Modern Optics
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Lecture - 44
Acousto-optic Effect (Contd.)

We were discussing Acousto-optic Effect. So, far we have discussed this the elastic deformation in terms of strain and rotation. We will see that the effect of rotation is negligible and it is not included in that discussion of this. And, we will discuss more in details about the acoustic wave propagation in isotropic and anisotropic medium. We will see that the nature of the refractive index change will be different in the case of an isotropic medium. But in case of isotropic medium it is the same, but it will depend only on the direction of propagation of the wave.

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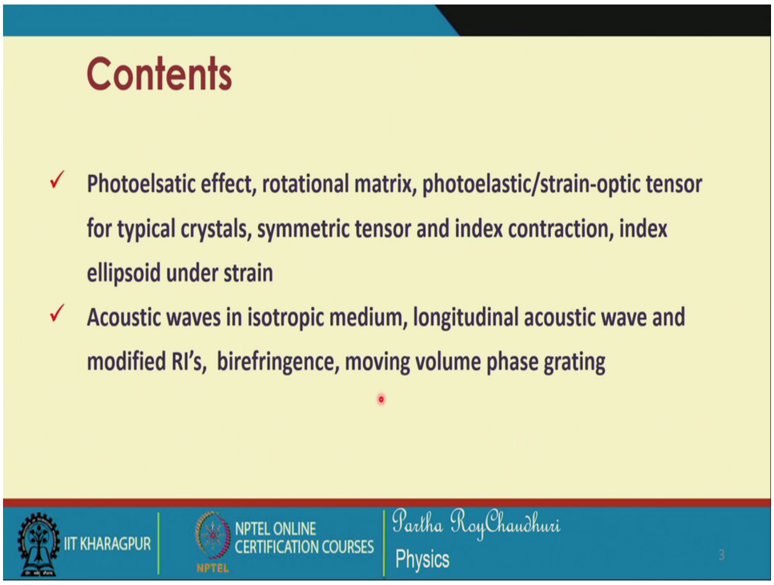

So, depending on that fact we have classified the discussion under this, that this rotational matrix why it is not included in the discussion. Then we will talk about the symmetric nature of the tensor which we have seen it is a follow up of that. And, then index ellipsoid under strain in the contracted form and from there we have also learned in between that how we can calculate the change in the permittivity from the change in the impermeability; by a relation which we will be using directly for the different case studies regarding the acoustic wave propagation, longitudinal wave and transverse wave for isotropic medium as well as for anisotropic medium.

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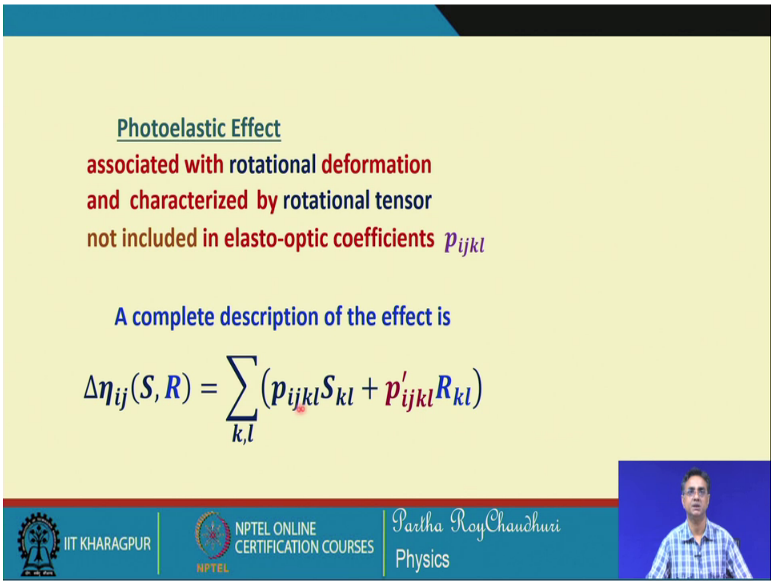

We will also look at the changes in the refractive indices, birefringence and the formation of the moving volume phase grating. So, this part will be more towards the optical interaction. So, photoelastic effect associated with rotational deformation which is characterized by this rotational tensor is not included in this expression because, this we expressed as equal to this only but, there is an additional term.

This is the complete description that the change in the impermeability, because of the deformation. Deformation is due to both strain and rotation or the other way the deformation can be described by both strain and rotation. As a function of the strain and rotation this change in the impermeability will be completely described by the change that is by the strain tensor and separately by the rotation tensor. So, this coefficient it is not included.

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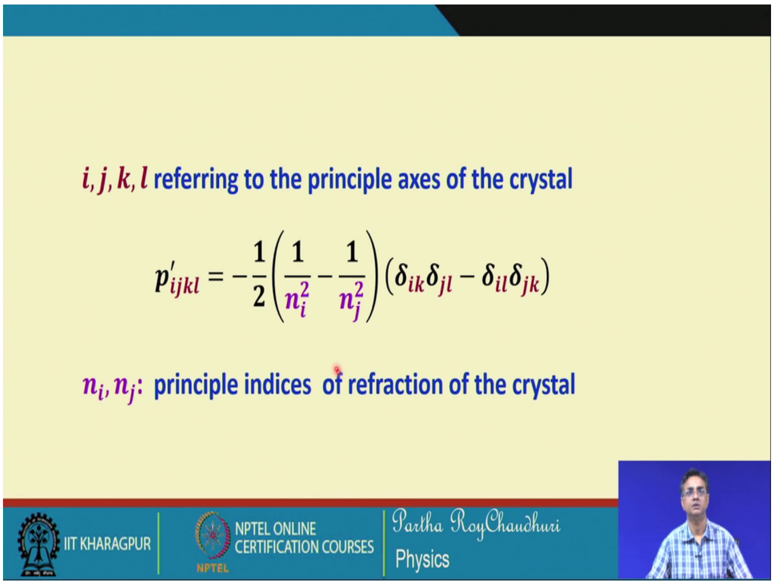

We will just quickly look at this fact. These are because, you have seen this tensor is not a symmetric, this is this quantity is not symmetric. And, p i j k l referring to this principle axis of the crystal we can write in this form; in this form n i j these are the and so, delta ik delta ji. So, this is the form we can represent this and n i n j they are the principle indices of refraction of the crystal. So, this is how this refractive index, rotation due to rotation this tensor will take the form.

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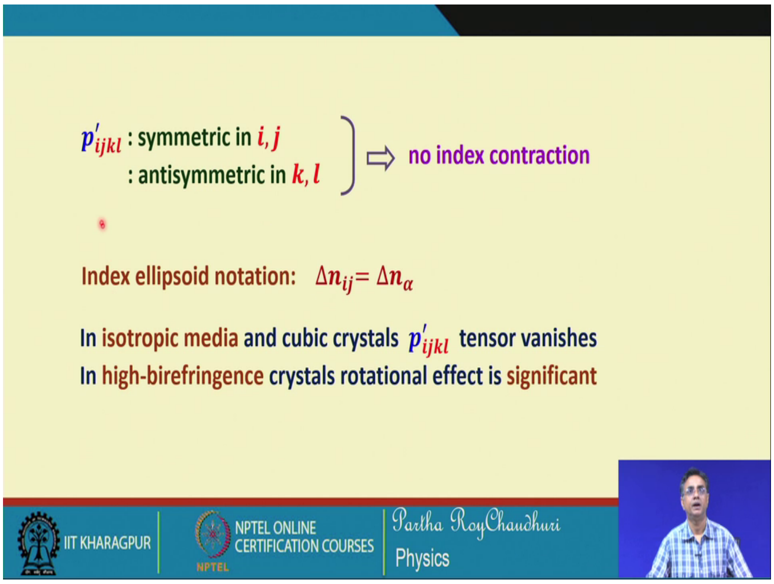

And this is symmetric in i j, but anti symmetric in k l. You can see from this equation this is symmetric in i j, but not symmetric in k l. So, the index contraction is not possible here. Even though the index ellipsoid notation this and in an isotropic medium and cubic crystal, this tensor actually vanishes. This does not come into play and rigorously for isotropic and cubic crystals medium this tensor is not included in the photo elastic effect.

