Modern Optics
Prof. Partha Roy Chaudhuri
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture – 41
Acousto-optic Effect

In the previous sections, we have discussed Electro optic Effect and we discussed various aspects in terms of the basic physics, the configuration, and different modulation schemes of Electro optic devices. And now we will discuss another very promising aspect that is this Acousto optic Effect, the light matter interaction through acoustic waves in the host of material medium.

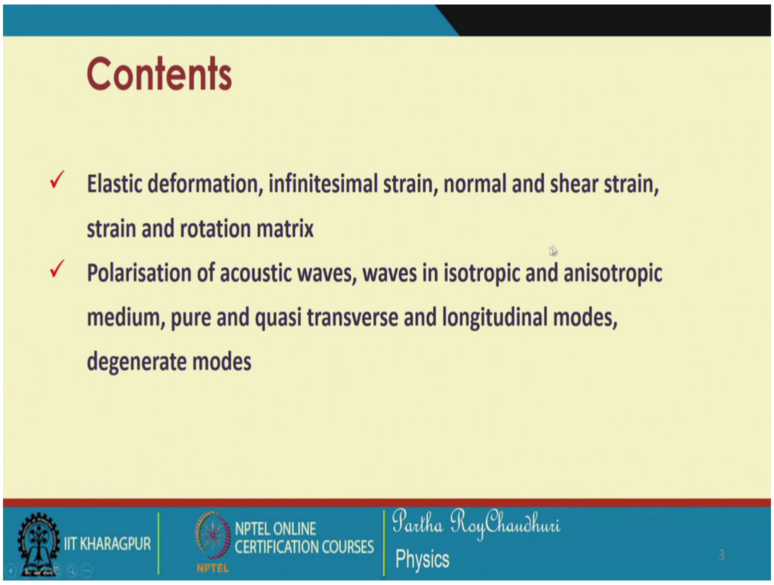
The Acousto optic Effect it basically consists of the interaction of the acoustic wave with the light wave through a medium where the medium will be deformed by the acoustic wave which is traveling through it. We know that when there is an acoustic wave, it creates a compression and rare refraction in the medium and the places where which is periodic and periodic with the frequency of the acoustic wave.

(Refer Slide Time: 01:28)

And the places where it is compressed and the places where it is rarefied, this the compression and rarefaction zones the material undergoes deformation up to different types. But this is a phenomenon which is which should happen even when there is a static strain if there is a force on a medium then the medium the bulk of the medium will undergo some deformation, and this deformation will create in turns some strain. The strain will be inducing will be creating some changes in the optical properties in terms of the permittivity and the refractive indices of the material.

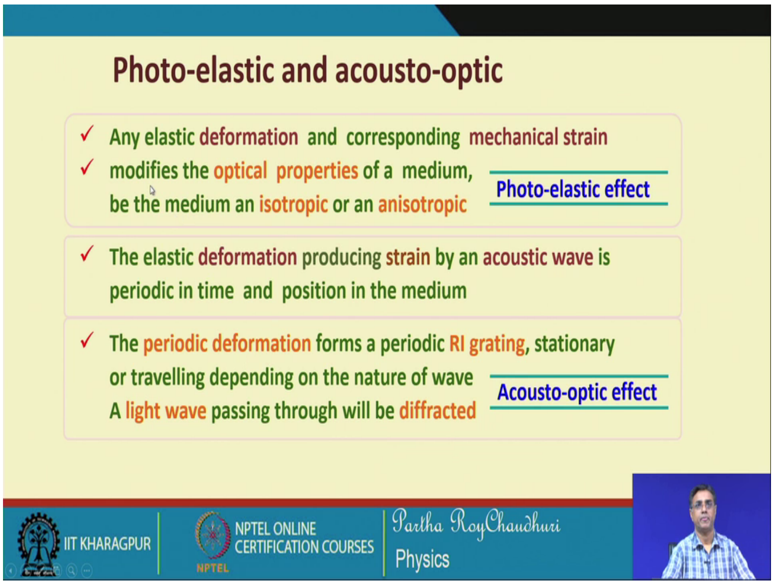
So, we do we will discuss this Acousto optic Effect initially with the elastic deformation which is the foundation to understand how this interaction takes place. We will study this elastic deformation in terms of the infinitesimal strain, the strain we will see that it is composed of normal strain and shear strain. Then there will be a strain and rotation matrix, which is there in the case of a deformation of any material medium. Then we will pick up this polarization optics a polarization of the acoustic waves.

(Refer Slide Time: 03:13)



Then the acoustic waves in an isotropic and anisotropic medium, pure and quasi transverse and longitudinal modes and the degenerate modes. So, first is to understand this elastic deformation.

(Refer Slide Time: 03:30)



So, any elastic deformation and corresponding mechanical strain modifies the optical properties of a medium. This strain this deformation which could be fixed or which could be time varying, will create a strain in the medium. Whether the medium is isotropic and isotropic because of the strain, there will be change in the optical properties. This fact is the photo elastic effect. That is the modification of the optical properties as a result of the elastic deformation.

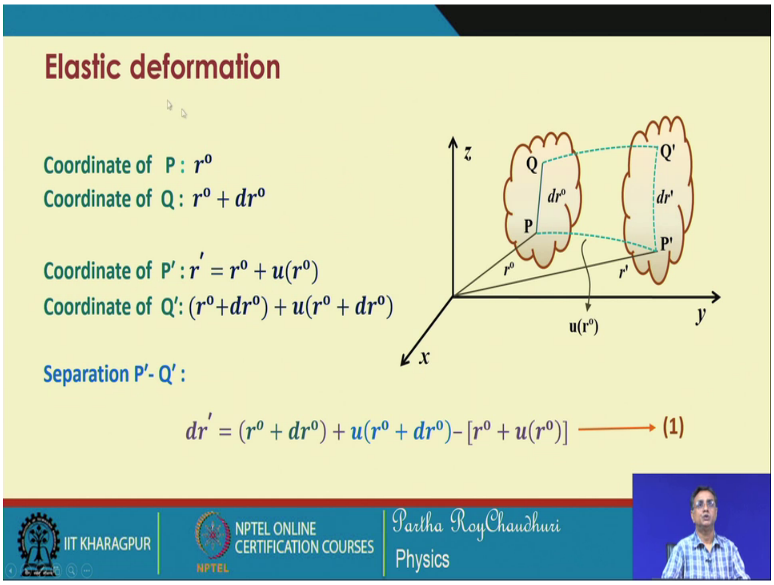
Now, when this elastic deformation that produce a strain, it could happen by a travelling acoustic wave, as the wave travels through the medium, there will be a periodic compression and rarefaction in the medium. That means, there will be a periodic deformation in the medium and because of the periodic deformation, there will be a periodic strain in the medium through which this acoustic wave is traveling because of this periodic strain, there will be periodic changes in the permittivity properties of the medium, which will be manifested in terms of the changes in the periodic changes in the refractive indices at the medium, which will result in a periodic refractive index grating.

