Modern Optics
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Lecture - 03
Maxwell's equations and electromagnetic waves (Contd.)

In this section we will discuss the Plane Electromagnetic Waves in Free Space.

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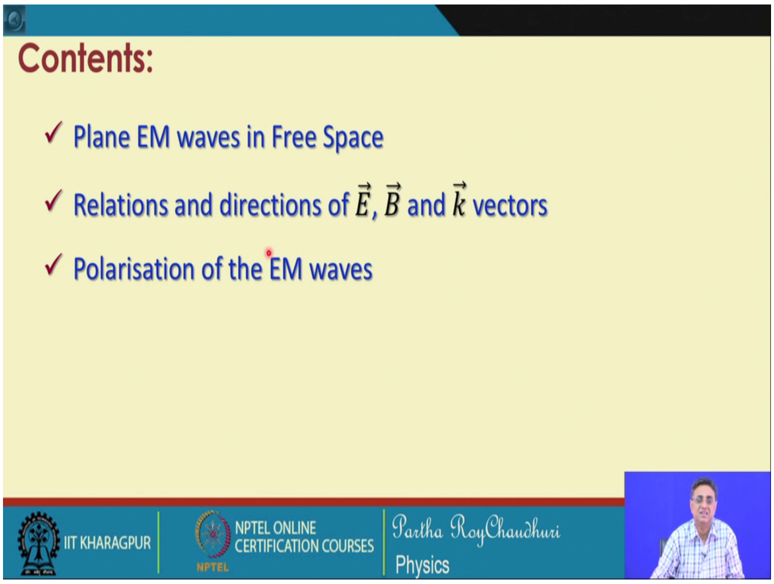

The contents are as follows: first we will consider plane electromagnetic waves equation in free space which is derived from the basic set of Maxwell's equations. Then we will look at the electric field, the magnetic field and the propagation vector, their directions, and how they are oriented in the space and their relative orientation also. Followed by this we will discuss the polarisation properties of electromagnetic waves in the free space.

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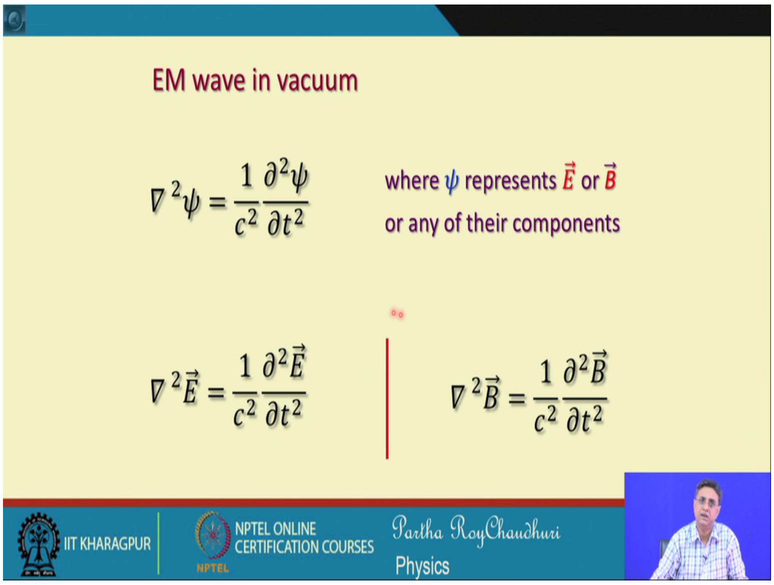

So, we first consider the basic wave equation which is in this form that del square of psi equal to 1 upon c square del square psi del t square. Here, the psi represents the electric field or the magnetic field or any of their components. So, if we consider the electric field or their component or any of their components then we can write this equation in this form. Whereas, if we considered the magnetic field or their components we will consider this equation, basically both of them are of the same form that is this equation.

