Modern Optics
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Lecture – 31
Electro - optic Effect (Contd.)

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So, by now we have seen the Electro-optics Effect in terms of the electro optic coefficients the changes in the permittivity, impermeability tensor, the refractive indices.

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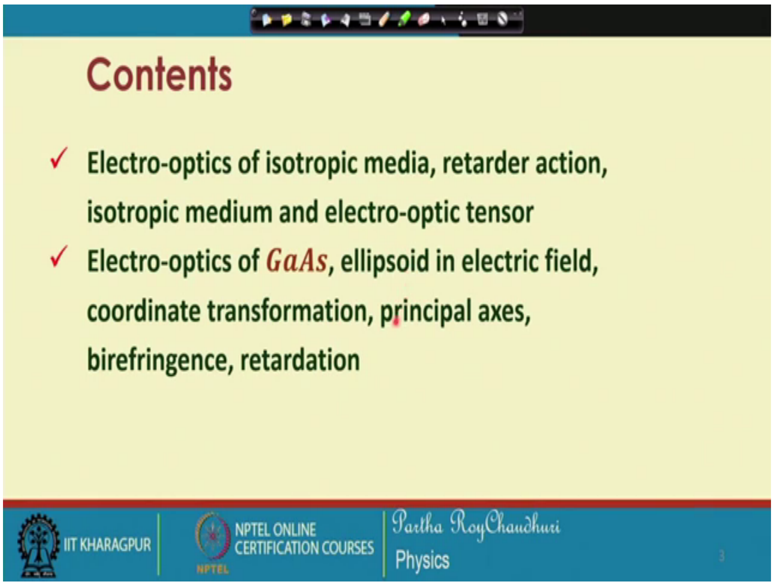

And, we have categorized the effect in terms of this Pockels effect and Kerr effect which are well known as linear electro optic effect and quadratic electro optic effect. Now, we will take examples of the very well known crystals; first we will consider the electro optic effect in isotropic medium. One example of isotropic medium is the gallium arsenide and for this gallium arsenide crystal, we will look at the index ellipsoid and under the electric field the discussion of the ellipsoid from their knowing the electro optic tensor, we will try to evaluate the new refractive indices. And then, we will try to look for the applications and for that we will first estimate the birefringence and retardation; then we will talk about the different modes of configuration of the crystal for modulator applications.

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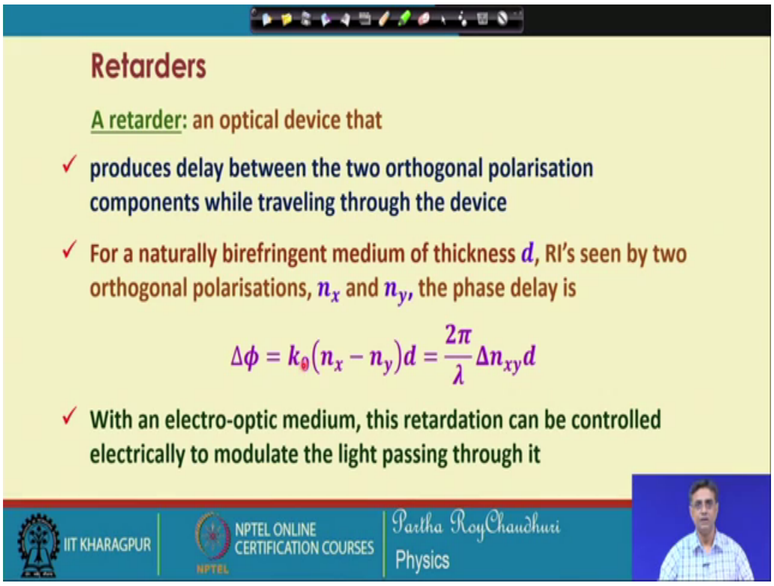

So, retarder: A retarder is an optical device that produces a delay between the two orthogonal polarization, orthogonally polarised light travelling in the medium, one of the polarization is faster moving while the other is slower moving. So, as a result in course of propagation through the medium they will develop a path difference and hence a phase difference and there will be a delay.

This phase difference will be the so, that accounts for the birefringence and the delay for a naturally birefringent medium; let us suppose of thickness d and the refractive index that is seen by the two polarization. In the direction along which the light is propagating let us suppose designated by n x and n y, then the phase delay that is delta phi will be equal to twice phi by lambda n x minus n y. This is the birefringence that is n x minus n y into d will be the phase delay between these two orthogonal polarized light.

And before the electric field and after the electric field there will be a change in the values of n x and n y. If so then we will be able to calculate the differenced n x minus n y or n y minus n x which will be delta n xy, then we can calculate the delay between the two polarized waves. And we will place the polarizer in such a way that the both the polarizations corresponding to the refractive indices n x and n y will be excited, and if they are excited in equal amounts then it will be a circularly polarized light. It will be orthogonal polarized light and if the delay is such that phi by 2 phase delay.

In that case it will be a circularly polarized light if the amplitudes are not different then it will be electrical; you have seen those examples in the earlier discussion in an isotropic medium ok. So, in an electro optic medium this retardation that is the delay in the phase this can be obviously, controlled by the externally applied electric field and this will result in the modulation of the light passing through the medium. So, that is the basic mechanism of modulation, light modulation by external electric field.

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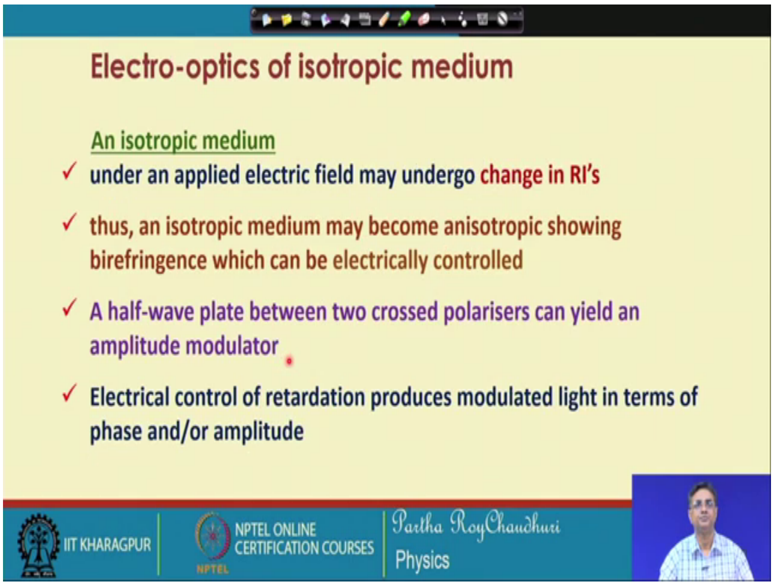

So, for an isotropic medium under an applied electric field may undergo changes in the refractive indices; this we have seen and we have understood very clearly. That as far as there is no electric field it is isotropic, but there may be a change in the refractive index properties when the external electric field is imposed. So, now you are going to take up an example of an isotropic medium and we will see that the refractive index properties are modified for a certain orientation of the external electric field applied across the crystal.