That means, when an acoustic wave is travelling through a medium because of the dynamic photo elastic property, there will be periodic deformation, periodic strain, and periodic changes in the permittivity, periodic changes in the refractive indices, and therefore, there will be a formation of periodic grating as long as the acoustic wave is traveling through the medium.

Now, if a light wave is passing through such a medium, this wave we will be see a dynamic periodic refractive index grating and as a result this light wave will be diffracted. This effect we will call this Acousto optic effect. That is the changes in the optical properties by an acoustic wave traveling through a medium, whether the medium is isotropic or anisotropic, this effect is always there, ee will study the effect in the case of isotropic medium and anisotropic medium and in terms of various configurations of the acoustic wave through the medium. Whether it is a longitudinal wave or it is a shear wave we will study those cases.

But, by now we realize that it is important to understand the elastic deformation and how they are connected to the changes in the optical properties through a change in the permittivity of the medium periodic change in the permittivity of the medium when it is associated with an acoustic wave with a mechanical wave traveling through the medium. But otherwise for a static deformation, we will see how this deformation is related to strain and how the strain is related to the modification of the permittivity properties.

(Refer Slide Time: 07:17)

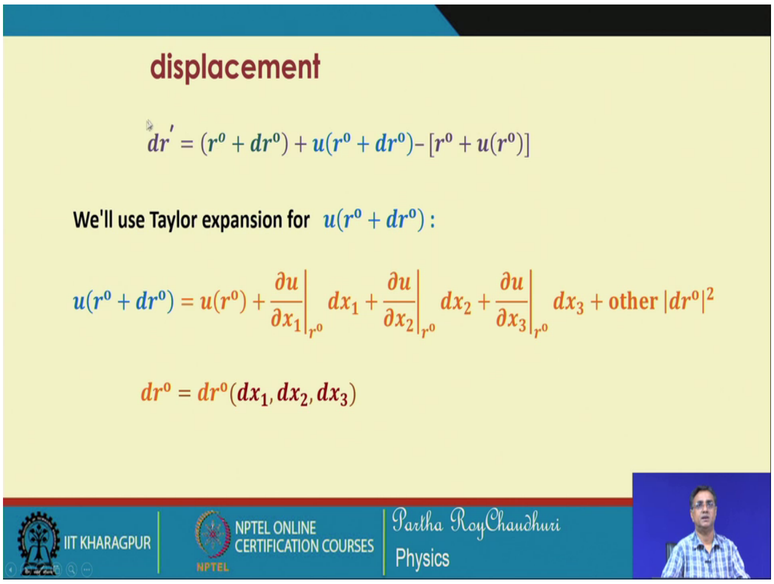


So, elastic deformation let us first consider the deformation of an element within the volume of a medium. Let us suppose that, this is the original undeformed state of an element there are two points P and Q whose coordinates with respect to this lab frame is given by this r 0 and r 0 plus dr 0. Now under deformation it moves to this place this is the deformed element where the point P has moved to P dash and the point Q has moved to Q dash and the resulting separation is now dr dash. So, this is the original element and this is the deformed element.

Now let us see that this coordinate of P dash is now r 0 plus u r 0 a function of u; u a function of r 0 whereas, the coordinate of Q dash is originally to r 0 plus dr 0. So, u as a function of r 0 plus dr 0 will be the new position characterized by this position. So, the deformed separation between these two points the original separation between P and Q was dr 0, but the separation under deformation is dr dash which is just the difference of these two.

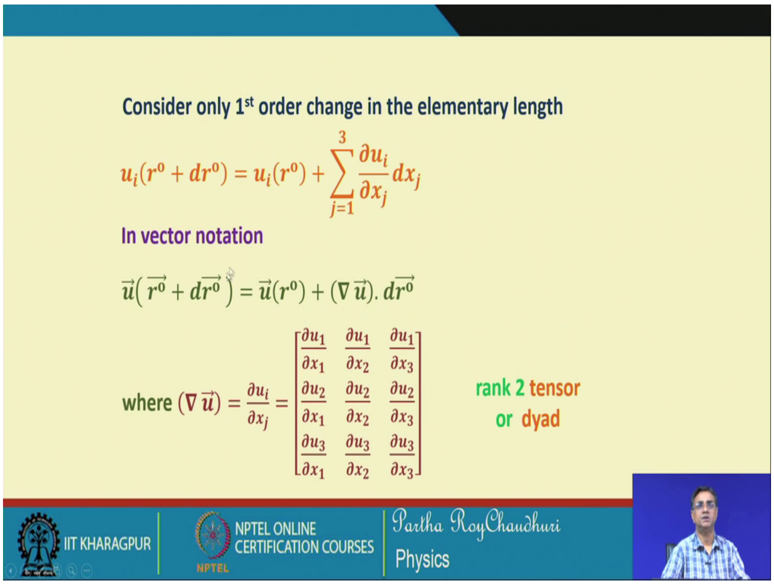
So, we can write r 0 plus dr 0; then minus of this quantity. So, therefore, we can write this equation we can write this equation because this u of r 0 u of r 0. So, that they will cancel these two will cancel and this will be the you know Q dash minus P dash if you do you will obtain this equation, which is to define the separation between P dash and Q dash the deformed separation the separation between the points under deformation.

(Refer Slide Time: 09:37)



So, dr dash that separation is equal to this we will use this Taylor expansion of this. Now, these two terms we will be cancelled. So, u r 0 plus dr 0 can be expanded in the Taylor series. So, if you written up to only three terms up to dx 3. We can write this d r 0 as a function of dx 1, dx 2, dx 3. Then we can expand this in this form. This dr 0 if we write as a function of is dx 1, dx 2, and dx 3 these coordinates we can expand in this from.

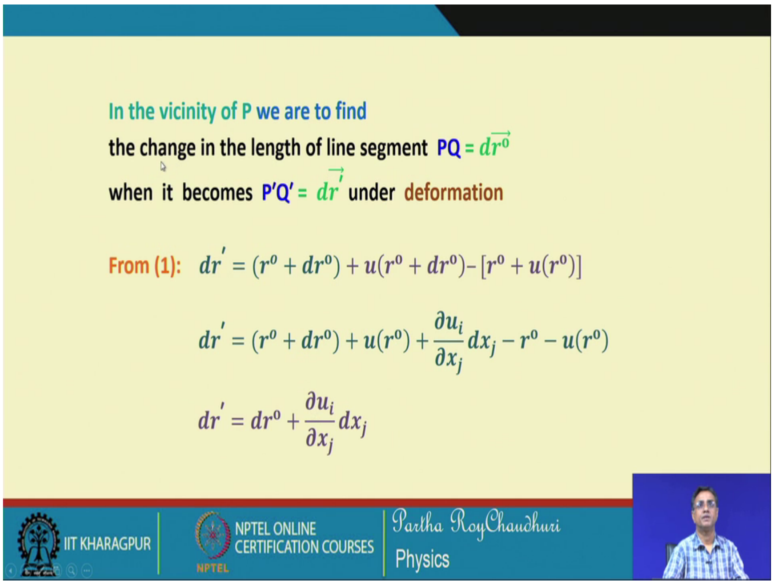
(Refer Slide Time: 10:22)



So, this expansion can be written in the compact form returning up to three terms that u of r 0 plus d r 0 equal to u i r 0 plus this and then some of these terms. In the vector notation we can write this connection as u r 0 and then del u dot dr 0 which where this del u we know that it is the displacement del u by del x j. So, where i and j can take up values up to 1, 2, 3 and this gives you this matrix which is a tensor and at a tensor of rank 2 or is also a dyad.