But for high birefringence crystals this rotational effect is significant, but we are not considering that case. So, it is not included in our discussion. So, from now onwards we will be only working with this quantity and that is good enough to describe the acousto-optic effect for the cases that we will be considering.

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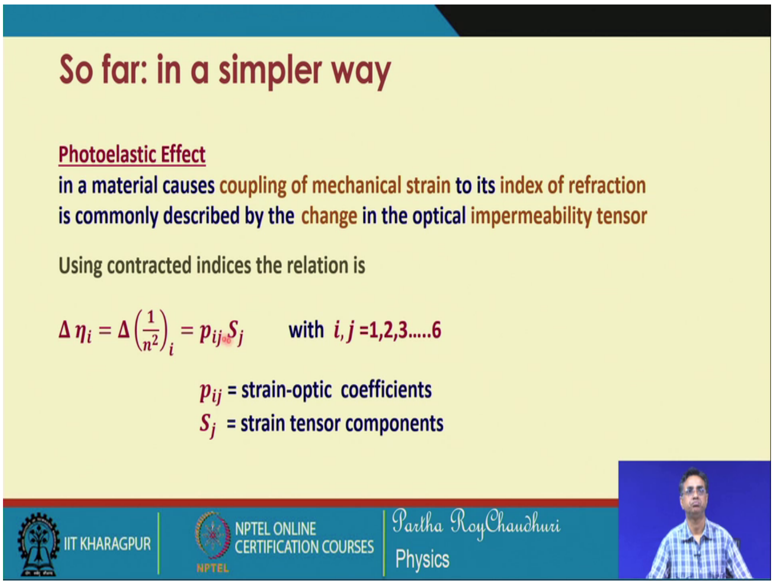

So, this using contracted notation we have seen that the impermeability can be represented by this. This is the strain optic tensor, this is the strain matrix strain tensor and these are used that repeated index convention. So, this is the strain optic coefficients, you have seen this is a 6 by 6 matrix and this is your 1 by 6 strain component matrix.

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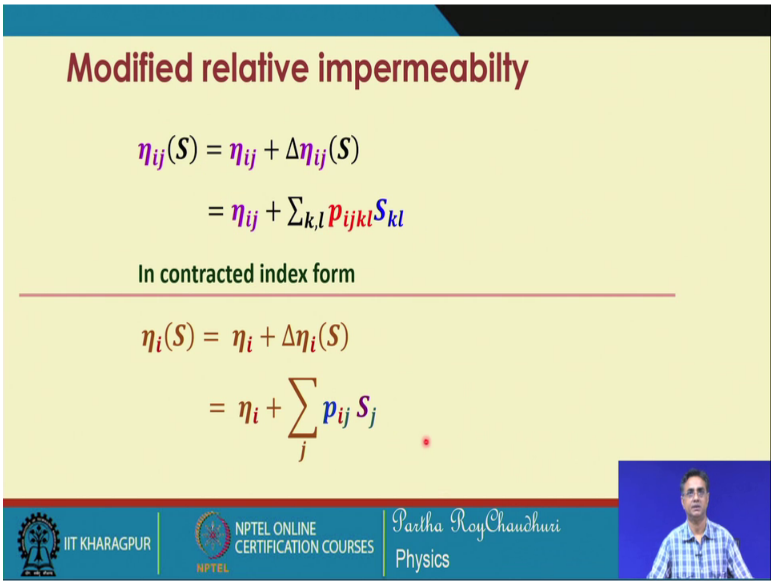

So, this equation now because we have we are only considering the strain matrix; so, you have eta ij equal to this impermeability without any field. And, this is the, if change in the in permeability which is coming due to the strain and in association with the strain optic tensor. In the contracted form we can write this eta alpha equal to eta alpha and this alpha beta S beta or ij whichever notation we would like to.

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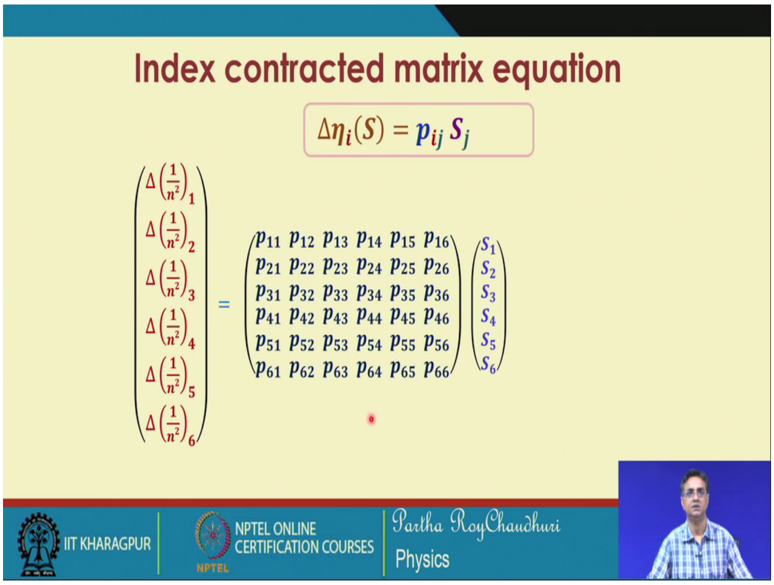

And this is the form that we have seen, that you have these diagonal elements here. And these are the off diagonal elements and because, they are occurring twice there will be a factor of 2, which is adjusted here. Here also these are the 3 diagonal elements of the strain matrix and these are 6, these are the representative of the 6 off diagonal elements.

And, now they are (Refer Time: 06:47) we have seen this thing earlier also and now we will apply this for the specific case of isotropic medium whose this photo elastic or strain optic tensor is known in terms of the non-vanishing, non-zero components of that matrix.

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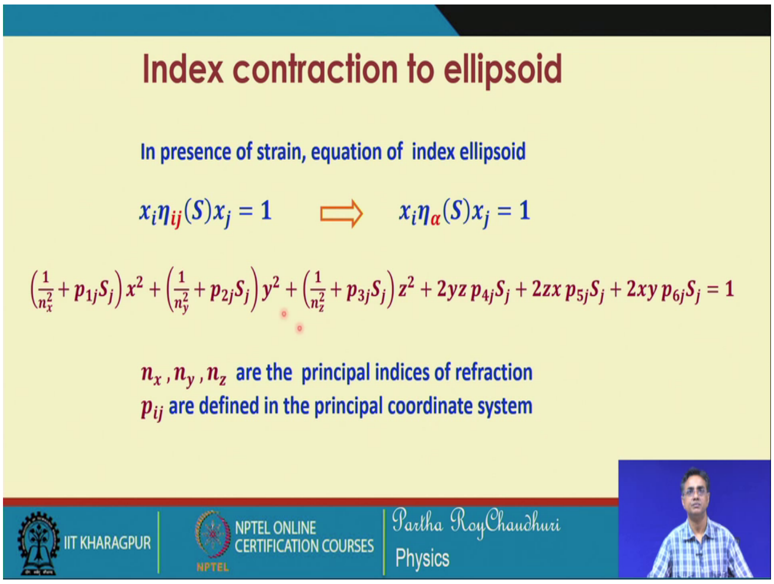

So, in presence of the strain you have seen that we can write this index ellipsoid in this form. Index ellipsoid in this form which is again contracted compact from of this and in the equation from ellipsoid equation from 1 by n x square plus this change in the impermeability delta eta. And, here is the change in the impermeability which includes all the components here also.

And, these are the change due to the off diagonal terms of the strain matrix that is the shear strain components. These are the longitudinal these are the normal strain component. So, these are the shear strain components. So, n x n y n z are the principal refractive indices here, these are the and p ij are defined in the principal coordinate system, this p ij.