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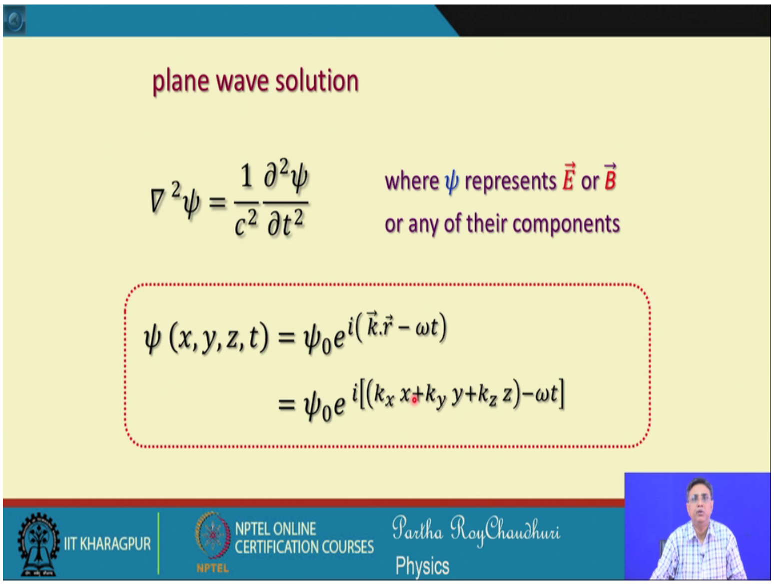

Then we look for the solution for this equation, as we know we have already seen that the plane wave solution is given by this equation in the Cartesian coordinate system that the total psi solution is equal to psi 0 and e to the power of k dot r minus omega t. Which, if you decompose this k dot r into the Cartesian coordinates then you can write that psi 0 equal to e to the power of i k x x plus k y y plus k z z minus omega t.

Your omega t it is the frequency of the electromagnetic waves, k x k y and k z are the components of the propagation constants in the three orthogonal Cartesian coordinate directions. So, this plane wave solution for the electromagnetic waves in vacuum is the same where, we will use this psi for a particular electric field or a magnetic field component.

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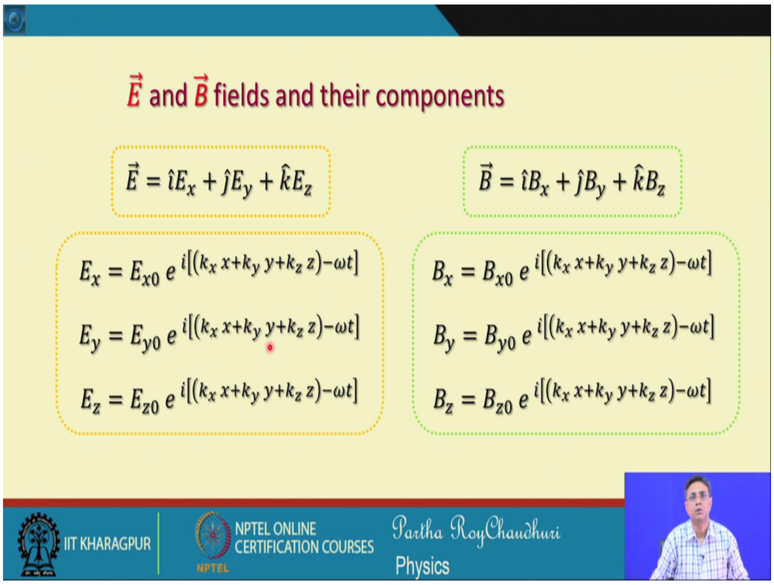

So, if I considered that the electric field in the Cartesian space it will be represented by this quantity that is iE x plus jE y and kE z. So, these are the three cartesian components of the electric field E x, E y and E z. And similarly, for the magnetic field we write that the components of the magnetic field along x is equal to B x, along y it is B y and along z this B z.

Now, each of the Cartesian components that is E x, E y and E z will follow the plane wave solution, as we have seen the plane wave equation will be satisfied by each of the Cartesian components of the electric field as well as the magnetic field. So, the electric field component E x will be equal to some amplitude of the electric field E x 0 into e to the power of this space factor. And similarly, for E y we have an amplitude of the electric field for the y component which is E y 0 and the space factor and similarly for E z.