So, this is the del u this quantity which is represented by this can look at the diagonal elements del u and deformation along the same direction deformation along the same direction. So, the force the strain that is along the applied stress applied force. So, we will see how it works. So, in the vicinity of P we are to find we want to find the separation the change in the length of the segment PQ.

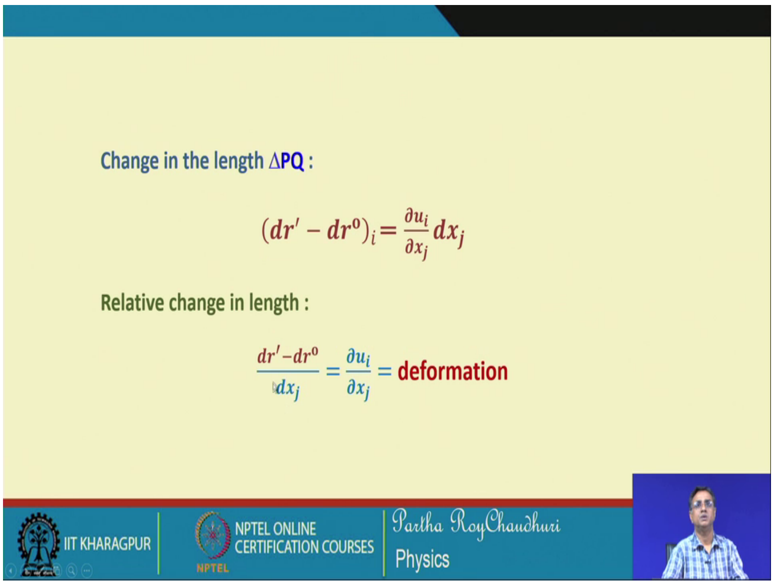
(Refer Slide Time: 11:46)



So, we started with a length PQ and now under deformation it has gone to P dash and Q dash. So, now, you would like to know how much is the change that has that this PQ element has suffered when it has become P dash and Q dash.

So, P dash and Q dash P dash Q dash minus P Q will be the change. And now this dr dash can be represented as this we have seen. So, this is the effective element therefore, d r dash which can be written as this quantity which can be written as this d r 0 plus dui dj dx j this also we have seen here d u d r 0 this quantity ok.

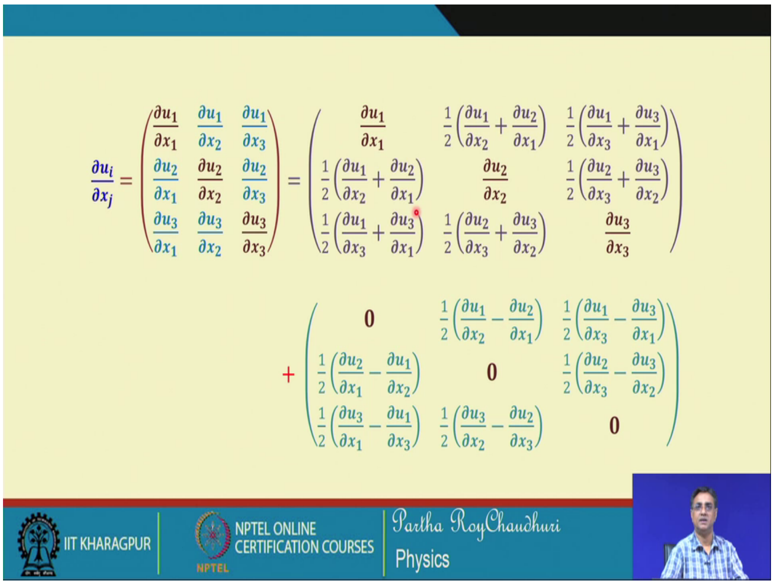
(Refer Slide Time: 12:46)



So, now that change in the length P Q that is P dash Q dash minus P Q will be dr dash minus dr 0 therefore, in a compact notation we can write that this is equal to du ui dj dx j. So, this is the change and the change per unit length we will define the strain. So, if we divide this by d xj. So, this change has occurred for a length for a length over a length of d xj. So, if I divide this by dx j like this then it gives you the deformation that is dui.

So, change in the length per unit length will be dui by dj which is the deformation. So, we arrive at this that we have a deformation which is del u i d del x j where i and j can take up 1, 2, 3 values which will give you all nine elements deformation y with respect to x x with respect to y, z with respect to x and so on.

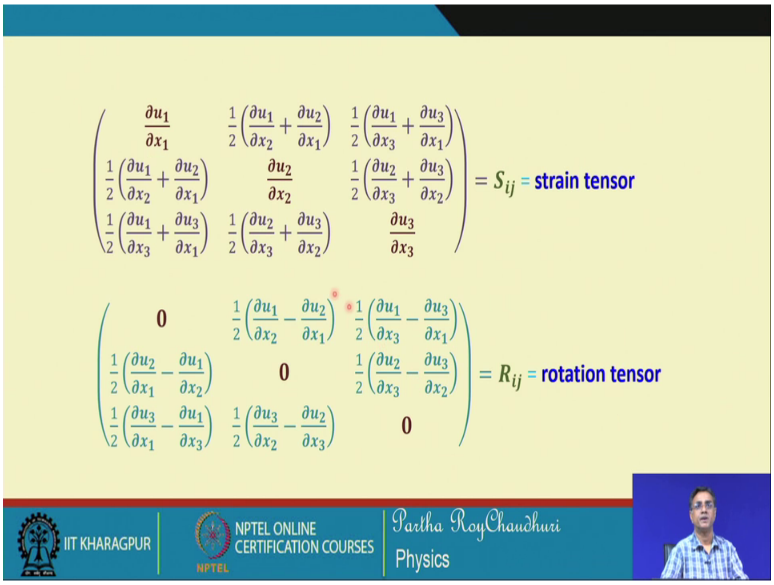
(Refer Slide Time: 13:57)



So, that can be put in this form del u 1 del x 1. So, this is the deformation tensor which can be written in the form of two matrices, I retain the diagonal elements as it is, but it can also be written in this form half of del u 1 del x 1 plus del u 1 and del x 2 again so, that sums of the two elements half of that twice we can write in this form.

So, these diagonal elements will be 0 in the case of the second (Refer Time: 14:32) matrix and if I write this off diagonal elements in this form. So, the sum of these two will be the same as it is will be the same as the off diagonal elements. And so we can write this deformation matrix as a sum of two matrixes and let us see what does this matrix indicate and what does this matrix represent?

