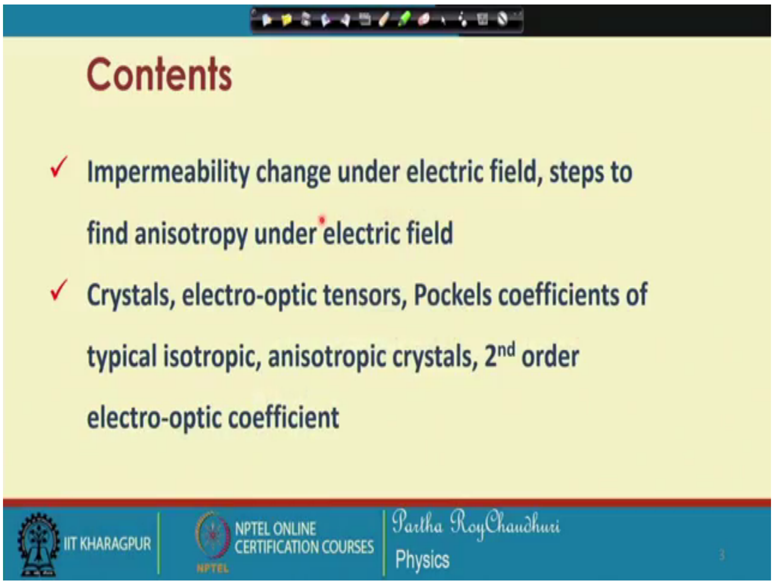
Modern Optics
Prof. Partha Roy Chaudhuri
Department of Physics
Indian Institute of Technology, Kharagpur

Lecture - 30
Electro-optic Effect (Contd.)

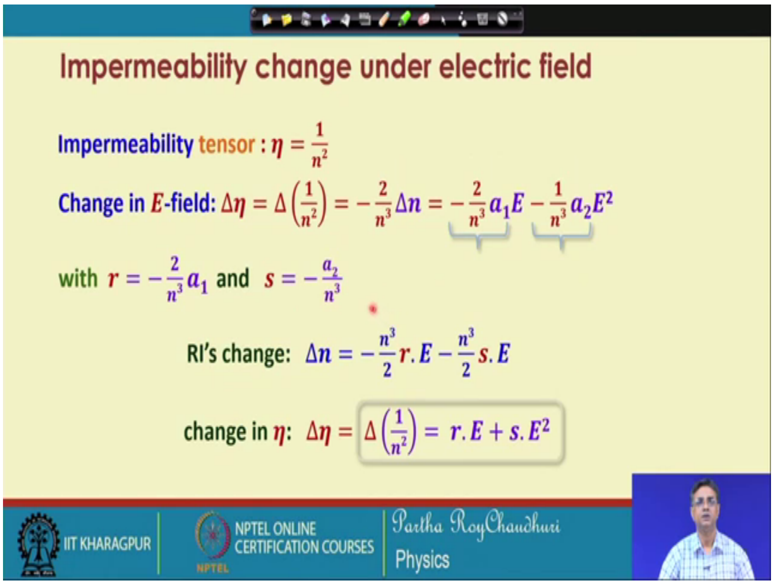
So, we have seen that Electro optic Effect in the 2 groups, that is the linear electro optic effect and the Kerr effect that is the quadratic electro optic effect.

(Refer Slide Time: 00:23)



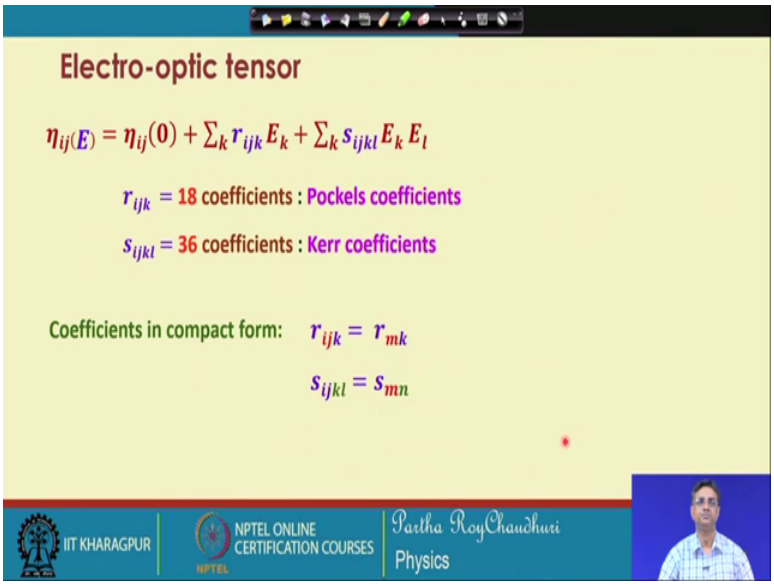
Kerr And the subsequent discussion is under this following topics that is impermeability change under electric field. And we will see the steps how to find anisotropy under the electric field and look for the principal refractive index system. Then various crystals various electro optic tensors associated with this to make a general overview and to have an idea about how this electro optic systems are used for various applications, typical isotropic anisotropic crystal, and then second order electro optic effect that is the quadratic electro optic effect.

(Refer Slide Time: 01:10)



So, let us look at this we have seen that impermeability tensor, that is eta is 1 by n square. So, change in the electric field, change in presence of electric field delta eta is represented by this, and we have seen that r the Pockels coefficient given by this quantity and the Kerr coefficients tensor quadratic electro optic tensor is given by this. So, delta eta is represented by this equation.