So, this is a complete set of the field components representing the total electric field, which is given by this equation. In the same way for the magnetic field we have this magnetic field components B x, B y and B z. So, each of them is related to the amplitude with the space factor by this equation and for B y by this equation and so on. So, we have seen explicitly the electric field components in terms of its amplitude and the space factors.

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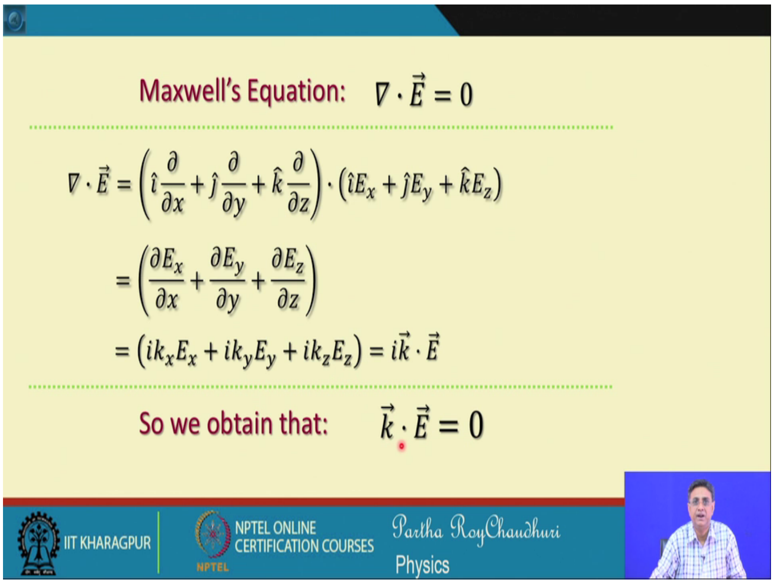

Now, our objective is to look at the orientation, the relative orientation of the electric field and the magnetic field and the direction of propagation of the electromagnetic waves; when you considered the wave is propagating in the free space. So, for that first will bring in the first Maxwell's equation for free space that is del dot E equal to 0. Having seen that, we can see that this del dot E is equal to this del operator and this is the electric field. So, del dot E can be represented by this del E x del x plus del E y del y plus del E z del z.

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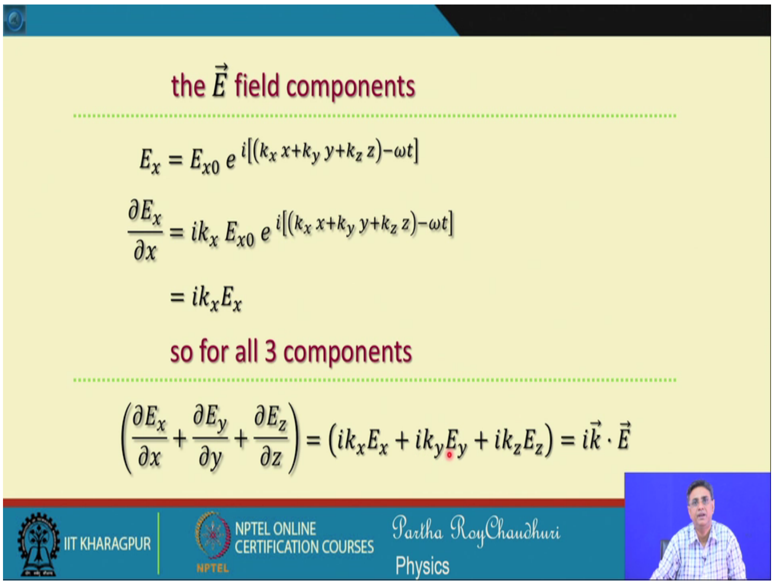

Now, since we know that the electric field x satisfies this equation, the solution of electric field E x is equal to E x 0 e to the power of i k x x k y y k z z minus omega t. Where omega is the frequency of the electromagnetic waves, k x, k y and k z as I have mentioned these are the three Cartesian components of the propagation vectors in the free space. So, del E x del x when you operate on E x del del x when you operate on E x then it will detect this i k x and will leave the E x 0 into this part, that is E x as it is.

