Modern Optics
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Lecture - 35
Electro-optic Modulators and Devices (Contd.)

So, we are discussing the Electro-optics of Anisotropic Medium and we considered this very important crystal KDP and we were looking at the birefringence retardation.

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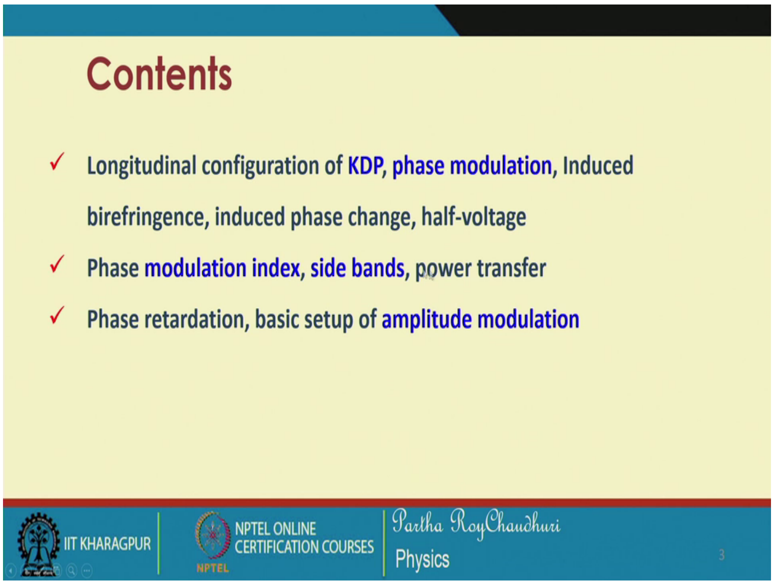

Now, in this longitudinal configuration of KDP, we will be discussing this phase modulation and in phase modulation we will look at the induced birefringence, induced phase change and half voltage once again. Then we will look at the theoretical consideration of the phase modulation, then how it generates the sidebands, and how power is transferred to the sidebands depending on the applied voltage. Then we will look at the phase retardation and the basic setup corresponding to the amplitude modulation because the same configuration, but with polarizer and analyzer one can make use of this setup as a amplitude modulation device.

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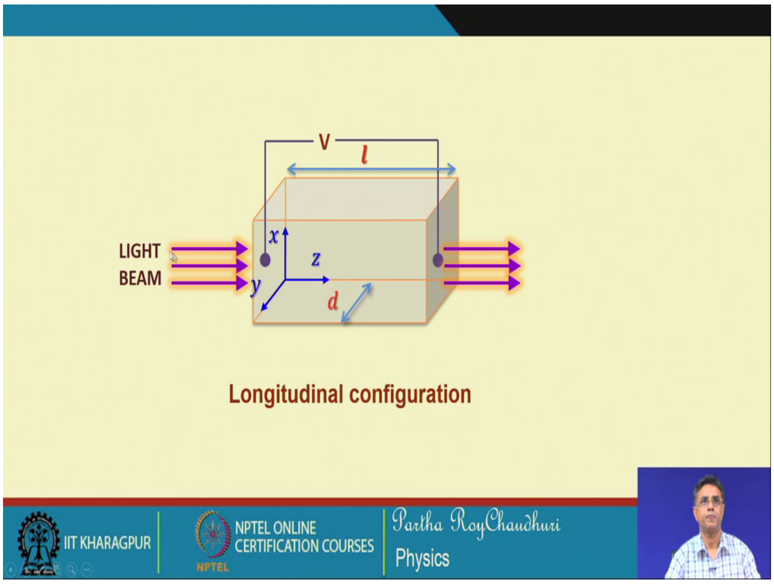

So, for longitudinal configuration phase modulation, once again this is the setup you have a light beam which is traveling through the crystal of length l and have applied a voltage V variable voltage. So, the electric field is V upon l; and this is the schematic of this you have an applied beam.

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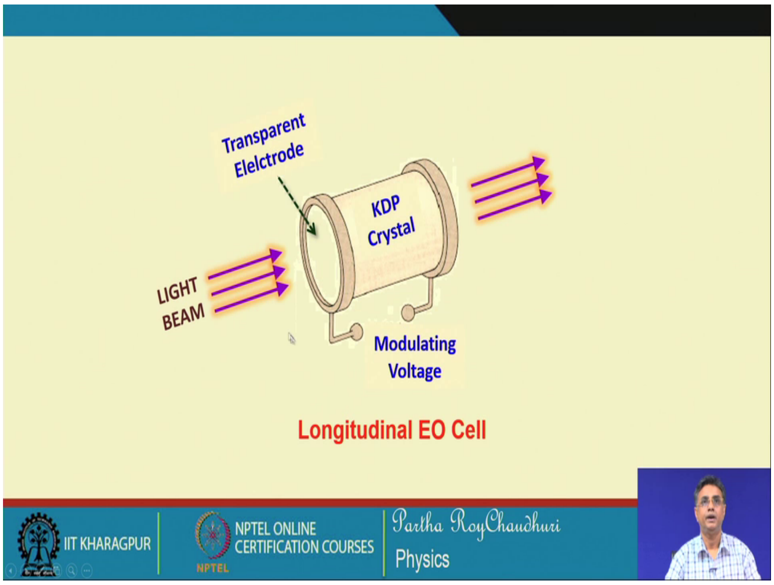

You can see that because the direction of the applied the field voltage application and the beam propagation they are same, so you have to use a transparent electrode. There are electrodes with a narrow hole that is also used and commercially available.

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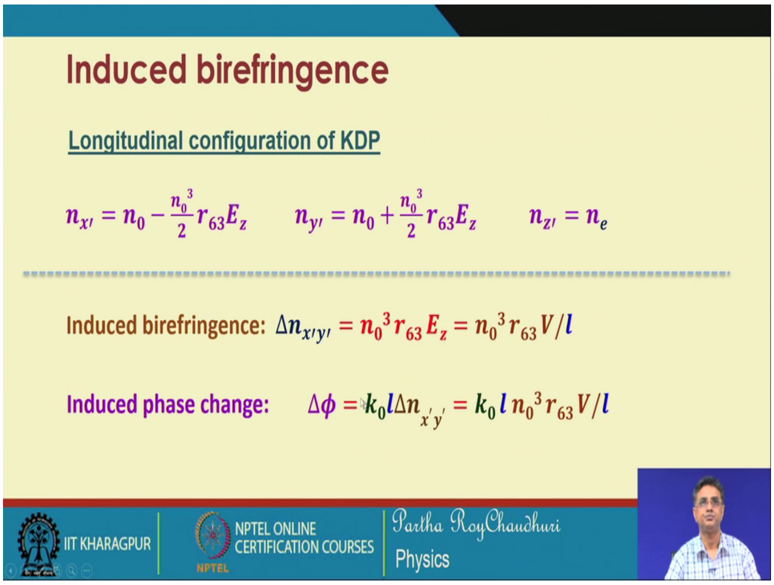

So, induced birefringence in this case we have seen that this new principal refractive index for x that is n x prime is equal to this and the same for n y prime is equal to this, z prime remains the same that is extraordinary refractive index value of the crystal. Therefore, the birefringence between this x prime and y prime in presence of the external electric field which is applied along the longitudinal direction that is E z. So, that is equal to delta n x prime y prime is equal to this sum of this quantity and this quantity.