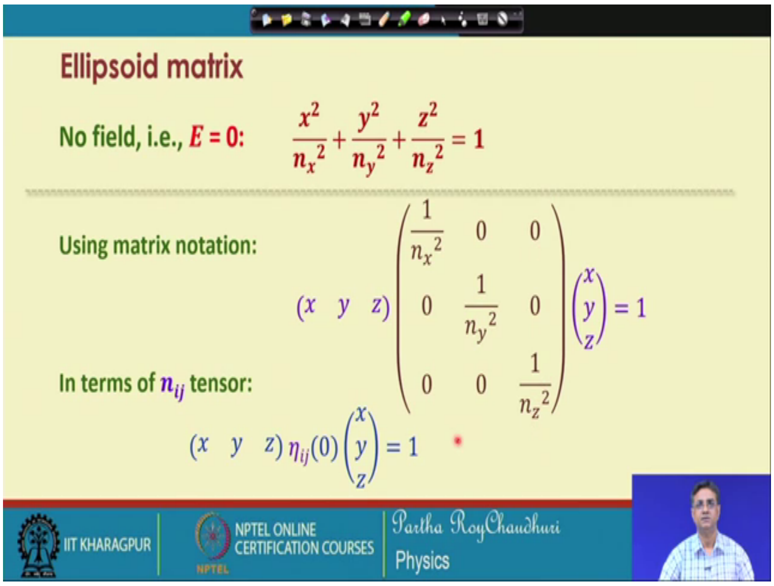
(Refer Slide Time: 01:47)



And this also we have seen that this impermeability, new impermeability tensor in presence of electric field is connected to the impermeability in absence of the electric field, plus the electric field and Pockel coefficients product term, the quadratic electro optic tensor and the electric field product term.

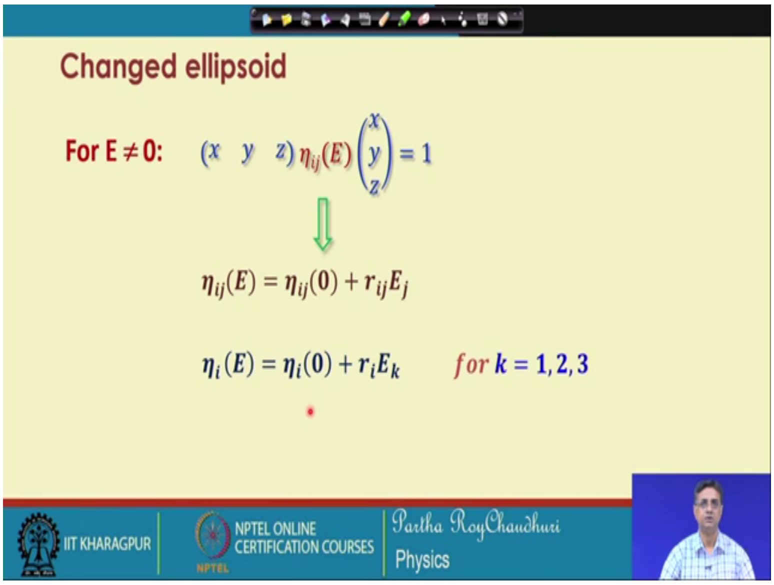
So, this we have seen that this Pockels coefficients are 18 in number; whereas, there are 36 coefficients for the Kerr coefficients. And this is the short form of compact form of writing the Pockel and Kerr coefficients.

(Refer Slide Time: 02:40)



Now let us take an example for example, if we do not have any external field that is E equal to 0, we write this index ellipsoid equation in this form. And using the matrix notation we can write this equation like this x, y, z and then this ellipsoid matrix then we have column vector x, y, z equal to 1. And in terms of eta ij tensor, we can write this equation in this form.

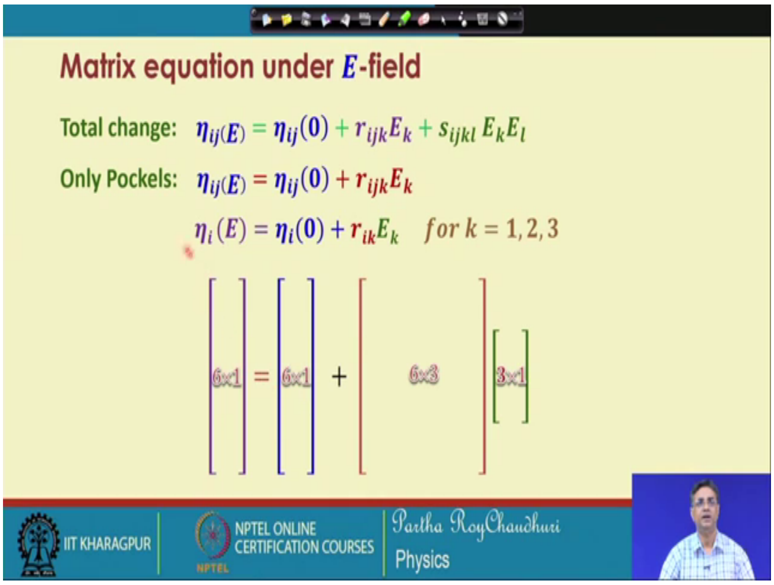
(Refer Slide Time: 03:21)



For E not equal to 0, that is this was in presence of in absence of any external field, but now that in presence of the external field it is not eta ij of 0, but it is eta ij of E and this represents the compact equation.

So, but this eta ij is nothing but the sum of eta ij plus this quantity that is Pockels coefficient tensor and the electric field; if we restrict our study only to the Pockels effect. So, eta ij in the short form in compact notation eta i is now eta i 0 r i E k, where k can assume 1, 2, 3 that is E x, E y and E z these 3 field values. So, let us try to understand that how electric field modifies the tensor and the components.

