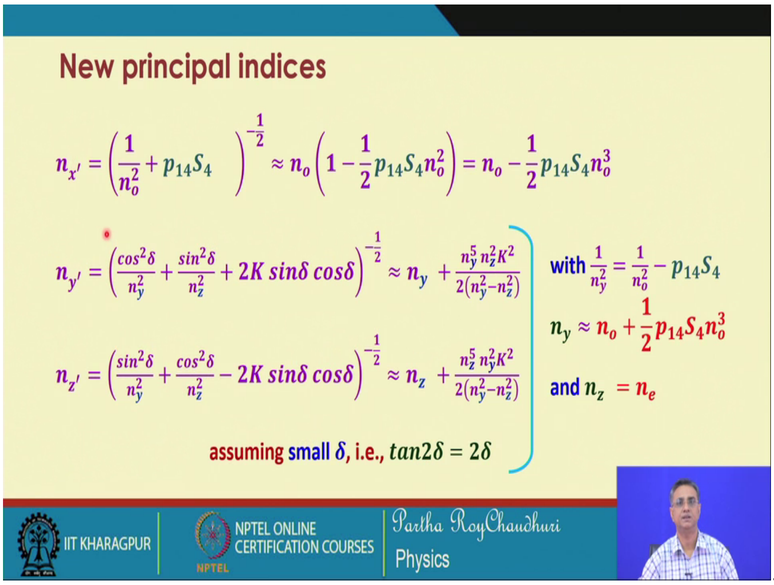

So, therefore, from the first one this one it directly gives you this relation n x dash equal to you can write to the power of half and making a binomial expansion because P 1 4 S 4 is as a small quantity and then we can write it and this example we have seen earlier. So, we will remember this and when we apply this diagonalisation of the matrix we will end up with this equation and this one is little involved expression n y dash and n z dash. So, this after this binomial expansion we can write in this form this is the form and once we know the values of n and n o n e P 14, S 4, P 44, S 4 we can evaluate this.

New n y dash and n z from here and this has been possible only after substituting that tan twice delta equal to 2 delta it requires a little involved calculation where, we actually this delta is very small for this particular case and we can show that it is of the order of a few degrees and therefore, we can make may be less than degree. So, we can make this substitution and we end up with this n x this is for n y and this is for n y and this is for n z; n z remains the same this is to represent n y, which will be represented by this and n z is nothing, but n extraordinary this we have seen earlier.

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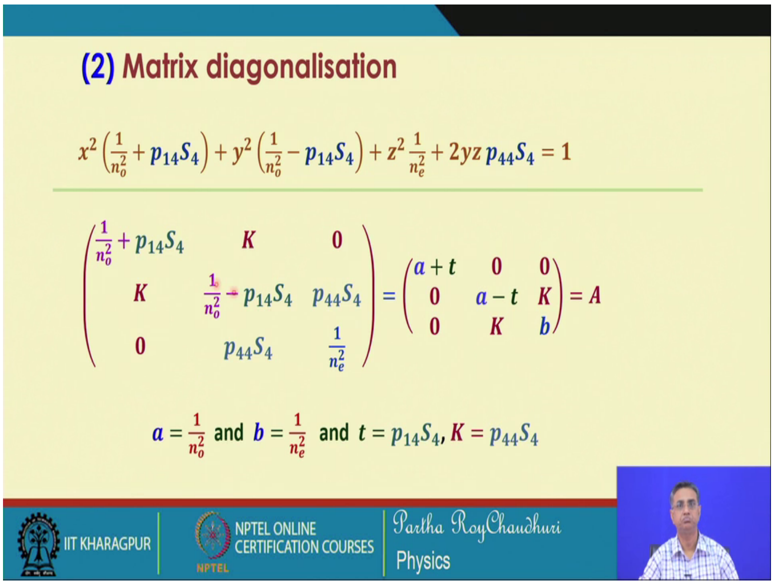

So, now we can evaluate with these results in hand now we can evaluate the same, using this matrix diagonalisation. So, this Ellipsoid equation will lead to this matrix impermeability matrix, which will be 1 by n o square plus p 14 S 4 this one will come from here these are the diagonal elements this would be also the diagonal element and this p 44 S 4 will occupy this position of the that is that is your y z position of the off diagonal element y z or z y position.