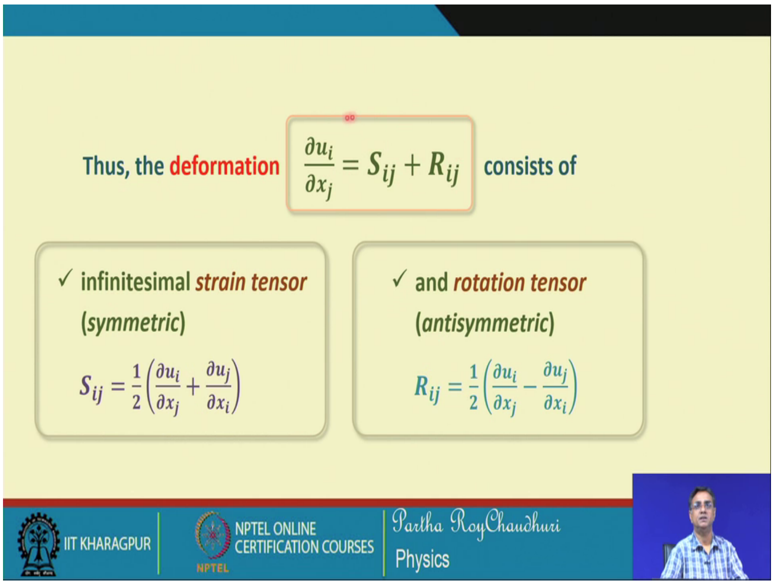
(Refer Slide Time: 15:09)



So, this is you can see that del u 1 del x 1 this will define the strain because if you apply if there is a change in the length in the same direction per unit length that will define the normal strain this will be the normal strain. Whereas, this quantity del u 1 del x 2 plus del u 2 del x 1 will define the shear strain.

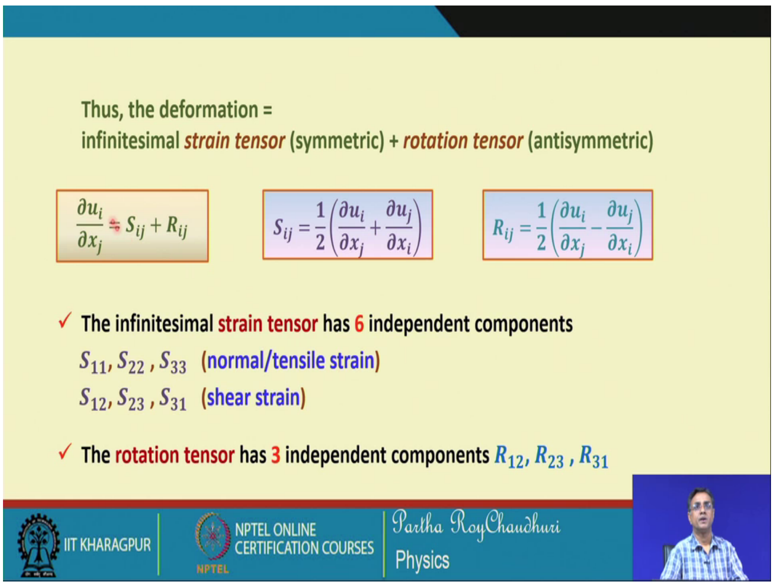
So, first matrix this one will be the it corresponds to the strain tensor whereas, this one del u 1del x 2 minus del u 2 del x 1 and so on will represent a rotation tensor. Therefore, we find that any deformation can be expressed as a combination as a sum of a strain tensor as well as a rotation tensor. We will try to understand, how these terms represent rotation and these terms represent strain?

(Refer Slide Time: 16:20)



So, any deformation is the sum of a strain tensor and a rotation tensor where this S ij stands for infinitesimal strain. If i equal to j then we get the diagonal elements, we get the diagonal elements i and j as the same. So, this will represent the normal strain and if they are different then we get the shear strain that is this tangential strain which are represented by this off diagonal elements. And for this Rij this will be a the elements of the rotation tensor. We will see that this is anti symmetric, because theta x will be mine in the off diagonal positions it will be minus theta x.

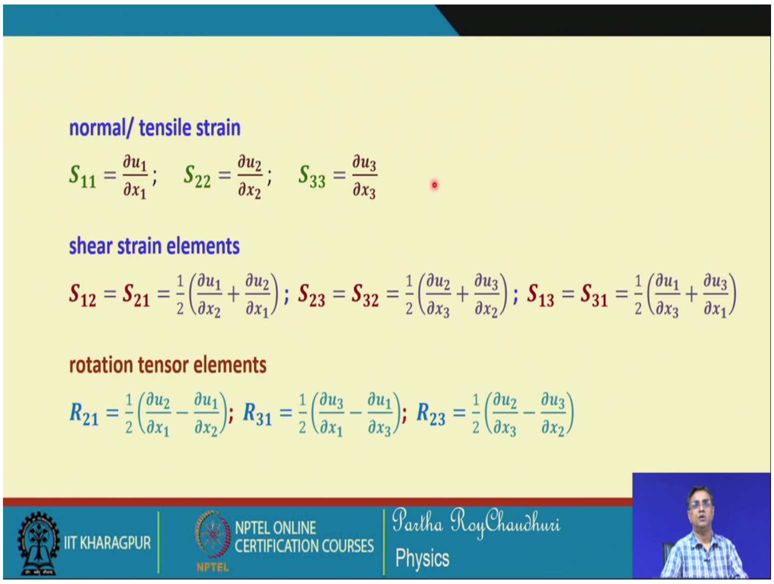
(Refer Slide Time: 17:10)



So, the deformation can be expressed as this strain infinitesimal strain tensor and an infinitesimal rotation tensor and they are represented by these two tensors. These two matrices the infinitesimal strain tensor has six independent components; we have seen they have six independent components, out of them these three diagonal components. The three diagonal components S11, S22 and S 33 they are the normal and tensile strain it represents a strain along the same direction whereas, it represents the strain along the perpendicular directions 12, 23 and 31.

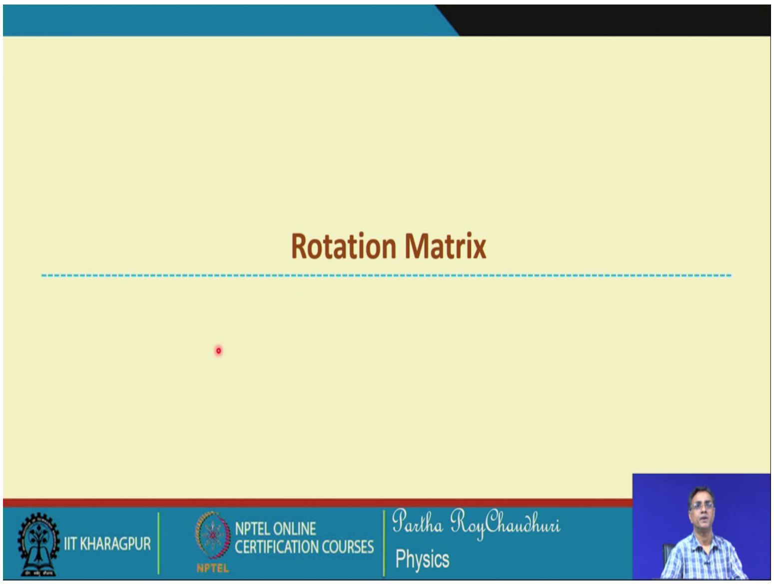
So, the strain in that tangential direction is represented by this off diagonal elements. For the case of rotation tensor R 11, R22, R33 that is the diagonal elements are always 0, but there will be three independent components R12 23 and 31, we will see they are anti symmetric in nature.

