Modern Optics
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Lecture – 43
Acousto-optic Effect (Contd.)

So we have seen that how this acoustic wave will be represented in the mathematically in the form of rotation and strain matrices. And it was the discussion was only about the acoustic wave, how which of the elements will be presented will be present there in the elements and also that the nature of the acoustic waves in terms of whether it is a longitudinal acoustic wave or it is a transverse acoustic wave. If it is a transverse acoustic wave which is actually the which has 2 polarizations, that is if it is traveling along x, it can have a y polarization or it can have z polarization.

But these 2 are you have seen they are degenerate polarizations. And in the case of mixed polarization that part also we have analyzed. And now we will bring in we will connect this nature of the acoustic wave in the medium that is in terms of the amplitude. And that we will we will connect with the impermeability tenser and how these optical properties will be changing in the medium that part we will now discuss.

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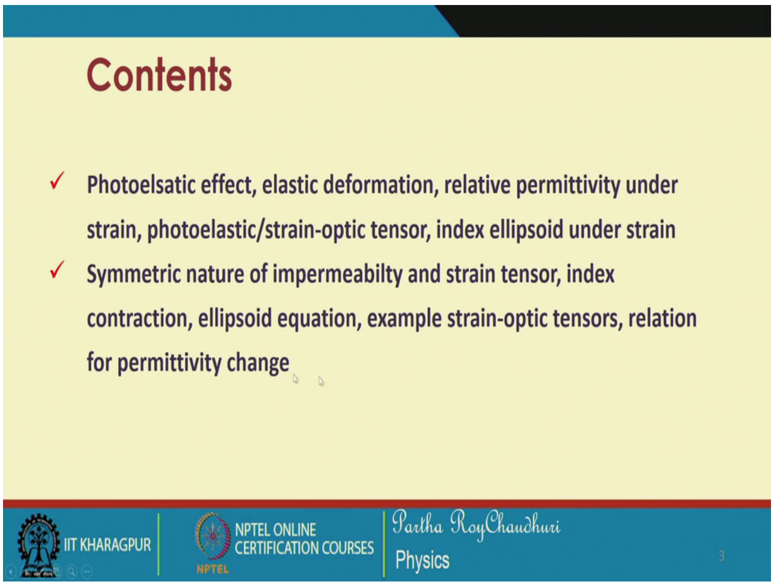

So, what we will look at this photoelastic effect and this elastic deformation, how this relative permeability is modified under the propagation of acoustic wave due to the periodic strain; present in the medium. And then we will talk about we will see the form of the photoelastic and that is strain optic tensor.

Then we will look at the index ellipsoid, without strain, without the presence of the acoustic wave that is the natural refractive index of the medium. And then we will see how this strain is modified or how this index ellipsoid is modified under the strain. We see that symmetric nature of the impermeability tensor and then the strain tensor put together we will apply this index contraction rule to make it only a 6 element column vector. Then we will represent the index ellipsoid, we will take the example of strain optic tensors and the relation of the permittivity changes.

So, that will complete the understanding of this acoustic wave with the impermeability tensor of the medium. So, we will try to bring in how this acoustic wave is changing the refractive index properties periodically, along the length of the direction of propagation of the acoustic wave.

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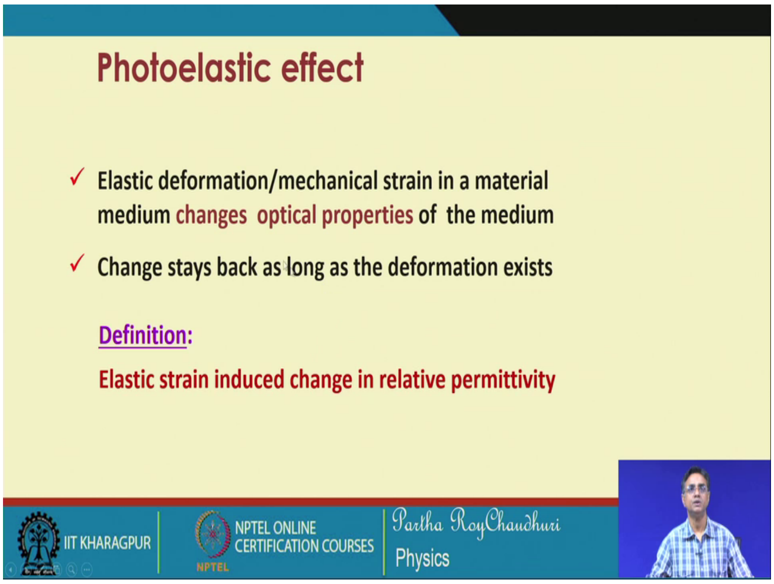

So, elastic deformation that is the mechanical strain in the medium that changes the optical properties. This is because it creates a periodic compression periodic deformation in the medium along the length of the propagation of the acoustic wave.

So, and this change in the change in the refractive index or the optical properties that stays back as long as the deformation exists, that is so long as the wave is propagating in the medium this deformation will be there the moment it is not there it comes back to that an elastic nature, it comes back to the original natural step; which will be represented by the natural the normal index ellipsoid.

So, elastic strain induced change in the relative permittivity that is how we look at this photoelastic effect.