Now, in place of just two just for the sake of simplicity we substitute for this as a and for this as b and this is for t that is p 14 S 4 will come here p 14 S 4 will come here and K equal to this, these are the substitutions if we write. So, you can just get this matrix which has to be diagonal let us call this is equal to a this matrix.

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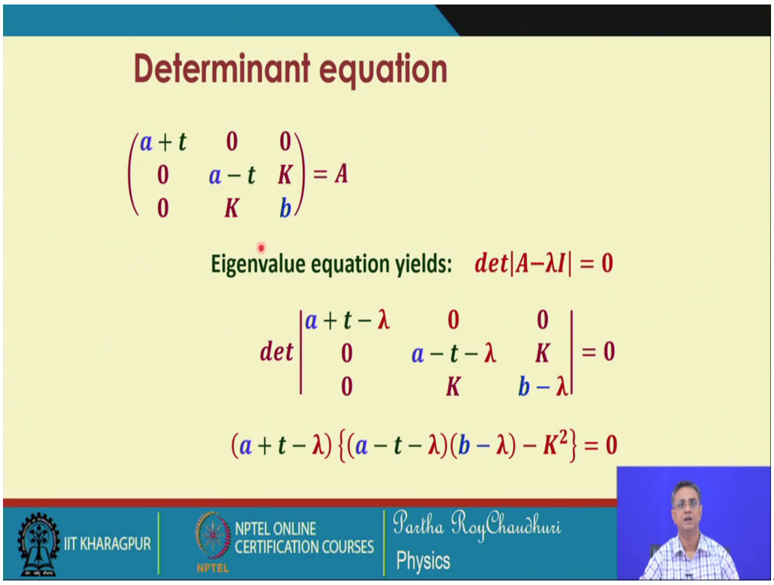

And the Eigenvalue equation will be this polynomial determinant of this equation will be equal to 0 that is well known and from here we can write this determinant equation this quantity into this into this minus K square, that is what is written here and from here this part is straightforward because if this quantity has to be equal to 0 which will represent one of the eigenvalues let us call that is equal to lambda 1.

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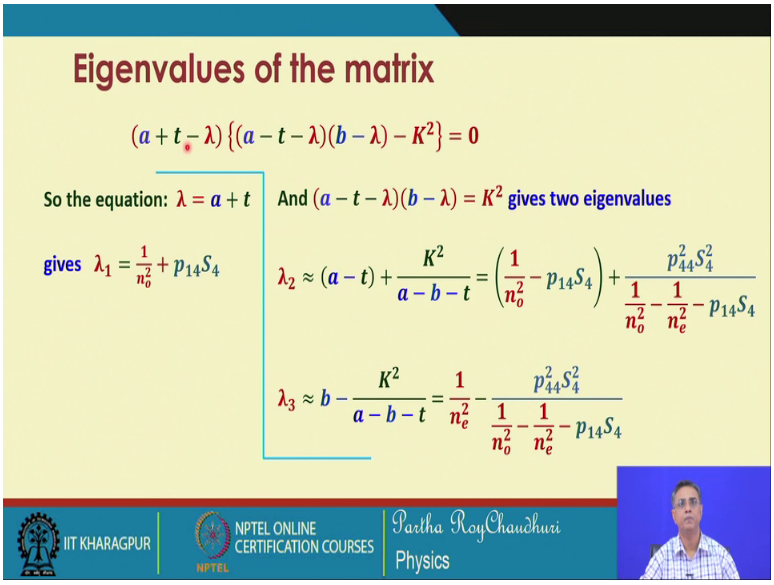

So, this lambda 1 will be a plus t from here. So, this a plus t will give you 1 by n o square plus and that is exactly what we have seen earlier, the other one will yield two eigenvalues lambda 2 and lambda 3 and if we substitute this values we can do a small binomial expansion for this quantity, because if I write the solution of this equation then we can see that we can write in this form with and therefore, this a minus b is to represent this 1 by n square minus p 14 S 4 plus this quantity K square by a minus b t will come from here will come from here and for lambda 3 this will be equal to approximately equal to beta b minus K square by a minus b into t. So, this will lead to this result.