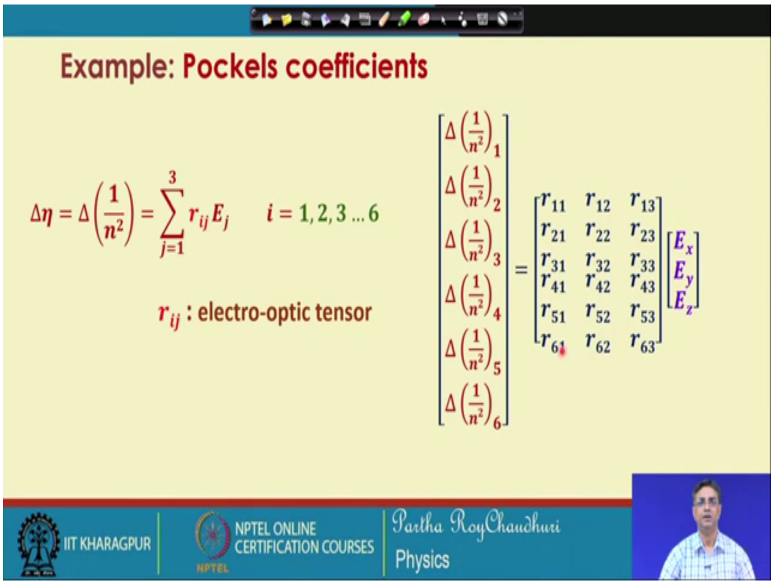
(Refer Slide Time: 04:23)



So, the total change now if you consider both the quadratic effect and the linear effect, then the impermeability in presence of the electric field will be the sum of the impermeability tensor. In the absence of the electric field, plus these 2 additional terms electric field dependent additional terms for only for Pockel this equation that is we just restrict only to this part. And then in that case eta i using that synced indices eta i of E equal to eta i 0 plus r i k E k.

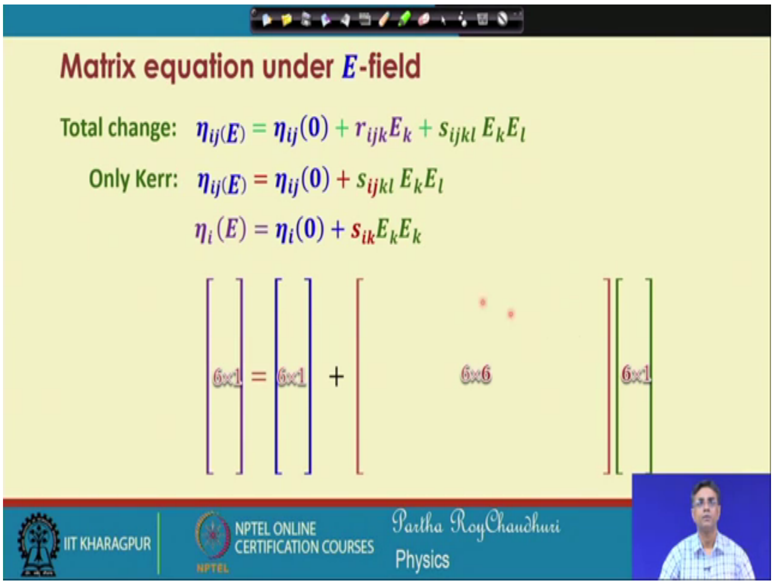
When if you look at the matrix equation explicitly, then this will be a 6 by 1 matrix which is this quantity will be represented by this because we have put ij equal to i only. So, this is 6 into 1, this is again 6 by 1 matrix, but this one will be 6 by 3 k equal to 1, 2, 3 and this is 3 by 1. So, this is because this one is E x, E y and E z. So, this is the explicit representation of this impermeability equation in presence of the electric field connected to the equation, connected to the impermeability in absence of the electric field.

(Refer Slide Time: 05:56)



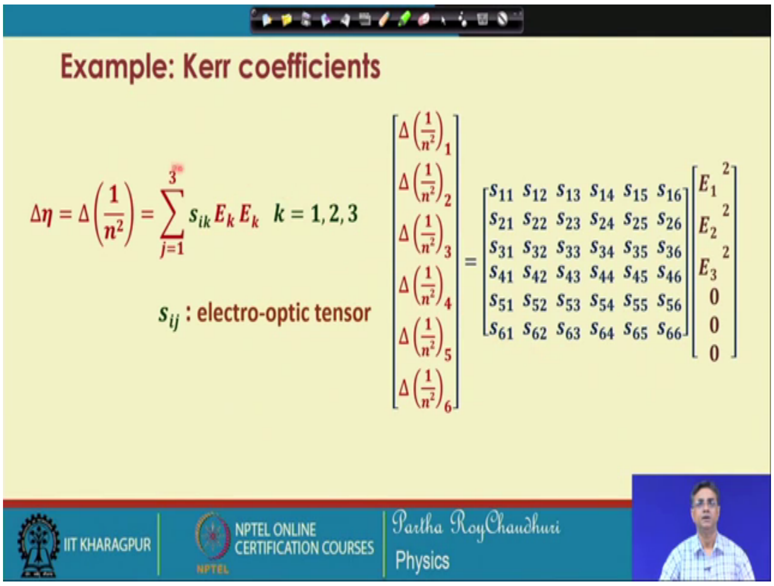
Now example of Pockels coefficient delta eta equal to this we have seen. And this one also we have seen we have identified that this represents this 18 coefficients for the Pockels coefficients. And this r ij is called the electro optic tensor the linear electro optic tensor.