So, by doing so, we get del E x del x is equal to i k x 0 i k x into E x and this is for only the x component del E x del x. If we considered the other 3 component, other 2 components that is del E y del y and del E z del z then we will end up with similar expressions for del E y del x del E y del y and del E z del z which you can put together in this form. So, we get that del E x del x plus del E y del y plus del E z del z is equal to this quantity i k x E x plus i k y E y plus i k z E z, which is which can be written as i k dot E.

Now, from this del dot E we have seen that i k del dot E is equal to i k dot E, but del dot E itself is equal to 0. Therefore, you readily obtain that k dot E is equal to 0. This tells you that k and E, because there dot product is 0. So, they must be perpendicular to each other. So, we conclude by saying that del dot E is equal to k dot E is equal to 0, which means that the wave vector k is perpendicular to the electric field E when the electromagnetic wave is propagating in the free space.

Now, if we consider instead of del dot E equal to 0 the first equation, the second equation that is del dot B equal to 0 and we go back with this equation del dot B equal to 0, you can again write this equation del dot B will be equal to i del del x j del del y plus k del del z dot i B x plus j B y plus k B z. As a result we will obtain this del dot B will be equal to del del x of B x del del y of B y and del del z of B z.

Now, in the same way if we calculate this del del x of B x we will obtain i k x B x and so on. Therefore, this del del x of B x plus del del x del del y of B y plus del del z of B z will be equal to i k x B x plus i k y B y plus i k z B z, which will be equal to i k dot B in the same way.

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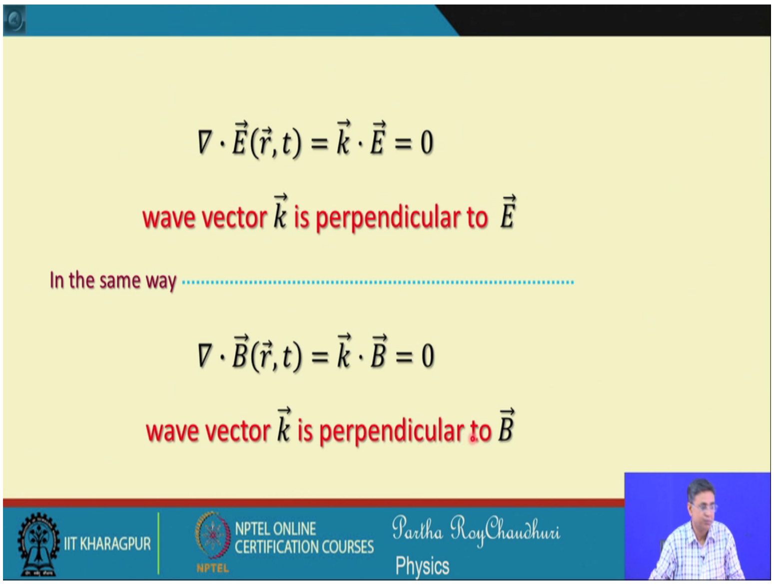

Therefore, we obtain in the same way the del dot B since, del dot B is equal 0 k dot B is also equal to 0; which will lead to the conclusion that the wave vector k is perpendicular to B. So, we have been able to achieve two relations that is k is perpendicular to E and k is also perpendicular to B.

This means that k and k and B are perpendicular, E and k their also perpendicular. But, that does not ensure that E and B whether they are perpendicular or not. So, there is a there is a more rigorous way of looking at the relation between k B and E; so to do that we consider Maxwell’s third equation the curl equation.