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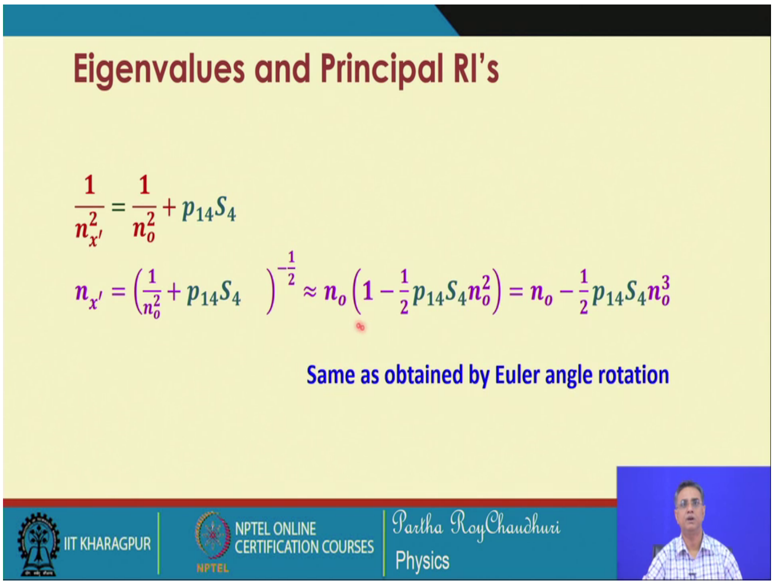

So, we can actually calculate all the values. So, 1 by n x square this quantity we have seen earlier after doing this binomial expansion and retaining only the first term, we can show that the change in the n x the refractive index along the new principal axis along x is this which is the same as obtained by this Euler angle rotations and for n y dash and n z dash we have this expression.

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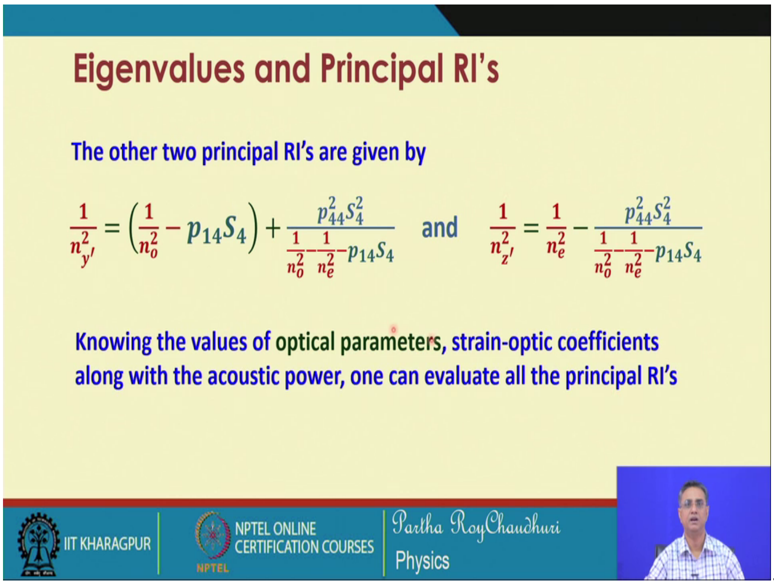

So, we have been able to calculate all the new principal refractive indices.

So, knowing these values of the optical parameters that is n o and n e the strain optic coefficients of the medium that is lithium niobate and together with the acoustic power we can calculate this S 4 and we can evaluate all the new induced refractive indices of the crystal in presence of the acoustic wave.

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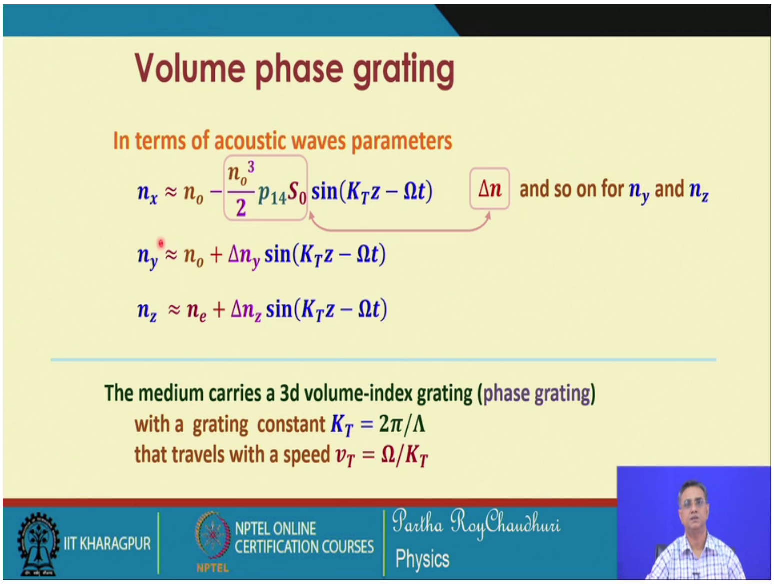

So, in terms of the acoustic wave parameters we can now write the first one which is already we have shown. So, this one this quantity is to represent the change in the refractive index on top of the of the refractive index the base value, which is there in absence of any acoustic wave.

