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Lecture – 39 Electro-optic Modulators and Devices (Contd.)

So far we were discussing various crystals, the electro optic properties of gallium arsenide isotropic material and isotropic materials like lithium niobate KDP, ADP. And we have seen with all details how these crystals can be used for light modulation, phase modulation of light amplitude modulation and also polarization modulation the basic principles. And those things we have learnt in all details.

But this discussion was limited to the electro optic linear electro optic. We assumed all throughout that linear electro optic effect that is the Pockels effect that electric field and your the phase dependents they are linear the refractive index dependence was linear. Then the counterpart that is the remaining part that is the electro optic quadratic electro optic effect, we will just consider one case to understand that this effect is also equally important and lot of devices are being used for instrumentation and light signal processing.



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So, this effect is Kerr electro-optic effect, we have already introduced with this that this is a second order that is quadratic electro optic effect.

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And there to discuss this a quadratic electro optic effect, we have we are going to a discuss under this topic that how this electro optic tensor looks like in this case and how this in the ellipsoid undergoes change in the presence of electric field. Then Kerr effect in an isotropic medium, then because we will see that it is even though it is isotropic or centro symmetric material, still because of the second order dependence that is quadratic dependence this effect is always there.

And then we will talk about that the Kerr cell which are practically used in the laboratory or in the signal processing instrument. Then there are solid state that there are few disadvantages with the Kerr cell in the form of liquids. There are very competing very useful and promising electro optic crystals second order quadratic electro optic crystals.

Then we will discuss a laboratory experimental setup. We try to show how this your were light is modulated as a function of the external electric field. So, a typical graph to show you that, ok.

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So, this Kerr effect going by this by the name of the Scottish physicist John Kerr and he first came across this effect in the year 1878. And in an electric field any isotropic medium becomes birefringent; as this is because even isotropic, but this is because of the second order electro optic effect which is always there that is the quadratic dependence.

And it is normally neglected when linear electro optic effect is present. Because linear electro optic effect is just dependent on the linearly dependent on the electric field whereas, this is proportional to the square of the electric field. So, usually it is neglected because of the strength of the effect. And so, this unlike this linear electro optic effect this exists in any medium of any symmetry. Independently regardless of the symmetry of the material this effect is always there.

And an electric field alters the dimension orientation of the index ellipsoid. So, it is it could change the length of the semi major minor semi axis of the ellipsoid, or it can rotate it can change the orientation of the index ellipsoid because of this effect the quadratic effect. The change depends on the orientation of the applied field, but this electro optic coefficients appear in the form of a tensor which is a 6 by c, 6 by 6 36 element matrix. And we have already seen that this is represented by S ik, where I equal to 1, 2, 3, 6; k is also ok.

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So, in centro symmetric and amorphous materials linear electro optic that is the Pockels effect vanishes. We have seen that because of the symmetry, because of the directionality this Pockels effect linear effect does not arise. But second order electro optic effect comes into play. The second order electro optic tensor we have used this original notation ijkl and when it is contracted then you can write this equal to ik and this is again 36 elements.

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RECALL the impermendility tensor			
RECALL THE IMPERMENDING TENSOR			
$\eta_{ij}(\mathbf{E}) = \eta_{ij}(0) + \sum_k r_{ijk} E_k + \sum_k s_{ijkl} E_k E_k$			
- Sec.			
replace (<i>i</i> , <i>j</i>) with single index <i>i</i>	replace (<i>k</i> , <i>l</i>) with s	single index k	
$(1,1) \rightarrow 1$ $(2,3) = (3,2) = 4$	$r_{m} = r_{m}$	$S_{iikl} = S_{ik}$	
$(2,2) \rightarrow 2$ $(1,3) = (3,1) = 5$	- ijk - ik	-ijki -ik	
$(3,3) \rightarrow 3$ $(1,2) = (2,1) = 6$	$r_{440} = r_{40}$	$r_{\rm rot} = r_{\rm ct}$	
×	$r_{112} - r_{12}$	$r_{12k} - r_{6k}$	
	$s_{1112} - s_{16}$	$s_{1231} - s_{65}$	
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Let us look at how you can do this contraction if we recall the impermeability tensor, which is actually a function of the electric field and the quadratic dependence of the electric field.

