Modern Optics
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Lecture - 04
Maxwell's equations and electromagnetic waves (Contd.)

So, we have seen that the electromagnetic waves in free space the electric and magnetic field vectors how they are related, how they are oriented, what is the relation amongst them.

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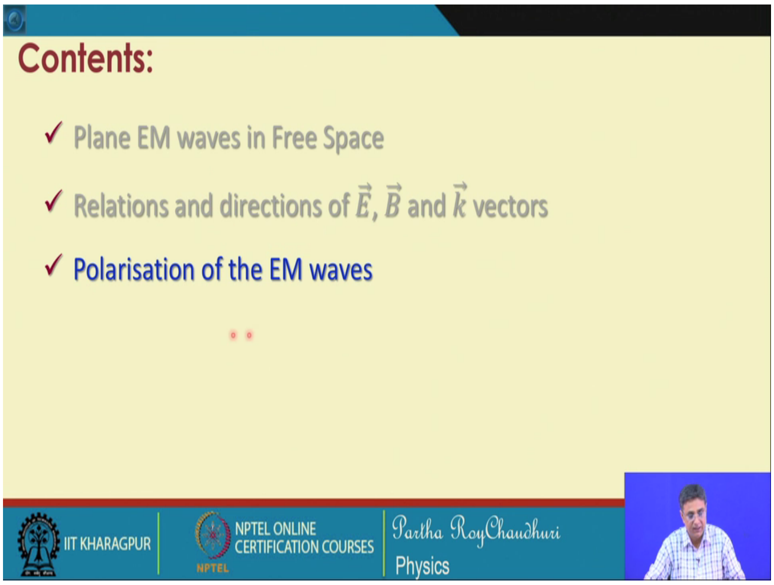

And now, we will see the polarisation property, very interesting property of the electromagnetic waves and that will first take up in the case of electromagnetic waves in free space.

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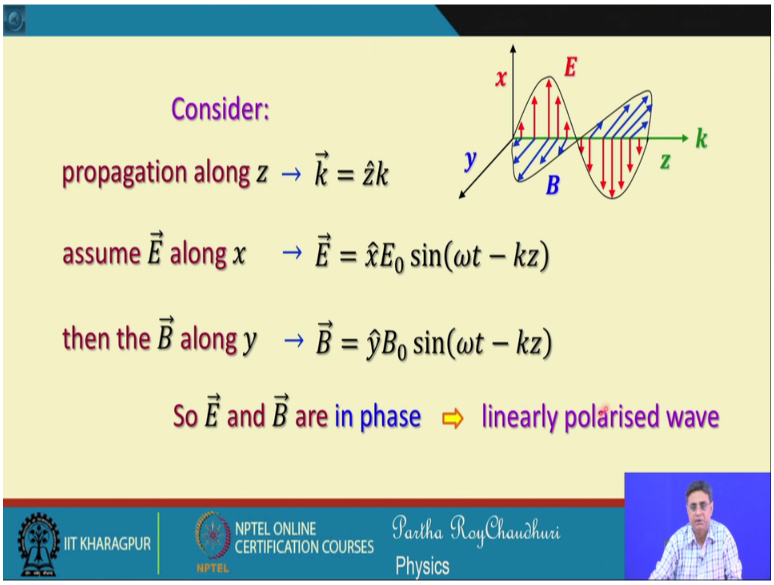

So, let us consider a situation that the wave is propagating along z direction. So, you can write that the k vector as z unit vector into k, where k is the magnitude of the propagation vector and you also assume that the electric field E is along x. So, in the earlier occasion also we assume the same convention that the propagation is along z direction and the electric field is oriented along x. So, having known that we can write the electric field in this way E equal to E 0 sin omega t x unit vector because, the electric field vectors are along the x direction.

Now, by writing this equation we have considered the imaginary part of the complex vector solution that is E equal to E 0 e to the power of i omega t minus kz. So, in the rest part of the discussion we will consider only in the sin cosine form. If I have considered E equal to x unit vector E 0 sin omega t minus kz, then the B will be oriented along the y direction we have seen as well. So, B equal to y unit vector B 0 sin omega t minus kz and you can see from these two equations that E and B are in phase. This situation, the orientation when the electron electric is restricted only in one particular coordinate direction and magnetic field is also restricted in one particular coordinate direction; such a situation of the electromagnetic waves called that linearly polarised wave.