(Refer Slide Time: 18:20)

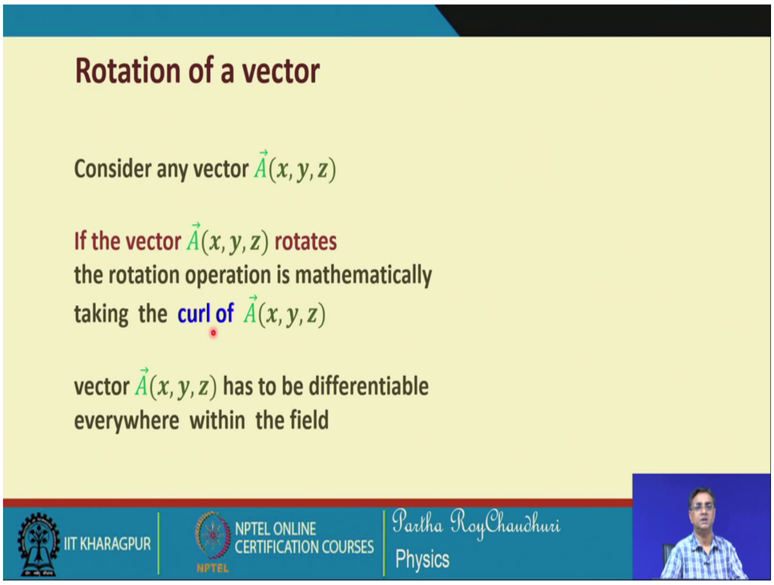


So, these are the normal strain, these are the shear strain elements of the strain matrix for the rotation matrix these are the three independent elements, 21 12 31 13 23 32. So, these are the shear elements.

(Refer Slide Time: 18:38)

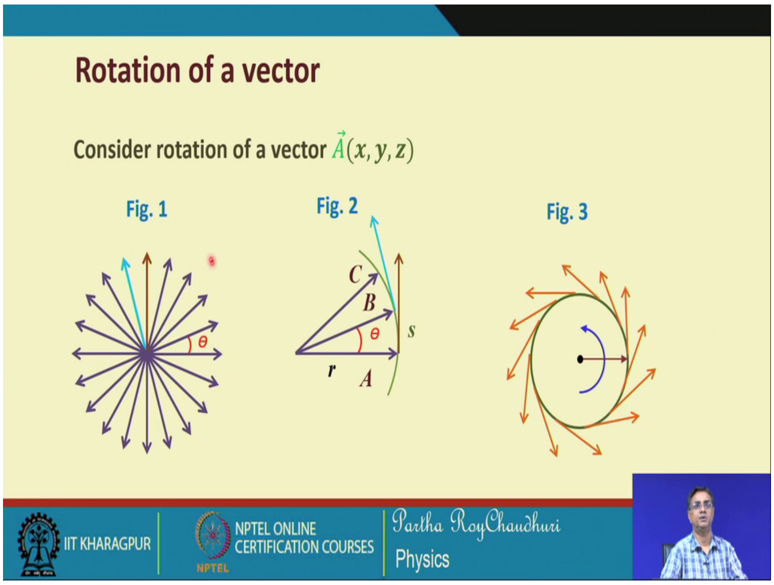


(Refer Slide Time: 18:40)



Now, let us understand how it represents the rotation matrix. To understand that let us consider any vector and the rotation operation is nothing, but taking the curl of this vector and the property that is required that this vector A has to be differentiable everywhere within the field where we are looking for the rotation.

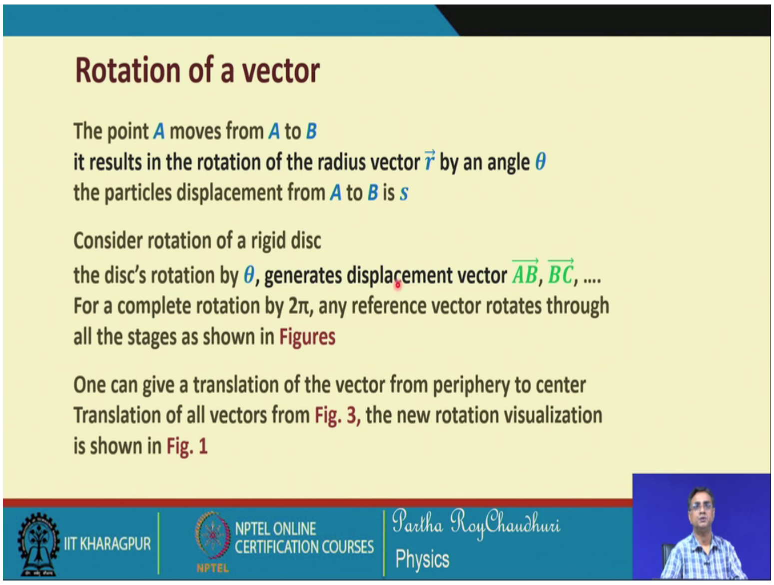
(Refer Slide Time: 19:00)



So, any rotation of a vector if we take this vector if we would to give a translation of this vector from here to here or from here to here, from here to here and from here to here the colors represent the positions, there parallel translation. So, being along the spoke this rotation of the vector is nothing, but being at the periphery of the rotation of the circumference the vectors like.