So, this k and k it can assume 1 to 6, any values between 1 to 6. So, this part is the Pockels part and this part is the Kerr effect part. So, r ijk we have seen that if you sink this k and L with a single index, you can write j and k by k and ijkl k and L by only k. So, therefore, for example, 1 1 2 will become 1 2; 1 1 1 2 will become 1 6. And similarly 1 1 2 if I replace this 2 by general index k, then it becomes you see 1 2 will become 6, because it will involve x and y. So, that goes to the 6th position of the element; 1 2 will correspond to 6, 2 3 3 1, 3 is z 1 is x. So, z x will correspond to 5.

So, if you remember the impermeability tensor, right.

$$\begin{split} & \Delta \eta = \Delta \left(\frac{1}{n^2}\right) = \sum_{j=1}^3 s_{ik} E_k E_k \\ & k = 1, 2, 3, \dots, 6 \\ & s_{ij} = \text{electro-optic tensor} \end{split} \qquad \begin{bmatrix} \Delta \left(\frac{1}{n^2}\right)_1 \\ \Delta \left(\frac{1}{n^2}\right)_2 \\ \Delta \left(\frac{1}{n^2}\right)_3 \\ \Delta \left(\frac{1}{n^2}\right)_4 \\ \Delta \left(\frac{1}{n^2}\right)_4 \\ \Delta \left(\frac{1}{n^2}\right)_5 \\ \Delta \left(\frac{1}{n^2}\right)_6 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} \\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35} & s_{36} \\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} & s_{56} \\ s_{61} & s_{62} & s_{63} & s_{64} & s_{65} & s_{66} \end{bmatrix} \begin{bmatrix} E_1 & 2 \\ E_2 & 2 \\ E_3 & 2 \\ E_2 & E_3 \\ E_2 & E_3 \\ E_1 & E_2 \end{bmatrix} \end{split}$$

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So, this is how so, this is this part now we are looking at that is the Kerr effect, not this part; this part we have discussed in all great details for various crystals. Now, we will be working with this. So, this is the change in the impermeability and k can take up this value 1 to 6 is electro optic tensor. And change in the impermeability will now depend on the external field if I assume that E x E y and E z; all of them are present then the then the changes will be all along assuming that all these parameters are non-0.

But we will see that for various crystals, these parameters are most of the places they are 0, and the non-zero components will only provide the effect in terms of the changes in the permeability, impermeability.

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Ellipsoid ed	quation
The equation of	index ellipsoid in presence of electric field
$\left(\frac{1}{n_x^2} + s_{1j}E_j^2\right)x^2 + \left(\frac{1}{n_y^2} + \right)x^2$	$s_{2j}E_j^2 y^2 + \left(\frac{1}{n_z^2} + s_{3j}E_j^2\right)z^2 + 2yzs_{4j}E_j^2 + 2zxs_{5j}E_j^2 + 2xys_{6j}E_j^2 = 1$
where $E_j^2 \Rightarrow$	$E_1^2 = E_x^2 \qquad E_4^2 = 2E_y E_z \\ E_2^2 = E_y^2 \qquad E_5^2 = 2E_z E_x$
	$E_3^2 = E_z^2 \qquad E_6^2 = 2E_x E_y$
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So, the equation of the index ellipsoid in presence of the electric field now, because in the earlier case it was only E now it is E j square, E j square, E j square. So, this j can now take up the values from 1 2. And this 1, this is S 2, this is S 3, this is S 4, because it involves y and z, this will be zx 5 x and y 12 to 6. 1 to will give 6, this is your 3 1 will give 5, this is 2 3 will give 4.