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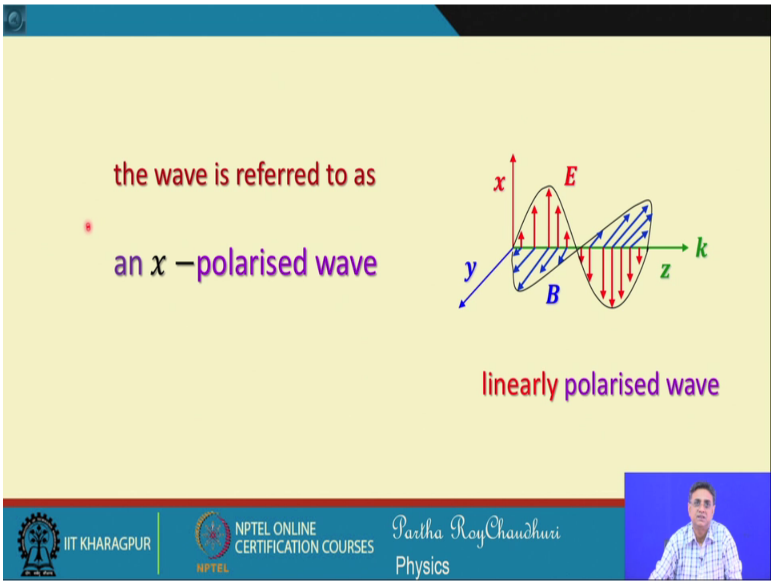

So, this is how we represent an x polarized wave. So, the situation where the electric field vectors are along the x direction we call such a wave as a x polarised wave, x polarised linearly polarised wave.

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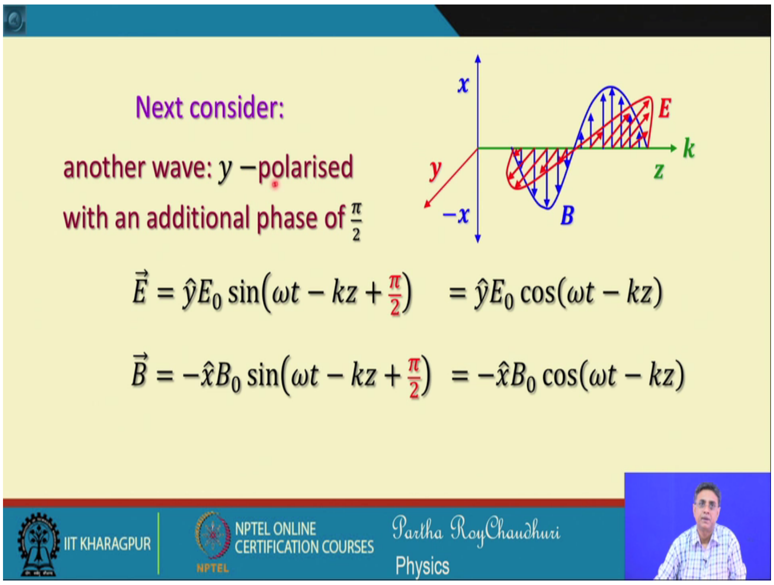

Then we consider a situation where because, our intention is to study the other variation the other properties polarisation properties. So, we consider another situation where the wave is y polarized, in that case how do I write this wave equation? E should be equal to y unit vector E 0 sin omega t minus kz and B will be equal to minus k minus x unit vector B 0 sin omega t minus kz. Look at this figure you see y along y the electric field is oriented and along minus x you have the electrical field oriented. So that means, this y polarised light means the electric fields will be along the y direction and magnetic fields will be along minus x direction.

So, by that convention if we consider this situation then we can take up this particular situation that we add an additional phase of pi by 2 to this wave because, we started off with this wave which has a 0 at z equal to 0. So, these a sin wave, but if I add a phase of pi by 2 to the power each of the waves, the wave will advance by a phase of pi by 2 and the other orientation of the electric and magnetic fields will remain same.

So, in that case how do I write this equation? For electric field everything remains same I just add a phase of pi by 2 and for the magnetic field as well as add a phase of pi by 2. As a result you can write this equation in this form y unit vector E 0 cosine of omega t kz. And for the magnetic field we can express as minus of x unit vector B 0, indeed these as become a cosine function earlier it used to be a sin function.