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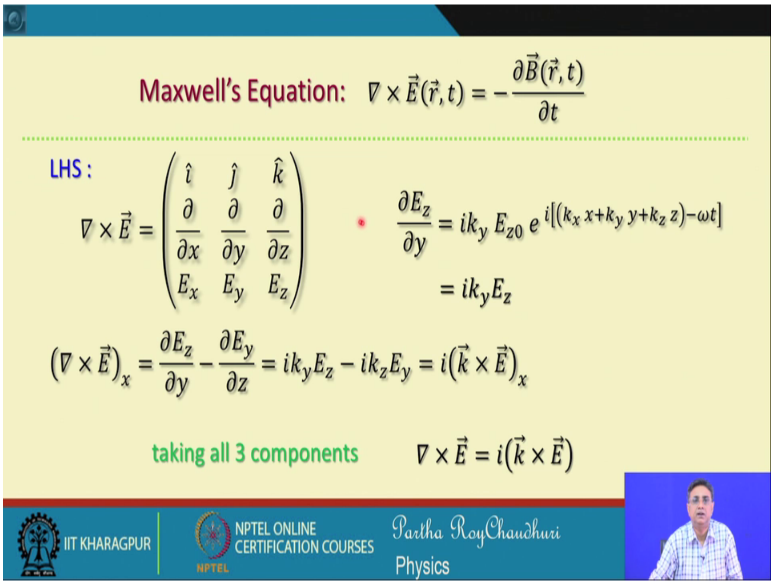

Now, the curl of E if we have to represent this we write in this way that i del del x j del del y k del del z E x E y E z. So, this is the curl operator operating on E which if we take the x component of this curl then it will be del del x del del y E z del del z of E y. So, this quantity is the x component of the curl which will give you i k x i k y E z minus i k z E y because, del E z del y will detach only the y component that is k y will be out. So, i k y E z 0 and this factor will remain as it is which is equal to i k y E z.

So, del cross E the x component of this quantity will give you i k cross E the x component of that and if we consider all the other 2 components, namely the del cross E y component and del cross E z component we will end up with two more expressions; i cross i into k cross E y component, i into k cross E z component. And then, if we put together all the 3 components we can write that del cross E is equal to i k cross E.

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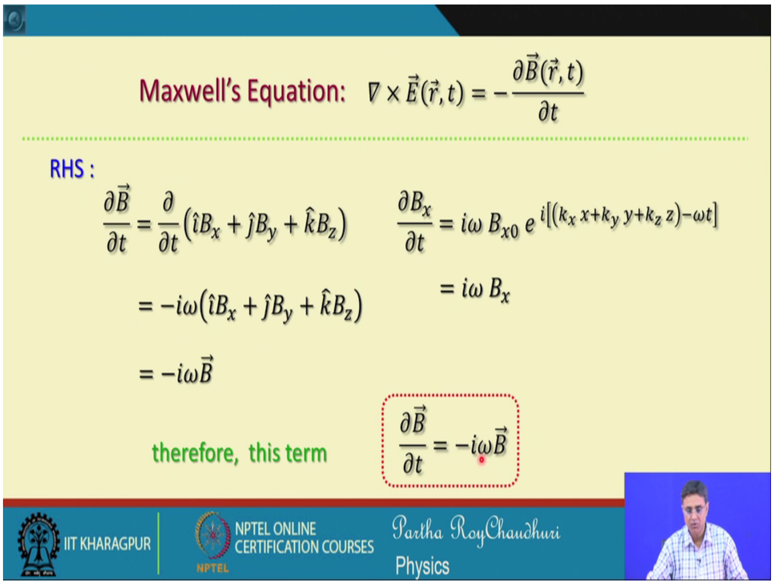

So, this is again a very interesting relation now, we will have to evaluate the right hand side that is del B del t. So, because you know that B vector can be represented with the Cartesian forms like i B x j B y and k B z. And if you take the time derivative of the Cartesian component, then we will see that del B x del t we will take out this omega and i factor outside. And this equation, expression for the magnetic field x component will remain as it is. As a result we can write del B x del t is equal to i omega B x.