So, this is how the index will be used. So, u one square is actually E x square E 2 square and so on, and E 4 square it means that it is actually ui and E z, E z, E x. So, this equation is complete and it tells you that this is actually contacted. So, if you have E z so, that is actually because it is 4 S 4. So, it will be a combination of this x and y y or z z or x together, right.

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Now, let us discuss this effect in isotropic medium and that is one representative to understand the effect very clearly.

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An optically isotropic medium in a static electric field becomes birefringent. Because of this effect even though it is naturally isotropic, but it may become anisotropic. The effect mostly goes to the alignment of the molecules that is the mechanism internal change in the structure deformation acted by the electric field.

So, in presence of the field the medium assumes optically uniaxial anisotropy. This is very important we will see. It becomes anisotropy it becomes anisotropic, but it is again uniaxial because 2 of the refractive indices will remain the same. And electric field direction defines the optical axis. This is also again very interesting to know that it is the direction of the electric field which will define the optic optical axis; that is about which they the axis about which the that is the extraordinary refractive index axis the optical axis.

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So, that we will see with the example S 1 2 equal to S 1 3, this is for isotropic medium and S 3 3 equal to S 1 1. So, from this the general representation of this electro optic second order electro optic this tensor. And from here I use this for the isotropic medium, that then it becomes this S 1 1 S 1 2, but this is again S 1 2 and S 1 they are same, these 2 are also same. This is also same and all the diagonal elements they are also same, but this 4 4 5 5 6 6 they are also the same you can say. So, this is the non0 elements arising here in the electro optic tensor second order tensor.

Now we will use the electric field orientation along with this with this tensor to define the changes in the impermeability. (Refer Slide Time: 11:15)



So, you have this S ij which is represented by this part. E k and E k in this case oh we have considered the apply electric field is applied along only z direction, so, along z axis of anisotropic crystal. You can see that this will be the only active component of the of the electric field. As a result, if you multiply, so, only this column will be multiplied and these 3 will give you 0 effect. So, these are only the square term effect not the cross term effect.

So, 1 2, 1 2, 1 3 will arise in the x square y square and z square term. So, that is what is it is here, with the x square term you have this with the y square term 1 2, these 2 are the same. So, that is why I was saying that it becomes uniaxially anisotropic; that means the change in the refractive in the x and z, sorry, this is in the y, they are the same. Whereas, the change in the refractive index in the z direction will be different and if you apply the electric field along z direction, then only you get this index ellipsoid. And this is very straightforward you have nx and ny they are identical, but it is different from.

So, we can call this is n o this is also n o, but this is n e, that is extraordinary refractive index this is ordinary. So, it is under the field isotropic material becomes anisotropic and this anisotropic is of uniaxial nature.

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That means that, under the electric field when this isotropic material was a sphere ellipsoid was actually a sphere. So, you have all the 3 axis same nx equal to ny equal to nz. But if you have applied an electric field along the z direction, then because of this z direction electric field, this has changed the values instead of 1 by n square it has now become 1 by n square plus this quantity and so on and so forth.

Therefore, the only the lengths of the lengths of the ellipsoid has lengths of the lengths of this sphere along these 3 mutually perpendicular directions have changed, as a result this sphere has become an ellipsoid. And these 2 that is x and y they have undergone a changed by equal amount whereas, this nz has changed by a different amount. So, that is why nx prime is equal to n y prime, but not equal to nz prime.

And you can see that this new ellipsoid has taken the form of this so, this is your n z. So, in a nutshell that you have a sphere under the electric field along the z direction it has become an ellipsoid; that means, all the 3 refractive indices were identical. Now we find that 2 of them are the same, but it has increased from the ones which were in absence of the field, but the other one it has also increased, but this is different from the x and y values. So, it has become a uniaxial crystal.