So, this is your induced birefringence and induced phase change will be this multiplied by your k 0 that is twice pi by lambda which will be effectively l cancels, so k 0, n 0 cube r 6 3 into V. So, we can see that induced phase change is proportional to the applied voltage. We will recall that. If we restrict our polarization to one of these x prime or y prime direction by placing the polarizer making an angle of 45 degree with the old coordinate axis that is normal natural coordinate axis of the crystal then it will excite either x axis, x prime axis or we can orient along y prime axis.

Let us suppose if we excite only x prime axis then with voltage only this much of the change in the refractive index in addition to n 0 will be modulated by this external voltage and the light that will be emerging out of the crystal will be phase modulated by this quantity. Whereas for y polarized y prime polarized light this quantity will be the modulation in the phase this will correspond to this will give rise to the modulation in the phase. And we will see this here we have calculated the birefringence between this x prime and y prime as if the polarization is along 45 degree with x prime and y prime.

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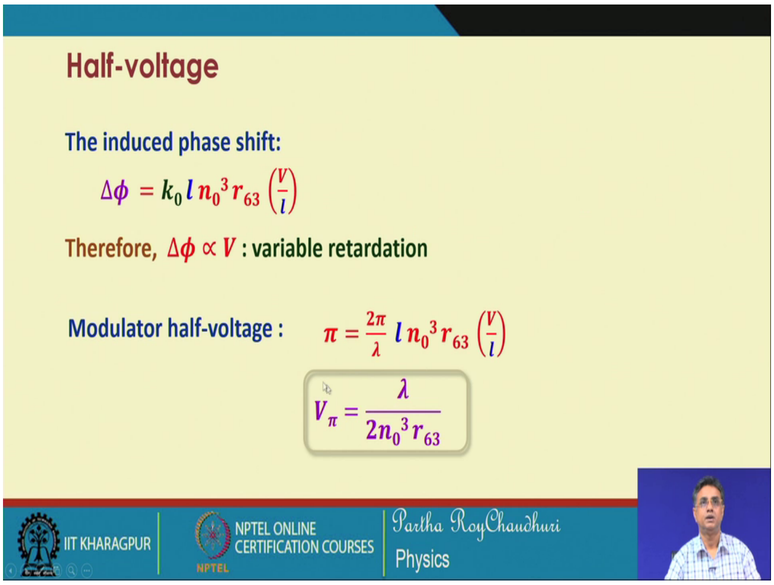

So, in that case this with this induced birefringence delta phi again we find that delta phi the change in the phase is proportional to V. Again this gives you a variable retardation and modulator half voltage is this we just have to which is the same we just have to use the value of the r 63 coefficient to get the half voltage of the crystal. It does not involve any length or any width of the crystal, it is simply proportional to the wavelength and inversely proportional to the to the electro-optic coefficient.

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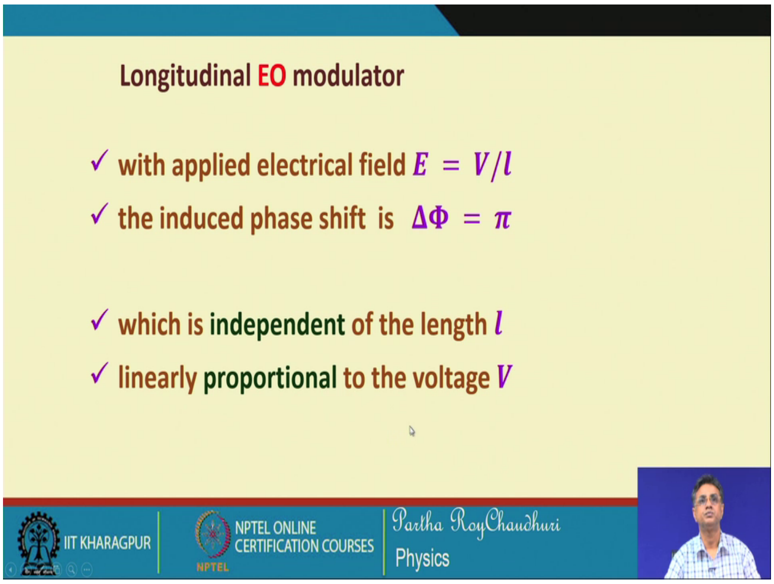

So, with applied voltage V upon l the induced phase shift for this half voltage is equal to pi which is independent of length l and is linearly proportional to the applied externally applied voltage.

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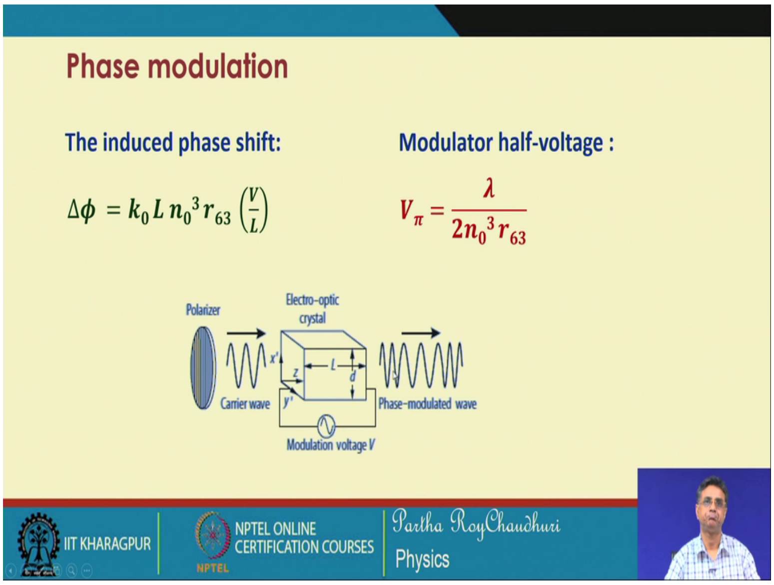

Now, for phase modulation we have seen that this delta phi will be equal to this and V pi modulator half voltage will be this, and this is the configuration that you have to excite the polarization input polarization which will be coinciding with either x axis or y axis and light will be travelling along z axis. Then the light which is coming out of the crystal will be a only x prime polarized, but the phase will be modulated because the phase the refractive index is being modulated by the external applied voltage and we will see this.