So, this is only for the x component of the magnetic field when you take the time derivative. And similarly, if you do the same thing for the other components that is del del t of B y del del t of B z then we can write this equation all put together in this form which is equal to i omega and B. And this is a very interesting way of looking at things that the right hand side just boils down to i omega B. So, del B del t equal to i omega B. Now, we have ready expression for del cross E as well as del B del t.

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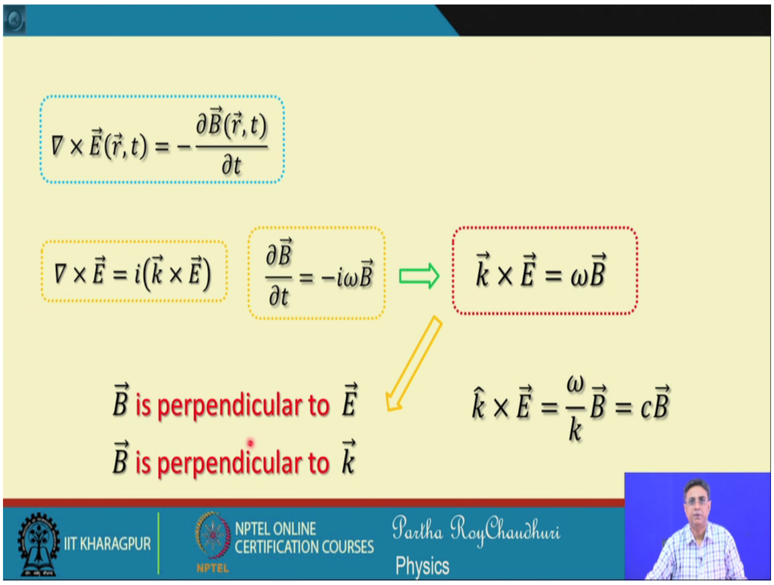

If we know put together so, del cross E has given us i k cross E and del cross B has given as del B del t equal to minus i omega B. So, we can use these two expressions to arrive at the conclusion that k cross E k cross E must be equal to omega B. So, that is a very beautiful relation which says that the B vector is perpendicular to E and B vector is perpendicular to k. That means B vector must be perpendicular to the plane containing the vector k and the vector E. So, B is perpendicular to E and B is perpendicular to k.

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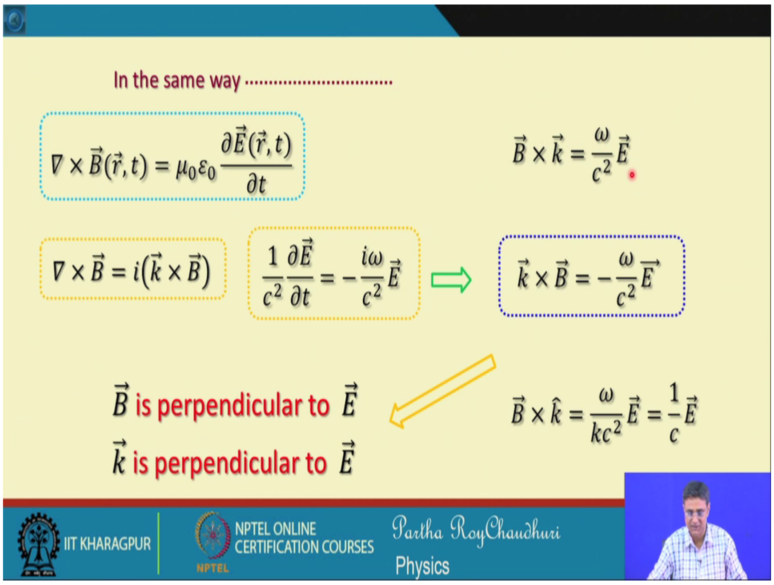

Now, in the same way if we proceed for the fourth equation that is del cross B is equal to is equal to mu naught epsilon naught del E del t. If we evaluate this equation and in the same way, if we evaluate this left hand part of the calculation that is del cross E del cross B is equal to i k cross B. And for the right hand side if I do this derivative time derivative of the electric field, I arrive at 1 by c square del E del t because, mu naught epsilon naught is 1 upon c square. So, this quantity will become i omega by c square and the vector E. So, I still get the equation for del cross E in this form del cross B and del E del t in this from.