So, and an isotropic medium may become anisotropic showing birefringence and because this birefringence is due to electrical field, the electric field can control the strength of the birefringence. That is by changing the electric field we can actually control we can actually change the birefringence property and hence the delay also the delay and all other property is related to this birefringence.

A half wave plate between two crossed polarisers can yield an amplitude modulator. This we have seen that if you have a variable delay element between two crossed polarisers one is polarizer at the frontend and the analyser at the backend, then by changing the delay actually the light which is emerging out of the analyser can be modulated. So, electrical control of retardation produces modulated light in terms of phase or amplitude that depends on the configuration; the suitable positioning of the optical elements with the optically active crystal.

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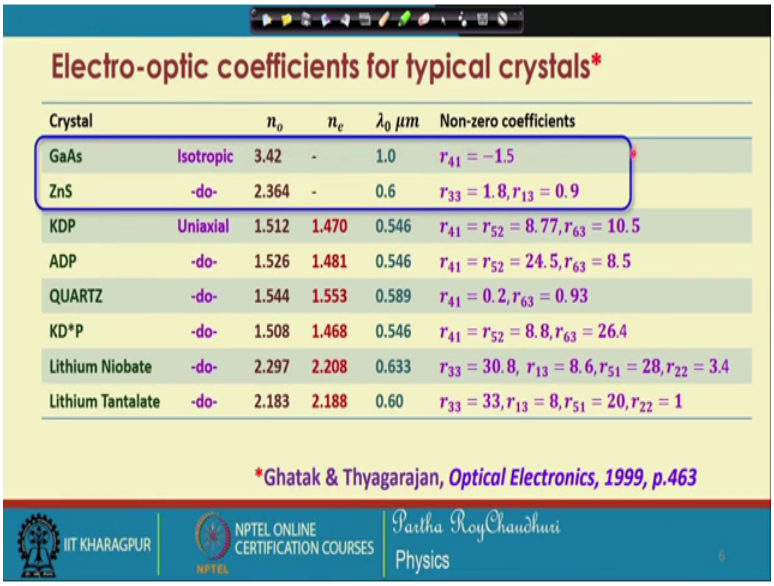
Electro-optic coefficients for typical crystals*

Crystal		n_o	n_e	$\lambda_0\ \mu m$	Non-zero coefficients
GaAs	Isotropic	3.42	-	1.0	$r_{41} = -1.5$
ZnS	-do-	2.364	-	0.6	$r_{33} = 1.8, r_{13} = 0.9$
KDP	Uniaxial	1.512	1.470	0.546	$r_{41} = r_{52} = 8.77, r_{63} = 10.5$
ADP	-do-	1.526	1.481	0.546	$r_{41} = r_{52} = 24.5, r_{63} = 8.5$
QUARTZ	-do-	1.544	1.553	0.589	$r_{41} = 0.2, r_{63} = 0.93$
KD*P	-do-	1.508	1.468	0.546	$r_{41} = r_{52} = 8.8, r_{63} = 26.4$
Lithium Niobate	-do-	2.297	2.208	0.633	$r_{33} = 30.8,\ r_{13} = 8.6, r_{51} = 28, r_{22} = 3.4$
Lithium Tantalate	-do-	2.183	2.188	0.60	$r_{33} = 33, r_{13} = 8, r_{51} = 20, r_{22} = 1$

*Ghatak & Thyagarajan, *Optical Electronics, 1999, p.463*

Now, we will refer to the list which are we have shown earlier we will focus to this isotropic elements. And we will pick up this gallium arsenide whose properties elect in terms of the electro optic tensor coefficients are known, refractive indices are also known. Then we will see that how this isotropic material can become an anisotropic under certain orientation of the applied electric field.

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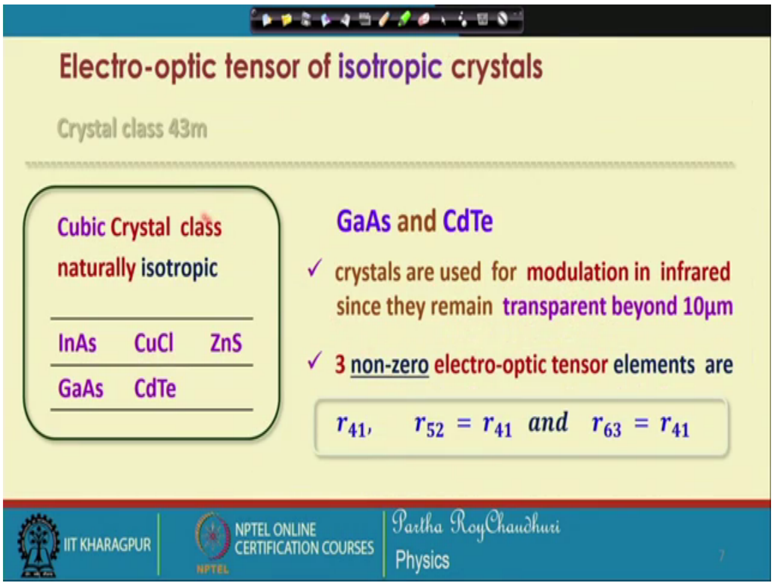

So, these isotropic crystals are there are few isotropic crystals like indium arsenide, copper chloride, zinc sulphide, gallium arsenide, cadmium telluride. These crystals are used very widely for modulation in the infrared region, because these crystals gallium arsenide and cadmium telluride they are transparent beyond this 10 micrometer wavelength. So, you have seen that for this isotropic material gallium arsenide the 3 non-zero electro optic tensor elements are r 41 52 and 63, but incidentally all these three are equal and is equal to the value of r 41.

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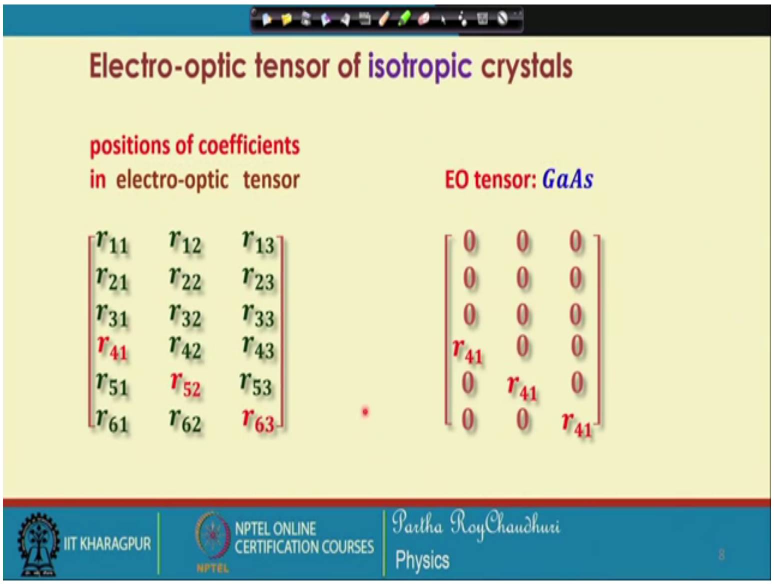

So, we have this tensor which reduces to this form r 41 r 41 and r 41 rest all other elements are 0. So, we need this tensor in terms of the values to evaluate the induced birefringence the new refractive indices in the principle axis system.