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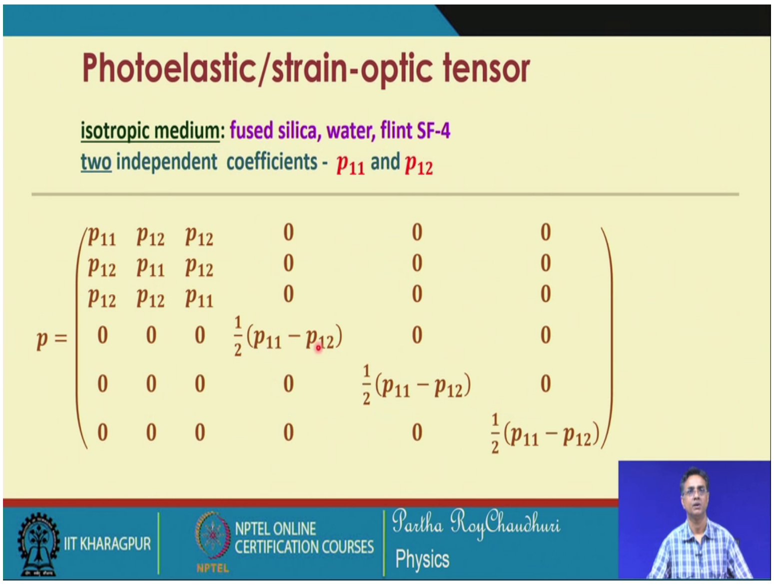

Now, that for isotropic medium these are mainly we recall that this fused silica, water, flint SF-4 these are the they follow this they have this kind of electro optic; sorry a acousto-optic tensor of this form. And, there are only p 1 1 and p 1 2 these are only two independent coefficients there. And, this is actually obtained from the symmetry consideration of the medium. Looking at the symmetry these terms are actually estimated; this form of the tensor is estimated and experimentally these values are calculated.

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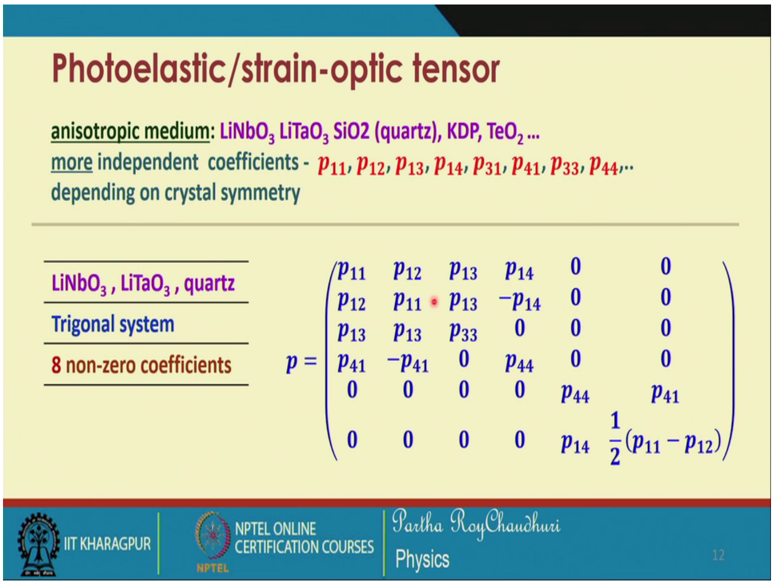

We have a list of the, we will refer to that list of the values of the coefficients. For an isotropic medium very well known is lithium niobate, lithium tantalate, then quartz, KDP, telluride oxide and, more dependent independent coefficients are required to represent the photo elastic tensor. In this case you can see a typical tensor which is which involves that 1 1, 1 2, then 1 3, 1 4, 3 1, 4 1, 3 3, 4 4. You have more number of independent coefficients in the strain optic tensor.

So, and it gives you more freedom to have different effect and influence of the acoustic wave propagating in different directions. So, that we will see we will consider some typical situations. So, these are the crystals: lithium niobate, lithium tantalate, quartz they are diagonal tri diagonal system and there are 8 non-zero coefficients in the strain optic tensors. And, these values are well known now in the literature.

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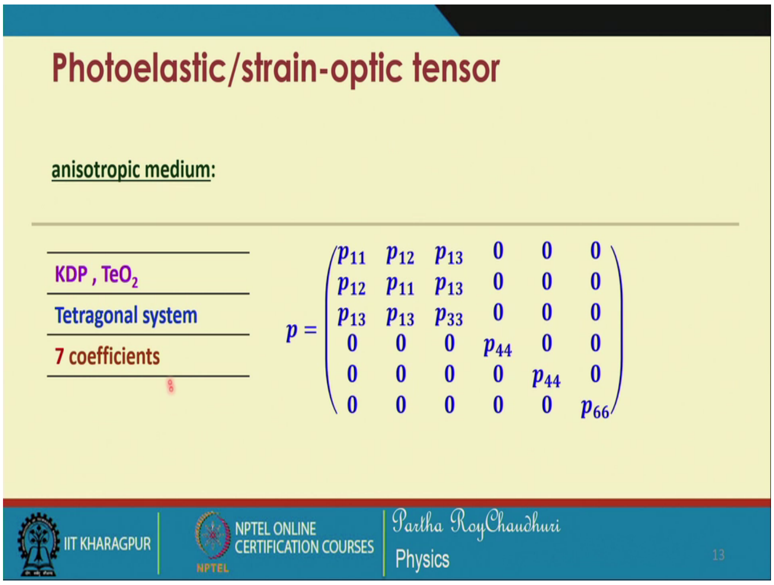

There are 7 coefficients in this in the case of this KDP and telluride oxide their tetragonal system and there are only 7 independent coefficients; you can see 1 2 1 1 1 3 3 3 4 4 4 4 6 6 3 3. So, there are only 7 independent components.

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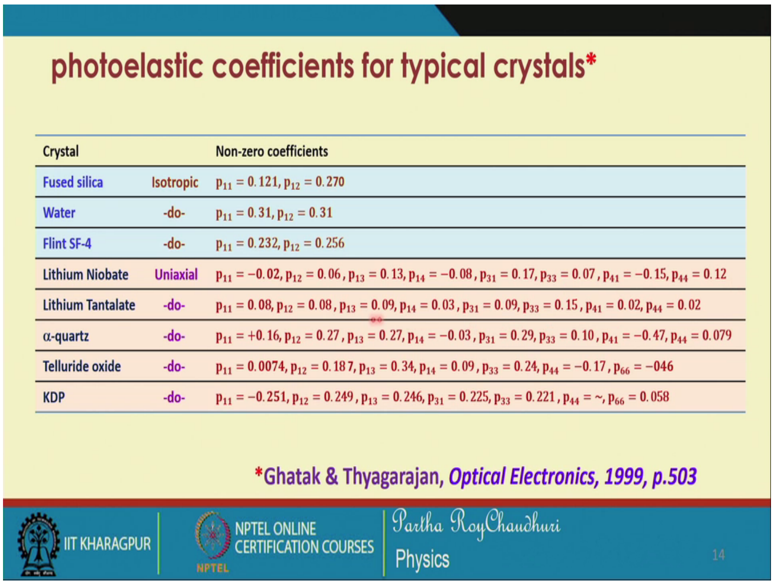