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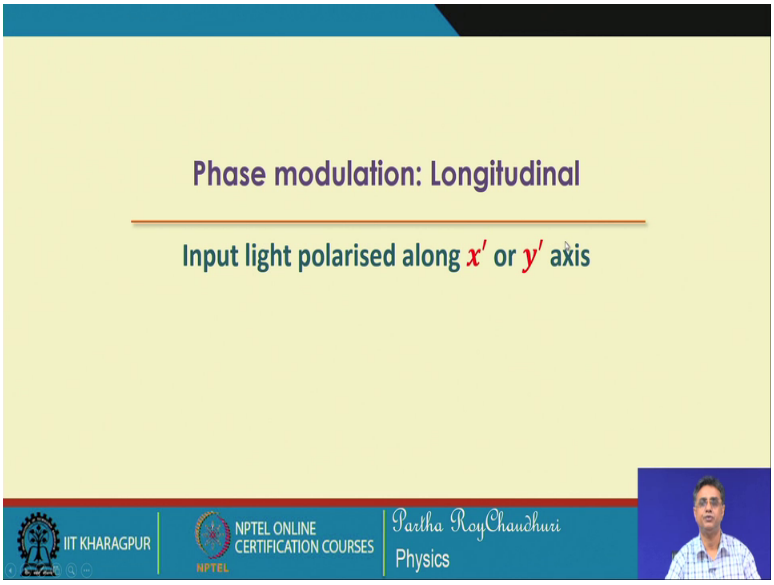

So, in the case of phase modulation for the longitudinal case input polarized light is either x prime or y prime axis. So, this is the thing that we will always have to remember that if you are looking for if you want to configure the device for phase modulation then the input polarization will be either x prime or y prime, not with 45 degree with x and y.

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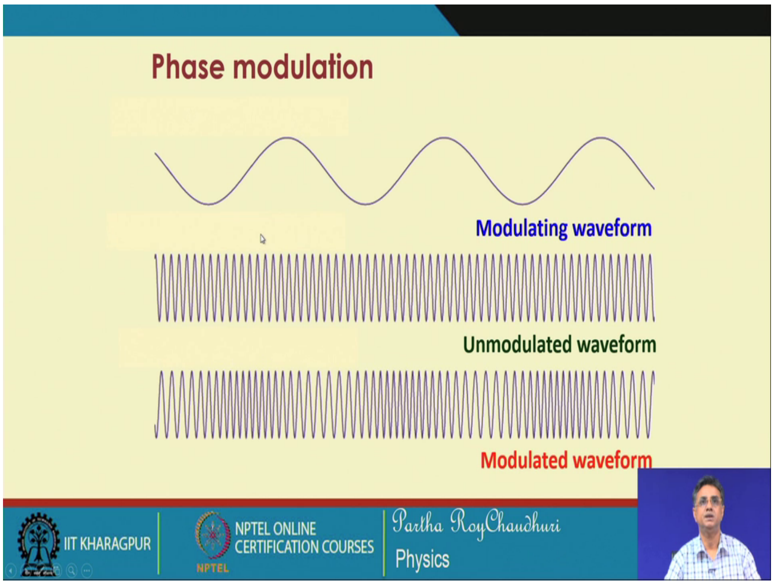

So, let us first consider and in the case of phase polarized phase modulation the light which will be available emerging out of the crystal, when there is no modulation. So, this will be the phase you can see that there is an uniform phase propagation, but if you have a modulating voltage which is sinusoidal then you can see that at the periods of the modulating voltage the waveform is compressed and verified.

So, this modulation information is impressed on the on the optical wave which is coming out of the crystal, ok.

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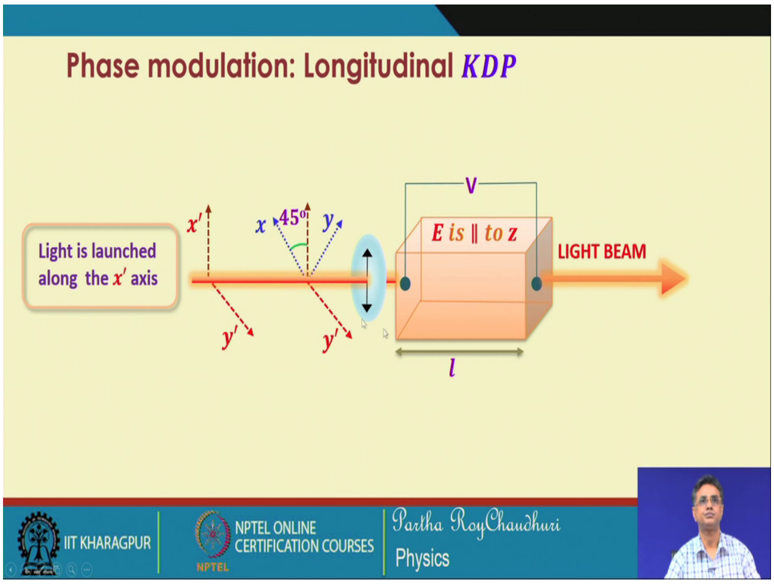

So, for phase modulation as I have mentioned that this light has to be launched with polarization with x prime or y prime. Let us suppose we consider that light is polarized along x prime axis and that is easy for this crystal because we know that it makes an angle of 45 degree with the old x or y axis. So, it is a very easy to locate that this the x prime axis which will be half way between the x and y axis of the crystal and z axis is the propagation direction voltage is applied in the longitudinal direction of the crystal of length l.

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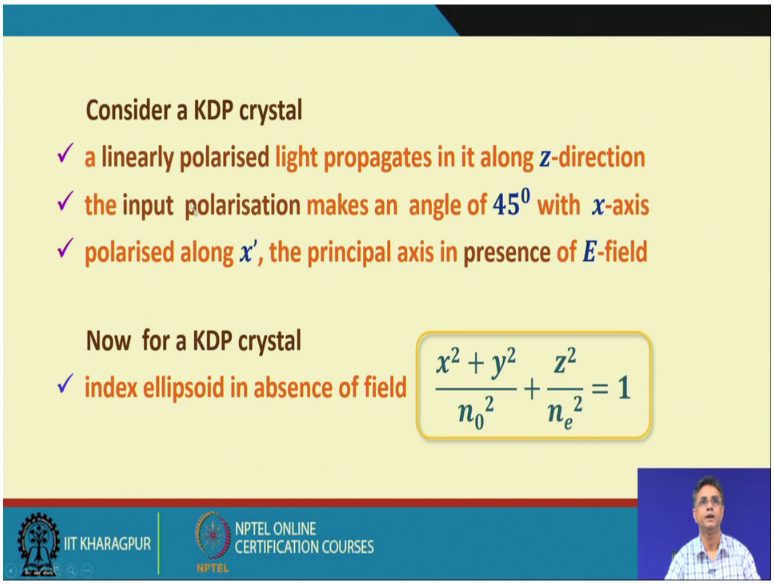

So, first case we considered that the input light is x polarized, then 45 degree with the x that is x polarized and then for this setup index ellipsoid without the electric field is already known. Now, we will we will consider the electric field which is longitudinal, but now how it affects the x prime direction refractive index.