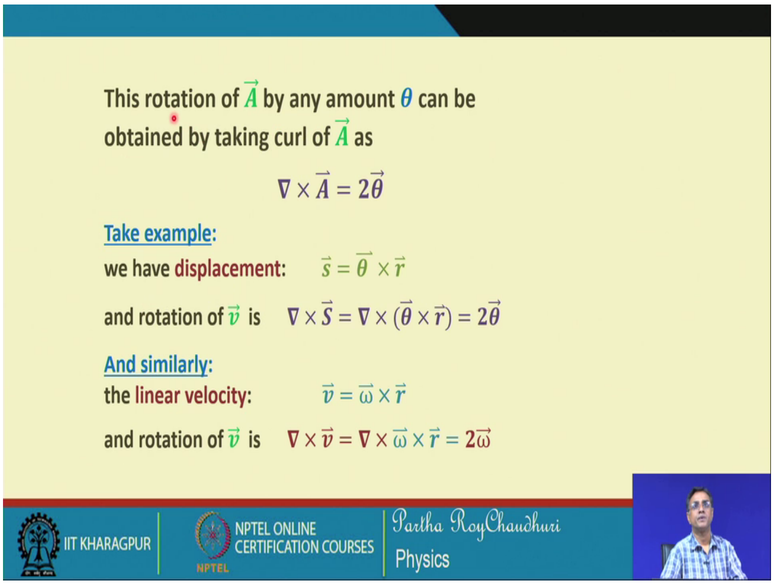
So, these two are identical situation these two are the same things. So, all the vectors can be picked up and placed at the periphery and they will look like this. So, this is the rotation of the vector a point A moves from A to B. A point A moves from A to B is nothing, but this vector this has undergone a change in it is position from here to here. A vector moves from this A to B and from B to C is nothing, but this vector has gone to this position and then internet has gone to this position. So, now we can change the, we can look at the change in the position of these two vectors.

(Refer Slide Time: 20:21)



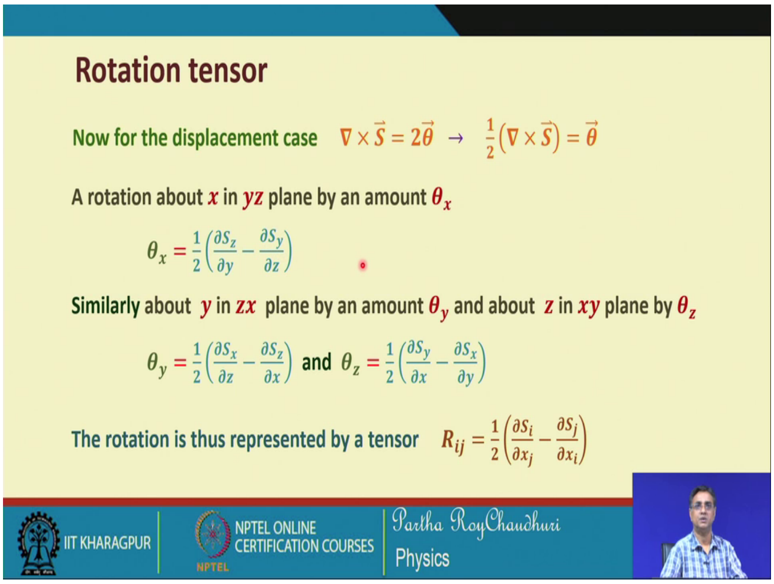
So, it is like the rotation of a disc and as shown the stages in the figure, one can give a translation of the vector from the periphery to the center or from the center to the periphery as well. That gives you the new rotation visualization. This rotation this rotation of the vector can be thought of the rotation of the vector like this.

(Refer Slide Time: 20:47)



So, therefore, this rotation of A by any amount theta can be obtained by taking the curl of this. So, del of A is equal to 2 theta, this we know from the elementary differential calculus and with mechanics. Then let us take the example if we take A is equal to displacement vector, then this will be connected with theta S equal to r cross theta cross r and del cross S will give you 2 theta. And similarly if you take this A as the velocity vector, then V equal to omega cross r, which will give you del cross V equal to twice omega. That means, if you take 2 to this side omega will be represented by half of del cross V. And likewise if we take theta on this side and this 2 to this side so half of del cross S will represent this theta. So, any rotation theta will be represented by half of this curl of this displacement vector. So, this is the point that that is connected to there I connected to the elements of the rotation matrix.

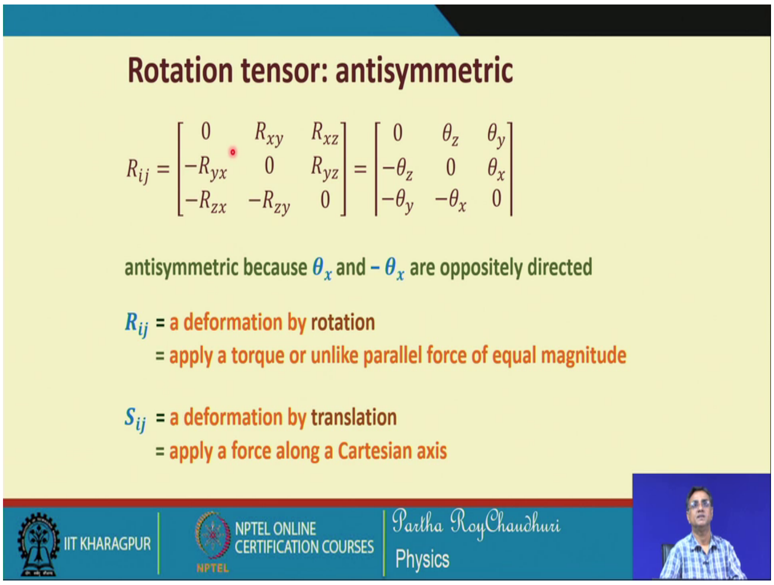
(Refer Slide Time: 21:58)



Therefore, half of del cross S is equal to theta. If I now write it component wise then the rotation about x in y z plane by an amount theta x can be represented by, look at this if I take the x component of theta. Then I will have to take the x component of this quantity which will be del S z del y minus del S y del z. So, this is your theta x, but if we take del S y del z minus this then this will represent minus theta x. Similarly, theta y is this the y component of this vector on either side will give me this quantity z component of this vector on either side will give me.

So, I have been able to see that this rotation theta x is nothing, but this displacement of the displacement components of the displacement vector along different directions and their differences. So, this therefore, this rotation tensor elements R ij can be written as half of del S i del xj. You can see that i and j can take up 1, 2, 3 these three values. So, it correctly represents the rotation of this vector. Therefore these elements are the R ij and R ji.

(Refer Slide Time: 23:33)



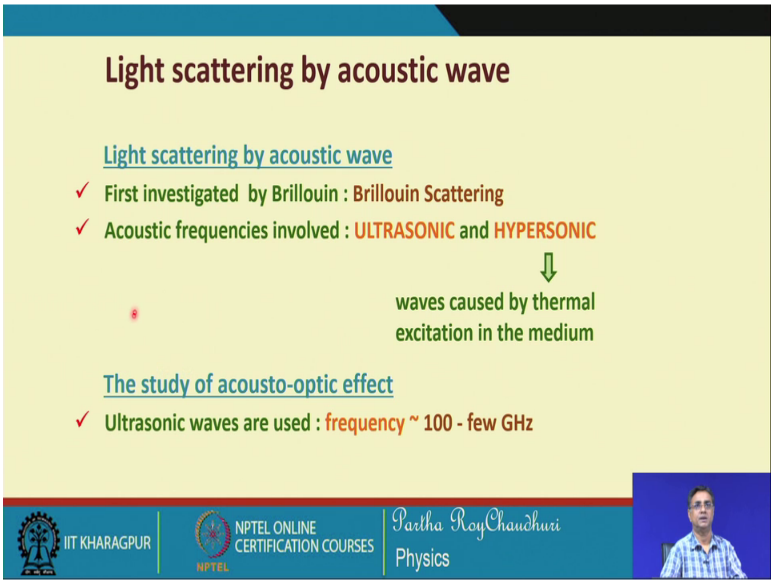
We have to just flip this minus this and as a result this R xy and R yx there not same there will be a minus sign there will be a minus sign R xz and R zx they are not same there will be a minus sign. Therefore this rotation matrix is always an anti symmetric matrix. So, this anti symmetric so if the R ij a deformation by rotation to visualize that to obtain that one has to apply a torque or unlike parallel force of equal magnitude which will create a rotation. And a deformation that occurs by translation to get that one has to apply a force along the Cartesian axis.