Crystal		Non-zero coefficients
Fused silica	Isotropic	$p_{11} = 0.121, p_{12} = 0.270$
Water	-do-	$p_{11} = 0.31, p_{12} = 0.31$
Flint SF-4	-do-	$p_{11} = 0.232, p_{12} = 0.256$
Lithium Niobate	Uniaxial	$p_{11} = -0.02, p_{12} = 0.06, p_{13} = 0.13, p_{14} = -0.08, p_{31} = 0.17, p_{33} = 0.07, p_{41} = -0.15, p_{44} = 0.12$
Lithium Tantalate	-do-	$p_{11} = 0.08, p_{12} = 0.08, p_{13} = 0.09, p_{14} = 0.03, p_{31} = 0.09, p_{33} = 0.15, p_{41} = 0.02, p_{44} = 0.02$
α-quartz	-do-	$p_{11} = +0.16, p_{12} = 0.27, p_{13} = 0.27, p_{14} = -0.03, p_{31} = 0.29, p_{33} = 0.10, p_{41} = -0.47, p_{44} = 0.079$
Telluride oxide	-do-	$p_{11} = 0.0074, p_{12} = 0.187, p_{13} = 0.34, p_{14} = 0.09, p_{33} = 0.24, p_{44} = -0.17, p_{66} = -046$
KDP	-do-	$p_{11} = -0.251, p_{12} = 0.249, p_{13} = 0.246, p_{31} = 0.225, p_{33} = 0.221, p_{44} = \sim, p_{66} = 0.058$

So, having known the numerical values of these coefficients we can actually estimate the effect of. This is the table which is taken from this reference Ghatak and Thyagarajan, Optical Electronics. This is the crystal, fused silica is isotropic isotropic; we have put it into one group water, flint SF-4. And, you have only two independent coefficients non-zero coefficients p 1 1 p 1 1 and p 1 2 p 1 2.

But, in the case of this uniaxial crystals lithium niobate, lithium tantalate, lithium niobate is very most used crystal for this purpose. You see you have 8 components, you have 8 components and you have 7 components. So, this table which gives you the coefficients is always useful to calculate and to design the devices, modulators using these crystals these materials. So, we will refer to these numbers while calculating the numerical values.

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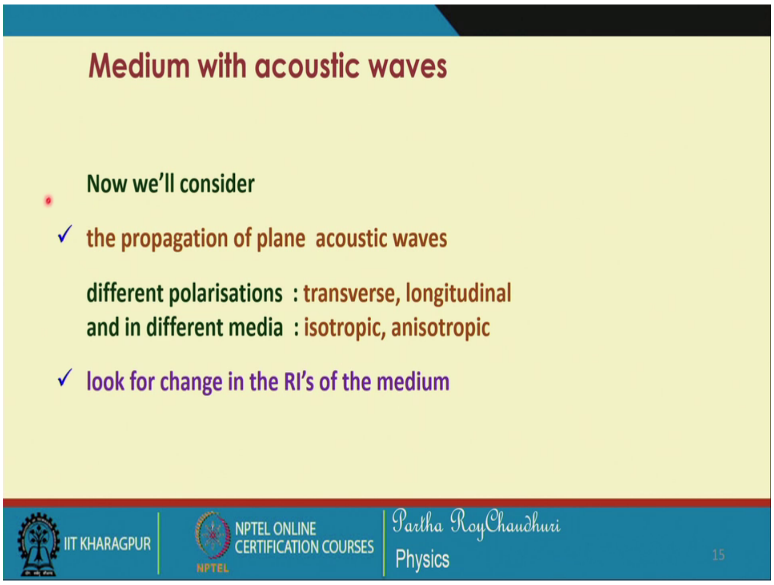

And now, we will consider the propagation of a plane acoustic wave. We have seen that the waves can have different polarizations depending on this whether it is transverse or longitudinal. In the case of transverse wave that is shear acoustic wave there could be two polarizations, which are degenerate that is will be represented by the same. And, for the longitudinal this is purely one longitudinal polarization and we will also consider the different kinds of media like isotropic, anisotropic.

In the case of isotropic we will consider the propagation along at least two directions and we will see that they are the same, except the propagation direction only defines the optic axis in the in presence of the because, it becomes an isotropic and uniaxial or anisotropic and this anisotropy is defined through this optic axis, which we will be straight forward the direction of the propagation of the acoustic wave. Whereas, in the case of anisotropic the orientation of the acoustic wave propagation, the direction of the propagation of acoustic wave with respect to the medium principal axis will change and can give you freedom of having different effects on the refractive indices. We look for the change in refractive indices in the medium.

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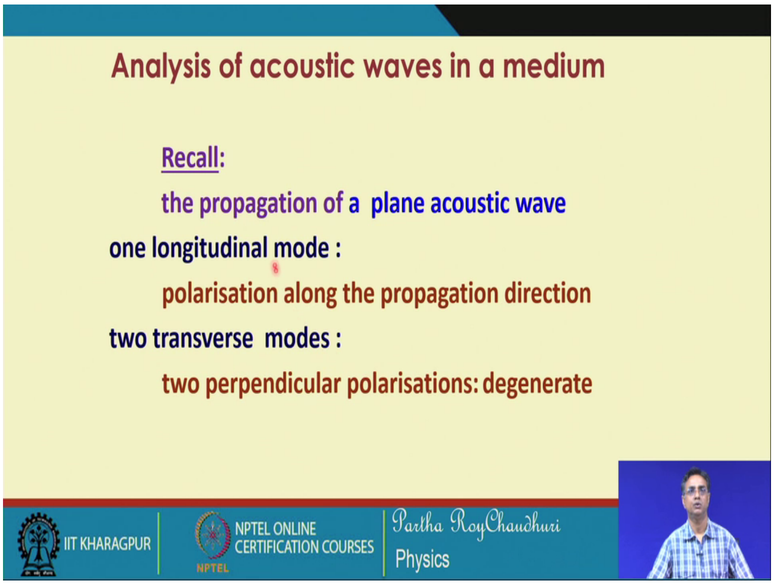

So, in order to do that let us first recall the that for the longitudinal mode you have polarization along the direction of propagation and for the transverse modes you have two perpendicular polarizations which are degenerate and we have seen it.

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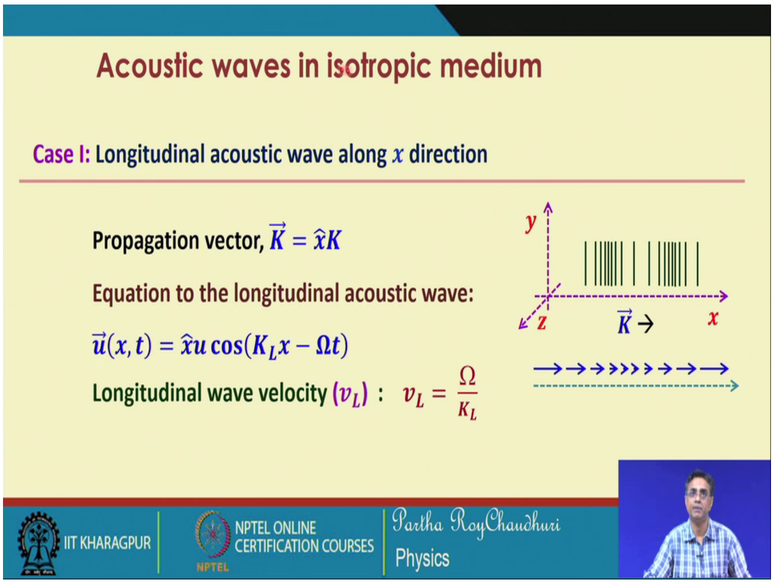

The clearly this, let us consider the case of longitudinal acoustic wave which is propagating along x direction. Followed by this we will consider the propagation of this acoustic wave along some other direction y or z. And, in the case of isotropic medium we will see that they are the same, but it only defines the optic axis about which where the axis the direction along which there is no change in the permittivity.

So, this will be an interesting outcome of this discussion. So, the equation to the longitudinal acoustic wave we have seen; this is propagating along the x direction. So, this is the only polarization and we can represent this equation by this where, K L is the propagation vector, propagation constant. And, this capital omega is the frequency of the acoustic wave, u is the amplitude which is u x and this wave velocity is given by this. So, all these parameters will be known for a given acoustic wave.