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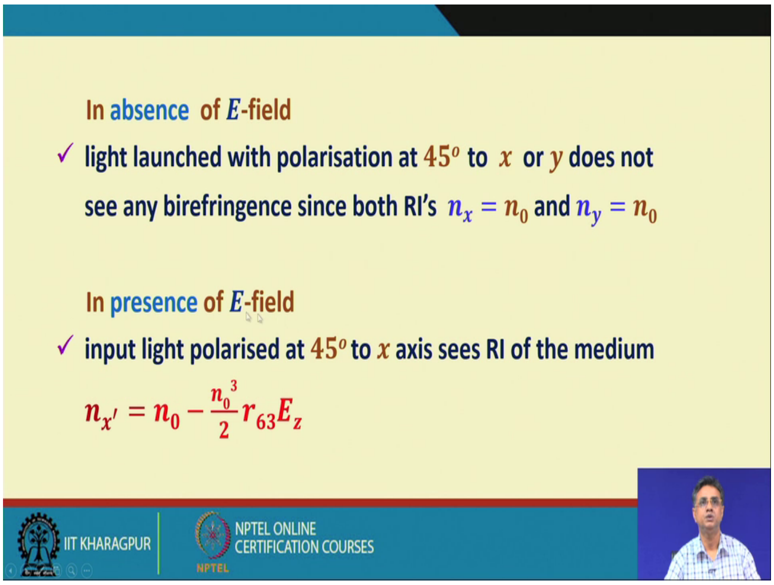

So, in absence of the field n x equal to n 0, n y equal to n 0, n z is anyway equal to n e and that is not useful here. In presence of the electric field input light is polarized at 45 degree with x that is along x prime direction that is the new principal refractive index direction. So, it is the crystal we will see only a light which is polarized along x prime and there is no other component.

So, as a result this refractive index that is experienced by the light of x prime polarization we see this quantity as because of the presence of the electric field. So, this is the quantity which will be in excess of the n 0 that is the natural refractive index and as E z changes this quantity will be modulated.

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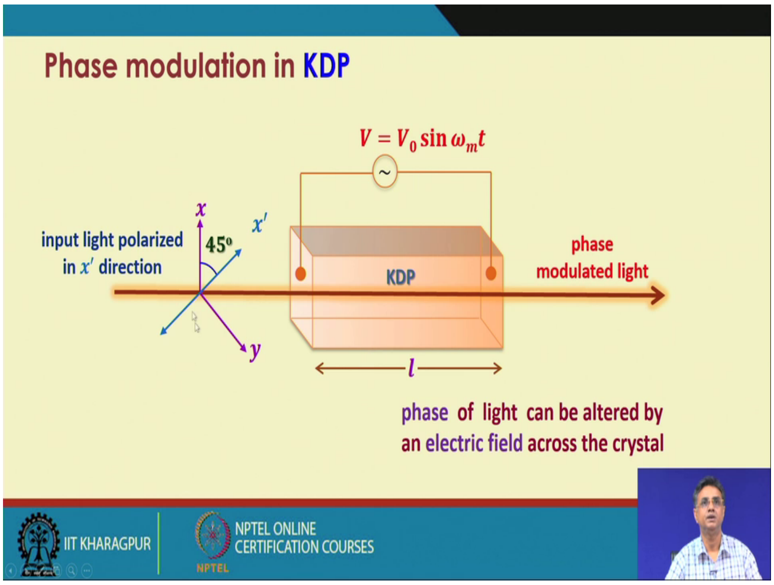

So, for this configuration phase of the light can be altered by an electric field across the crystal if I apply a modulating voltage which is V 0 sin sinusoidal voltage of omega m, m stands for modulating frequency. So, with this frequency if I modulate the external voltage which is applied along the length of the crystal then the phase of the light which is which is traveling through the crystal will be modulated.

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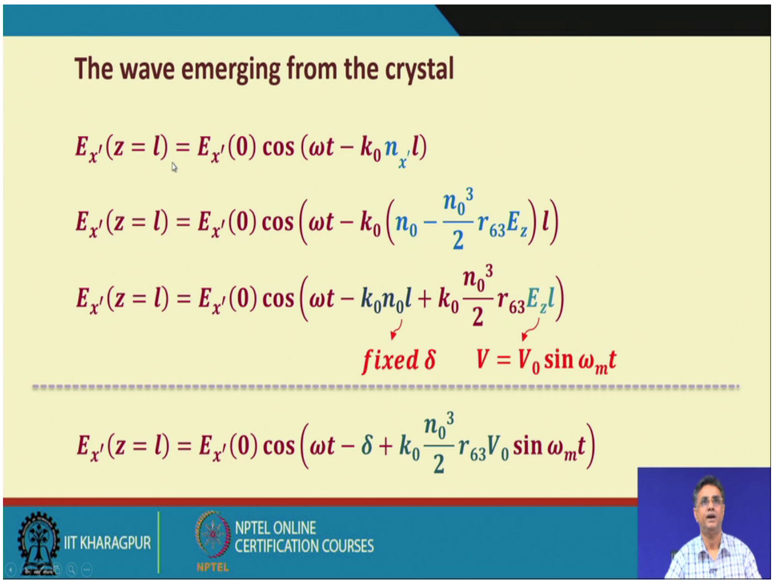

So, in that case E x dash at z equal to l that is at this point will be will be modified at z equal to 0, at z equal to 0 E x dash and then cosine omega t. So, this is the this is the new refractive index multiplied by l into k 0, will be the additional phase because of the presence of the voltage otherwise it would have been k 0 and n 0 only into l of course, and so this one can write in this form that k 0 n 0 because n x dash is anyway known to us. So, we can write in this form. Then from here if you open the bracket k 0, n 0 l this quantity is fixed and this is the natural phase because of the propagation of the light through a length l, but this one is the phase which is because of the electric field E z for over and for propagation of length l.

So, here this E z into l is nothing but E 0 sin omega t as per our assumption. So, this from this we can write this equation as E 0 dash cosine omega t minus delta is fixed phase and this is the phase which is undergoing the modulation because of this voltage. So, we can write this equation in this form.