So, this is the change in the peak value of the refractive index because of the presence of the acoustic wave. So, it can vary up to plus minus delta n and similarly from the other two we can also after doing necessary calculation we can show that this will be n o plus delta n y and this will be n e plus delta n z these are the original refractive indices in absence of the acoustic wave and these are the changes due to the presence of the acoustic wave.

So, as a result we can see very clearly that along all the three mutually perpendicular directions x y and z in the new principal axis system we see that there has been a change in the there has been a modulation of the refractive indices which is at par which is inconsistence with the with the acoustic wave this travelling acoustic wave. So, the medium carries a three dimensional volume index grating phase grating which is given by which the peak value of the refractive index, peak change in the refractive index is given by this is a sinusoidal grating, travelling grating and the grating constant will come from this K t equal to twice pi by lambda where lambda is the wavelength of the acoustic wave.

And the grating travels with a speed of this. So, we have been able to characterize the induced refractive indices change in the principal axis system in presence of the acoustic wave, and that completes this discussion we will be using this delta n sine of omega t or sine of K into z as the periodic perturbation in the case of Raman Nath and Bragg type of diffraction. So, this result will be carried forward to the discussion of the optical beam diffraction in the next appearance.

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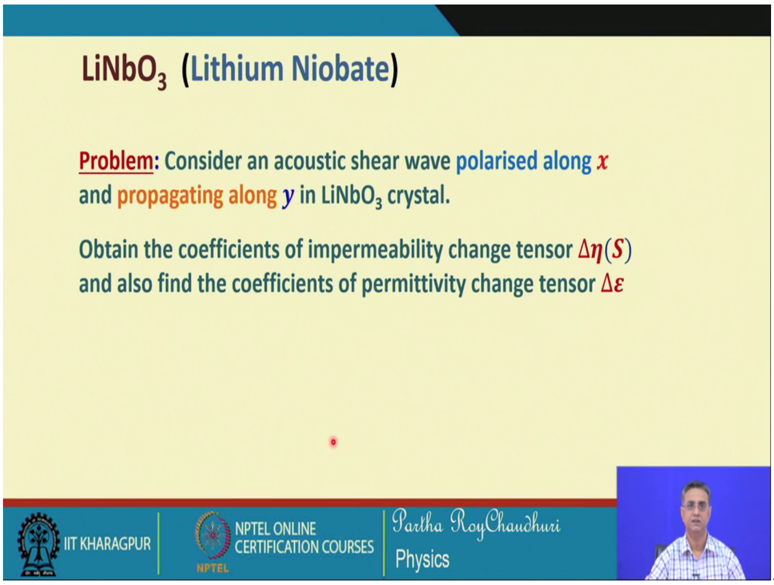

Now, it appears in the form of a problem let us consider, but this is in a different configuration slightly different configuration consider an acoustic shear wave that is polarized along x and propagating along y in the lithium niobate crystal, and to obtain this coefficient of impermeability change.

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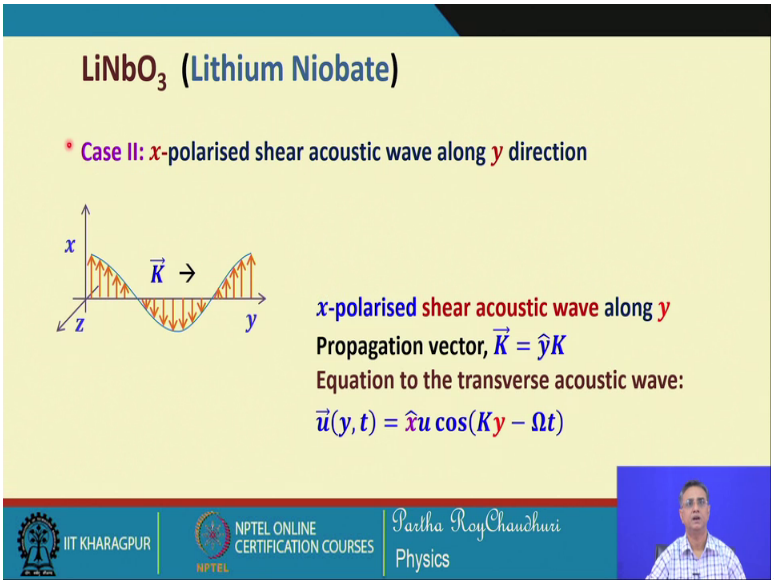