(Refer Slide Time: 06:30)



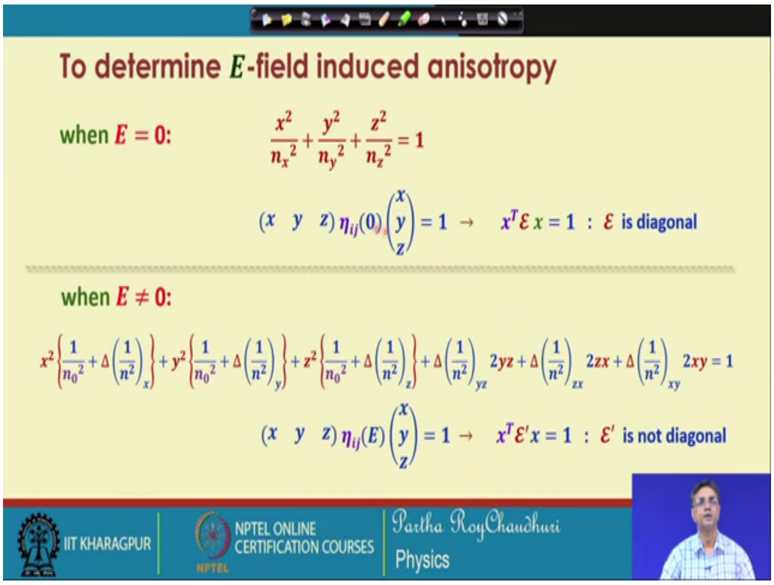
So, the matrix equation total change in this case is again the same equation, but for Kerr I now consider only the third term, not the second term. So, eta ij equal to eta ij 0 plus this quadratic electro optic coefficient tensor E k and E l so, in the compact notation eta i E eta j. So, this is a very good way of looking at the explicit form of the matrix equation. You have 6 into 1, here to represent this 6 into 1 in absence of the electric field. This is the total impermeability in presence of the electric field, which will come from the from this 6 by 6; that is, a s ik tensor, and this electric field components E 1 square E 2 square E 3 square and 0, 0, 0 for this matrix equation ok.

(Refer Slide Time: 07:35)



So, let us look at it explicitly, you have this equation for the quadratic electro optic tensor. The change in the impermeability, and this column, this is the tensor that is s ik now 36 components, 36 coefficients and this right hand side E 1 square E 2 square and E 3 square. So, this is again the an example of writing this matrix equation for the Kerr coefficients. These are useful in evaluating and estimating the changes in the refractive indices or impermeability in presence of the electric field by knowing the values of the electro optic tensors, the numerical values.

(Refer Slide Time: 08:30)

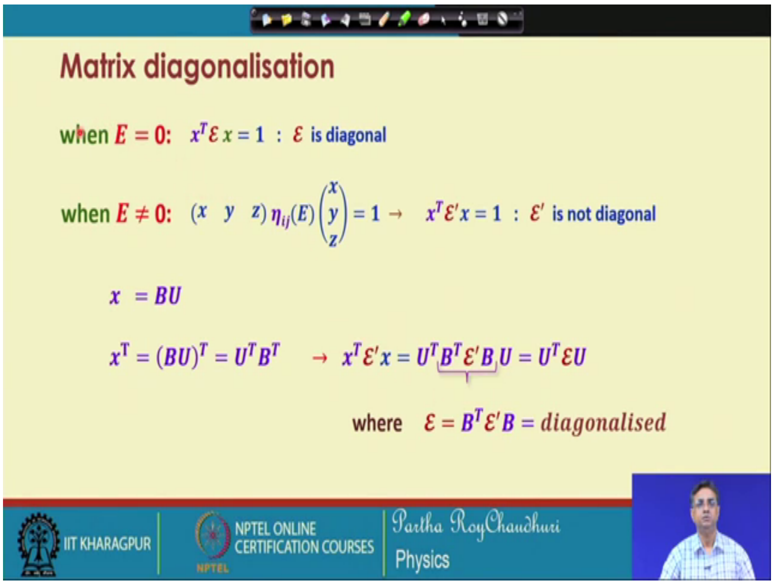


So, determine to determine the electric field induced anisotropy, this is the way we will do it if E equal to 0, we have seen that this equation will represent the index ellipsoid in terms of the matrix equation. And x transpose this is your x and this is x transpose this is x.

So, you can write this equation as this, and this will definitely represent that E is a diagonal matrix. Truly, if eta equal to in absence of field principal axis system, only you have 1 by n x square, 1 by n n y square 1 by n z square these are the 3 diagonal elements. So, you have E is a diagonal. In presence of electric field that is when E is not equal to 0, you have all the changes at there associated with all 6 components.

And then we can write this equation in this form. But this time this E matrix, E matrix to represent this equation this eta ij is not different, because it is eta ij of e. So, we write E dash x transpose E dash x equal to 1. E dash is not diagonal in this case; because we have all these all the 6 elements present here, all the 9 elements rather actually 6 because the symmetric terms are identical.

(Refer Slide Time: 10:07)

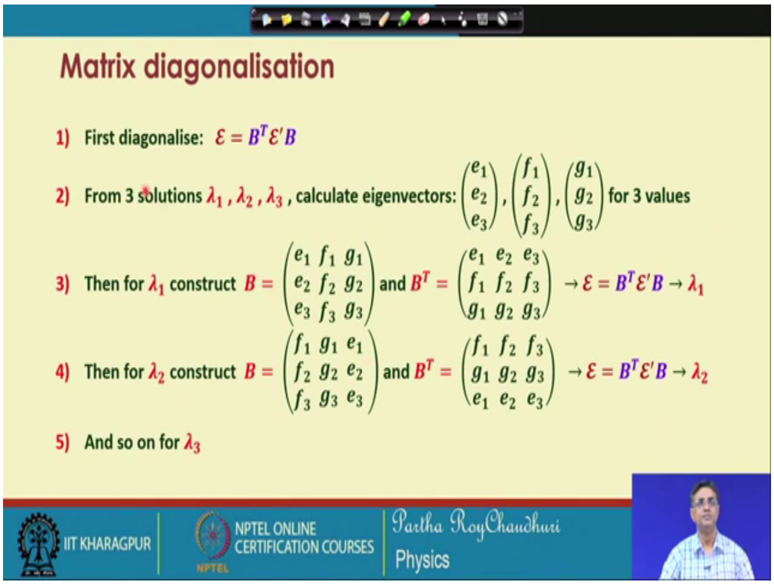


So, it is a way to find out the principal axis system just by diagonalzing this equation. We have seen one example earlier; that when E equal to 0, we have this equation; when E is not equal to 0 we have this equation. But this E prime is not diagonal so, we set x equal to B into u, then x transpose will be B into u transpose. So, that is equal to u transpose B transpose.