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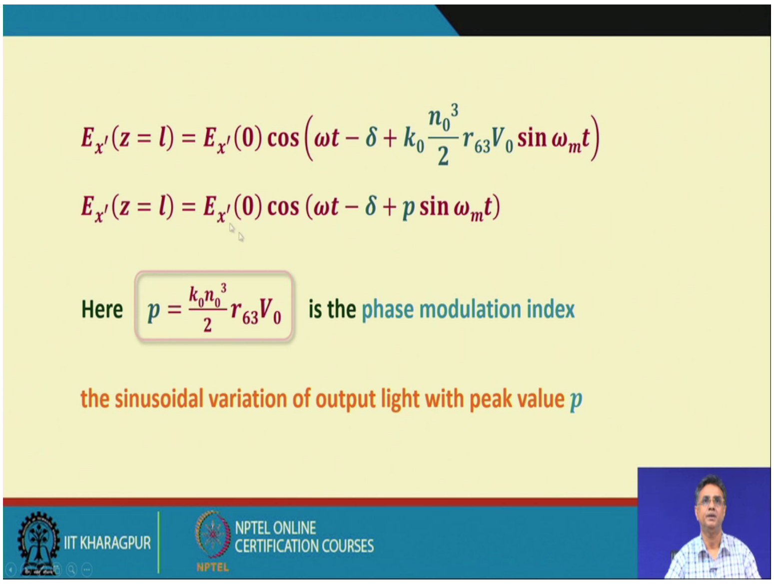

And therefore, because this quantity is anyway constant V 0 is the peak voltage this is constant this is constant wavelength of operation is constant. So, in place of this we can write a new constant p, then this equation becomes omega t minus delta plus p sin omega t. This p is the phase modulation index which is again well known, this is the quantity the peak value of the modulation, the sinusoidal variation of the output light with the peak value of p, so that will be the modulation index, right.

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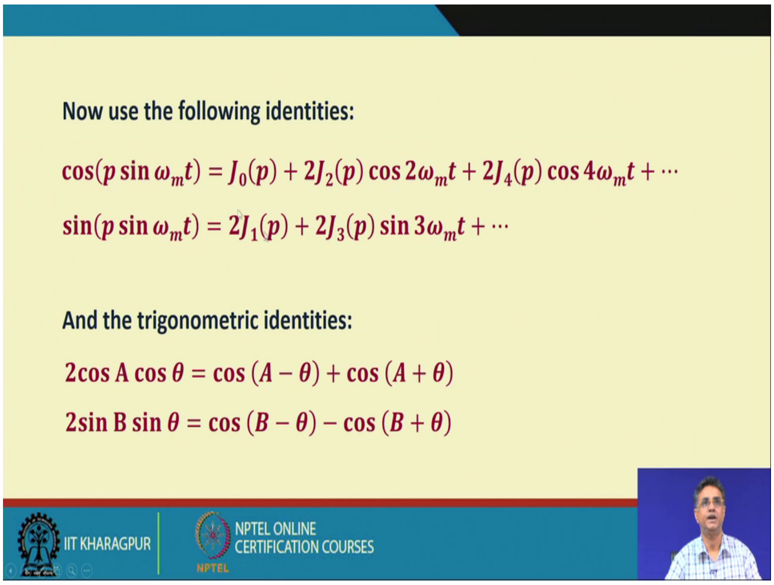

Now, use the following identities that is for because you have p sin omega t we can write this cosine of p sin omega t. This is again a very well known identity and for p repair of identities p sin of p sin omega t which is equal to this. Then we will also use this trigonometric identities that is twice cosine A cosine theta can be written as the sum and difference of A and theta, similarly B and theta can be expressed as a sum and difference of B and theta.

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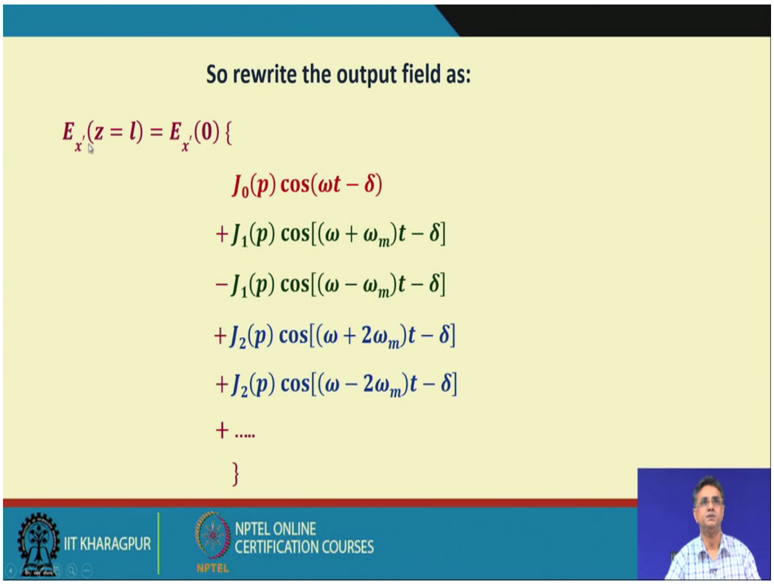

Therefore, this equation this equation can now be written in this form that E x at z equal to l will be E 0 x and then p cosine omega t can be expressed as the superposition of these various order Bessel functions. If we collect the collect the Bessel functions n 1, j 1, j 1 together j 2, j 2 together and so on we can write this in this form.

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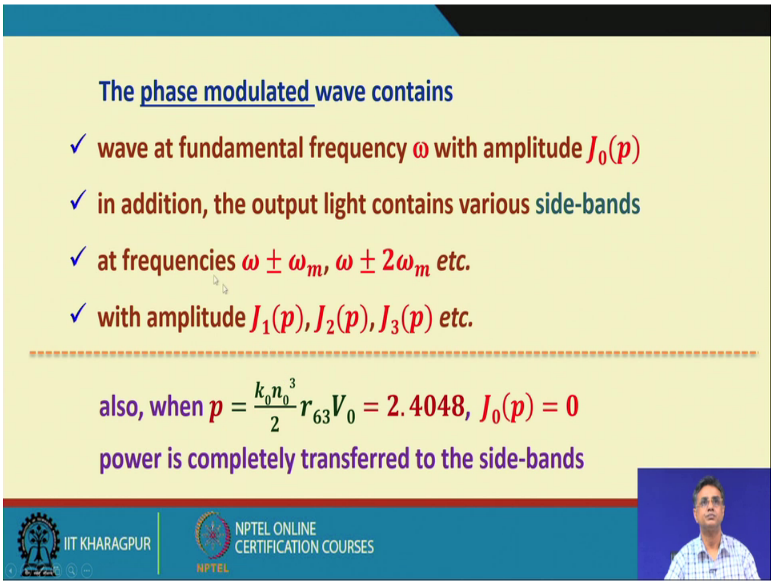

The phase modulated wave then contains wave at frequency omega with amplitude j 0 p. Look at this after doing this algebra we can see that it contains the modulated output wave at z equal to 0, it contain a wave whose amplitude is Bessel j 0 of p with fundamental frequency omega. But it also contains j 1 omega plus omega 1 the modulated frequency modulating frequency has got into this fundamental frequency the original frequency and here so this is the sum of the of the frequency of the light plus the modulating frequency this is the difference they are associated with j 1. For j 2 this the amplitude of the light with j 2 will have frequency omega plus twice omega m and omega minus twice omega m.