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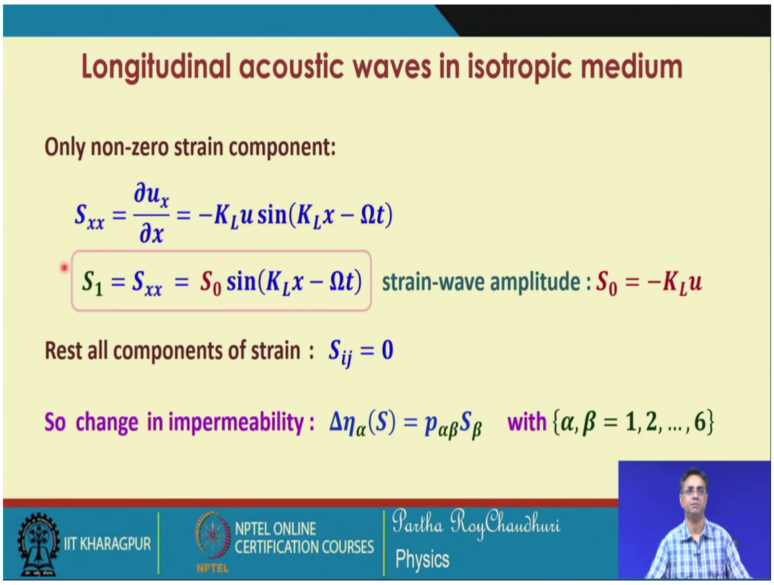

Now, the only non-zero strain component and we have seen that in the previous discussion we have seen that all other elements; even all the diagonal elements except this S 1 are 0. Off diagonal elements are as well 0, anywhere rotation matrix we are not considering here. And, in this isotropic case we have seen that truly the rotation matrix does not come into the picture.

So, rest all other strain components are 0 here therefore, very easy and straightforward to calculate the change in the impermeability. So, delta eta alpha of S that is as a function of strain will be represented by this. We are using this equation several times.

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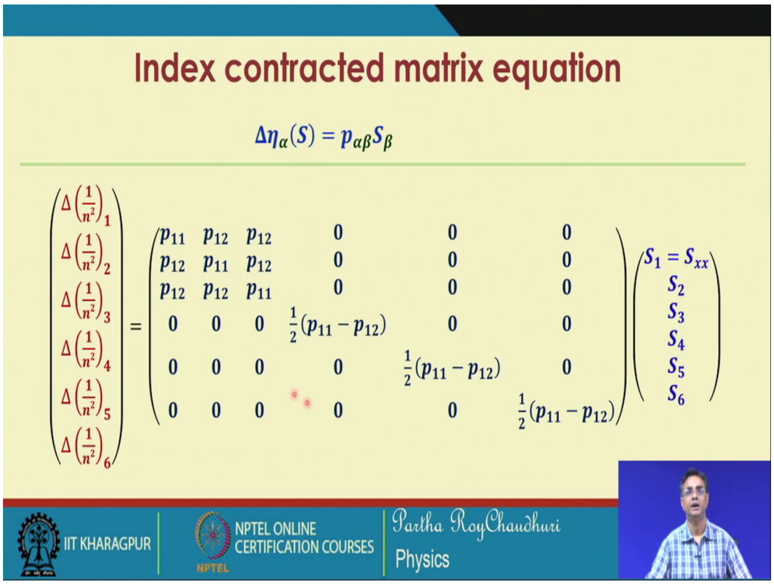

And this for the case of isotropic medium, we know the form and this is the change in the impermeability and, for this particular case when the wave is propagating longitudinal wave and that is propagating along x direction. So, it represents that the propagation direction and this is xx that is it is the direction of the vibration. Direction of vibration that is in terms of this direction of the changes that is also x. So, S 1 this is the only non-zero element and rest all of them will be 0.

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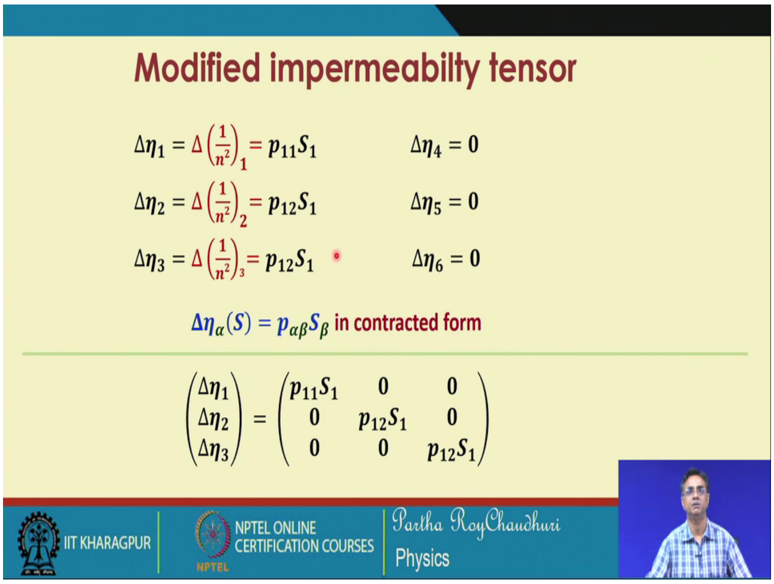

So, as a result you see these are all the other elements will be 0 in this case and so, this S 1 will be multiplied with these 3 quantities because, rest all of them are 0. So, S 1 p 1 1 S 1 p 1 2 S 1 p 1 2 S 2 will be the changes for delta eta 1 delta eta 2 and delta eta 3. So, that is what is followed by this. So, delta eta 1 equal to p 1 S 1 etcetera and rest all other elements because, of the 0 element present here rest eta 4 eta 5 and eta 6 they are all equal to 0. So, in the contracted form because you have all the 3 other elements 0; so, eta 1 eta 2 eta 3 we can write in this form rest all of them because, these are eta 1 eta 2 these are the diagonal elements and this is also as well the diagonal elements.

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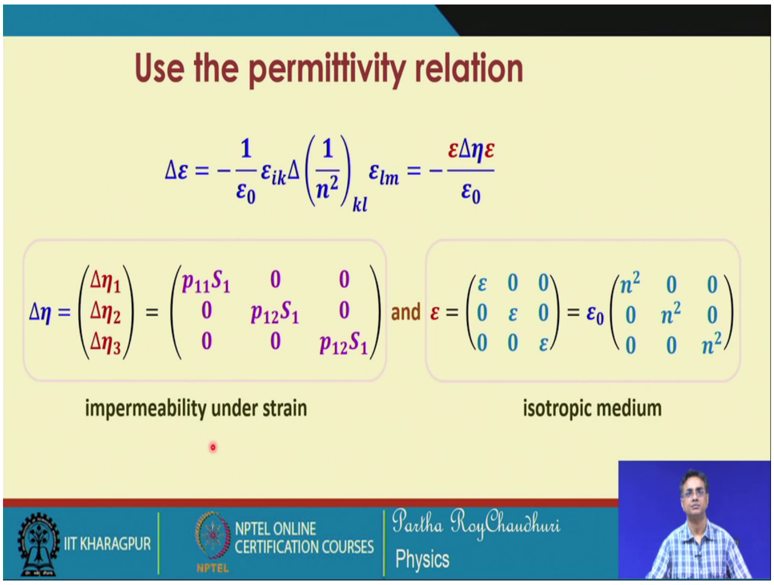

Therefore we can write, we can use this equation which we have learnt how this change in the permittivity can be expressed in terms of the in terms of the impermeability change multiplied by the permittivity. So, this from on from either side of this delta eta so, this equation we will use to calculate this delta eta delta eta 1 delta eta 2 delta eta 3. We have seen that this is the values these are the values of p 1 1 S 1 p 1 2 S 1 etcetera and eta is this.

So, now because everything is known, impermeability under strain that we have been able to calculate by now and for isotropic medium this is the known form of the permittivity tensor. So, epsilon 0 n 1 square n 1 square n 1 square only the diagonal elements, here also you have the diagonal elements. Now, the task is very easy we will directly plug into this equation, this equation permittivity will come here, permittivity will come here in the transform form and this we will go into this place.