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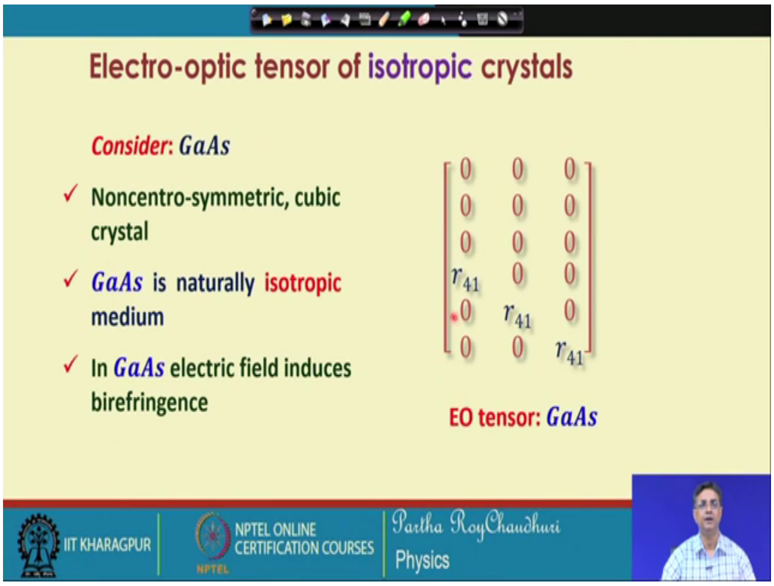

And we will use that for gallium arsenide which is non centro symmetric cubic crystal these are the non-zero elements and this is naturally isotropic and by in by applying external electric field it can create field induces birefringence. So, this is a tensor and the values are also known we will make use of this electro optic tensor to calculate the.

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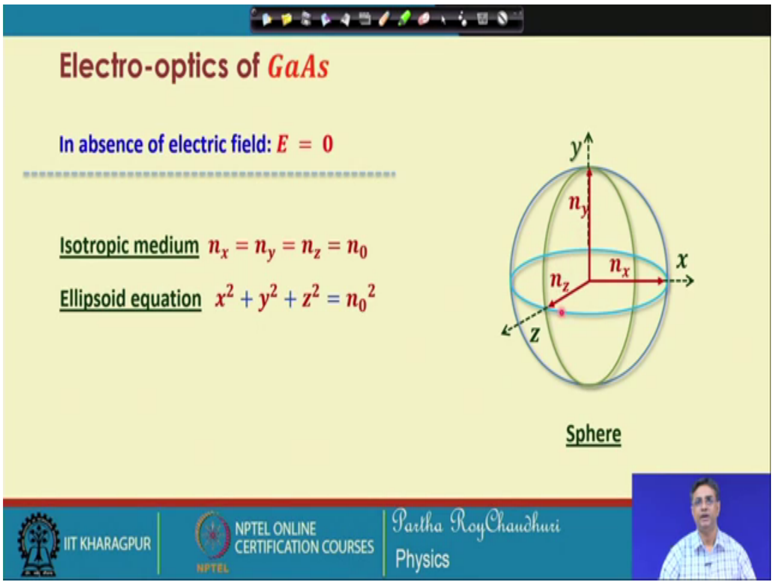

So, in absence of the electric field E equal to 0 because it is an isotropic medium. So, n x n y and n z all of them are equal and is equal to the ordinary refractive index n 0. And the ellipsoid equation becomes a square given by this and are represented by so, n x n y and n z all of them by magnitude they are all equal.

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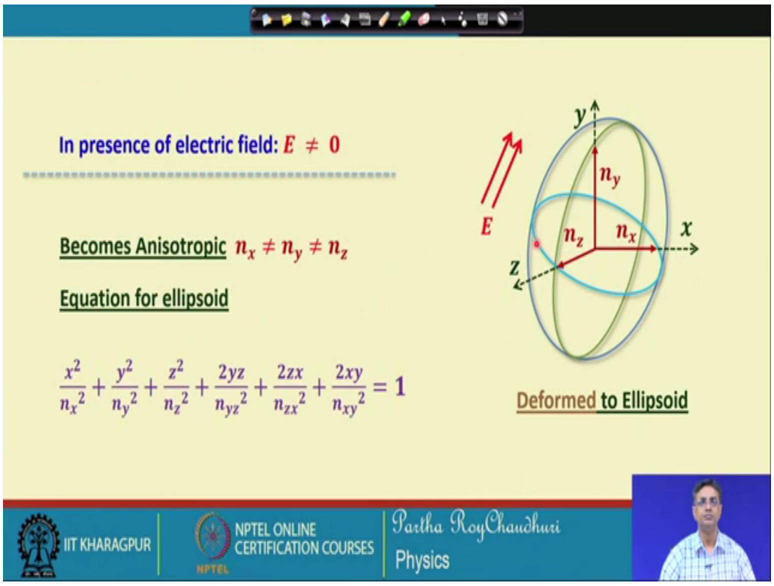

Now, in presence of the electric field when E is not equal to 0 then you can see that the ellipse is now deformed, ellipsoid is now deformed and the equation new equation to represent this ellipsoid maybe given by this equation, when where you can see that the other terms are also the cross terms are also appearing in the ellipsoid equation.