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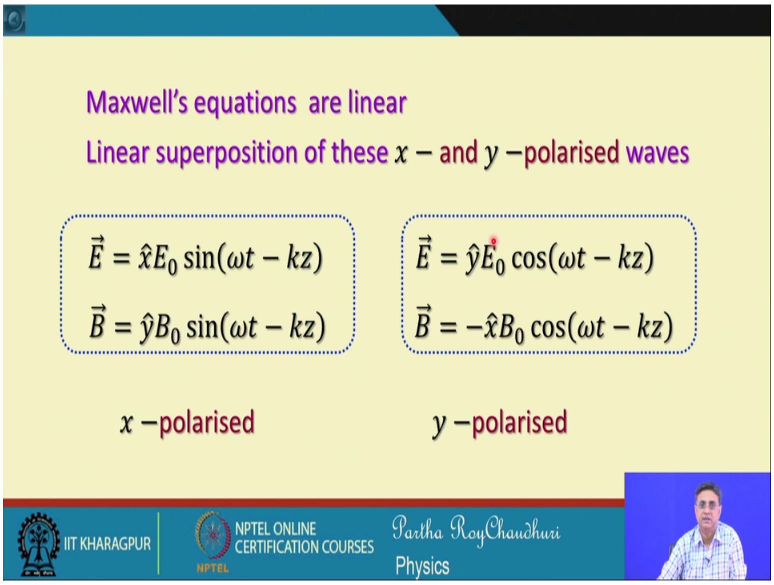

Now, recall that Maxwell's equations are linear and a linear superposition of their solutions the x polarised wave and y polarised waves will be considered. And that is also valid when I take this x polarised light which I have considered first, that is given by the set of these two equations. And the y polarised light which I have considered by which are given by the set of these two equations. This is y polarised light, but there has been an advancement of phase by pi by 2 from the starting point. So, at z equal to 0 we will consider what happens to the space, but before that you we will have to consider the superposition of these two.

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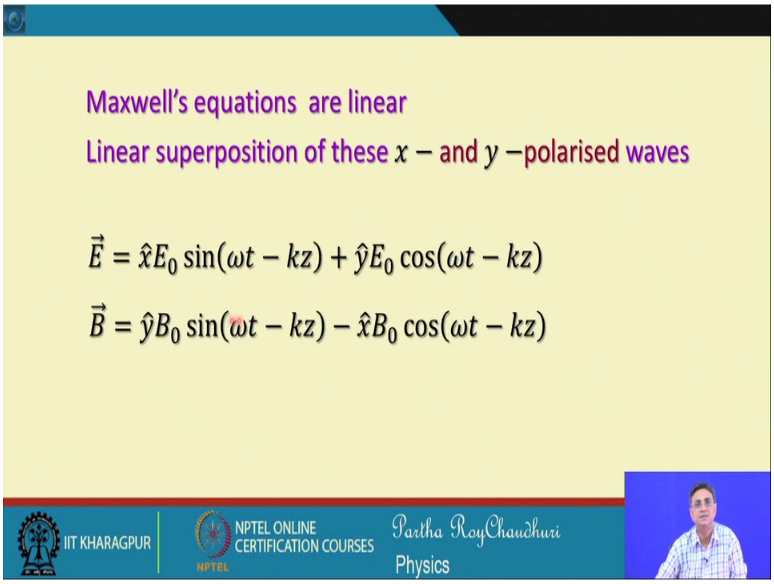

So, the linear superposition of this x and y polarised waves will give you the total electric field E equal to this x polarised part and x component of the x polarised part and y component of the polarize part. Similarly, for the magnetic field we can write these equations.

So basically, these are taken from these two, I just add take the superposition linear superposition of E and this E. So, this quantity plus this quantity put together and likewise for the magnetic field this quantity and this quantity will put together to get this equation in this form right.