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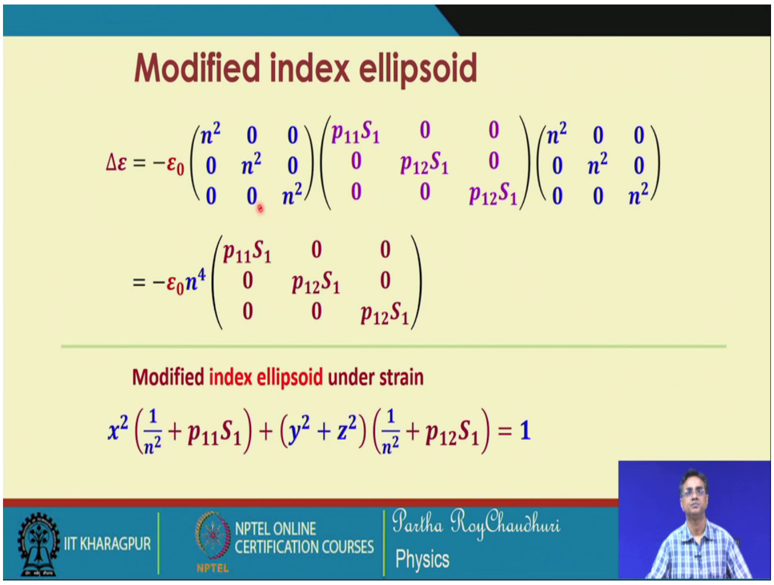

So, by doing that delta eta equal to minus epsilon 0 because, you have a minus epsilon 0 here and, then this quantity and then this will be here and this will be here These are coming from because this is this will come twice epsilon square and you have one epsilon naught in the denominator. So, effectively there will be one epsilon naught in the numerator. And, n square if you take outside this is possible in the case of isotropic medium. We will see that in the case of an isotropic medium we will not be able to bring out, take out this n to the power of 4 outside this bracket.

They will appear here in the form of n p square or n o square in this form. But in this case it is straightforward delta E is represented by this. So, this change in the permittivity we have been able to express in the form of this and that directly tells you that the modified index ellipsoid under the presence of the strain S 1 can be written in this form p 1 1 S 1 p 1 2 S 1 p 1 2 S 1 they are the same. So, they are going to the same group that is y square and z square, for y square and square you have the same change in the impermeability.

But for this is because, of the nature of the strain optic tensor for the isotropic medium because, of the nature of this. So, you have p 1 1 is attached to x 1 square whereas, p 1 2 is attached to y square plus z square. Therefore, this medium becomes uniaxial you can see and this is now this can be called as n x, but n y and n z they are the same. So, it becomes a uniaxial system and you can make out from here that this is the optic axis because, about this x axis you have y and z we will have the same indices of refraction.

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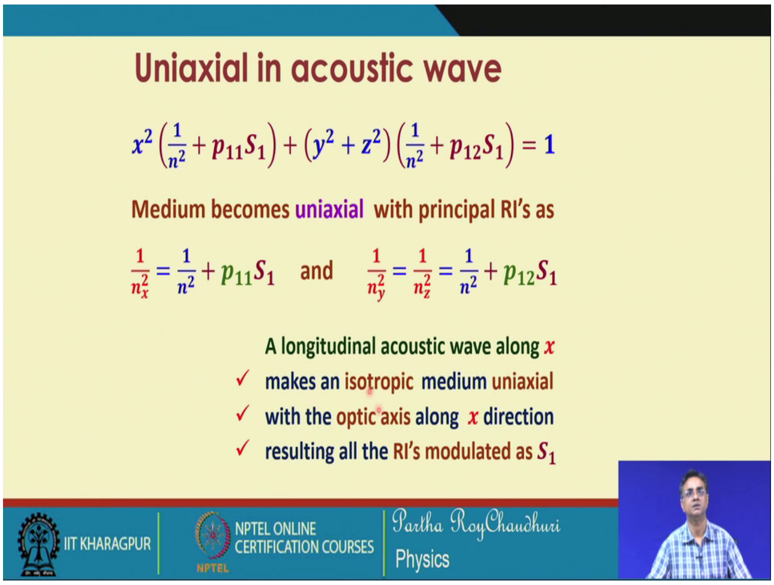

So, you can see that 1 by n x square can be represented by this whereas, 1 by n y square and 1 by n z square can be represented. So, two of them n y and n z are having the same value whereas, n x is different. So, that is why a longitudinal acoustic wave along x makes this isotropic medium a uniaxial one, it makes a uniaxial one if this strain is constant it is always there. So, because of the strain it becomes a uniaxial medium. So, if you apply a strain along the x direction as long as the strain exists then the medium will become uniaxial.

And, this along x will be the optic axis, but along y and z the indices of refraction that is the polarization of light along y and z we will see the same refractive indices. So, this resulting all refractive indices, which is now modulated by S 1. So, if S 1 is modulated they will modulate both x and n x n y and n z n y and n z modulation will be identical whereas, the modulation of n x will be different from this.

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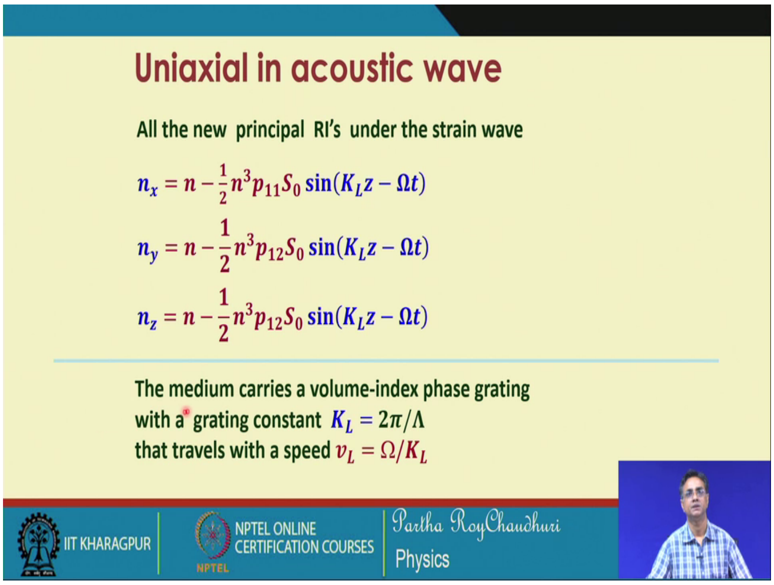

So, all new refractive indices will look like this n minus n 0 because this also we have learned. Now, if you met this is equal to so, we can represent n x square with some approximation that it can be represented by this. This we have seen several times the medium. Now, you can see that because n x is apart from the original refractive index that is the refractive index, in absence of any strain it sees there is an additional term which is the acoustic wave dependent term.

So, because this acoustic wave is as an amplitude which is changing the displacement therefore, this part of the refractive index which is to represent the n x is being modulated by this wave. So, this can take up sin of K L minus z minus omega t can have plus minus 1 maximum minimum value. So, this quantity will be from minus 1 to plus 1 multiplied by this quantity. So, if I call this is equal to delta n so, this delta n can have plus minus delta n value. So, in addition to n it can go up to n plus delta n, it can go up to n minus delta n and that is true for these two.

So, in the case of n y and n z the modulation is the same whereas, the modulation is different in the case of n x depending on the values of the tensor coefficient that is p 1 1 and p 1 2. So, you can see that this such a medium through which this acoustic wave is travelling in the longitudinal direction. So, this any light wave which is propagating along this medium to this medium, we will see a volume index phase grating throughout the volume of the medium where the acoustic wave exists.