And if you plug in this u transpose and B transpose or x transpose in this equation, then we can write this equation in this form. Which will eventually give you u transpose E into u, and then this quantity will represent E as the diagonalised; is a way to diagonalize this elementary diagonalization of matrices. And once I have this diagonal diagonalization diagonalised matrix that is only the diagonal elements, those elements will represent the new principal axis systems refractive indices.

So, this is one way of finding the principal axis system of the distorted ellipsoid in presence of the electric field.

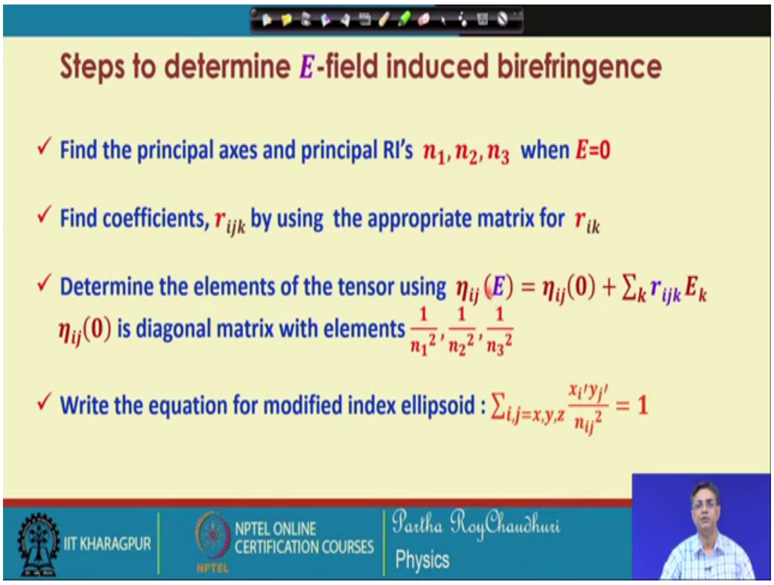
(Refer Slide Time: 11:25)



So, what we have represented here with indices? Now we just quickly discuss in terms of the steps. So, the first step is to diagonalize this E by doing this. And from the 3 solutions, once you have diagonalized E, you get 3 solutions lambda 1, lambda 2, lambda 3. Then we can calculate the eigenvectors knowing this from the polynomial equation of the matrix. We can calculate the eigenvectors that is e 1, e 2, e 3 f 1, f 2, f 3, g 1, g 2 for the 3 values of lambda 1 and lambda 2. Each value of lambda 1, lambda 2, lambda 3 will correspond to the respective eigenvectors.

Then we can construct this B here for lambda 1 like this. And then B transpose will be simply the transpose of this e 1, e 2, e 3 will go in the row from the column and so on ; which will give you this E equal to so that corresponds to this lambda 1. And similarly, for lambda 2 again we can construct this equation f 1, f 2, f 3 which is the corresponding eigenvector, and maintaining the same sequence we can again calculate B transpose which will correspond to lambda 2 and in the same way for lambda 3. So, we can calculate all the 3 eigenvectors corresponding to the 3 eigenvalues which will represent these Eigen values will represent the corresponding refractive indices in the principal axis system, under the influence of the external electric field.

(Refer Slide Time: 13:17)

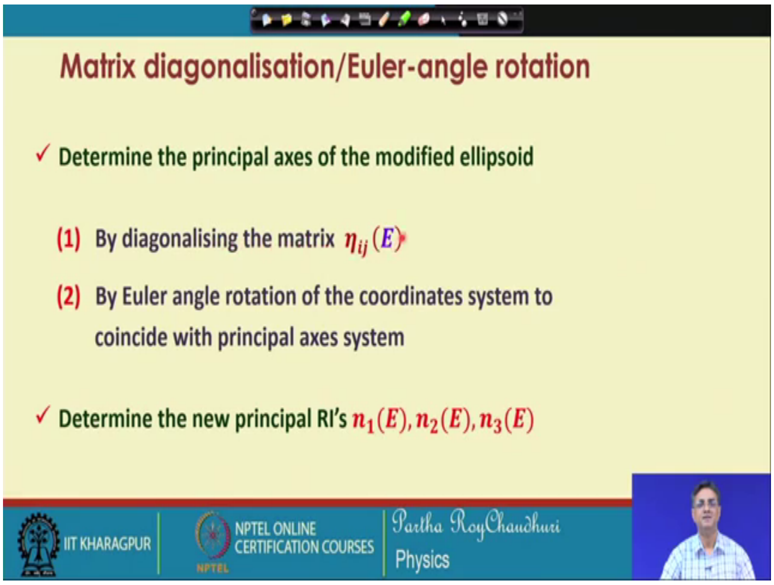


So, again to summarize the steps to determine the field induced birefringence; that is how the changes in the refractive indices can be quantified can be calculated. So, let us first find the principal axis and the principal refractive indices system; when E equal to 0 that is by writing the diagonal matrix. Corresponding to the index ellipsoid, then find the coefficients r ij k by using the appropriate matrix for r i k. Determine the elements of the tensor using this equation, this equation we have seen explicitly in terms of the matrix form. This is by 6 by 6 and this is 6 by 3.

So, this is a diagonal matrix elements with this one. This is already diagonal matrix element, but when you add these 2 unit no more remains diagonal then you have to diagnolise. This write the equation for the modified index ellipsoid by using this. So, first we will look at there electric field components who all are present. Then will take care of this quantity that r ij k and this. So, this will give you the matrix these 2 matrix will be added up, which will represent the new matrix. New matrix to represent this impermeability in presence of the electric field and then we can write the index ellipsoid in this form.