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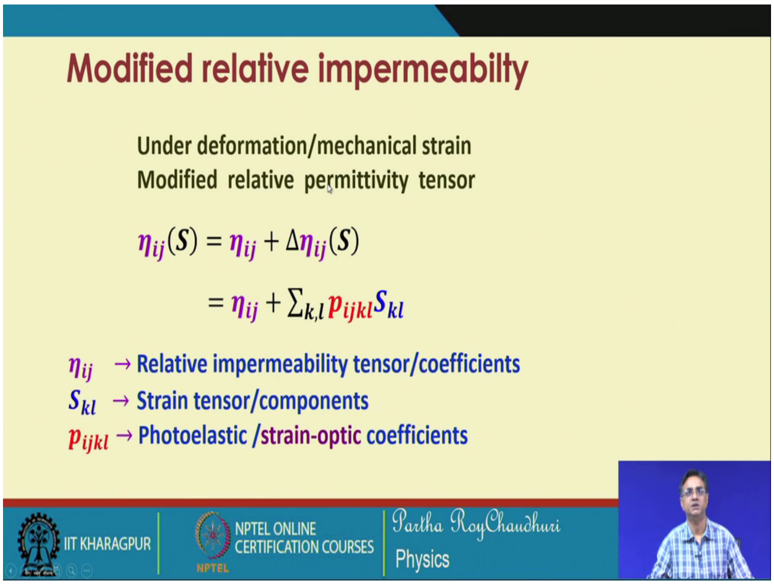

So, under deformation or this mechanical change, this modified relative permittivity tensor is given by this where this is impermeability; in presence of the strain this is impermeability without strain, and this is the change induced this is the strain induced change in the impermeability which can be represented by this tensor that is pij kl S kl. So, this is different from the one which we have discussed in the case of electro optic effect, this tensor is called the strain optic tensor.

So, eta ij the relative permeability tensor and S kl that is the strain tensor which we have discussed you have 2 component strain and another is rotation; for the rotation this p ij kl will appear in a different way we will not consider in this case. We assume all pure longitudinal wave and pure shear wave S pij kl that is the strain optic coefficients. So, we will see in details what are the forms of these of these tensors.

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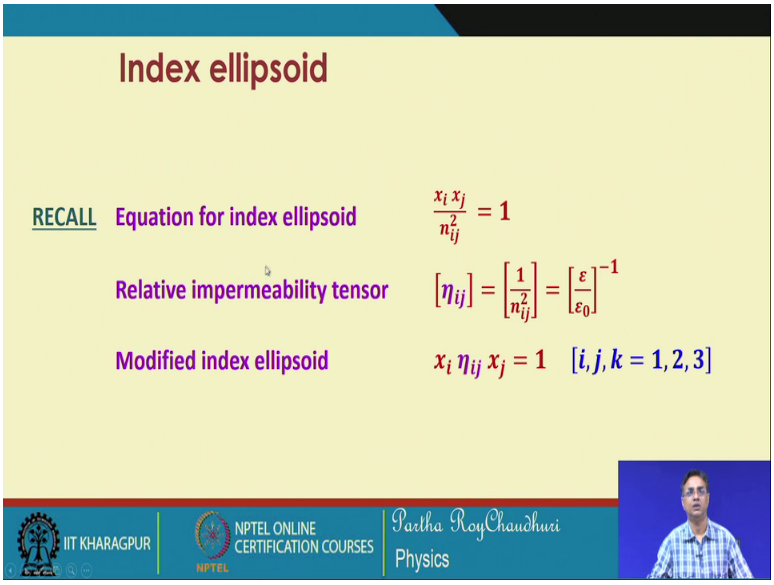

So, let us recall that the index ellipsoid in the compact form is xi j by nij scale. This is refractive index in terms of impermeability one can write in this form. And therefore, we can write this index ellipsoid in the compact xi eta ij xj.

So, this is column vector, this will be a this is a row vector this will be a column vector and this eta ij will be I cross j matrix elements. So, in this case it is 3 by 3; so 9 component element we have seen this earlier also.

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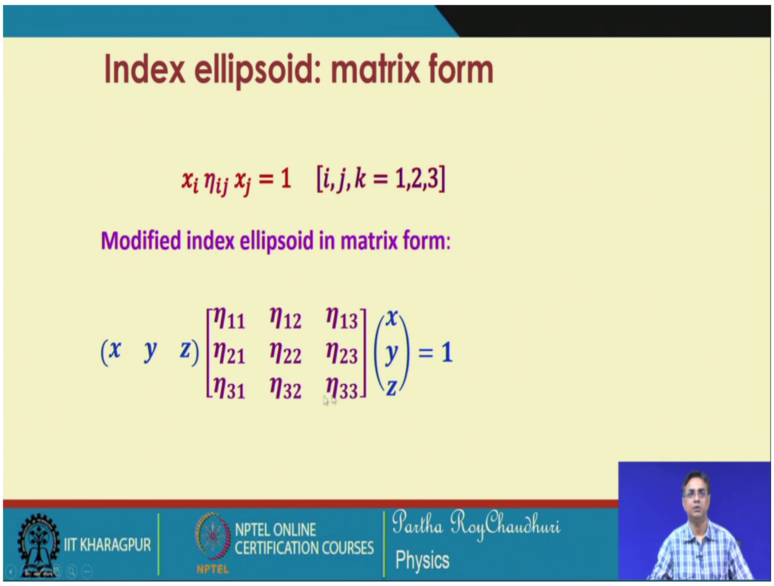

So, the exact form of this is x y z you have eta 1 1 eta 2 2 these are the diagonal elements these are the components of the impermeability tensor. So, the modified if it has to represent this, modified index ellipsoid then this eta 1 1 is all the elements. So, this matrix is also the modified matrix that is, this is the sum of this matrix with no acoustic wave plus the change in this matrix in presence of the acoustic wave, which is represented by this equation.

So, it is the this is the sum of the impermeability without the strain and the change in the impermeability in presence of the strain. So, it is the sum of these 2 is represented by this in the case of modified index ellipsoid.