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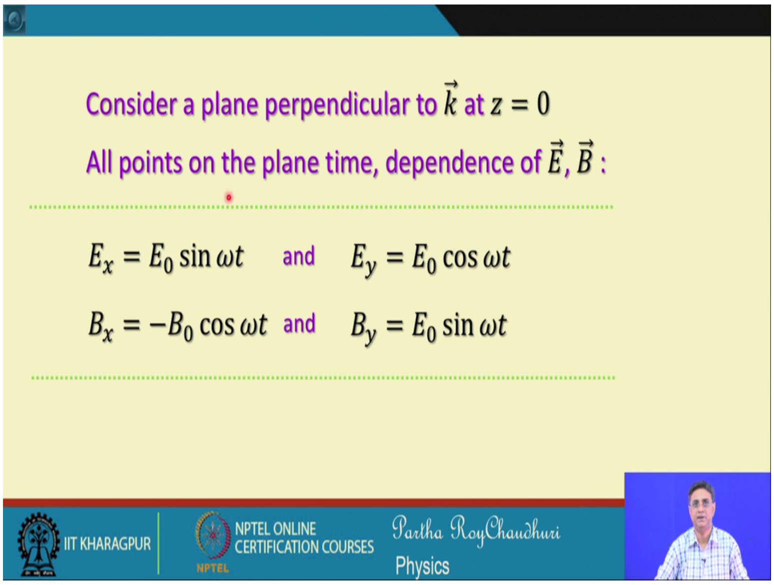

Now, we will consider what happens when we see the wave at z equal to 0. So, at z equal to 0 I put this z equal to 0 so, this become 0, this becomes 0 and all together all the in all 4 places it becomes 0. So, it becomes only a time dependent osculating electric field and indeed it is true at a given point for a travelling wave electric field vector suggest osculating at the same point and the osculation is purely simple harmonic type. So, E x can be represented as E 0 sin omega t and E y will be represented as E 0 cosine omega t. And similarly, for the magnetic field you can write this equation.

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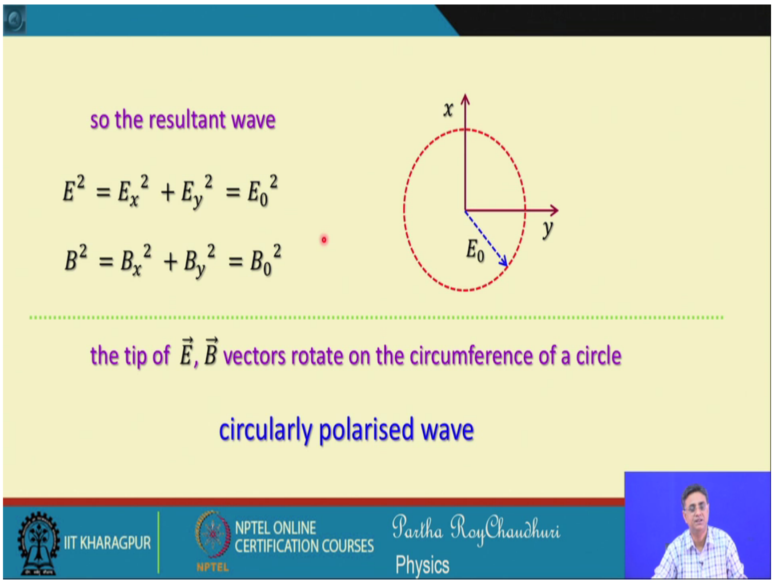

Now, look at this situation that if you take if you look for the amplitude of the total wave you can write, because, if you take the square and add them you get this equation E square, the resultant amplitude square will be equal to E 0 E 0 square which is constant. And this, equation basically gives you the it reminds you the equation of a circle E x square plus E y square equal to sum constant square. And the same thing will have seen in the case of rotating magnetic field and many other occasions E x square plus E y square equal to E z square will be represented by a circle, where the radius of the circle is the amplitude of the resultant wave.

For the case of magnetic field so, it will be a circle of different magnitude B 0 because, B 0 and E 0 they are related by a factor of c which is the velocity of the electromagnetic wave. So, they will describe a circle and the superposition will give you that the tip of the electric field and the magnetic field vectors rotate on the circumference of a circle.

This particular situation, this particular orientation of the electric field or the magnetic field in a travelling electromagnetic wave is known as the circularly polarised wave. And this is of tremendous importance for various parts of optics and also it is very interesting to analyze different situations for circularly polarised light, linearly polarised light, their production, their analysis and these are all very interesting findings.