We find that this is the case where your shear wave is polarized along y and it is propagating along polarized along x and propagating along y direction will be given by this equation.

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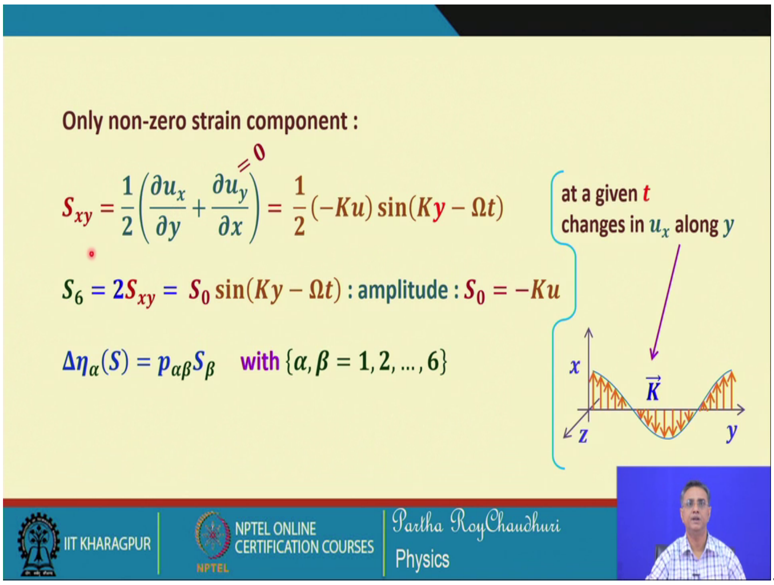

And then only the non zero so, looking at this because it involves x and y. So, the strain shear strain component will involve only this non 0 quantity rest all of them are 0.

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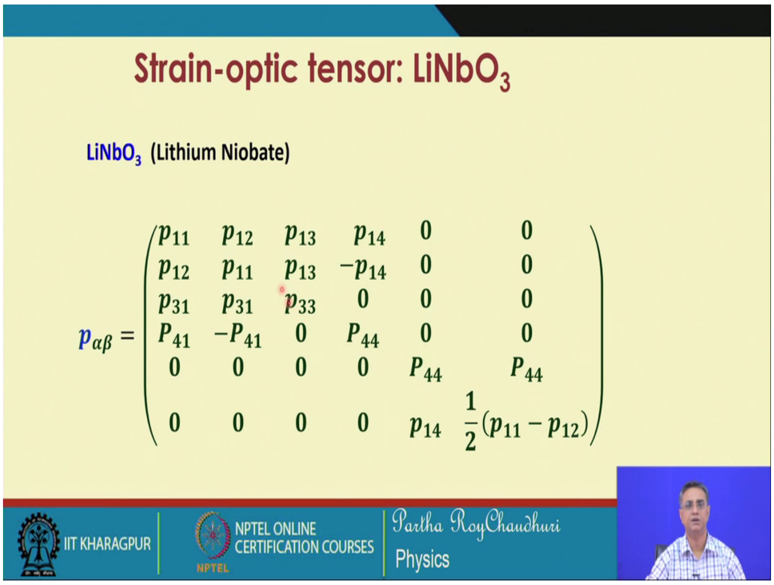

So, it is now again straightforward. So, we will apply this strain optic tensor of lithium niobate for this therefore, we get the changes.