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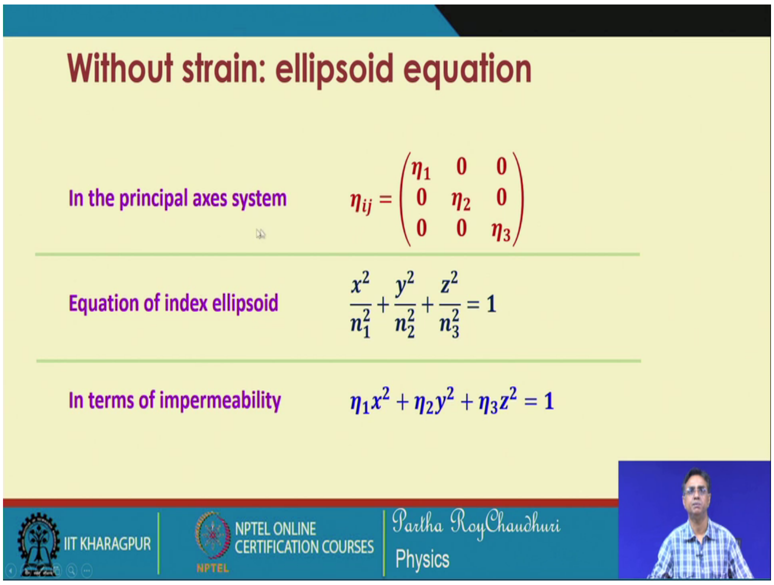

Now, in the principal axis system we have eta 1, eta 2, eta 3, these are the principal impermeability components equation of the index ellipsoid is this; which is in the principal axis system of the medium. And that is all when there is no strain there is no deformation, there is no acoustic wave or anything is the under the normal state.

In terms of impermeability this equation can be written in this form because 1 by n 1 square is eta 1, 1 by n 2 square is eta 2 and so on.

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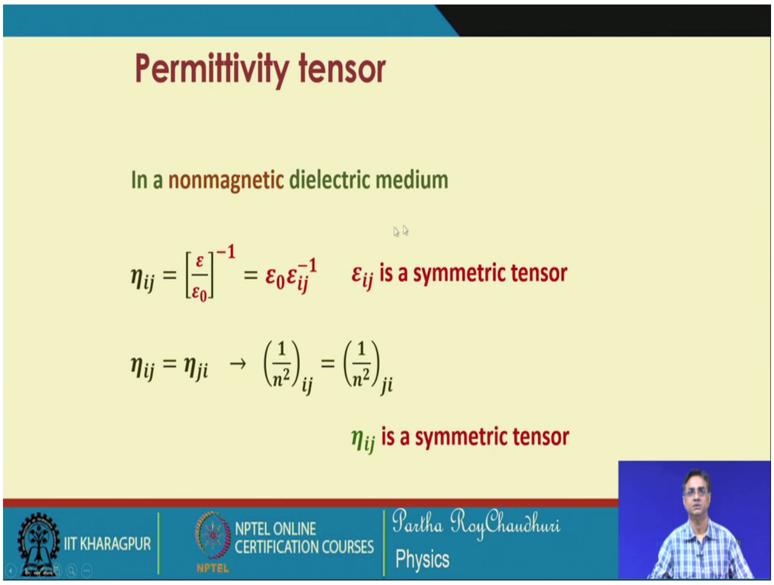

In a in a non magnetic dielectric medium this is a symmetric tenser and eta ij eta ji, because of the symmetric nature which actually helps in the index contraction. We will see how this symmetric nature reduces the task of this algebraic manipulation; 1 by eta square ij equal to 1 by n square ji ok.

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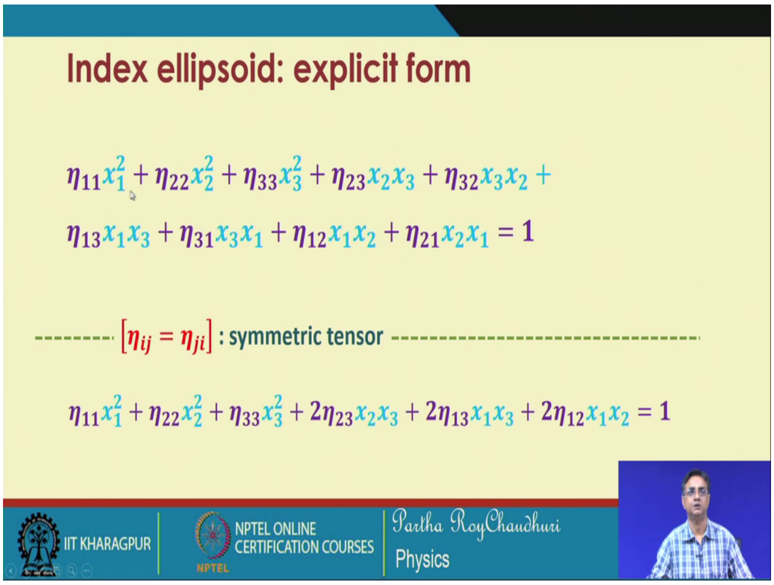

So, in the explicit form that is if I have to consider all the elements that in then it is x 1, square, x 2 square, x 3 square. So, again this is the complete form of the general form of the ellipsoid equation this equation and when it is modified so this is again a symmetric nature of the tenser. So, these 2 of the things that is eta 2 3 and eta 3 2 they will be put together with a 2. And similarly x 1 3 x 3 one will be put together because they are symmetric so that is again 2 and so on. So, you actually compress this 9 elements to 6 elements because it of the symmetric nature of this and that helps.

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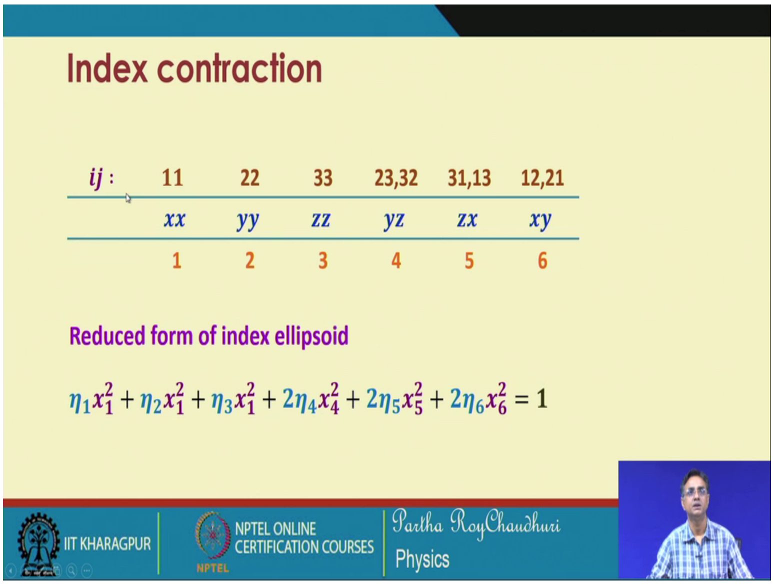

So, the index contraction rule is xx is 1, yy 2 these are the diagonal elements, yz zx and xy they are the off diagonal elements. So, this x 2 x 3 x 3 x 2 that is will be represented by this 4. So, using this we can write this equation in this form index ellipsoid form. In the compact form with the contracted index eta 1 x 1 eta 2 x 3 and so on is equal to 0.