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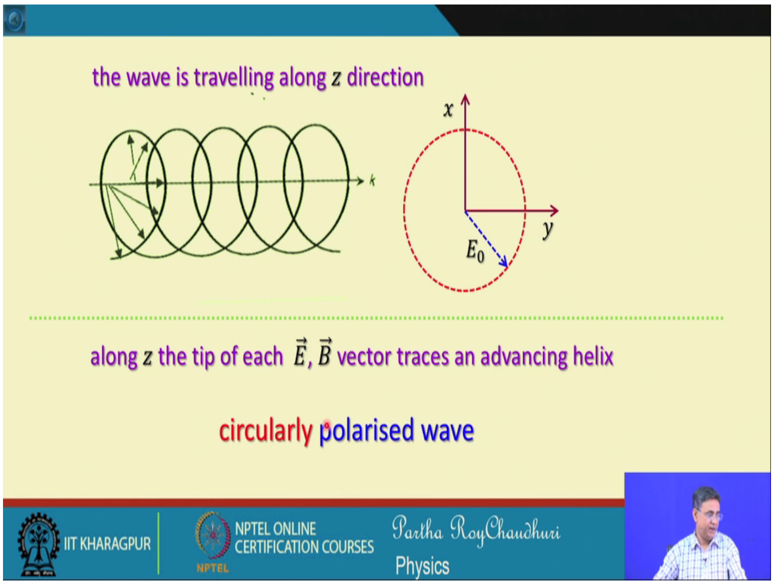

So, we have seen that the tip of the electric field vector is describing a circle, but there is an aspect that we have to consider whether it is rotating in the clockwise sense or in the anticlockwise sense. So, that should be decided by the orientation of the electric fields and the magnetic fields, their relative orientation of the electric and magnetic we will consider a particular situation. So, the electric filed vector the tip of the electric field vector, when you consider different positions along the z axis they will be advancing in the form of an helix.

So, E and B vectors traces an advancing helix as it is shown here and this situation is the truth three-dimensional picture for a circularly polarised wave. Even though it describes a circle at one plane at one position which is perpendicular one plane which is perpendicular to the z axis, but the tip is advancing because the wave itself is advancing. So, the tip will be described will be described in the trace of and the helix.

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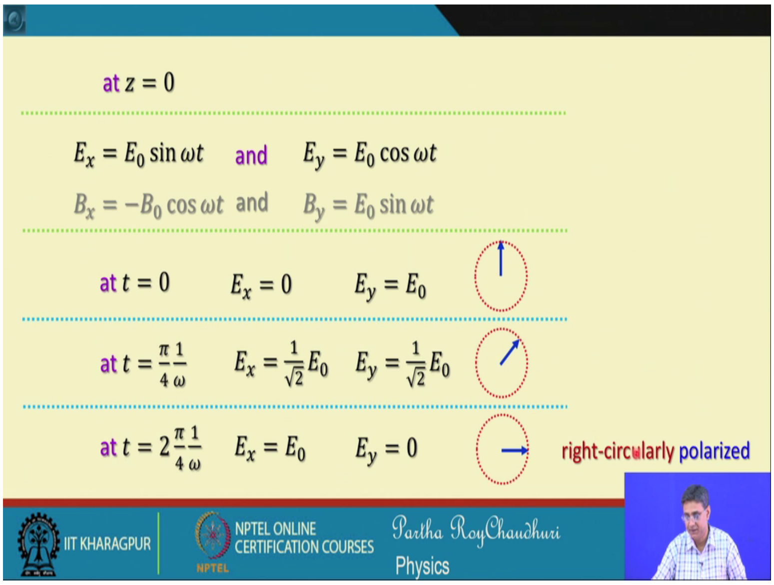

Next we consider this situation that, how we analyze whether it is the tip is rotating tip of the result and electronic result and electric field is rotating in the clockwise sense or anticlockwise sense. In order to do that, we considered that at z equal to 0, what happens to this electric field and the magnetic field. Let us consider the electric field E x equal to E 0 sin omega t and E y equal to E 0 cosine omega t.

Now, at time t equal to 0 that is at the initial time and the position is also at z equal to 0 for all the discussion, we have considered that the position is fixed at z equal to 0. That is why we would write the equation E x equal to E 0 sin omega t, E y equal to E 0 cosine omega t. So, at z equal to 0 you get that E x equal to 0, but E y equal to E 0 because, cosine omega t will become 1 and you get that because E y equal to the maximum amplitude E 0. So, this is your the orientation of the resultant electric field is along this

Then next we consider at time t equal to pi by 4 times 1 upon omega. So, at this time if I substitute this value of t in this equation, then this will give you E x equal to 1 by root 2 E 0 because, pi by 4 into 1 by omega will give you only pi by 4. So, sin pi by 4 we give you 1 upon root 2. So, E x will have the value 1 by root 2 E 0 and E y will have the value 1 by root 2 E 0; when you substitute this t value here, because, cosine and sin at 45 degree will give you the same value. That means, if you take the resultant of the electric field component and the magnetic field component that is E x square plus E y square which will be equal to E 0 square their position will now, be along this direction.