We will see a phase getting there will be a periodic change in the refractive indices. If the periodicity comes from the sin function and the grating constant will be given by K L we will see this. We will continue with this discussion twice pi by lambda, this is the periodicity and that travels with the grating itself. Because, this delta n is changing with it is moving with the acoustic wave so, it is a function of this z as well as time. So, in the same place it is changing as well as with different positions it is also changing. So, it is a moving grating in along the x direction.

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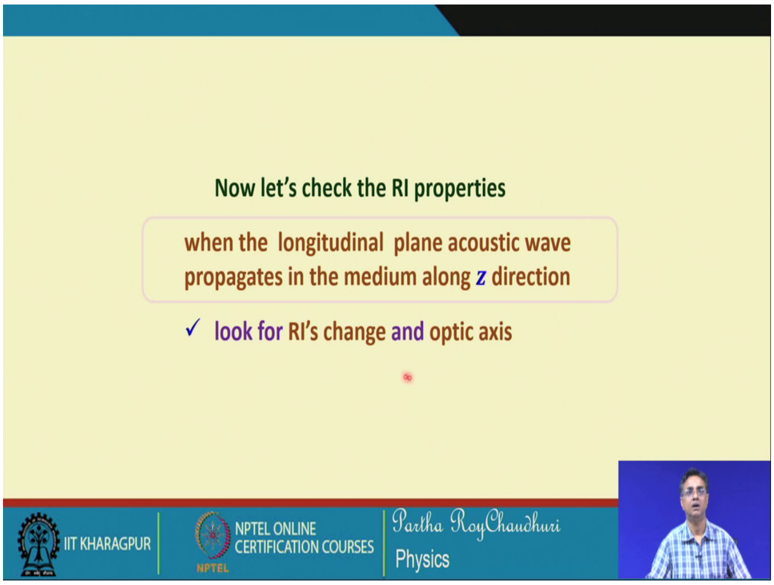

Now, we will consider a similar situation just to check that, if I allow the acoustic wave to propagate along the z direction instead of x direct, x propagating wave we will allow a z propagating acoustic wave in the medium. What are the changes in that case? Because, it is an isotropic medium so, let us look for the changes and look for the optic axis.

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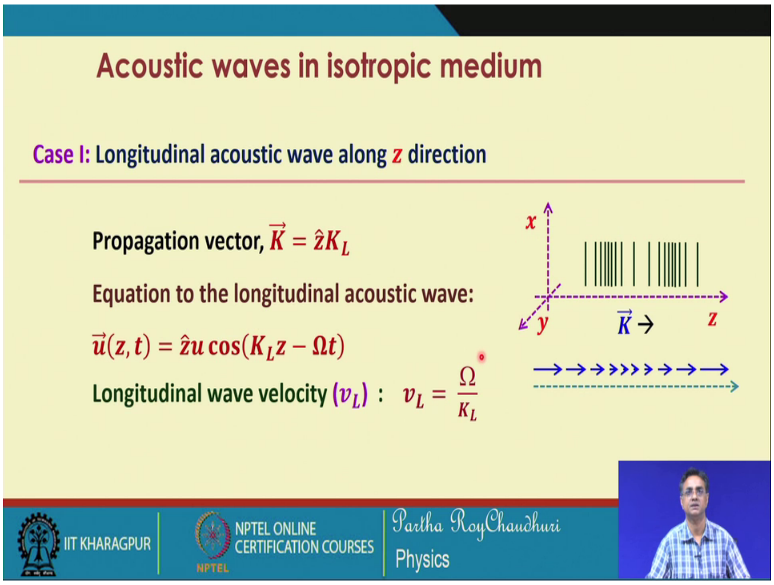

And then we can conclude that whatever it is so, longitudinal acoustic wave traveling along z direction, in that case when you straight forward. We have done this for the case of x propagating acoustic wave longitudinal wave. So, very straight forward so, we just replace this x by z and we end up with this equation.

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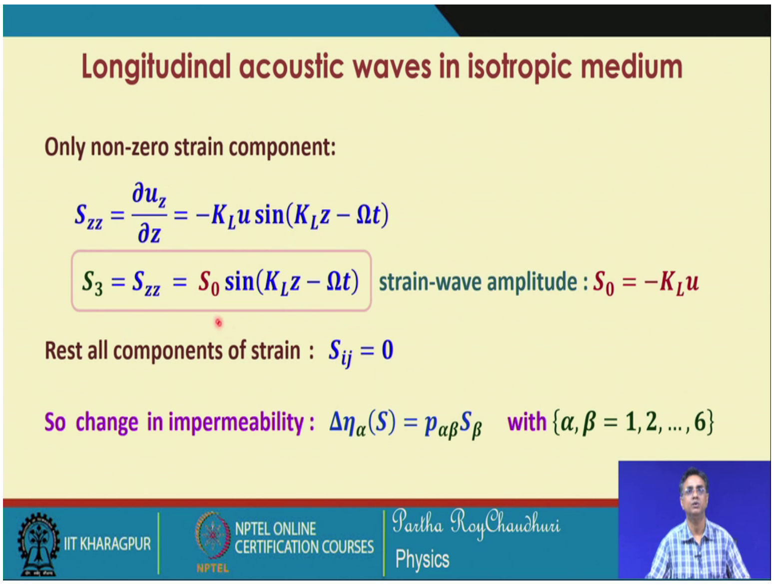

Now, that let us look at this strain tensor. The strain only the diagonal element S 3 is non-zero rest all other will be 0 because, now it is z propagating and variation in the refractive index is also along z because, it is a longitudinal wave. So, we consider only S zz which is equal to this and if I call this quantity equal to the amplitude of the sin wave, then I can write S 0 sin K L z minus omega t which is a strain wave and rest all components are 0.

Now, we can directly put it into the impermeability tensor which is, which comes from which can be calculated from the strain optic tensor and the strain tensor. Because, this S B beta is now known which has only one component S 3 and now it is straightforward rest all elements are 0 only S 3.

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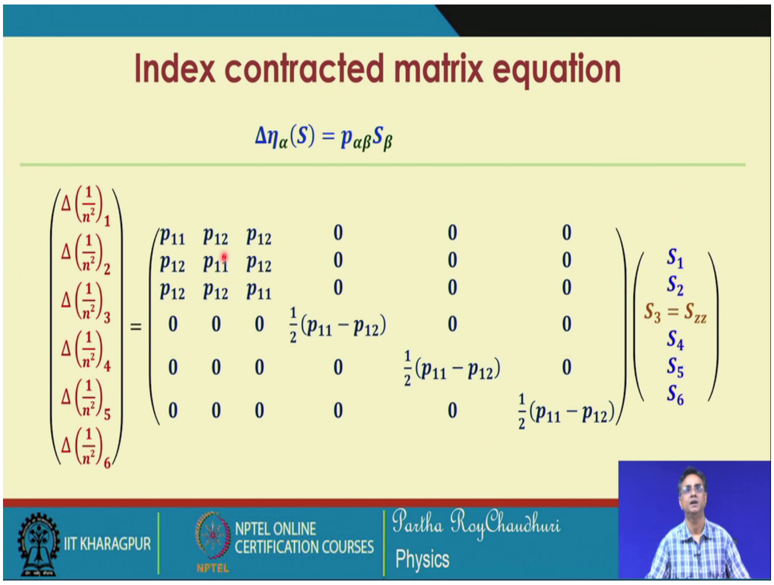

So, it will be multiplied with only these 3 components and rest all of them are 0 which is again straightforward.