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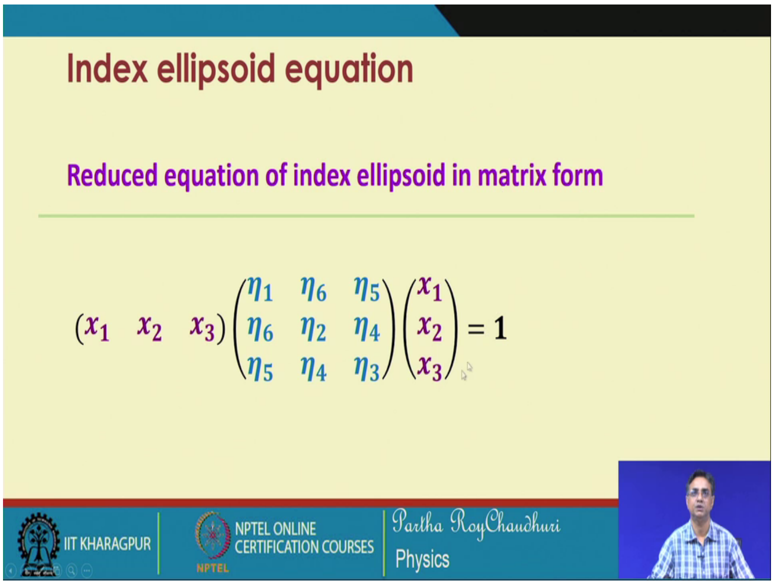

So, this reduced equation now it takes this form because now we are using this x 1 x 2 x 3 in place of xyz and it becomes eta one eta 2 eta 3 which is again very straightforward and very simple algebra to understand.

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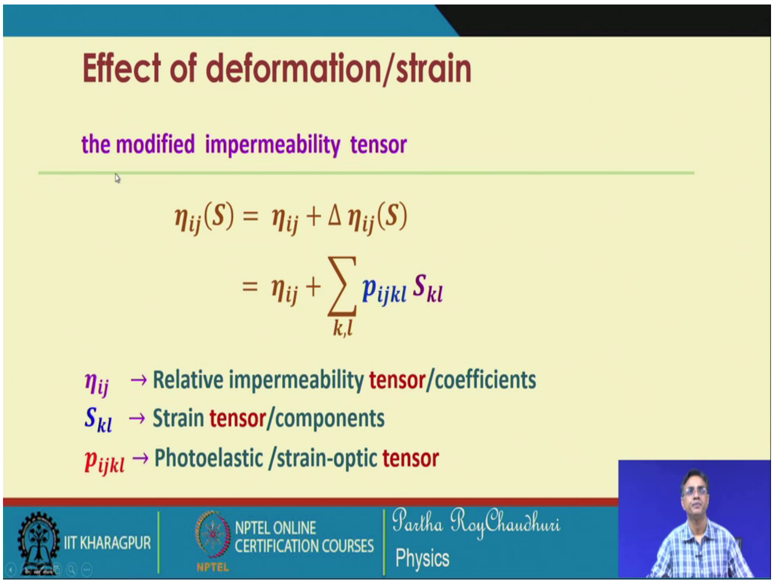

Now, the modified tensor which we have mentioned that this is in presence of the strain, this is in a absence of the normal state, and this is the change in the a per impermeability because of the strain.

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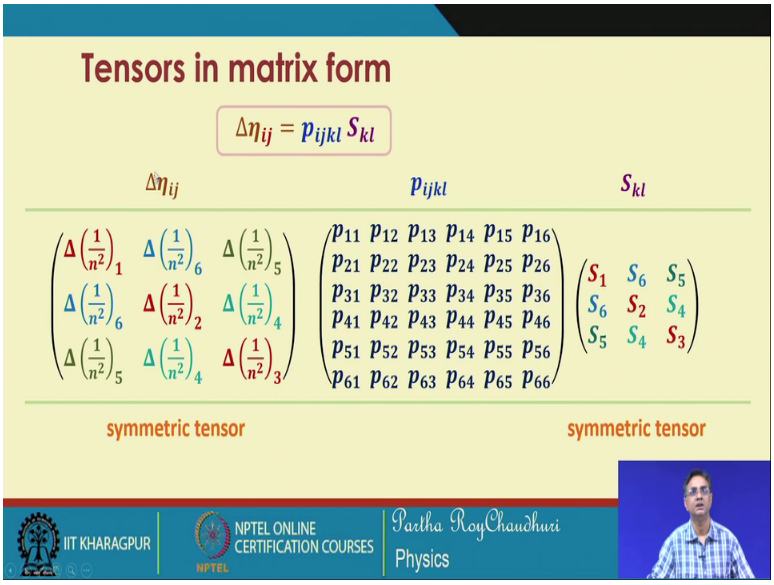

So, this the has the has the form of this. So, because delta eta because of the symmetric nature we can write this as this diagonal term 1, 2, 3 this is your 4 5 and 6. 4 and 4 they are the symmetric 5 and 5, 6 and 6 these are the symmetric off diagonal components. So, this is your delta look at this delta eta ij is represented by this 3 by 3 matrix, pij kl under compression 1 1 1 2 I and j put together equal to 1, j and k equal to 1 2 3 etcetera. So, you have this 6 by 6 element strain optic tensor.