Next, if you consider twice this time at an interval of pi by 4 into 1 by omega t then E x because, now this quantity will become 1 whereas this quantity will become 0. So, the result is that we will get E x equal to E 0, but E y equal to 0; that means, E x the entire amplitude is along the x axis that is along these direction. So, it tells if you proceed further with the same analysis, with the values for intervals of pi by 4 1 by omega t, you can see that the electric field will next time orient along this direction and next time it will come along this direction and so on.

So that means, the tip of the vector is now rotating in this sense. So, this is called a right circularly polarised light. So, we could very nicely explain that how the tip of the electric field is rotating on the circumference of a circle and representing a right circularly polarised light.

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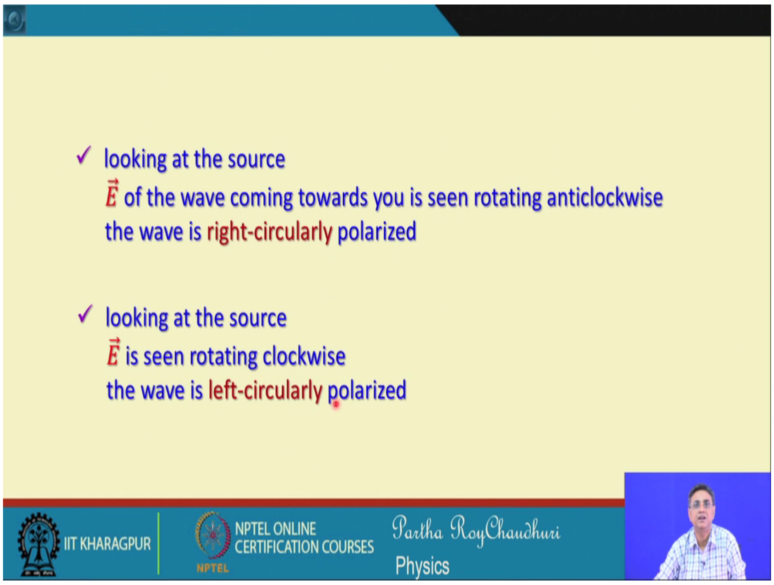

So, looking at the source this is how we distinguish the right-circularly polarised light and the left-circularly polarised light. So, looking at the source E of the wave coming towards you is seen rotating anticlockwise the wave is circularly polarised light; and looking against the direction of propagation that is looking at the source, if E is seen rotating in the clockwise sense the wave will be left-circularly polarised light.

So, in the earlier case what we have seen the electric field vects vector is rotating in the in the clockwise sense. But this is wave from the source looking against the source, looking towards the source this will be right-circularly polarised light.

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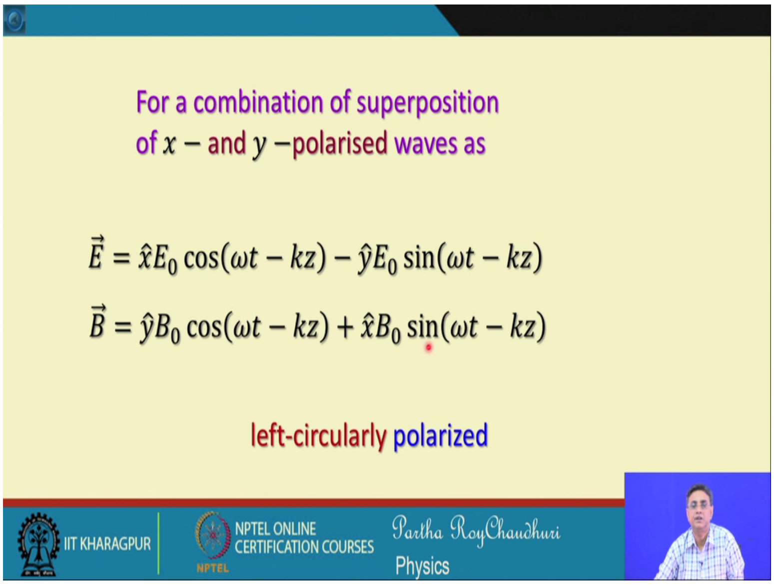