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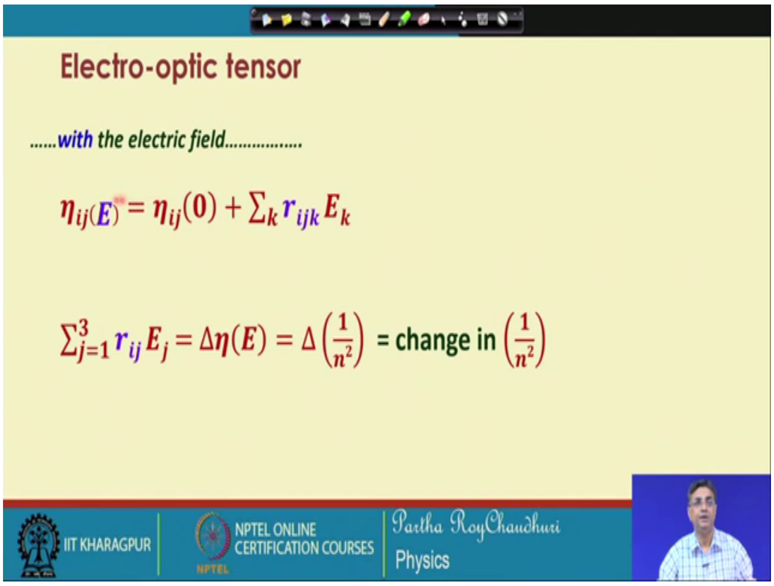

With the electric field we have seen that in terms of the impermeability tensor impermeability in presence of electric field is equal to impermeability in absence of electric field plus this total coefficient multiplied by this electric field. So, this will give me the new impermeability tensor, but we need the change in the impermeability that is the difference that is n i j E minus n i j O this difference we need then we can calculate this quantity. So, by knowing this quantity because electric field is known to us the coefficients are known to us. So, we can evaluate this if we evaluate this then this will give me the difference the change in the impermeability tensor.

Once I have the change in the impermeability tensor I can couple this change to the original to get the new ellipsoid, and from the new ellipsoid by diagonalisation or by Euler angle rotation we can calculate the new principal refractive indices. And from there we can calculate the birefringence retardation and all other associated properties that is in a summary the task that has to be done.

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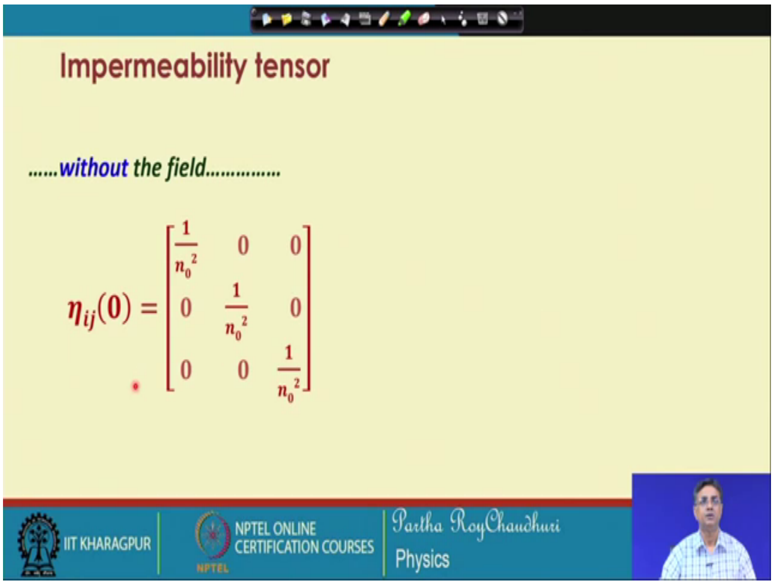

So, without field I have the form of the impermeability tensor which are all the same diagonal elements because it is an isotropic material.

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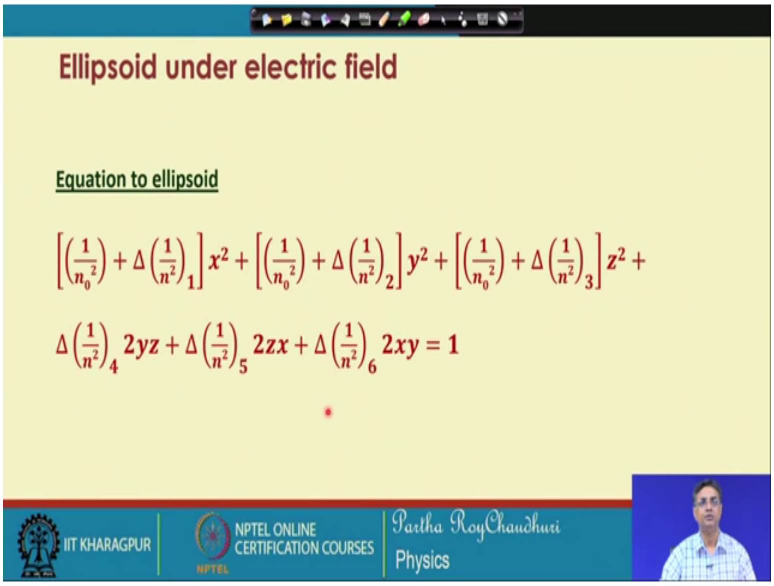

Now, in presence of the electric field all the other off diagonal elements they are also appearing this and the cross terms are also appearing, because all the 6 components in general maybe affected. We will see specifically which components are actually affected depending on the orientation and the property of the crystal.

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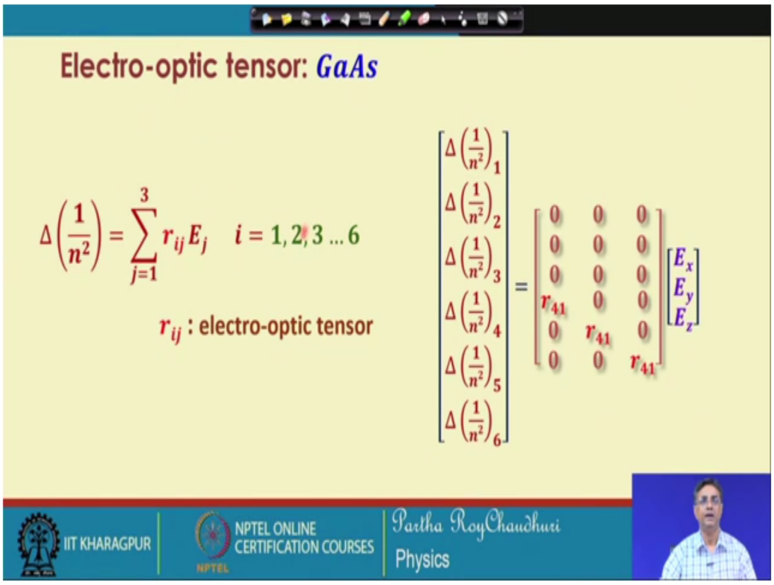

So, for gallium arsenide I write the change in the impervious the well known equation we have several times before that delta of 1 by n i n square that is impermeability change is equal to this quantity and the explicit form is this all the 6 elements are now affect are now evaluated in terms of this. So, this will decide who all elements will be non 0 after I take the matrix multiplication of E x E y and E z, so we will do that to understand this.