And for S kl which is also symmetric we have seen for the strain matrix, strain matrix is also a symmetric. So, S 6 S 6 S 4 S 4 S 5 S 5 and this is the diagonal element. So, they are very consistent and very confirmable. So, we can put an equal to sign and then we can arrange to calculate the individual change in the refractive impermeability or the refractive indices.

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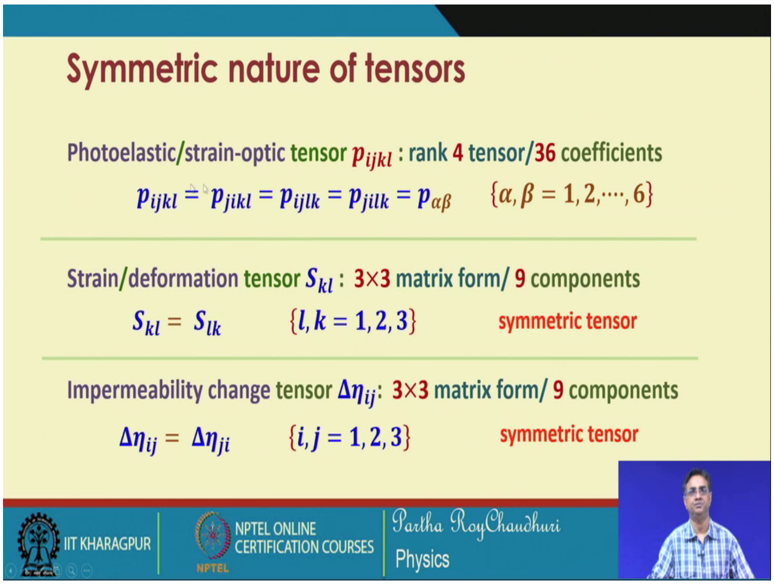

Therefore this is a 4 rank 36 element tensor, we have seen that this is 6 by 6 matrix and you can apply this index contract.

So, this is very interesting that if I contract this indices then we can represent this kl, lk putting 1 2 3 for both of them is a symmetric tensor, ij ji that is also 1 2 3 is the symmetric tensor. So, we can we can compress the indices to represent this.

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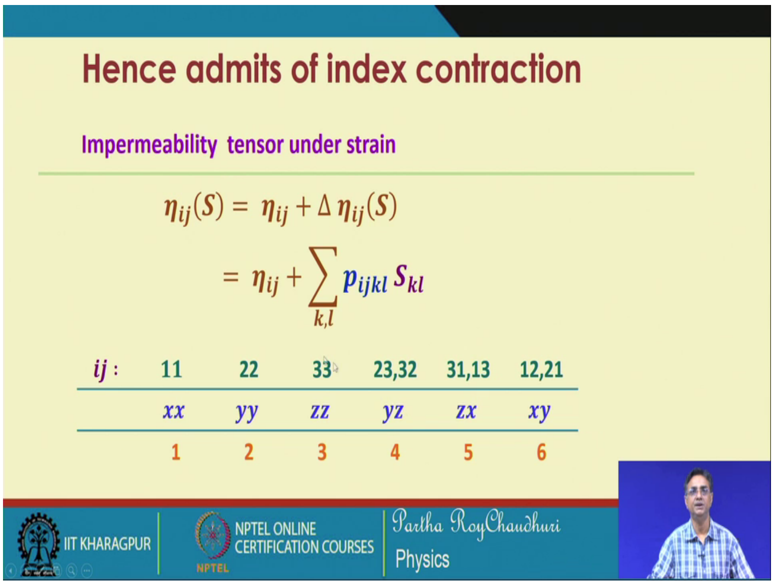

Therefore, now we apply this index contraction to this equation which will give me, which will give me this; 1 1 xx equal to 1 2 2.

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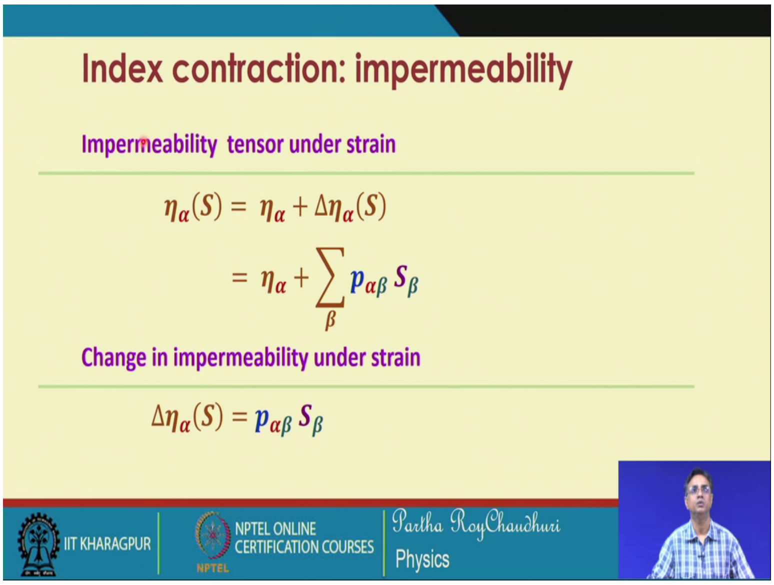

So, using this principle we can now write this impermeability tensor under tense with one index that is eta alpha, eta alpha and delta eta alpha which will give me.

Now, I have been able to contract the indices alpha beta S alpha beta. So, change in impermeability this quantity is now delta eta S, which is we have a convention of repeated index for summation, so, using that this quantity we want to calculate. This quantity we will come from the strain or the propagation of the acoustic wave and then connected with this for the which is the material property, that is the strain optic tensor. This and this put together will give us the change in the impermeability that is the change in the refractive indices.