So, a torque is required to create a rotation and a translation a linear force is to be applied along the Cartesian axis to get the. But in the case of deformation both the things are coming into play and any deformation is a sum of these two things that is the strain and the rotation, which could be represented as the sum of the two tensors the strain tensor and the rotation tensor.

(Refer Slide Time: 24:48)

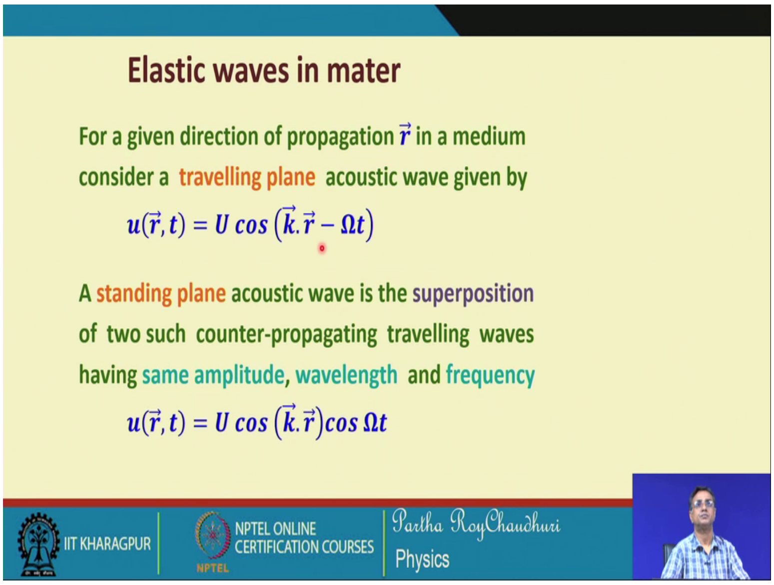
And now we will see the deformation that is induced that is created in a material medium when the acoustic wave is traveling through it.

(Refer Slide Time: 25:00).



So, this was first observed light scattering by acoustic wave by Brillouin and that is Brillouin scattering and acoustic frequencies involved ultrasonic and hypersonic waves. That is caused by thermal excitation in the medium. The study of acousto optic effect, is usually by ultrasonic waves are used within the frequency of 100 hertz to a few Gigahertz. So, this is the range of study of this acousto-optic effect.

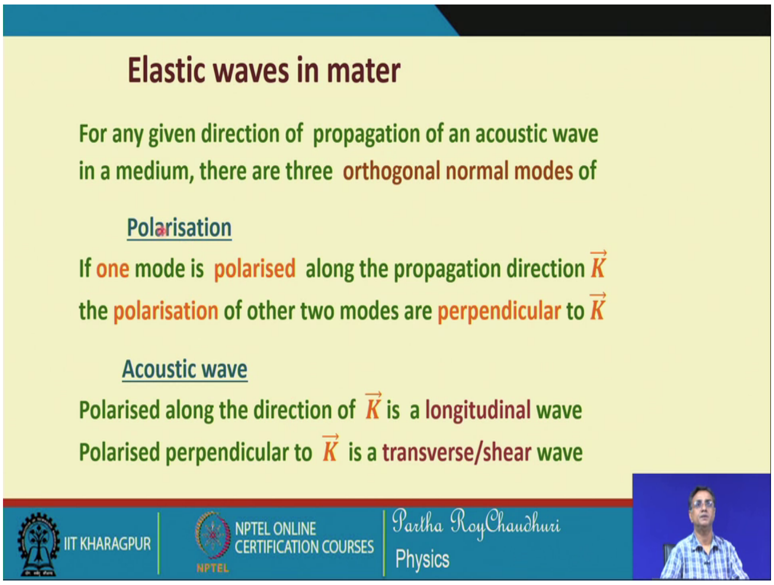
(Refer Slide Time: 25:33)



Now, elastic waves in matter for a given direction of propagation r in a medium any plane wave traveling plane acoustic wave can be represented by this. We have seen that this represents the phase of the wave, which is a plane wave front k dot r minus omega t; this capital omega is the frequency of the acoustic wave. This should be capital k in the case of acoustic wave if we want to differentiate from the electromagnetic waves capital k which will be the propagation vector U is the amplitude of the acoustic wave.

So, a standing acoustic wave is the superposition of two such forward propagating and backward propagating acoustic waves of the same amplitude. Then we get a standing wave which is also used to create a standing static type of deformation in the medium, which is also used as a fixed grating static grating in a medium. When we will see that if we allow the acoustic wave to be reflected back into the same medium in the same direction the superposition will create a standing wave and the position where it is compressed and the position where it is rarefied that will remain fixed with time with position, but not with time so, that will give rise to a static grating. So, this standing wave is also equally important standing acoustic wave to form a static refractive index grating.

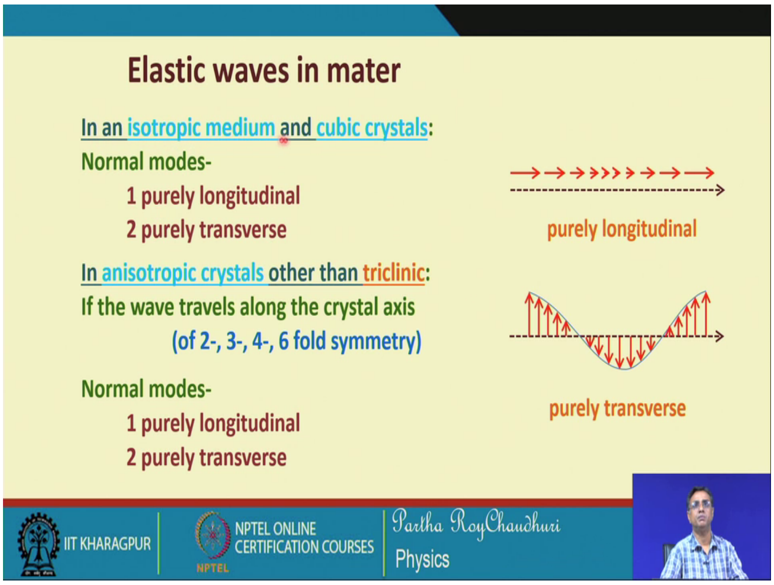
(Refer Slide Time: 27:07)



Now this looking at the elastic waves this property in the matter, there are three orthogonal modes of elastic waves in a in material medium. The polarization is one mode is polarized along the propagation direction that is what is called the longitudinal parallel polarization mode. The polarization of the other two modes are perpendicular to the direction of propagation K.