Now, once we have the new matrix in terms of a in presence of the external electric field, then we can actually a diagonalise this matrix to find lambda 1, lambda 2, lambda 3 as we have discussed.

(Refer Slide Time: 15:09)

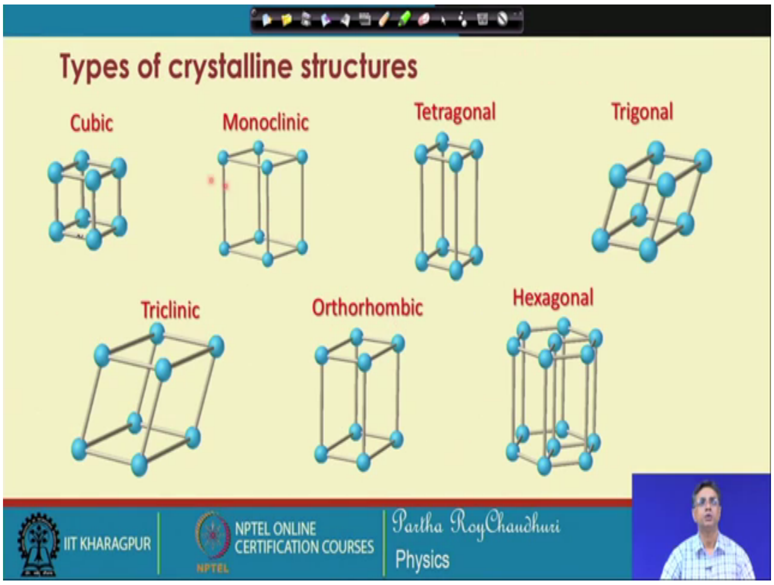


So, this is one way of doing the principle. So, this lambda 1, lambda 2, lambda 3 the 3 Eigen values will directly represent the corresponding refractive indices in the principal axis system after the ellipsoid is distorted undergone some rotation. The other way is by using an appropriate Euler angle rotation of the coordinate system; which will coincide with the principal axis system of the medium. And this example we have seen and it is also very useful by looking at the cross terms and the direct terms in the index ellipsoid.

Then by doing this any of these 2 operations we can determine n 1, n 2 and n 3 the new principal refractive indices in presence of the electric field. And these 3 will be will be if they are different then we can actually calculate the birefringence for the way which are having these 2 refractive indices as the 2 polarization components; that is, if a wave is travelling along the traveling through the medium.

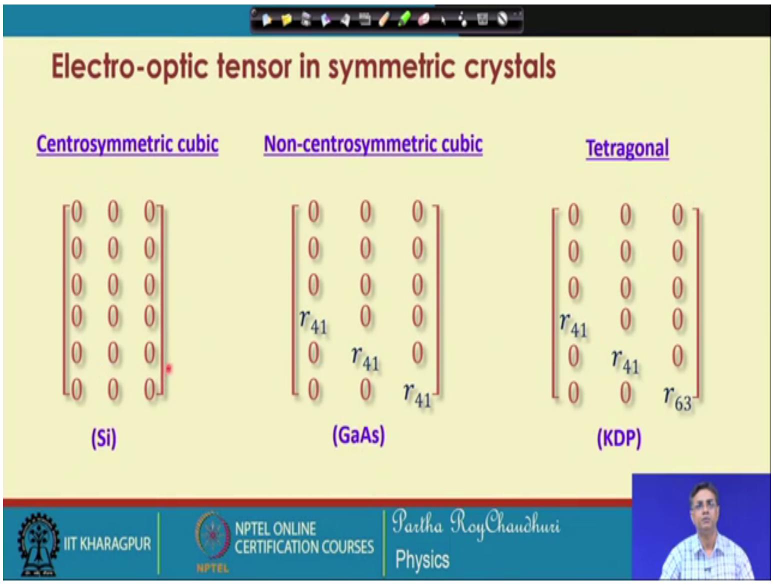
So, that it is electric field vector corresponds to this direction principal axis direction, and the another linearly polarized wave which corresponds to the direction represented by the direction of this which will come from the eigenvectors of this of this principal refractive indices, then there will be a if in general n 1 and n 2 are different. There will be a birefringence between these 2. And similarly if the orientation is of the polarizations of the wave is along these 2, then there will be a birefringence between these 2. It could be as well from this and this ok.

(Refer Slide Time: 17:19)



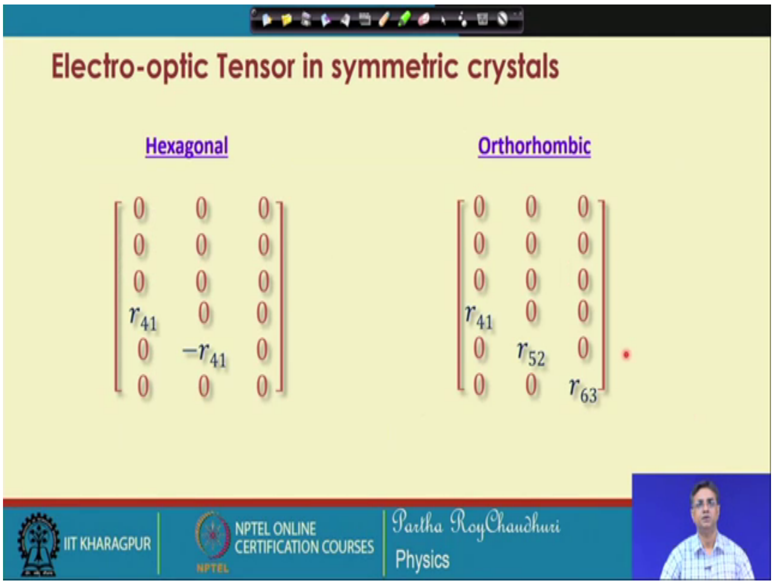
Now, we will look at few crystalline structures cubic monoclinic, there is a you know very detailed study of the crystalline structure as regards the electro optic effects and there are groups. So, this is monoclinic, tetragonal, trigonal, hexagonal, orthorhombic. And looking at the symmetry 2-fold 3-fold 4-fold there are groups of the crystals.