And how this change in the refractive index indices attack takes place in presence of the strain, in presence of the acoustic wave or in presence of any deformation that can be 1 to 1 calculated.

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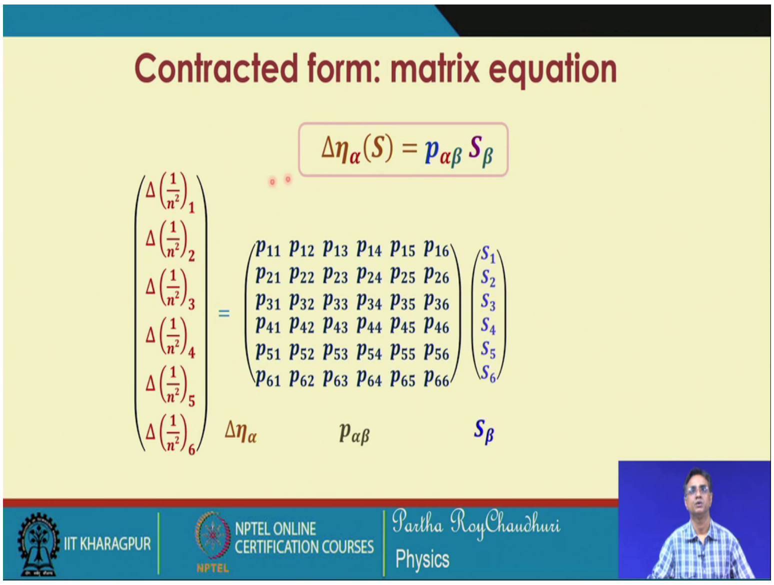

So, this is under this contracted form you have all the elements because earlier the form was like this, this is 3 by 3 this is 3 by 3, but this is your 6 by 6. So, if I represent it as 1 by 6 and this is 6 by 1 then it is conformable and we can calculate the individual changes.

So, this one you see under index contraction this is also under index contraction. And this is also represented under index contraction. So, we have this equation in the contracted form with matrix notation.

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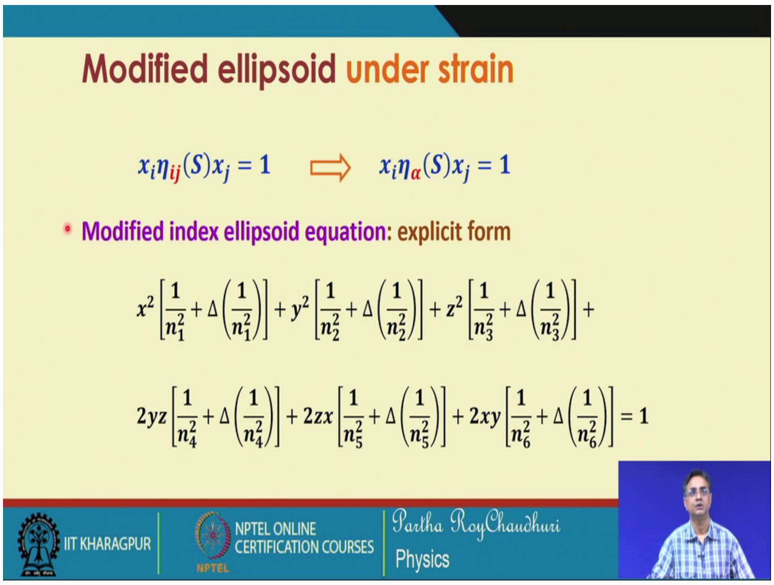

So, the modified index ellipsoid equation in the explicit form now takes this form because this is we are well familiar. This is the change in the impermeability, which is attached to this x square. Similarly, y square and z square which will come from the diagonal elements of the impermeability and these are the off diagonal elements.

But there is no diagonal off diagonal by now because we have been able to put them into 1 column vector. So, this is these 3 are the diagonal elements these are the off diagonal elements this will appear twice, this will appear twice, and this also will appear twice. Therefore, this 2 has come outside and this is the general form of the modified index ellipsoid.

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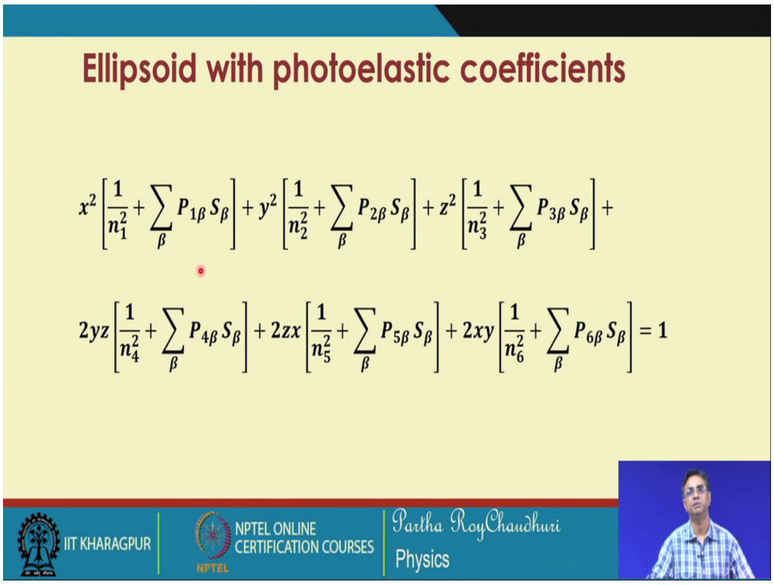

We will see which of the components will be nonzero, will be non menacing and which of them will be 0; depending on the condition of the acoustic wave and the a medium property in the form of the strain optic coefficients.

So, this is in the in the strain optic form because this is delta impermeability 1 can now be represented by this is the again in the compressed notation. So, this is how we can write this equation.