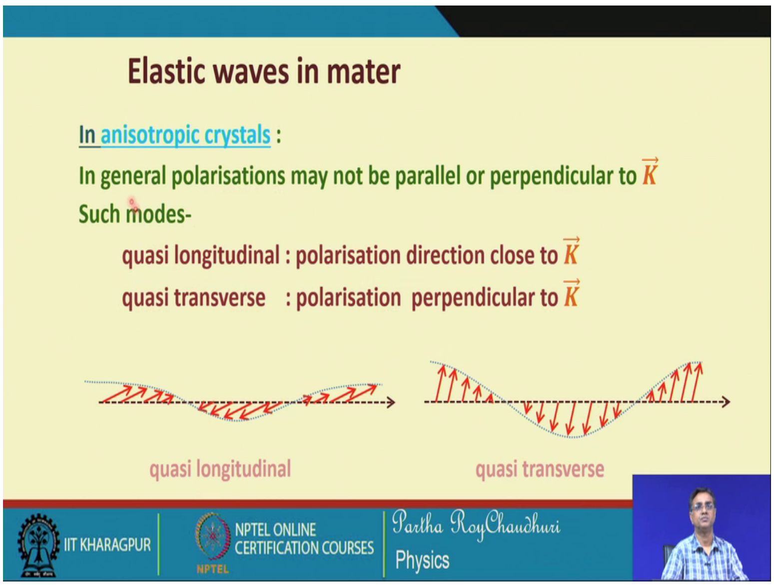
So, when there is an acoustic wave this could lead to a polarization that is along the direction of K and that is what is called the longitudinal wave. And when the polarization that is the medium particles the mechanical vibration of the particles occurs in the direction perpendicular direction to the direction of propagation K that is called the shear wave. And this shear wave we will be resulting that shear strain longitudinal wave will result in the longitudinal the normal strain that is the usual strain.

(Refer Slide Time: 28:15)



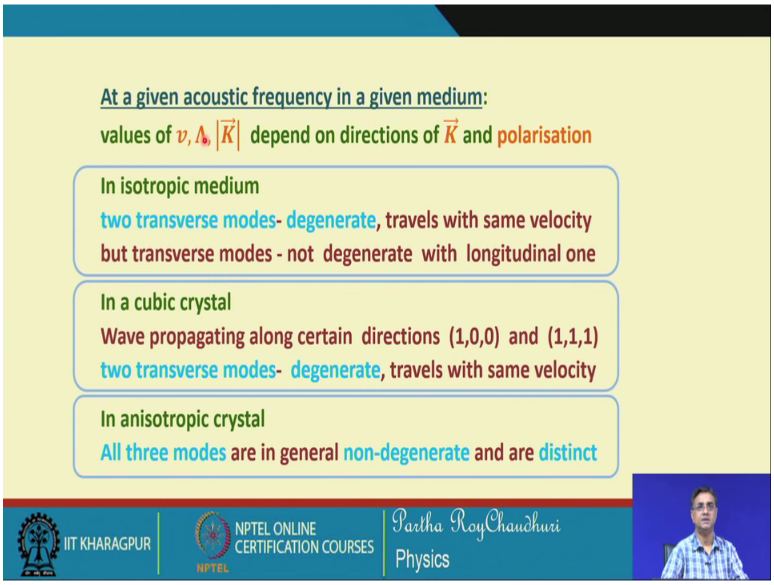
So, in anisotropic medium and cubic crystals this normal mode; one is purely longitudinal mode. You can see the medium will be compressed and rarified along the longitudinal direction; you can see the lengths of the vectors representing the compression and rarefaction in the medium. So, 1 purely longitudinal mode and there could be 2 purely transverse modes. So, this could be let us suppose this is directed along x, then there could be y polarization and there could be z polarization as well. So, there are two tangential modes to two transverse modes these two modes are degenerate they are identical.

(Refer Slide Time: 28:53)



So, in anisotropic medium anisotropic medium; however, in general the polarizations may not be parallel or perpendicular to K. In general it may be that the it all depends on the material properties. So, it is very close to close to longitudinal modes that is why it is called the quasi longitudinal mode and whereas, this one is more inclined to the transverse tangential polarization. So, this is why it is called quasi transverse quasi transverse polarization.

(Refer Slide Time: 29:29)



So, this again can have two polarizations whereas, this is the one possible polarization longitudinal mode.

So, at a given acoustic frequency in a medium the values of V, the velocity of the medium the periodicity and the propagation vector the depend on the direction of K and the say all they put together they decide this polarization. So, in anisotropic medium you have two transverse modes two of them are, these two transverse mode are degenerate travel with the same velocity, but transverse modes are not degenerate with the longitudinal modes so they are different from the longitudinal mode. In a cubic crystal the wave propagating along certain directions defined by this direction or by 1,1,1 plane making an equaling inclination will all the three coordinate axis the two transverse modes are degenerate travel with the same velocity.

In anisotropic crystal all three modes are in general non degenerate and are distinct. In general if this is true for an isotropic medium because of the natural anisotropy of the medium. So, by this discussion we try to understand that when an acoustic wave travels through a medium, it deforms the medium and this deformation will be decided by the polarization of the acoustic wave. It can have longitudinal polarization, it can have the transverse polarization, transverse polarization can have two degenerate modes.

So, these are all going to deform the medium periodically and as a result of that the there will be a periodic periodic changes in the in the mechanical properties, the strain. And as a result there will be a periodic changes in the permittivity and refractive index properties. We will discuss each of the cases in details and try to understand that different configurations of this longitudinal modes, transverse modes, x polarized modes, y polarized modes; in the case of isotropic material medium, in the case of an isotropic medium.

How they are going to change the permittivity properties? Periodic permittivity properties in the medium as long as that particular acoustic wave is existing in the mediumwe will try to understand, and then we will apply those deformation, those changes in the periodic refractive index properties to define the various modulation scheme and various devices.

(Refer Slide Time: 32:23)

So, by this discussion we try to understand this basic elastic deformation, the mechanical deformation properties. In terms of the infinitesimal strain which this strain is again consists of this normal strain and shear strain. If the displacement we estimate along the change of the length along the same line same direction, then it is a normal strain. And if it is estimated in the along the perpendicular direction you apply a force along this direction and there is a change in a perpendicular direction. This all we know with all details that this is your shear strain.

So, both of these are going to constitute this mechanical strain. Whereas, the rotation matrix these two strain and rotation matrix these two put together defines the elastic deformation. Then we discuss this polarisation of acoustic waves, there are two different polarisation modes longitudinal modes and transverse modes. These transverse modes are also composed of two degenerate orthogonal modes; there could be pure and quasi transverse and longitudinal modes. We will continue with these acoustic waves in the medium.

Thank you.