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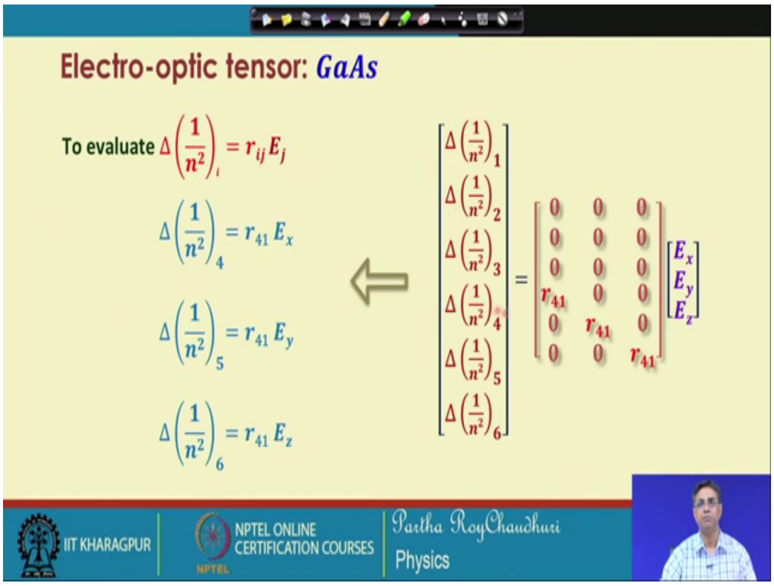

Let us take this gallium arsenide example, so by multiplying this with this equation I can isolate each of the components and you can see that delta of n square 4 that is this is non-zero, this is non-zero and this is non-zero and interestingly because r 41 is the only element non 0 element. So, all of them are identical, but depends on the strength of the electric field for the three mutually orthogonal directions. If all the three components of the electric fields are present then all the three incremental changes in the refractive index or the corresponding impermeability will be associated as a consequence of the electric field.

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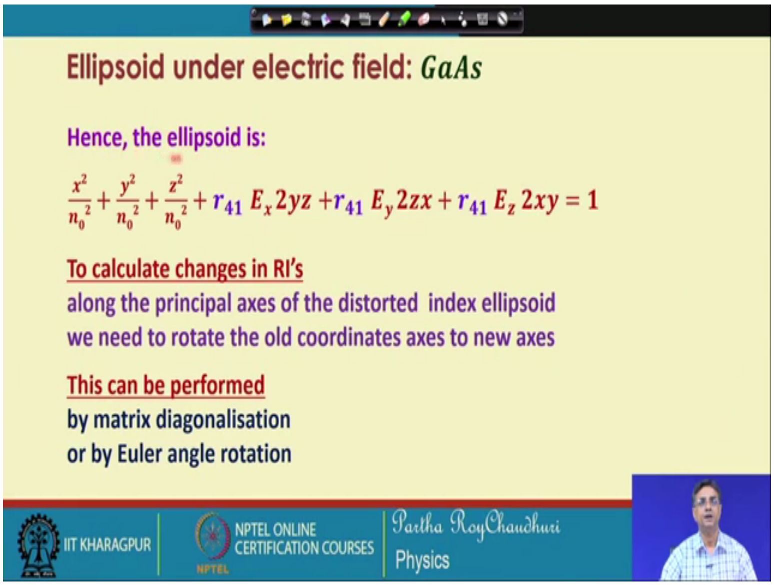

Therefore the ellipsoid equation from here we can write this ellipsoid equation because these elements which is associated with x square this element associated with y square and this three that is 1 2 and 3 they are all 0. That means, the square terms they remain unaffected, but there will be an coming of this cross terms that is y z z x and x y coming into play into the new ellipsoid equation.

So, this cross terms are now associated with this new ellipsoid equation, now you will have to calculate the changes in the refractive indices for the principal new principal axes system. So, this is now the distorted ellipsoid equation deformed ellipsoid equation from here you will have to calculate the changes in the reflected along the principal axes the distorted index ellipsoid.

We need to rotate the old coordinate axes to the new axes and this can be performed by matrix diagonalisation or Euler angle rotation. As we have mentioned earlier with the examples that is you can use a matrix diagonalisation or you can use and Euler angle rotation to go inside the new coordinate system with the principal axes system of the crystal, under the influence of the external electric field.

Now, having known this equation this where you have the effect of E x that is electric field along x along y and along z all the three components that is in general, when the electric field is acting on the crystal in a general direction then all the terms will be appearing will be there. But if I considered for simplicity that electric field is applied only along a certain direction for example, along z direction then what happens

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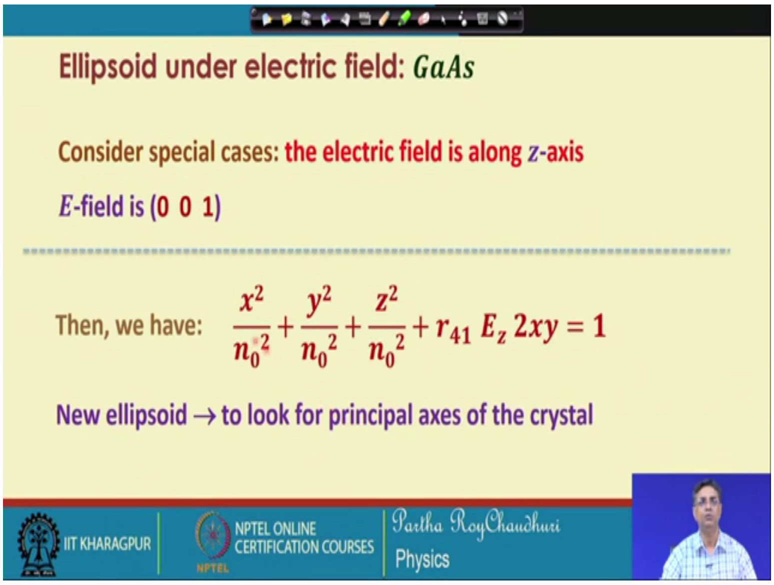

Because this is the way we can actually understand how this electric field is going to change the refractive index properties of the crystal. So, electric field is 0 0 1 direction that means, you have E z direction only. So, only you have this E z component E z electric field, so this component surviving in the ellipsoid equation and then we can do this diagonalisation of this matrix corresponding to this ellipsoid equation.

Or we can simply inspect that that there is a cross term associated with x and y that means, it requires a rotation about z and because x and y are symmetric if you change interchange x with y then the equation remains invariant. So that means, the rotation about z will be 45 degree, so that it is identical to x and y that is 45 degree rotation.