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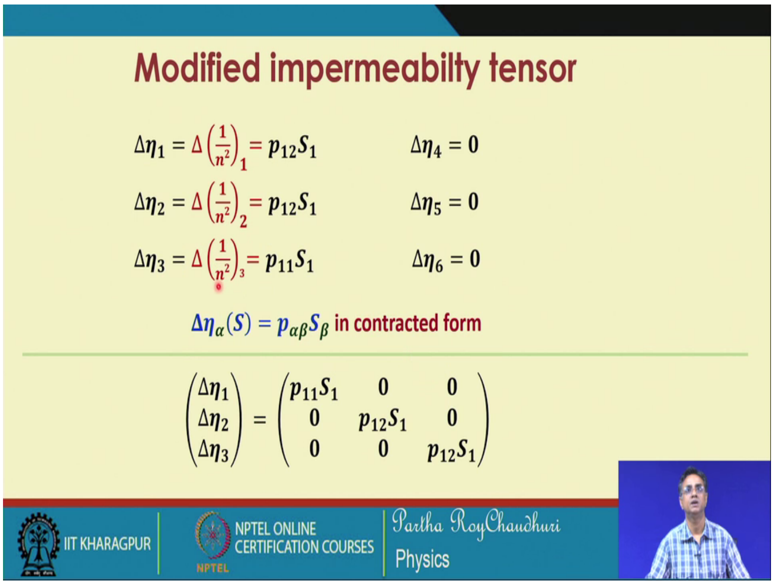

But, this time you can see that because it is z propagating wave so, this z along the z the change in the impermeability is different. But they are degenerate, they are same along the x and y direction they are same. So, that tells you that this x is only, I think this is a mistake, this will be this will be 1 2, this will be 1 2, but this will be 1 1, this will be 1 1 it is a mistake.

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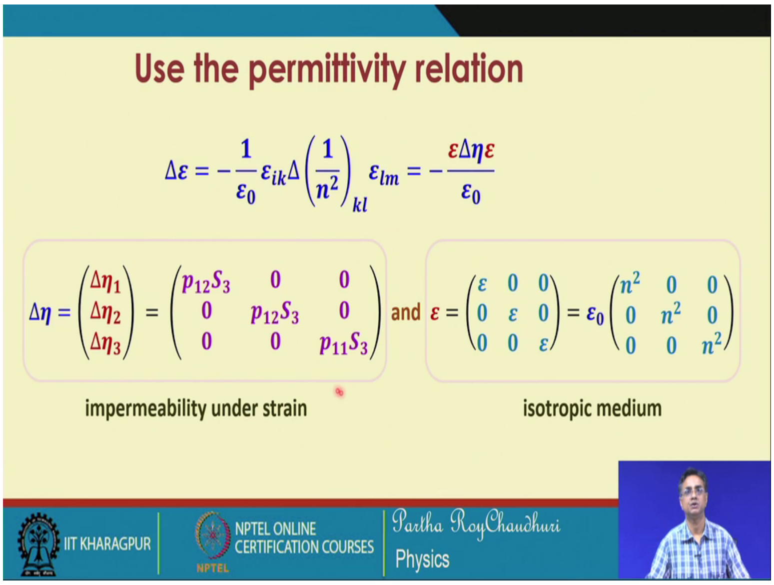

So, if I use that this is correct in this case I have used this equation here p 1 2 p 1 2, but this is p 1 1. So, this was this should be p 1 1, this is p 1 2, this should be p 1 2. Therefore, I get straightforward using the same relation we have done it earlier. Then we can end up with this index ellipsoid equation, this time it is because the wave is now propagating along the z direction. So, the modified refractive index along the z direction is different from the refractive indices along x and y which are identical.

So, the medium again becomes isotropic, anisotropic that is uniaxial anisotropic in which case in that case the optic axis is now defined along the z axis. So, we can conclude that the direction of the propagation of longitudinal wave defines the optic axis of the temporary of the strain induced uniaxial medium, which is normally an isotropic medium. And, it results in the results in the changes in the refractive indices which are same along x and y; if the wave is acoustic wave is propagating along the z direction.

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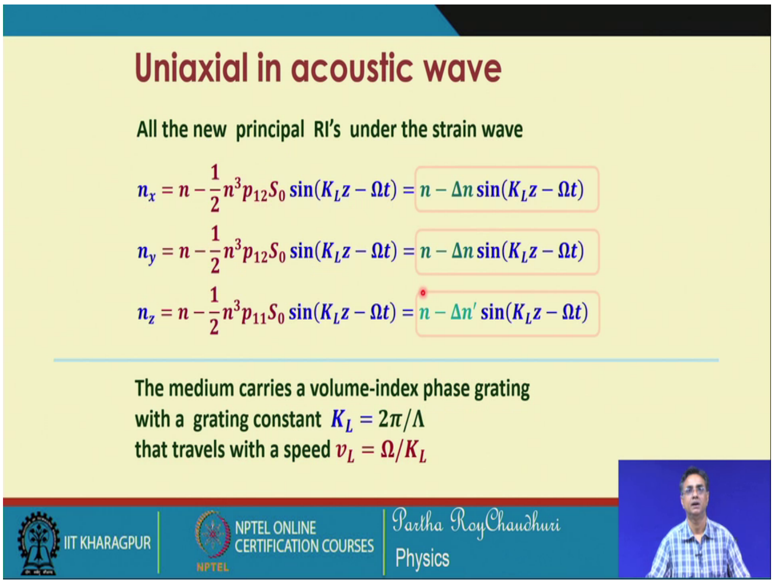

So, we can write in this case n x n y and n z equal to this which is similar to the one for the case of acoustic wave. But, you have this n x and n y are same whereas, the n z is different. In the earlier case n x was different n y and n z were same and you can see that it defines a periodic refractive index change whose amplitude of the refractive index is delta n. So, this is a periodic function sin function. So, that delta n can have plus minus delta n.

So, this n will be modulated by plus minus delta n as long as this wave is existing, which will create a periodic change in the refractive index all along the medium. And, we will see that it will behave like a volume phase grating and this phase grating any light which is propagating through such a medium will experience a phase grating volume grating and the light will be diffracted. We will see that this diffraction can be of two types depending on the periodicity and the frequency of the acoustic wave.

But, before that we will continue our discussion with the longitudinal waves in the isotropic medium. And, also we will consider the propagation of the shear waves in an isotropic medium to make all the possibilities well understood. So, that we can apply straightforward to the individual cases of the orientation of acoust (Refer Time: 31:16) acoustic wave and to look at the changes in the refractive indices.

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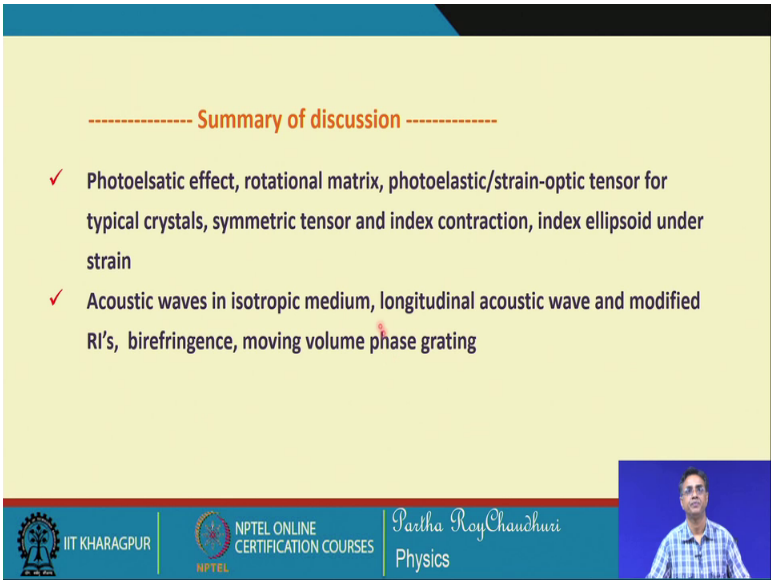

So, by this discussion we considered the case of acoustic longitudinal acoustic wave propagation in an isotropic medium. And, then we looked at the change in the impermeability and then using that equation impermeability and permittivity equation we calculated the changes in the refractive indices. And, we saw that the longitudinal wave in an isotropic medium gives the same effect whether it is x propagating or y propagating or z propagating. It only defines the optic axis of the uniaxial anisotropic medium which appears because, of the presence of the acoustic wave in the medium. We will continue this discussion.

Thank you.