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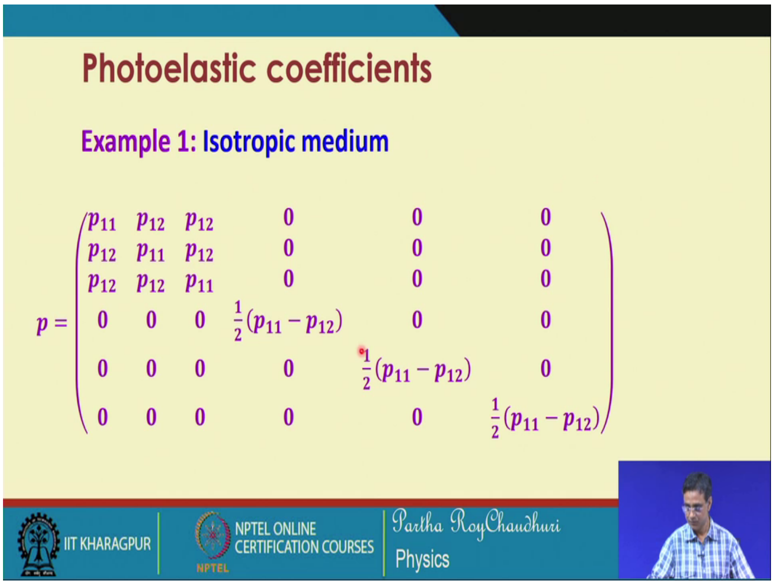

Now, let us take 1 concrete example any isotropic medium the photoelastic coefficients, that is a strain optic tensor is the typical one, that is in this case we have only p 1 1 and p 1 2. These are the 2 independent components and rest all other 0. So, we just know want to we just need to know that that 2 components p 1 1 and then p 1 2 from there we can construct this tensor, you can see and this is for the isotropic medium well.

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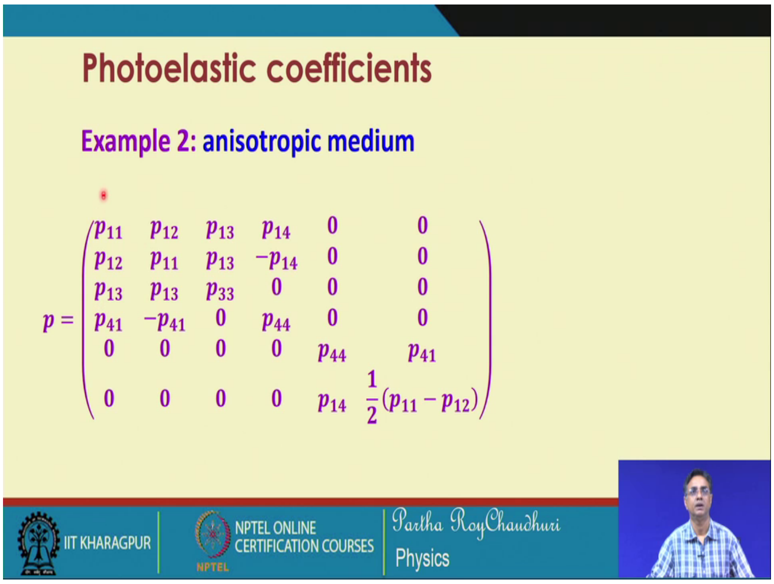

So, for anisotropic medium this is a typical electro optic tensor p 11 p 1 2 p 2 1 2 1 1 4 1. So, some of the elements some more elements are present independent elements. So, that we will see for the individual medium when we have the knowledge of these coefficients, then we can actually calculate the changes in the refractive indices.

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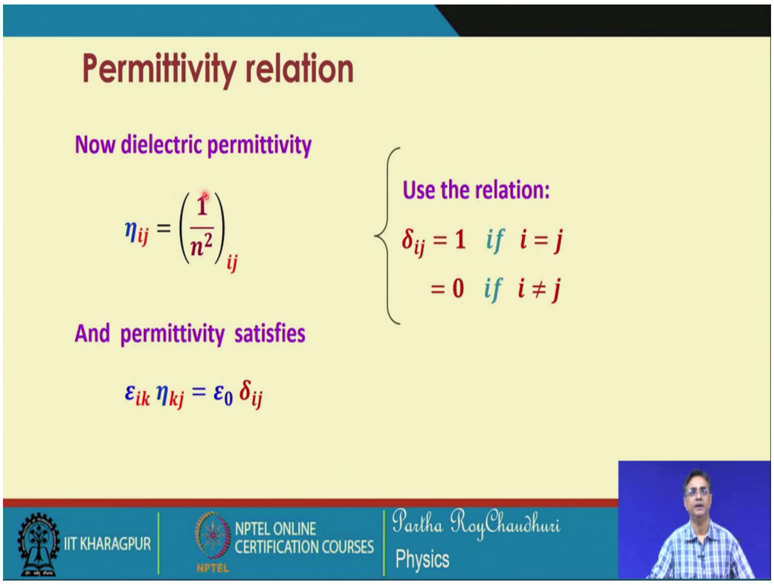

So, this again the dielectric property we know that delta ij is equal to 1 if this is the tonality condition and this condition is satisfied by this.

So, permittivity because this is your eta n square ij and this is 1 by n square ij. This and this eta epsilon ij put together it will be equal to when I equal to j j equal to k then only this it will have epsilon 0. So, this is the permittivity relation with impermeability.