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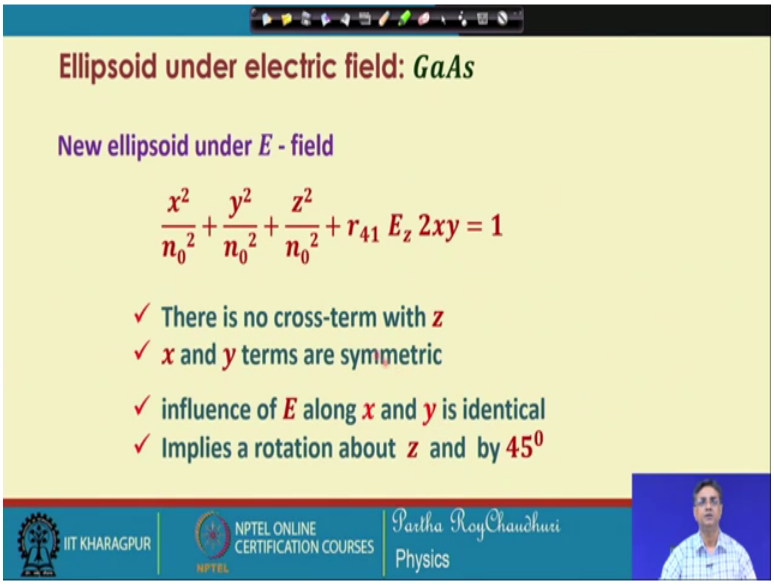

So, the new ellipsoids, so that is what just mentioned that require a rotation of rotation by 45 degree about the z axis and the influence of E along x and y is identical ok.

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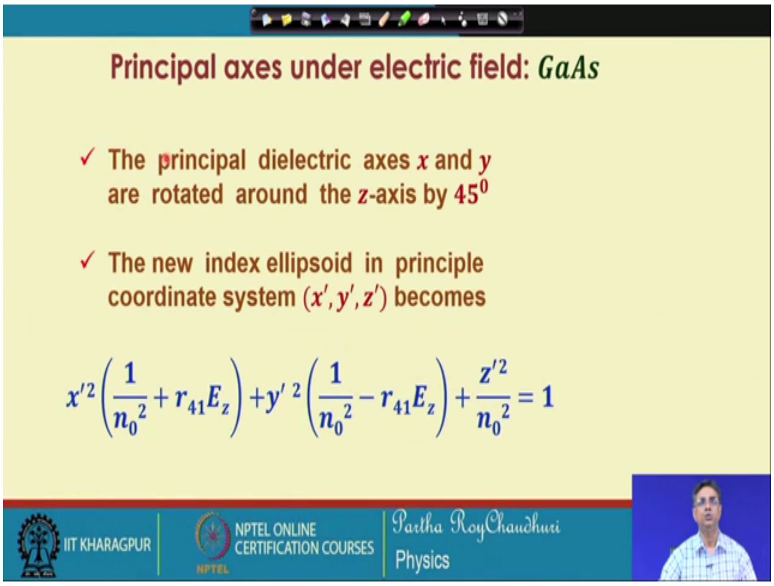

So, the new the principal dielectric axes x and y are rotated around the z axis 45 degree and by doing that we have seen that this for example, in the previous discussion that it can be represented by this equation giving Euler angle rotation.

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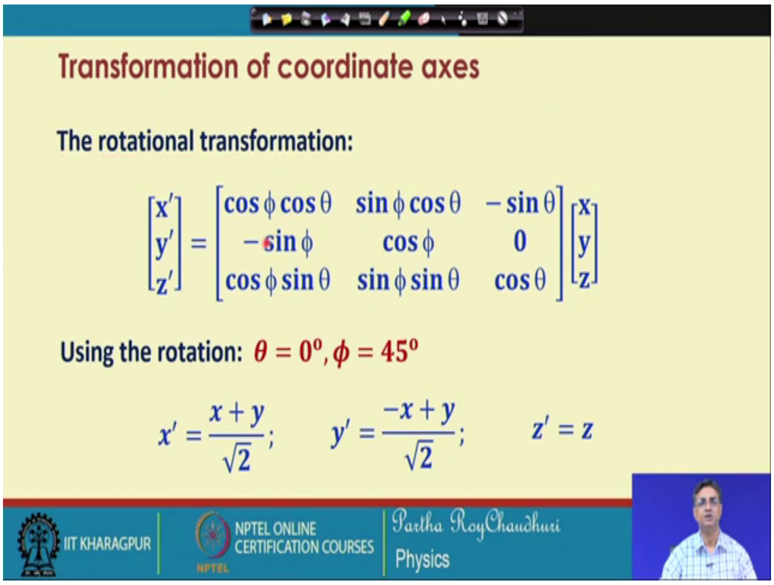

So, to understand that how we get this equation let us look at the rotational transformation that is the new coordinate system connected to the old coordinate system by this transformation matrix. Now for the particular case we are discussing we will put theta equal to 0, because that is the rotation corresponding to x and y and phi is the rotation corresponding to z.

So, then theta equal to 0 phi equal to 45 degree we can reduce this equation to x prime equal to x plus y by root 2 y prime equal to minus x plus y by root 2 and z prime equal to z. Now, this is the transformation that is required to make the coordinate system coincide with the principle axes system of the deformed ellipsoid.

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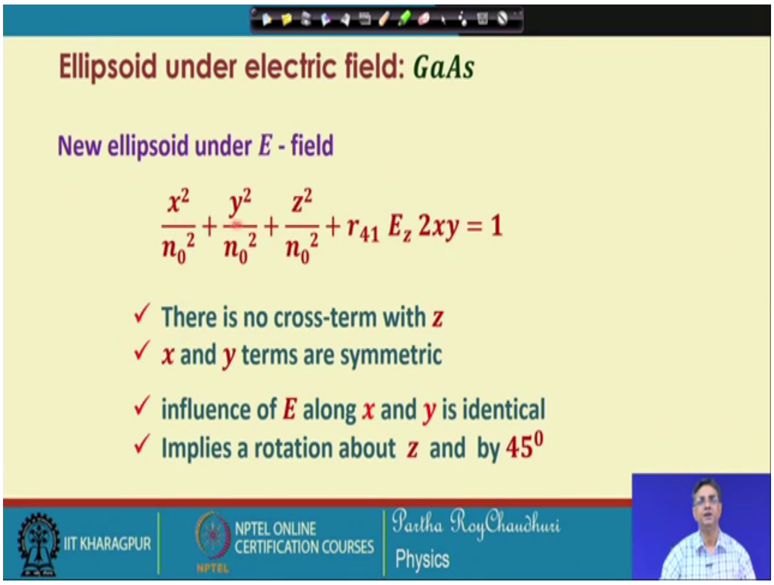

But the reverse transformation is required because we have this equation, in hand x y and z. So, we have to substitute for x y and z by x dash y dash and z dash.