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Fountion to the index ellipsoid		
(1, -2) = (1, -2) = (1, -2) = -2		
$\left(\frac{1}{n^2} + s_{12}E_z^2\right)x^2 + \left(\frac{1}{n^2} + s_{12}E_z^2\right)y^2 + \left(\frac{1}{n^2} + s_{11}E_z^2\right)z^2 = 1$		
in a more compact way		
$\frac{x^2+y^2}{z^2}+\frac{z^2}{z^2}=1$ where $n_o=n-\frac{1}{2}n^3s_{12}E_z^2$		
n_{e}^{2} ' n_{e}^{2} and $n_{e} = n - \frac{1}{2}n^{3}s_{11}E_{z}^{2}$		
in presence of electric field		
isotropic medium becomes a uniaxially anisotropic one		
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So, the equation of the ellipsoid is this, and in a more compact way we can write this is the form that we often encounter, x square plus y square by n 0 n o square is equal to z square. This is the form of uniaxial representation of this index ellipsoid. But the values are now this so, we can see that it has the 2 values no and n e are different. So, it can be used for phase modulation for amplitude modulation as well. So, in presence of this electric field it has becoming uniaxially anisotropic.

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The birefringence becomes.... $\Delta n = n_e - n_o = \frac{1}{2} n^3 (s_{12} - s_{11}) E_z^2$ with applied electric field $E = E_z$ Kerr birefringence is often $\Delta n = \frac{1}{2}n^3 \left(s_{12} - s_{11}\right) E^2 = K E^2$ written as $n_e - n_o = K \lambda E^2$ K= so-called Kerr constant Therefore $\Delta n \propto E^2$ - quadratic λ = the vacuum wavelength Partha RoyChaudhuri NPTEL ONLINE CERTIFICATION COURSES IIT KHARAGPUR Physics

So, the birefringence in this case n e and n o; that is, if I have if I have a light which is polarized in any direction in the xz plane, then it will see one reflects refractive index along x n x, and it will see another refractive index the 2 components we will see another refractive index the 2 mill see another refractive index which is n z. So, these 2 will travel along this length y with different velocities, and they will develop a phase delay.

So, that will result in the retardation. Other way also if I allow a light to pass through the crystal along the x axis, but it is polarization is anyway between this y and z axis, because this y will also see a refractive index, electric field dependent refractive index; which is different from the polarization that will see this refractive index n z these 2 are different. So, again while passing along these direction, these 2 components will see a will see a delay between them in terms of the phase.

So, therefore, your no that we have seen is equal to this whereas, n e is equal to this. So, in presence of the electric field isotropic medium has become uniaxial, we were discussing this birefringence that delta n has now become equal to this n cube S 1 2 minus S 3, if it is along xz or yz, if the polarization is at 45 degree between x and z or y and z with the applied field along E equal to e z.

So, and this gives you this interesting relation the delta n birefringence is actually proportional to E square. E z is now E that is why any field E along z direction can be represented by E. So, it delta n depends on is proportional to E square. So, the proportionality constant is k, but we will define that more often it is used that the proportionality constant involves this equation; that is, if I write the phase then we will write twice pi by lambda into this n e minus no into L that is equal to phase difference.

So, this lambda if it goes to the right hand side, that proportionality n e minus no is equal to k lambda E square usually this is called the Kerr constant. So, and this equation is very well used for in the laboratory purpose for calculation of the Kerr effect the rotation. So, this k equal to so called Kerr constant, lambda is the vacuum wavelength of this. And you can see from here that delta n is actually proportional to E square, and you get a quadratic effect of the change in the birefringence.

So, the change in the birefringence is proportional to the square of the electric field. So, this is a very beautiful way of looking at this Kerr effect through a isotropic crystal.