Now, if I use this left hand side and right hand side together I may right that k cross B must be equal to i omega upon c square into E. So, look at this equation this tells you that k is perpendicular to E and B is perpendicular to E; that means the electric field E is perpendicular to the plane containing the vectors k and B. So, this is the complementary part which we have seen earlier for the electric field.

But all three of them they are very nicely related can see that if I remove, if I want to get rid of this minus sign we can reverse this position that is B cross k will be equal to omega by c into E. So, B k and E they form this equation from here you can conclude that B is perpendicular to E and k is perpendicular to E also.

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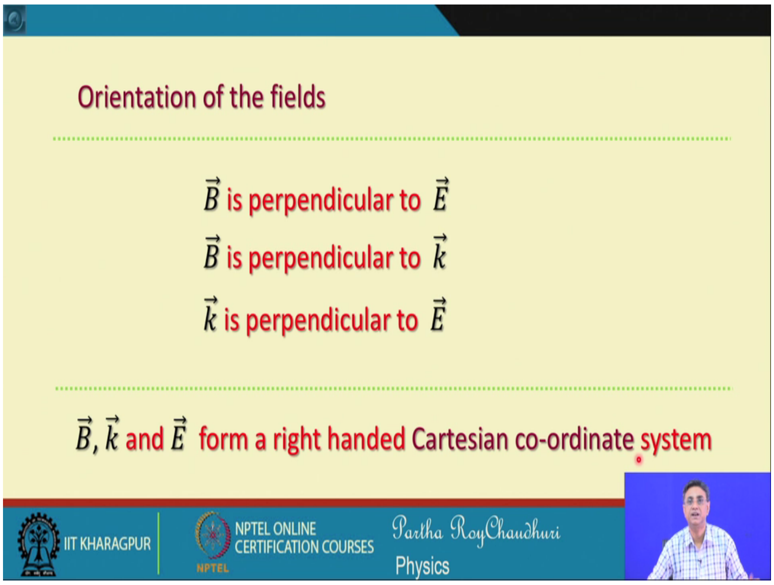

Now, we will look at the consequence of these two findings B is perpendicular to E, B is perpendicular to k and also k is perpendicular to E. This means that B, k, E all of them are perpendicular to each other, all of them are or at right angles to each other and they form a right handed Cartesian coordinate system. We will see that how they are they are connected to each other in terms of the direction, in terms of the coordinate access.

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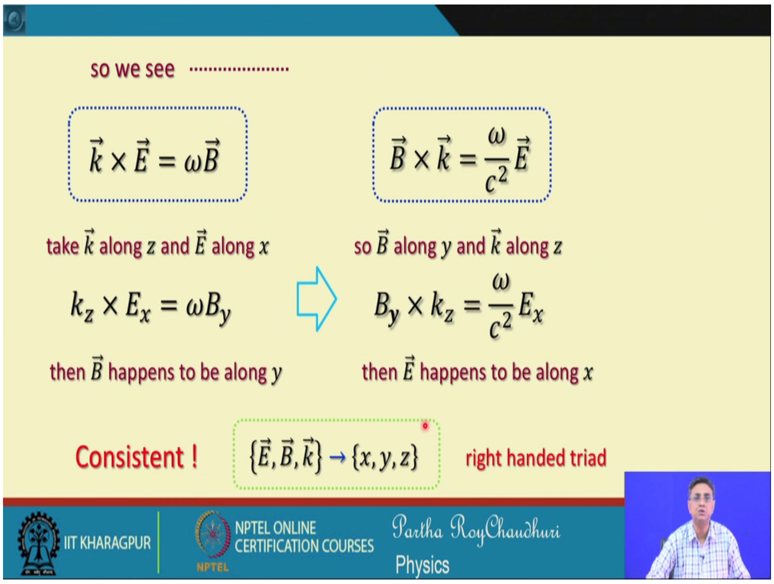