(Refer Slide Time: 17:45)



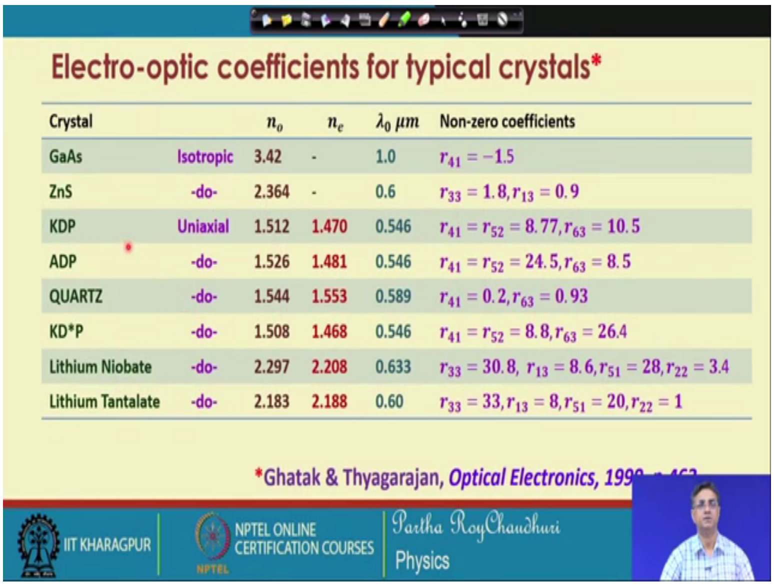
The centrosymmetric cubic crystal we have seen that this the tensor is 0, Pockel tensor is 0, Pockel coefficient tensor and for non-centrosymmetric. But isotropic like example is gallium arsenide, these 3 coefficients will be non 0, rest all of them as 0. For tetragonal like KDP potassium dihydrogen phosphate, this 3 are the non-0 coefficients we have a for hexagonal, we have distance r.

(Refer Slide Time: 18:21)



So, these are the tensors representing different crystalline structures corresponding to different media. And knowing the values of this r 4 1, 5 2, 6 3 etcetera we can calculate the change in the impermeability. By knowing the strength of the electric field, then we can calculate the birefringence the retardation and all other relevant properties. We will see various examples.

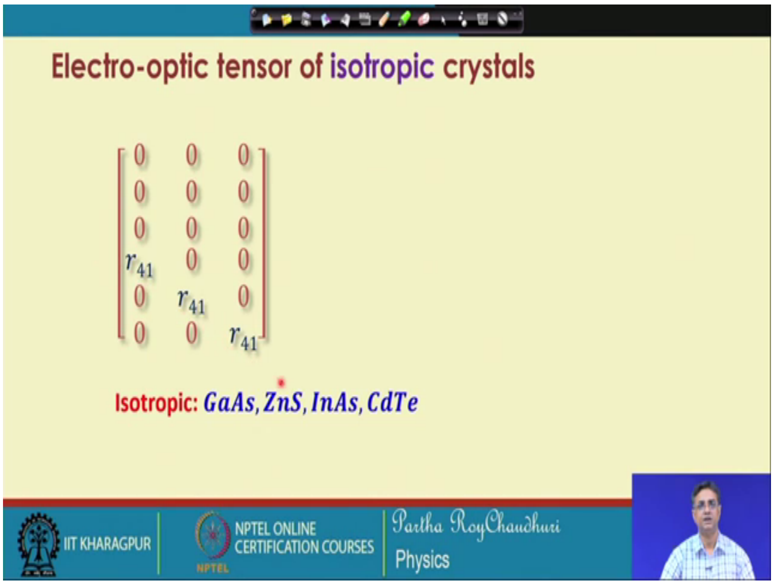
(Refer Slide Time: 18:56)



. So, here is a list of some very useful very commonly used crystals, isotropics, we have mentioned only 2 isotropic gallium arsenide, zinc sulphide. They have n 0, this quantity which is and because it is isotropic. So, there is no question of n e extraordinary refractive indices. And these are the operating wavelength at which this so, this refractive indices. And these are the coefficients for gallium arsenide you have only one non-0 coefficient r 4 1, which is equal to this and into 10 power of minus 12 pico meter per volt.

So, this these are the coefficients for various crystals KDP, ADP, quartz, KDDP, (Refer Time: 19:52) deuterium phosphate; this lithium niobate, lithium tantalate so, this is table is very useful. And these are the experimentally obtained values. And just using these values and knowing the electric field one can completely specify the refractive index and hence the polarization properties of the electromagnetic waves travelling through the particular medium. And this table is taken from optical electronics by mister Ghatak and also Thyagarajan this book optical electronics.