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Liquid	Formula	Wavelength $\lambda(\mu m)$	R I (<i>n</i>)	Kerr Constant $K(m/V^2)$
Benzene	C_6H_6	0.633	1.503	4.9 × 10 ⁻¹⁵
Carbon disulfide	C_6H_6	0.546	1.633	3.88 × 10 ⁻¹⁴
		0.633	1.619	3.18 × 10 ⁻¹⁴
Chloroform	CHCl ₃	0.589		-3.5 × 10 ⁻¹⁴
Water	H_2O	0.589		5.1 × 10 ⁻¹⁴
Nitrobenzene	$C_5H_5NO_2$	0.589		2.44 × 10 ⁻¹²
Nitrotoluene	$C_5H_7NO_2$	0.589		1.37 × 10 ⁻¹²
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And Kerr constants for some selected liquids, this is very useful to refer to this table. For benzene, there are some organic liquids which are very often used for this Kerr effect experiment. At this wavelength, you get the Kerr constant this equal to this; which is I think this is. And then you have this Kerr constant equal to this and nitrobenzene, water, chloroform they are all they all exhibit this Kerr effect. And you have a table of this Kerr constants and the right column. And this is taken from this one can refer to this book here even a optical waves in the crystals even in all details.

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	Some selected quadratic electro-optic coefficients #				
	b				
	Substance	Wavelength $\lambda (\mu m)$	$RI(n_o)$	EO Coefficients $n_o^3 s_{ij} \ [10^{-18} m^2/V^2]$	
	BaTiO ₃	0.500	2.42	$n_o^3(s_{11}-s_{12})$ = 72,000 $n_o^3 s_{44}$ = 44,000	
	KTa _{0.65} Nb _{0.35} O ₃	0.633	2.29	$n_o^3(s_{11}-s_{12})$ = 34,700	
	Pb _{0.69} La _{0.07} (Zr _{0.65} Ti _{0.35})O ₃	0.550	2.45	$n_o^3(s_{33}-s_{13})$ = 72,000	
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Some selected quadratic electro optic coefficients which are very often used. Barium, titanite and this crystal this crystal which are used at this wavelengths, and it shows the refractive indices this properties are this. And the electro optic coefficients that is S ij and 0 cube, calculated for this is they are actually experimentally obtained values which are coated here. This is again refer to this book Springer and Verlag. So, it is very useful table for designing and you know for experimental measurement of this Kerr effect.

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A Kerr modulator consists of a Kerr cell containing 2 transverse electrodes. The glass cell is filled with polar liquid placed between crossed polarizers with pass axes at 45 degree to the electric field. So, if you have an electric field applied along certain direction, then you apply at the you launch the light to pass through the medium a 45 degree the pass axes. And such a device can respond very effectively to frequencies up to up to 10 gigahertz. So, Kerr so, that is why it is very useful for modulating light at a very high speed. Then Kerr cells containing this nitrobenzene and carbon disulphide, they are used for many years for q switching and instruments of the pulse laser system.

The a Kerr cell of length L an electrode distance d that gives you a the phase retardation of this; which is proportional to the square of the voltage, and this will get into the equation very interestingly we will see.

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Typical nitrobenzene cells these are the typical numbers practical numbers attached to experimental where d equal to 1 centimeter. L is several centimeter, but that requires a voltage which is of the order of 30 kilovolt to respond as a half wave plate.

But the drawback is that nitrobenzene is both poisonous and explosive. So, this is quite useful and is being used for long, but the problem is it is poisonous and explosive. So, one has to be very careful take lot of precautions to use this. Therefore, the people have come up with alternative transparent solid crystals barium, titanate, KTN, PLZT, these PLZT is very well known in the laboratory. It is very often and used. They are largely used as Kerr modulators. And it because it is solid state so, there is it is very easy to handle and put up in the experiment also.

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So, this is a typical experimental setup for observing Kerr effect. You have a photodetector here. I think we should start from here you have a laser source. And this laser light falls into a polarizer the pass axes of this polarizer. And pass axes of this analyzer there at 90 degree that is there crossed polarizers. And you have this for this Kerr crystal PLZT crystal which is connected which is energized by a high power source that is high voltage source. So, you apply the voltage here, the property of the crystal changes as we have discussed looking at the electro optic tensors.