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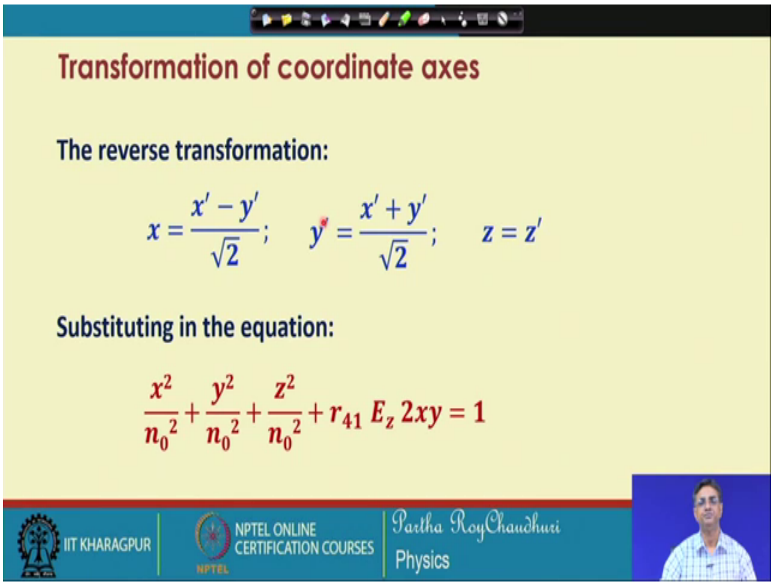

So, therefore, therefore we need a reverse transformation x will be represented by this y will be represented by this and z is anyway equal to z y. Therefore, by substituting this values of x for this should be y not y prime this should be y if you substitute back into this equation we can write this equation in this form.

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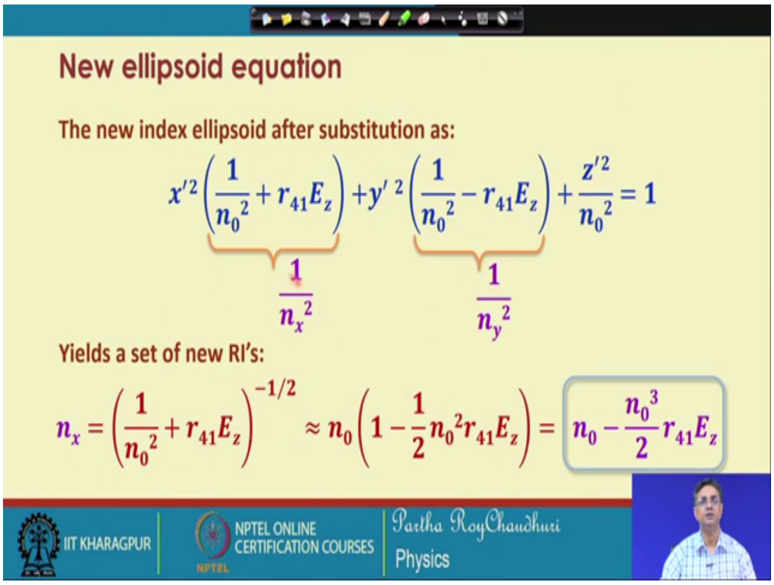

And then it yields a set of new refractive indices because this entire quantity is now 1 by n x square the new refractive index for x and this is the new refractive index for y. So, n x will be given by this which is by binomial expansion can be approximately written as this because this quantity is very small 10 to the power of minus 12 that is so. So this clearly gives the value of n x which is equal to n 0 minus n 0 cube by 2 r 41 is it so this is for n x.

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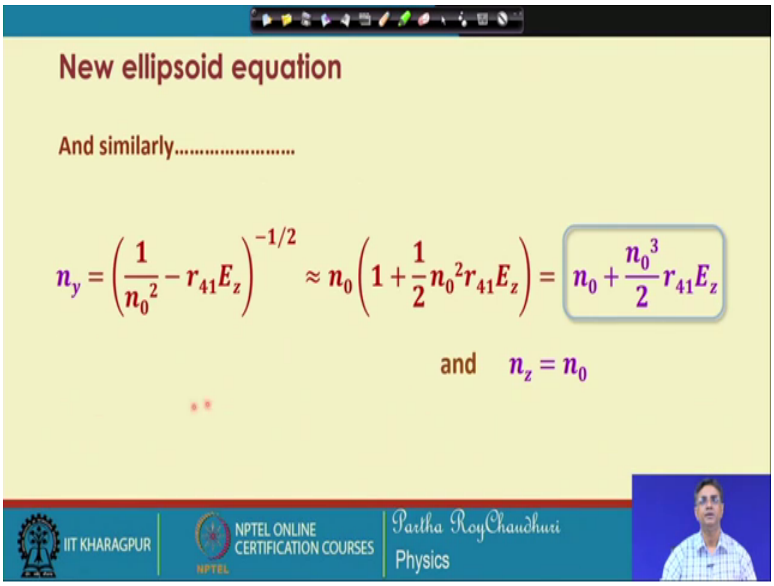

And similarly for n y we can calculate which will be plus n 0 plus n 0 cube and so on and n z remains n 0 itself. So, you can see that in presence of the electric field E z the refractive index which was n 0 has now become has become less depending on this r 41, whether it is positive or negative is it is positive it has become less from n 0 whereas, n y has become more by the same amount. So, 1 n x has decreased and n y has increased, but n z remains the same. That means, I have a new principal refractive indices where n x is more is less than n 0 n y is more than n 0, but n z remains the same.

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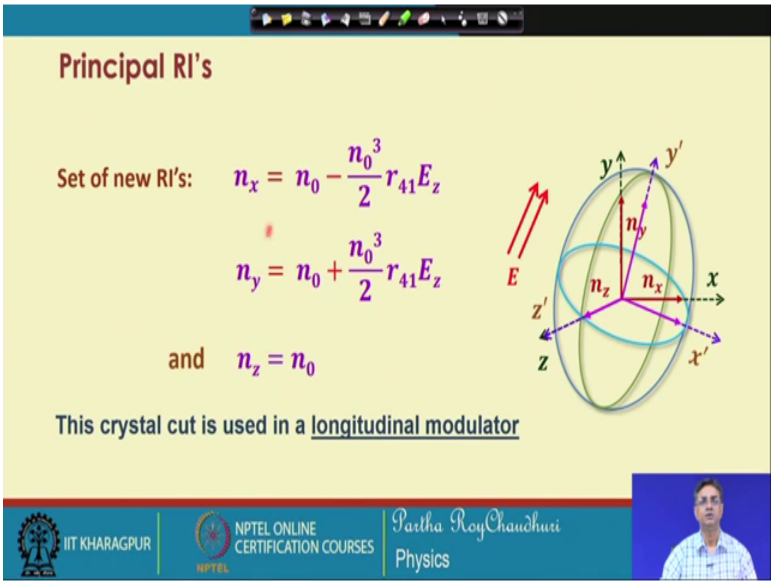

So, this is how we represent the refractive indices here so n z equal to n 0 and this crystal cut is used as this particular cut that when you apply an electric field along the z direction. Because for an isotropic material z direction is completely arbitrary any direction could be defined as z direction. And the result will be depending on the direction of the electric field there will we can define the x and y if we call this is equal to z and in the perpendicular directions in the orthogonal directions x and y there will be changed in the refractive indices.