So, we get we can find that in addition to this original frequency there are sidebands at equal intervals omega plus omega m, omega minus omega m omega plus twice omega m omega minus twice omega m and so on. So, it also excites the sidebands as well in addition to the original frequency. So, that is what is there that the output light contains various sidebands at frequencies omega plus minus omega m and so on and so forth with amplitude j 1 p, j 2 p, j 3 p etcetera.

And also that p because the value of p is known by knowing this k 0 twice pi by lambda for a given wavelength n 0 the ordinary refractive index that is also known, then this coefficient electro-optic coefficient is also known and assuming a certain value of the peak voltage that is applied across the crystal. This p if it coincides with these value 2.4048, this number is very important and very useful particularly in fiber optics where Bessel functions are there.

So, this is this will give you the 0 of the Bessel function of the Bessel function j 0. So, at this value p equal to 2.4048 j 0 will become equal to 0 and in that case there is no power in the j 0, but the complete power transfer will take place from this j 0 to the sideband that is j 1, j 2, j 3 etcetera. So, all the power will be transferred to the sidebands, but there is no power in the in the j 0 that is original frequency, ok.

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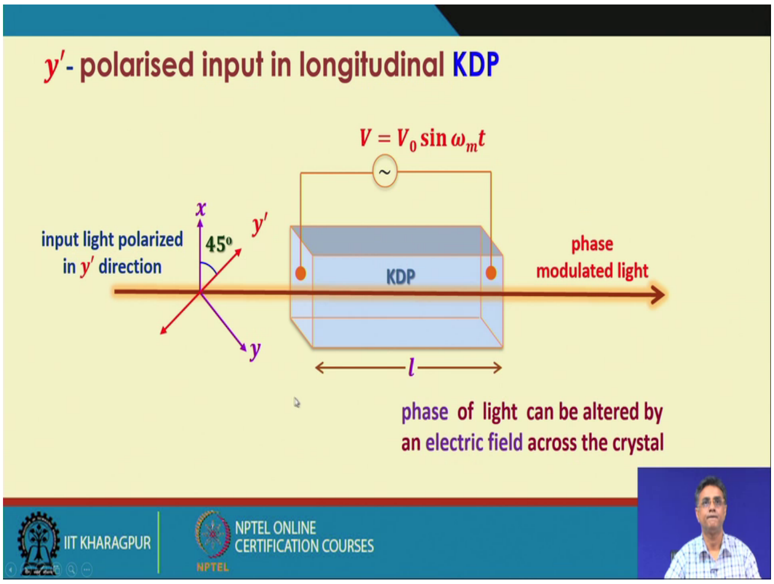

Now, let us suppose that if we assume the polarization of the input light is now along new y prime axis that is it is again 45 degree with the old that is the natural a principal axes x and y, but with a 45 degree, 90 degree, at 90 degree with the x axis.

So, this x y prime polarization if we launch light then the output light is again phase modulated, it will happen the same way as it had happened in the case of x y prime polarization light.

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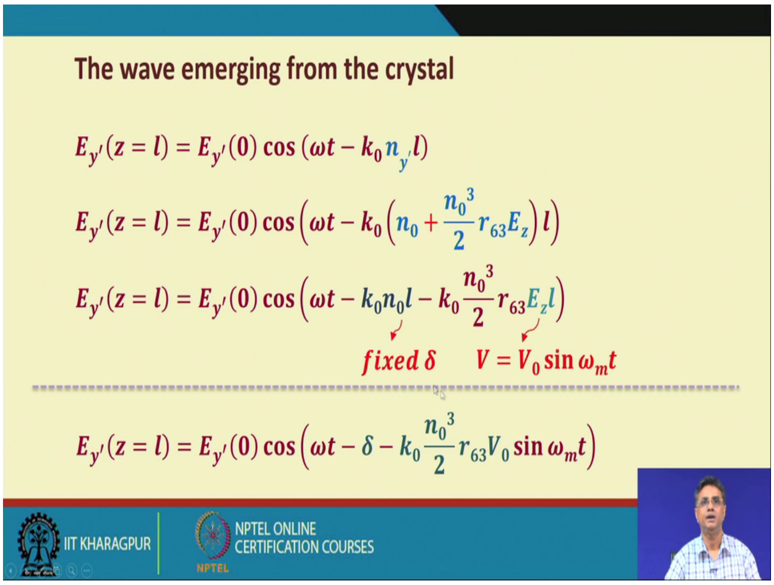

And then we can write this equation once again we can just repeat for y prime and that will lead to this fixed value of fixed value of the phase which is due to the natural due to the original the refractive index ordinary refractive index length and in the same way we can expect this.

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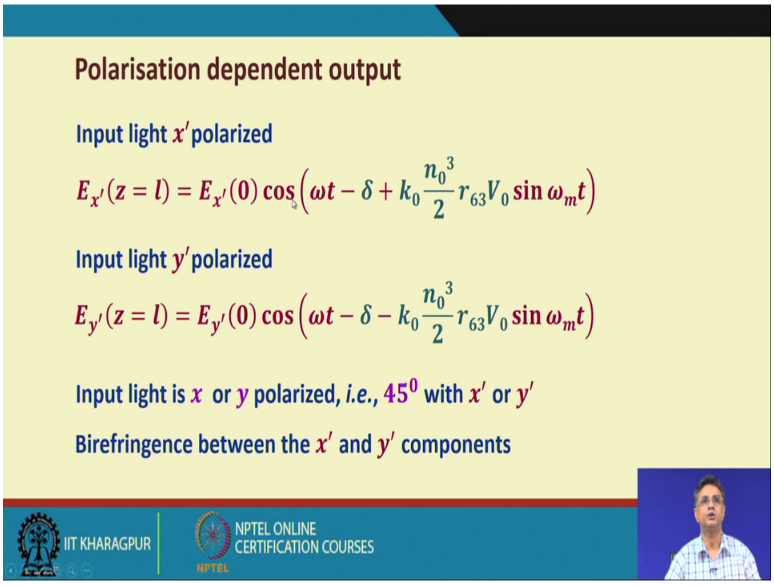

Therefore for input polarized for input light with polarization x prime we have this output E x 0 at the input and then cosine of this quantity, but if the input polarization is y polarized y dash polarized then the output polarization output waveform will have this expression this form.

Now, we have separately calculated the input polarization for x prime and also for y prime. Now, we will consider that if it is half wave through that if the input polarization is along x or y that is the old principal axes that is 45 degree with x prime or y prime axis in that case the birefringence between this x prime and y prime will now be calculated.