So, this light which falls on passing through this along the pass axes falls on the crystal. They will be passing, and they will undergo a retardation while moving through this crystal. And then these 2 polarized light, these 2 polarizations the delay depending on the delay, you will get an amplitude modulated light at this output. So, you can measure the intensity of the amplitude model; because your crystal is placed between 2 crossed polarizers and there is a delay.

So, that tells you that it can be used as an amplitude modulator, we will see that if I change the voltage the delay will also change, but this delay will be proportional to the square of the electric field. As a result, this square of the electric field and the modulated output that is the amplitude modulated output. They one can plot this, and can see the variation, and we will see that it really follows the equation that we have seen.

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Kerr effect experiment: analysis	
Phase shift between the orthogonal polarisations at output of crystal (<i>l</i>) $\Delta \phi = 2\pi \frac{l}{\lambda} \Delta n$ $\Delta n = n_e - n_o = K \lambda E^2$ $\Delta \phi = 2\pi K l E^2$ Also for this setup : $E = V/d$ $\Delta \phi = 2\pi K l V^2/d^2$ Crossed polarisers : $I = I_0 sin^2 \frac{\pi K l V^2}{d^2}$	Therefore, intensity-voltage: $\frac{I}{I_0} = \sin^2 \frac{\pi K l V^2}{d^2}$ $\sin^2 (cV^2) \text{ variation curve}$ $I^2 = \frac{d^2}{\pi K l} \sin^{-1} \sqrt{\frac{I}{I_0}}$ $I^2 - \Delta \phi \text{ : a straight line}$
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So, the analysis is that phase shift between the 2 orthogonal polarizations are the output of the crystal. Delta phi this you have seen. So, n e minus no which is proportional to this n e and n o they are actually r 3 1 and r 3 3, we have seen the r 1 2 and r 1 r 1 2 and r 1 1 r or S let me see, yeah S 1 1 and S 1 2. So, these 2, these 2 numbers are attached to this n e and n o. So, therefore, this delta n is proportional to E square, delta phi is also proportional to E square.

So, for this setup E equal to v by d, as a result you get delta phi which is equal to twice pi k, the length of the crystal then the square of the voltage by d square because this is to represent the E square. And this gives you the equation you can plot this v square and delta phi, v square versus delta phi will be a straight line; which you can plot and as you change v square the values of v square if you take you get the change in the which will be linearly proportional because all other quantities remain the same.

But more interestingly that is usually taken into care that is this expression that from here we can write this equation that I by I naught which will be equal to, which will be equal to this sin square pi kl v square by d square. So, this part because the sin square of this remaining part except v square is a constant. So, you have a sin square of this v square variation curve and the typical typically it looks like this.

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This is very well seen in the laboratory experiment in the undergraduate labs; that you have a voltage if you plot along this. Square of this voltage will give you this change in this. So, these are the positions 21, this is actually a set of data which we have taken from our laboratory experiment, 21 degree, 46 degree and so on. So, there is a phase shift for this which is recorded here, this is the change in the voltage and you have plotted I by I 0.

So, as your you increase the voltage there is a periodic modulation. It would have been a equal spacing same frequency modulation, if it depends on v, but because it is a v square dependence. So, that is why there is a bunch up of bunch up effect of this and it becomes more close to each other. And you can see from here that this equation I equal to I naught sin square of. So, v square and this proportion I by I naught if I plot I get a graph like this. This is very interesting to observe this Kerr effect through this experiment.

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And now, we summarize all that we have done so far through this section of the discussion. We consider this quadratic electro optic effect electro optical tensor electric field in presence of the electric field how this ellipsoid looks like then we consider the electro optic tensor for isotropic medium um.

Then we used the external electric field along z direction and seen that anisotropy how anisotropic material becomes an isotropic, because of the quadratic dependence of the refractive index on the field. Then this we discussed a typical Kerr effect experiment very often used in the laboratory and we will continue the discussion with the modulators in the next session.

Thank you.