So, if you apply an electric field along this direction and if you apply, if you inject a light to travel through the medium in this direction itself, then this light which has two orthogonal polarization. Let us suppose it has two orthogonal polarizations will be experiencing two different refractive indices along x and y when the electric field is there and as a result there will be a delay there will be a birefringence depending on the length traveled by this there will be a phase delay.

And this configuration and once you have the phase delay or the retardation then you can you can set up a modulator with this. So, this configuration that is the electric field and the light travelling in the same direction in an isotropic crystal gives you a longitudinal modulator.

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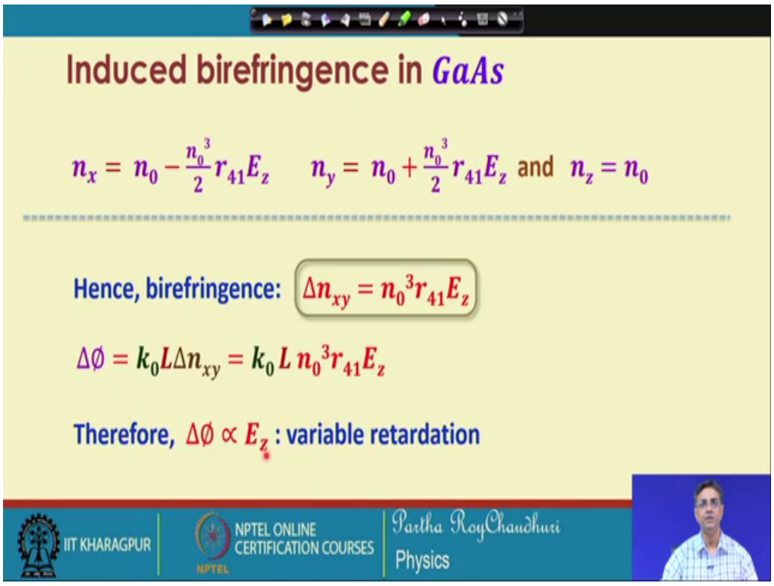

So, the induced birefringence now is n x equal to this which is less from n 0 and n y is more than n 0 by the same amount n z equal to n 0 the original refractive indices. Hence the birefringence that is the difference in n x and n y will be just this because half of this here half of this here. So, these will the difference will be this quantity.

So, the phase difference will be k 0 l delta n x y, so this phase difference you can see that is just proportional to the applied electric field strength. So, del phi is proportional to E z so it is this phase difference or the retardation is proportional to external field that means, that it is a variable retarder system where the retardation can be controlled by the external electric field that is E z.

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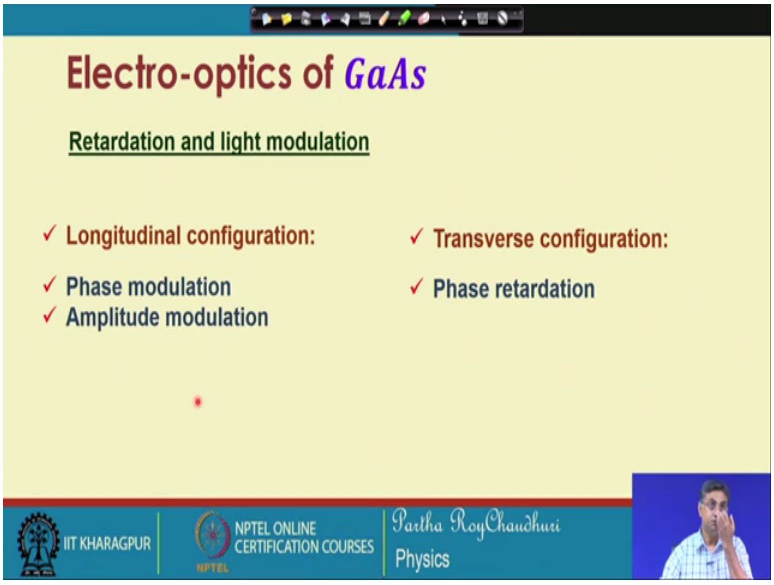

So, we have seen that because there is no other choice to talk about this isotropic system gallium arsenide whichever direction you apply an electric field can be treated as because it is isotropic can be treated as E z. And then depending on this as E z that is z direction you will have x and y and those x and y the new x and y system depending on the direction of the electric field. We see the birefringence and this birefringence can be used for modulator application, but there is no other choice apart from this when it is set as a longitudinal modulator system.

So, this for this system this retardation and light modulation and again be sub divided into two different ways of looking at the configuration is longitudinal. That is when the electric field and the light which has to be modulated are travelling in the same direction through the medium then that longitudinal configuration can be designed as a phase modulator as well as an amplitude modulator.

And we will see these through examples how the light can be modulated. The other way is that if you apply an electric field in the transverse direction in the transverse direction, then you can have the phase retardation which can again be used as a modulator. So, this configuration is also very useful when you apply the electric field across the crystal along a direction which is perpendicular to the direction of the propagation of the light waves through the crystal and we will discuss this in the next section.

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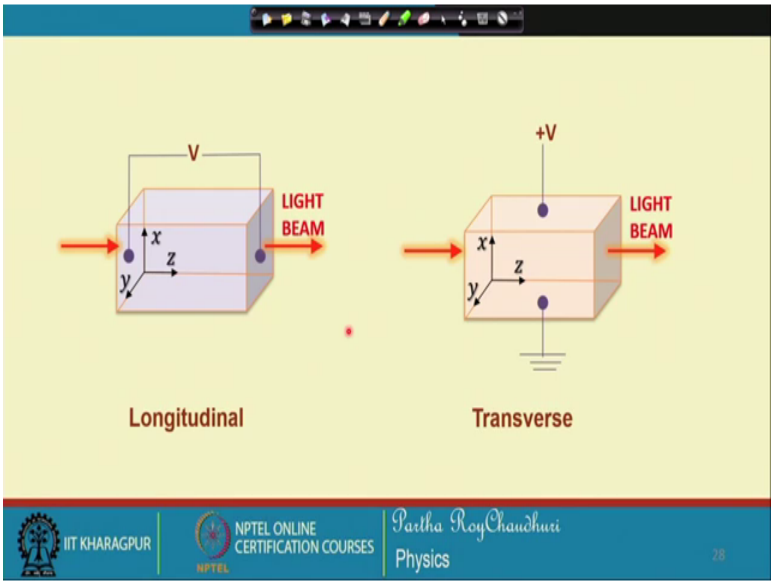

So, this is the longitudinal configuration of an electro optic modulator you have a beam which is traveling through the medium and you apply the electric field in the same direction, but this has some inconvenience as regards that the direction we have to put an electrode. So, usually a small hole is placed or sometimes transparent electrodes are mounted, so that the propagating electromagnetic wave is not abstracted.

The other configuration is the transverse configuration where the electric field is applied across the crystal and the light wave is travelling in the in these direction. So, these are the two different configurations and we will discuss all in details with examples in the next section.

Thank you.