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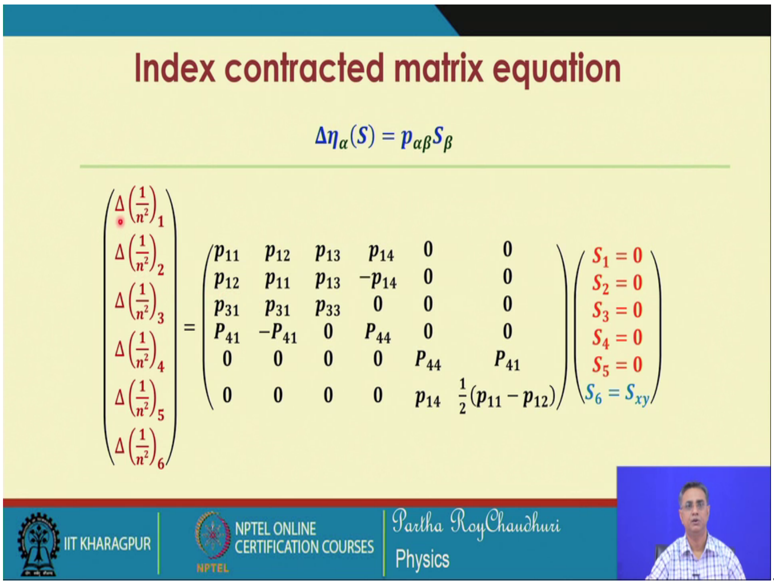

So, in this case only the last so, this last column will be non zero will contribute to non zero elements of this change in the impermeability.

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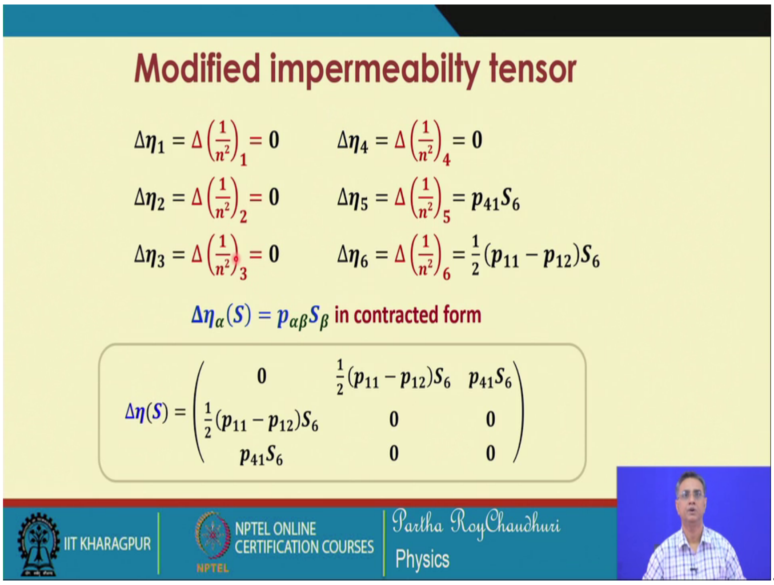

And therefore, we can write all of them are 0 only an eta delta eta 5 and delta eta 6 will be given by this and they occupy both of them occupy the off diagonal position this is fourth position. So, this is fifth position and this is the sixth position.

So, this is delta eta 6 this is delta eta 5 and interestingly one can see that both of them are symmetric and of course, this would be symmetric. So, therefore, this requires in this case there is no change in the square terms of the ellipsoid, only the cross terms which involve x y plus x z, but no y z. So, there will be 2 x y into this and 3 will be two x z into this in the index ellipsoid.

So, having known this we can use the permittivity relation to calculate the change in the.

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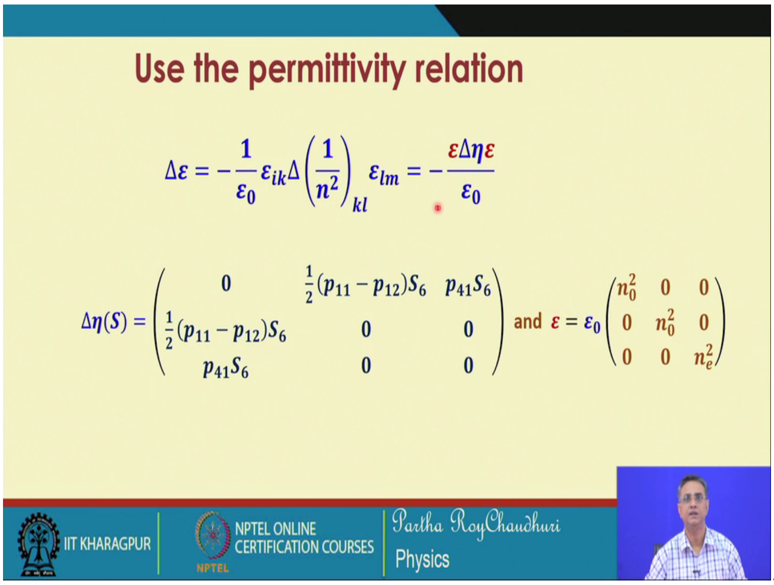