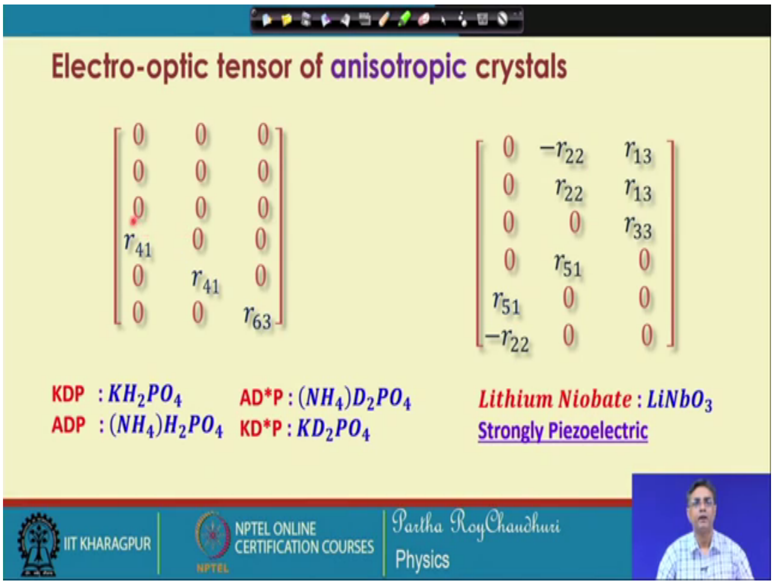
(Refer Slide Time: 20:29)



Electro optic tensor for isotropic crystal, gallium arsenide, zinc sulphide, indium arsenide, cadmium telluride.

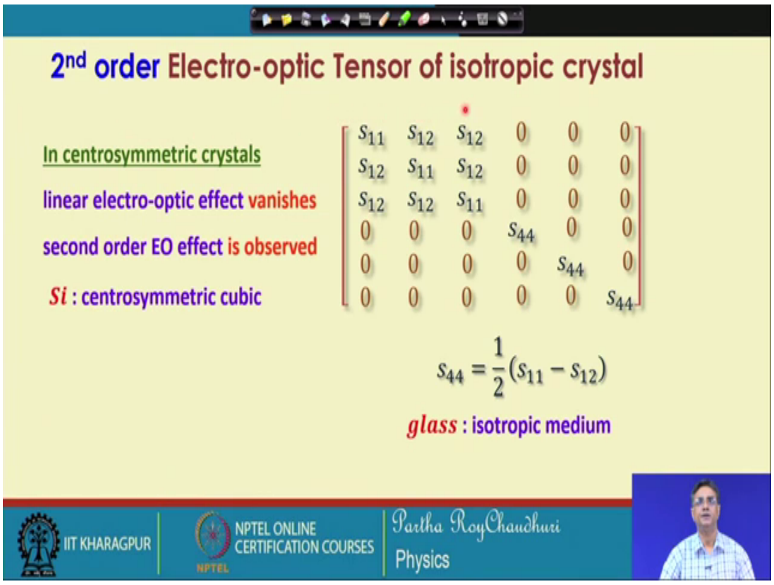
So, for all of them the firm is the same. That is all the 3 coefficients non 0 coefficients are the same that is r 4 1. Actually this is 5 2 and this is 6 3. But because the numbers are the same so, we write r 4 1. So, this is for this for zinc sulphide the values are different.

(Refer Slide Time: 21:03)



For KDP this r 4 1 and r 5 2, they are same, but this is different r 6 3. For lithium niobate and this is the group of medium group of crystals which exhibit exhibits this particular which is represented by this tensor. And this lithium niobate, which is a very strongly piezoelectric material, it shows this electro optic tensor like this.

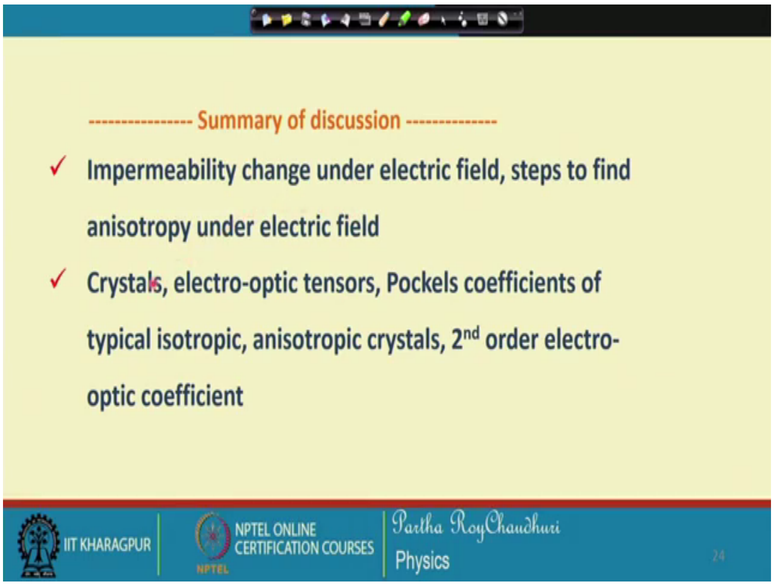
(Refer Slide Time: 21:43)



Ok Second order electro optic tensor of isotropic crystal that is quadratic electro optic tensor. It has this from, we can see this is this has 6 by 6 element matrix.

So, and the designations of this are s 1 1, s 1 1, s 1 1, s 4 4 so, all these 3 are the same. In the case of isotropic medium s 4 4 equal to half of s 1 1 minus s 1 4 these are all experimentally observed coefficients. And this tensor is then very useful will take up one very interesting example for this Pockel effect, at the end after we discuss for the Kerr effect. For the quadratic electro optic effect after we discuss the this thing. So, this silicon is a centrosymmetric cubic crystal, second order electro optic is observed linear electro optic effect does not arise as we have seen because it is a centrosymmetric crystal.

(Refer Slide Time: 22:59)



. So, in this discussion today, we have summarized this impermeability change under electric field, and then we looked at in details the steps how to find the anisotropy under electric field. Anisotropy in terms of the change in the permeability, impermeability tensor; that is, the coefficients attached to the impermeability tensor.

And from there we can calculate the values of the electric field dependent refractive indices in the principal axis system of the of the crystal. And from there we will see in the subsequent examples how we can calculate the birefringence retardation the delay of the orthogonal polarizations; which is very useful for designing different kind of modulators and devices in the devices for communication systems for switching and for various other applications. So, this anisotropic crystal second order electro optic coefficients, all these things we have discussed by now.

Thank you.