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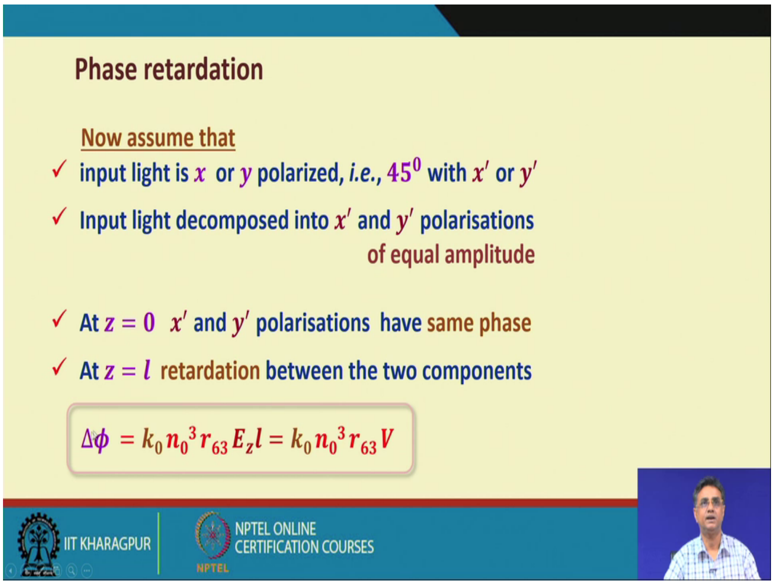

So, in that case you will have this birefringence delta phi which will be just the sum of these two quantities, the sum of these two quantities. This one is due to the y polarized y prime polarized light and this is for the x prime polarized light, right. So, we calculate the birefringence in this case.

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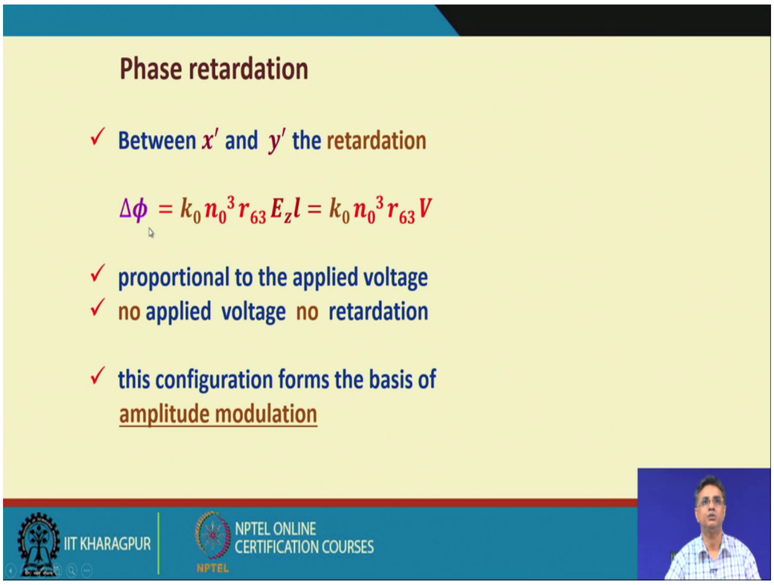

And between x dash and y dash the retardation will be equal to this which is again the proportional to the external the applied voltage V, and if there is no applied voltage there is no retardation. So, V equal to 0; that means delta phi equal to 0 which is very obvious and this configuration that is the basis of the amplitude modulation.

As I have mentioned that if you launch light only with x prime or y prime then the output light will be phase modulated. But if you launch light with input polarization which is 45 degree with x prime and y prime that is if the input polarization is with any of the old x or y axis along x or y axis in that case and if you place an analyzer at the output which is cross with the input then the system will give you amplitude modulation.

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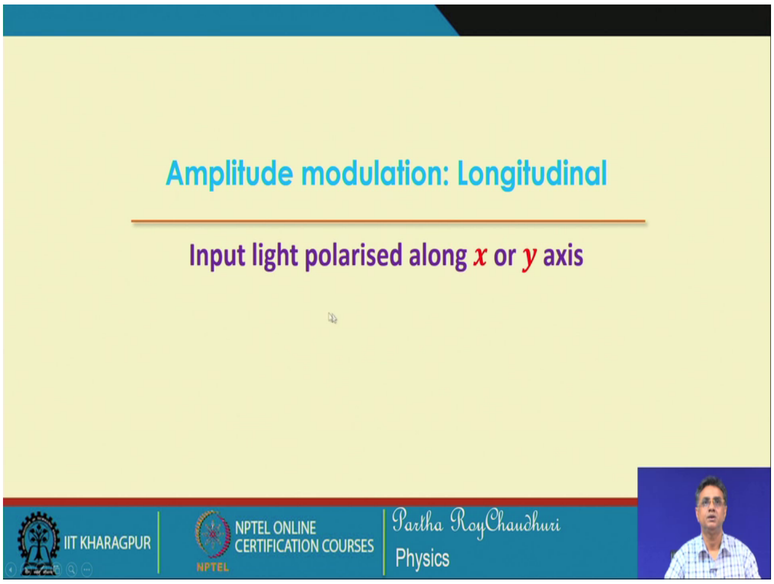

So, in the longitudinal configuration case the amplitude modulation requires that light will now be polarized along x or y which are the old coordinate systems that is the old the principal axes system of the crystal without any external field.

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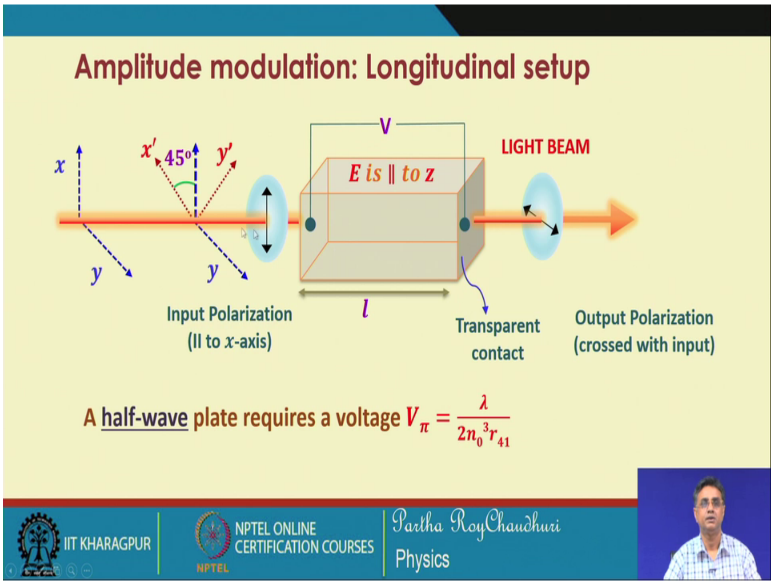

So, in that case we have light which is polarized with this thick blue dash line and you have an analyzer a polarizer which is along this direction and the light travels through this and you have a crossed analyzer a polarizer which is which is which is working as an analyzer in that case the output light will be amplitude modulated. And we will have a half wave plate voltage that is required for this is equal to V pi which is lambda by twice n 0 cube r. So, this would be 63 not 41, this would be r 63.