So, you see k cross E equal to omega B, k cross B cross k equal to omega upon c square E. If we consider this example that, let us suppose that I take the electromagnetic wave to be propagating along the z direction. Then I can write k equal to k z an if I consider the electric field is oriented along the x direction, then k z cross E x should give me omega mu B y. So, that should yield B y; that means, if I take k along z and E along x I should expect that B should be oriented along y, z x y.

And if I assume that B happens to be along y axis and then k is along z axis which I have already considered, then B cross k B y cross k z equal to omega by c square E x; that means, y z x. So, they again consistent then E happens to be along x. So that means, this is consistent, because I have started with the assumption that k is along z, E is along x, the result was that B is along y,

Now, I have considered that B is along y and k is along z so, it yields that E is along x. So, they are consistent and we can we can write in a compact form that is E B k will represent that x y z the axis of the Cartesian coordinate system and it forms a right handed triad of vectors. So, this is how we could see that the electric field, the magnetic field and the direction of propagation they are normal to each other. And because, the direction of propagation is such that the electric field and magnetic field they are always perpendicular and there are also perpendicular to each other than the wave is a transverse wave.

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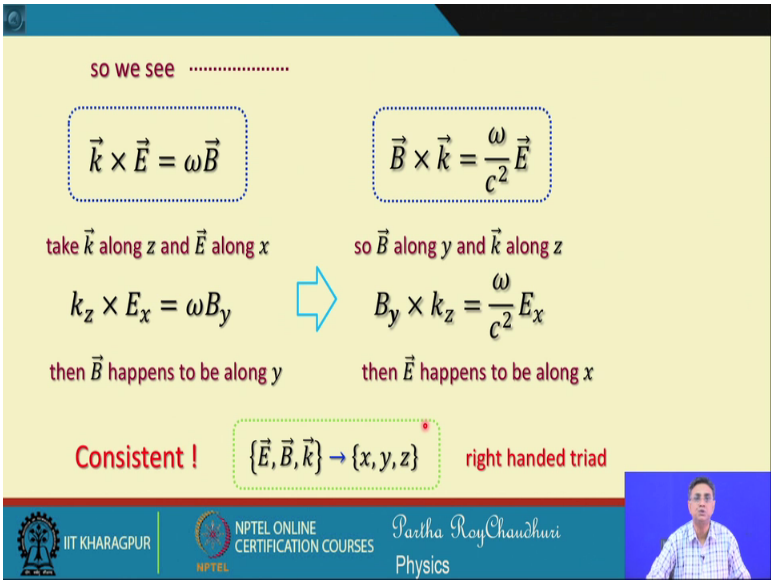

So, this is how we can also look at the nature of the electromagnetic waves, the transverse nature of the electromagnetic waves in the free space. So, pictorially if we represent this is your x axis, this is your y axis and this is your z axis. So, along the x axis if you consider the electric fields are oriented then along the y axis the magnetic fields will be oriented. And as a matter of fact that orthogonal triad of vectors will give you that the propagation direction will be along the z axis. So E, B, k forms a right handed Cartesian coordinate system.

Now, we discussed this fact for the case of electromagnetic waves in free space and you know not only for free space, but for most dielectrics this permeability mu is equal to mu 0. Those this is a nonmagnetic property and mostly mu you can approximate as mu 0. Therefore, this B becomes equal to mu naught into H, where H is the magnetic field, B was the magnetic induction vector. In that case as well for the rest part of all formalisms we can replace that B by H.

So, if you orient the electric field along the x direction and H along y direction then again the propagation can vector will be along z direction. So, in that case E, H and k they will form the triad of vectors. So, in free space we can see that E, H, k they form the triad of vectors and also E, B and k that forms the triad of vectors. Because E and B, H and k they are related in the same direction.