Let us next consider situation to look for left-circularly polarised light. For that if you considered the combination of x and y polarised waves in this form that is E equal to E 0 cosine omega t which is the x component and y component it will be minus E 0 sin omega t minus kz. For B similarly, we will have a cosine for the y component and the sin for the x component. Then this equation, if you do the same analysis that is at time at z at position z equal to 0. We write down this equation for this which will be E 0 cosine omega t and this will be E 0 sin omega t.

For the magnetic field this will be B 0 cosine omega t and B 0 sine omega t, but this is minus and this is plus. So, we can take the superposition. And we can find the resultant of this wave at different positions that is at t equal to 0, t equal to pi by 4 into 1 upon omega and 2 pi by 4 into 1 upon omega and so on and so forth. Then we can see that the tip of the of the electric field is rotating in the anticlockwise sense, in the counter clockwise sense and such a wave will be referred to as a left-circularly polarised light.

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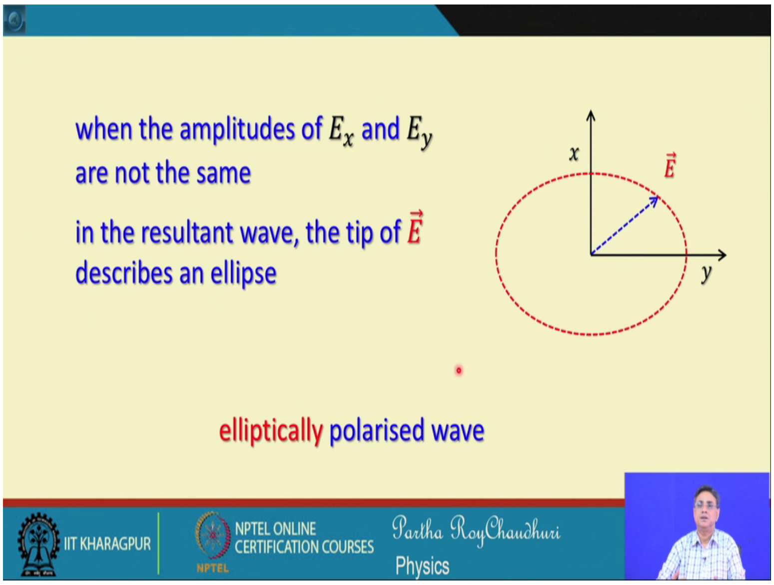

When the amplitude of E x and E y are not the same because so, for throughout the discussion we have considered that the electric field amplitude and magnetic field amplitudes; electric field ampli component the magnitude of the electric field for the x component and that for the y component they were same. Then it was a it was a circularly polarised light because, the contributions from E x and E y where same, but let us suppose in a situation where the E x and E y they are different then instead of a circle now, the tip of the electric field will describe an ellipse.

The ellipticity will depend on the relative magnitude of the E x and E y component and in the same way the tip of the electric field will advance as along the direction of propagation. So, the ellipse will down represent the circle in the case of elliptically polarised and such a situation is called elliptically polarised wave. So, we have seen in this discussion that starting from the basic electromagnetic wave equation for free space or vacuum how we write the solutions plane wave solutions.

And from the plane wave solutions we use the Maxwell's equation: the first equation, the second equation. From there we arrive at that the dot product of k and E, k and B they are 0. So, meaning that k and B will be perpendicular to each other, E and k there also perpendicular to each other. But, it is not sure till then that whether E and B are also perpendicular to each other. And for that, a more rigorous analysis we use that the Maxwell's third and fourth equation the curl equation; where we could constitute the cross product of the k and E. And also, the cross product of k and B to show that all the 3 components, all the 3 vectors that is the electric field, the magnetic field and the propagation vector three of them are oriented mutually perpendicular to each other.

And they, we also found the relative magnitude of the electric and magnetic field, they are they are connected by a factor c which is the speed of light in the case of free space. Then, we analyzed how the how the electric field is oriented, and how we achieve the linearly polarised light, circularly polarised light. And we have also seen, if the amplitude of the x and y component of the electric fields are different then we end up with and elliptically polarised wave.

Thank you.