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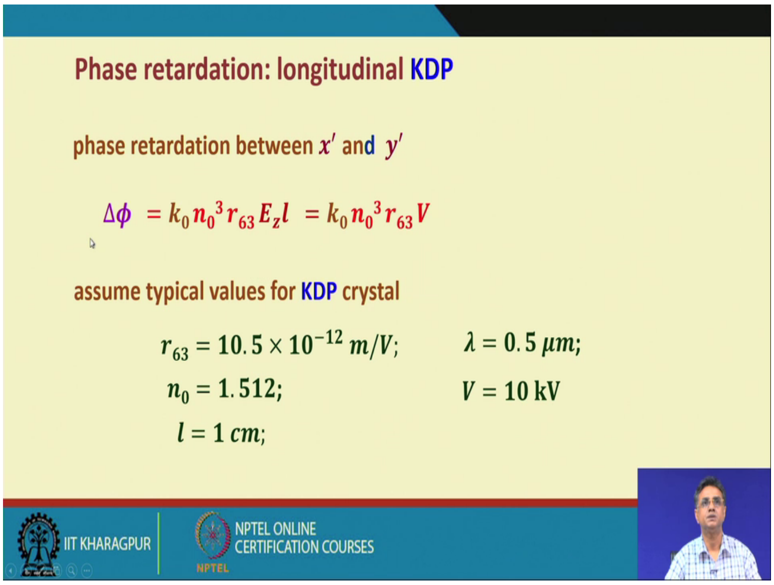

So, this phase retardation between x and y prime will be equal to this. Assume this typical values of this KDP crystal which we have used earlier also that is r 63 equal to this, n 0 equal to this, and length of the crystal which anyway does not come as per as the voltage is concerned, so, and V equal to this 10 kilo volt lambda 500 nanometer.

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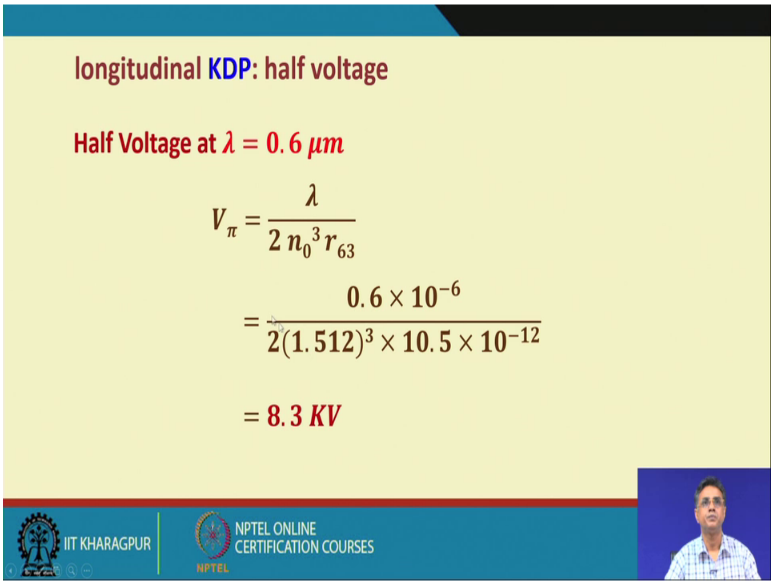

Then the half voltage for lambda equal to 5 but here we have used lambda equal to 6.

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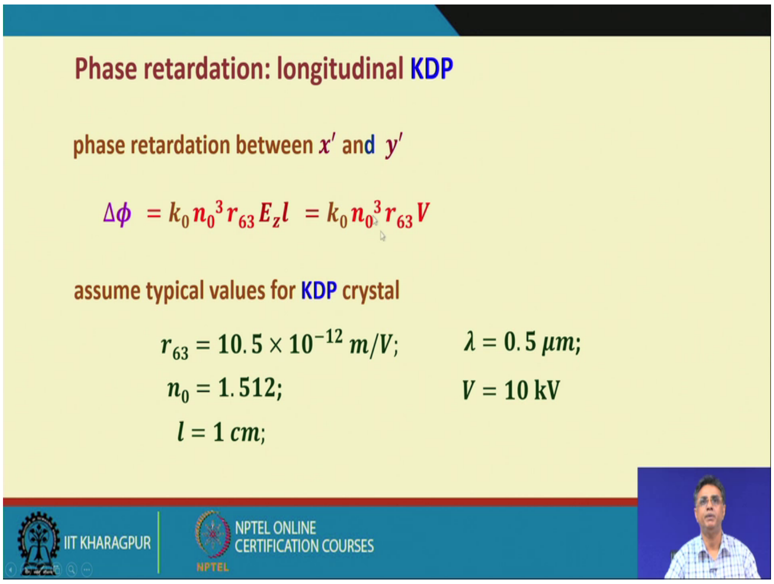

So, for 6, 0.6 micrometer lambda this V pi will be equal to 6.3 kilo volt and that we have seen also earlier.

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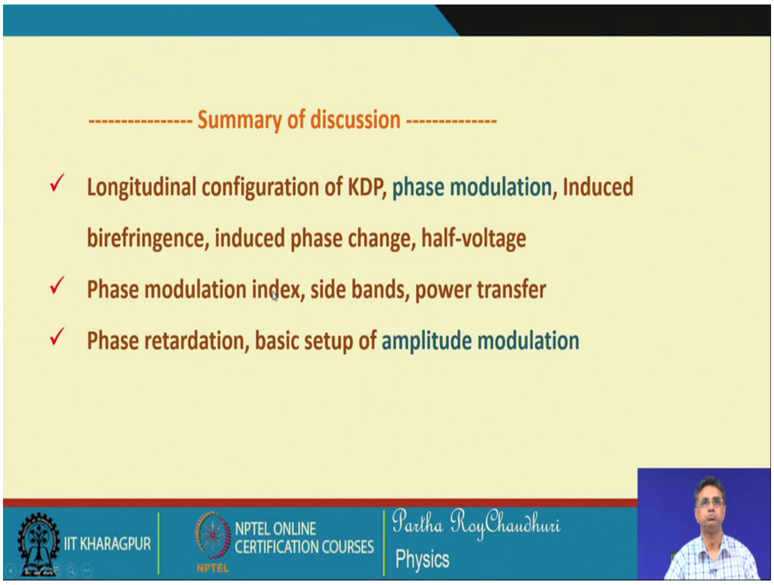

So, we discussed the longitudinal configuration of KDP in terms of its phase modulation, and we have seen that for phase modulation it requires the input polarization to be along the new principal refractive indices one of the new principal refractive indices direction.

Then, will calculated the induced birefringence, induced phase change and half voltage. Then in this configuration we also evaluated theoretically the phase modulation index sidebands and how the power is transferred to the sidebands for a given value of the modulation index p which is equal to 2.4048 that is the Bessel j 0 is 0. Then for amplitude modulation we will looked at the phase retardation and the basic setup, and how it can be used as an amplitude modulation which across polarizer at the input and output that also we have discussed.

Thank you.