To calculate the change in the permittivity so, this will lead to from here and here we post multiply and pre multiply this equation and from there, we can calculate the change in the permittivity as this.

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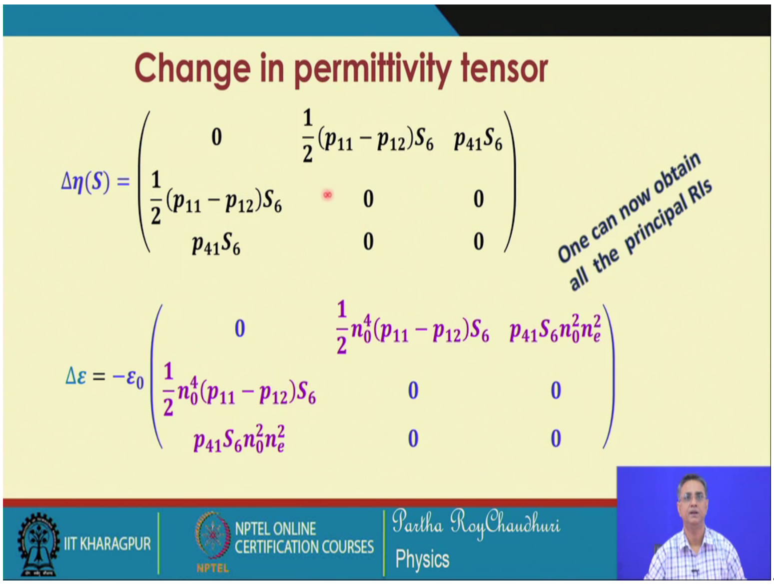

So, once we have this then we can diagonalize the matrix or we can provide that Euler angle rotation and we can calculate the new refractive indices in the principal axis system, because of this configuration the propagation of the acoustic wave in them in an anisotropic medium shear acoustic wave in the medium.

So, we have considered two such cases and these two case actually provide the guideline the recipe how to analyze the various orientations, which will be useful to design different you know target specifications of the of the acousto optic modulator which we will be discussing later.

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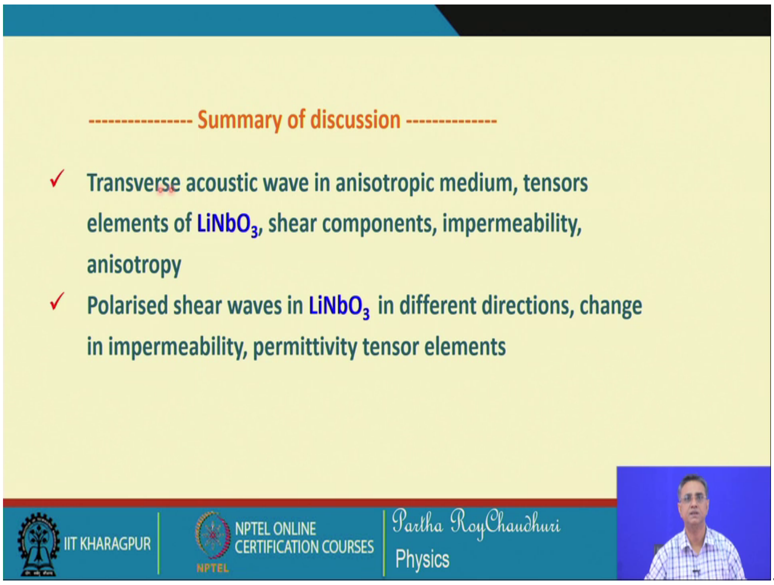

So, we have discussed this transverse acoustic wave in an isotropic medium, shear waves and two different configurations and we have learnt how to calculate the changes in the refractive indices once the acoustic wave is propagating in the wave for various configurations of the wave with respect to the Crystallined access system.

Thank you very much.