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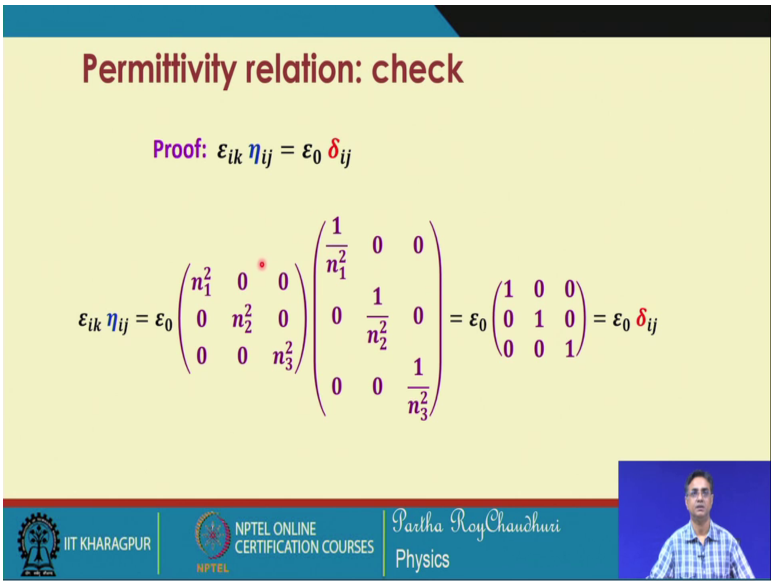

And just to check that how we get this relation there is a just one-line exercise eta I k and eta I j. So, this is your 3 by 3 this is again 3 by 3, if you take the product of this only this becomes 1 1 1 so this is eta ij and epsilon j so this is 1 1 1, so it is just one-line check.

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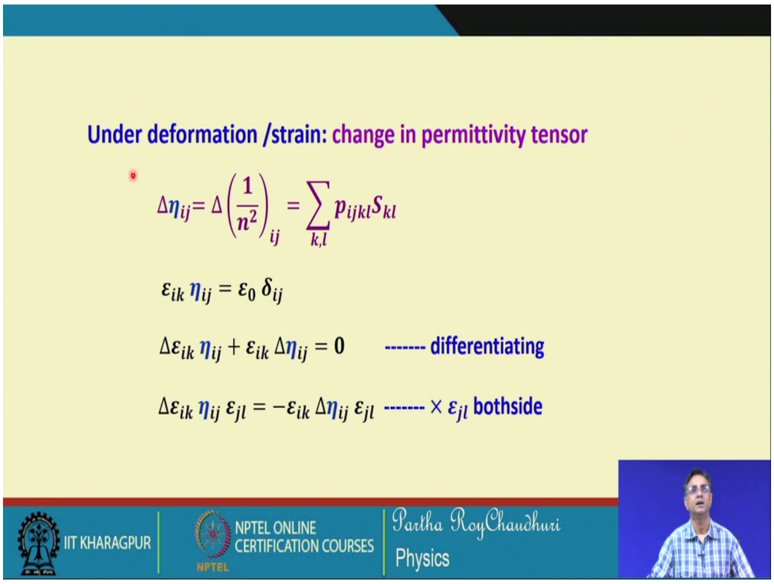

Now, if we represent that delta eta ij equal to this take this one. So, this now you differentiate then you can represent this delta eta ik, delta epsilon ik and delta eta I j plus epsilon ik delta eta ij this is equal to 0. So, if you transpose to the right hand side will carry a minus sign then you post multiply with eta ij both of them. Now this quantity eta ij epsilon I k eta ij, epsilon ik eta ij, epsilon ik eta ij if we just change this so this in place of this we can write epsilon 0 delta ij.

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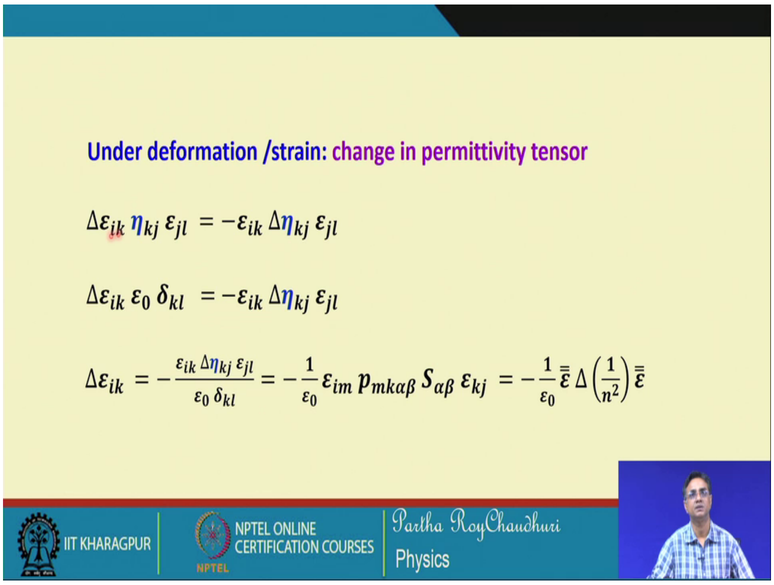

So, that is what we can represent that if we write in this form, then we can write this in this form and because this quantity eta kj eta jl can be written as this. Therefore, delta eta j this quantity comes to the denominator and you can write this in this in the expanded form with all the indices. We can write that epsilon I m this is the strain optic coefficient tensor, and this is a strain tensor, and this is a permittivity tensor. So, this is permittivity permit. So, this is a very useful relation to estimate to calculate the change in the permittivity from this relation that; this change in the permittivity is in the change in the impermeability multiplied by the permittivity old permittivity.

So and we will use this relation we will use this relation to calculate the different cases of the acoustic wave propagation. And we can directly use this to calculate the change in the impermeability this change in the impermeability. And from there we can again calculate the index ellipsoid.

And knowing that index ellipsoid we can calculate the change in the refractive indices individually in the principal axis system for all xy and z direction. And that will help us to construct the equation of nx ny and nz with the periodicity of the acoustic wave, that is this and we will see that nx will be equal to some n, which is constant plus or minus some delta n into some cosine of the periodicity of the of the acoustic wave.

So, we will be able to represent the change in the refractive indices by using this relation, which will go into the to explicitly express the change in the refractive indices; in the 3 mutually perpendicular direction in the principle axis system of the medium.

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So, we will continue with the different acoustic waves in the longitudinal direction and in the transverse direction for various cases and we will continue that.

Thank you.