Modern Optics
Prof. Partha Roy Chaudhuri
Department of Physics
Indian Institute of Technology, Kharagpur

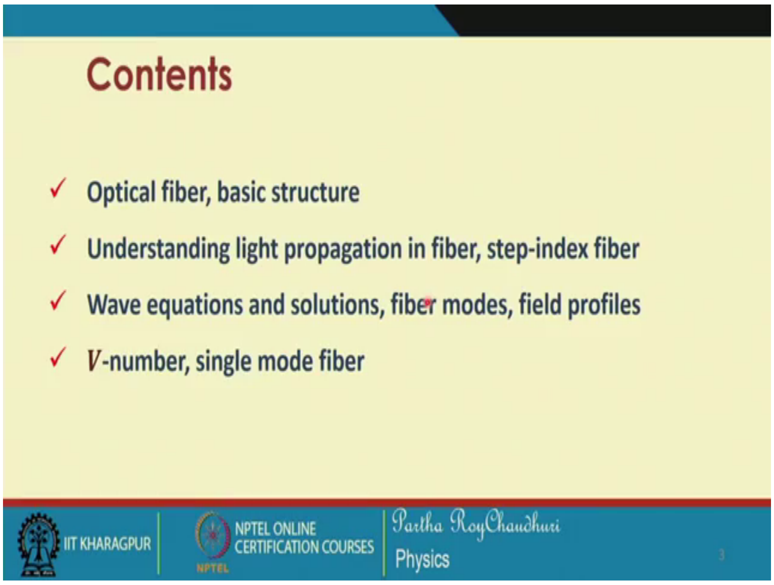
Lecture – 26
Coupling of waves and optical couplers (Contd.)

Well, we were discussing Coupling of waves and various optical waveguide couplers. And in the context we discussed that parallel waveguide couplers and the basic couple mode theory. But there is one very competing and very important member in photonics and optics is the optical fiber waveguides.

And coupling of the waves between the optical fibers is very important. And we will we try to understand how this coupling takes place.

But before that we will discuss a bit about the basics of this optical fiber and the mechanism how the light is guided into through the structure; a cylindrical structure.

(Refer Slide Time: 01:00)

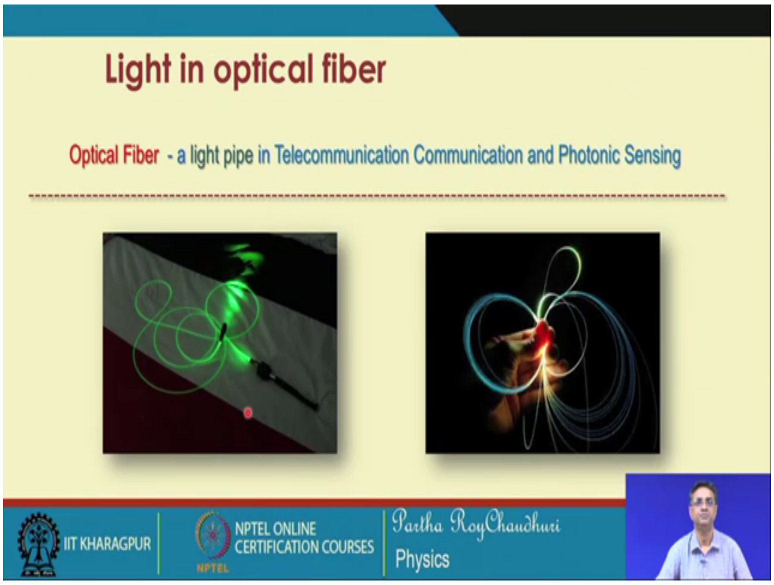


So, the topics that we will be discussing is the optical fiber, the basic structure, understanding of this light propagation in optical fiber. We will concentrate more on the step index fiber, but there are many other varieties of optical fibers like graded index fiber and so on.

Waveguide, then wave equation and solution that we will discuss about the step index fibers which will lead to the fiber modes. And in terms of this field profiles a very important parameter that is V number which we have discussed already in the context of planar optical waveguides. But here we will again define almost a similar quantity which will specify give all the specifications of about an optical fiber.

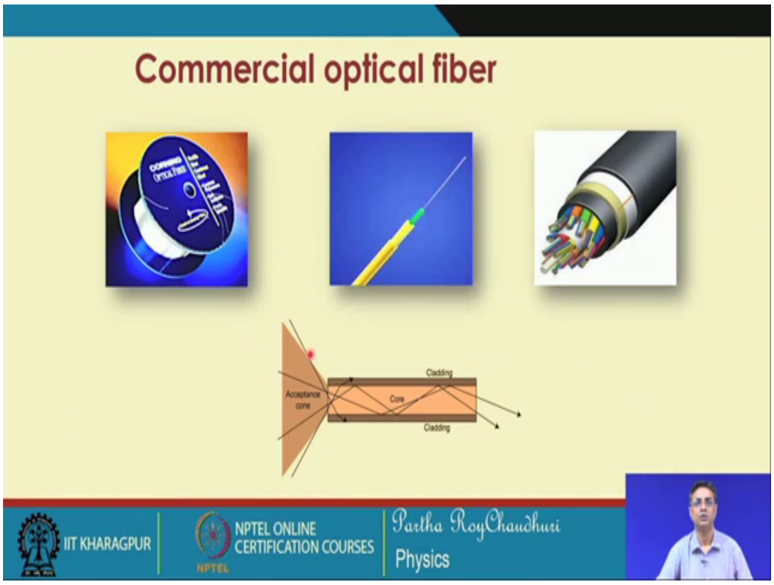
Then we will look at the condition under which fiber will be acting as an single mode fiber ok.

(Refer Slide Time: 02:13)



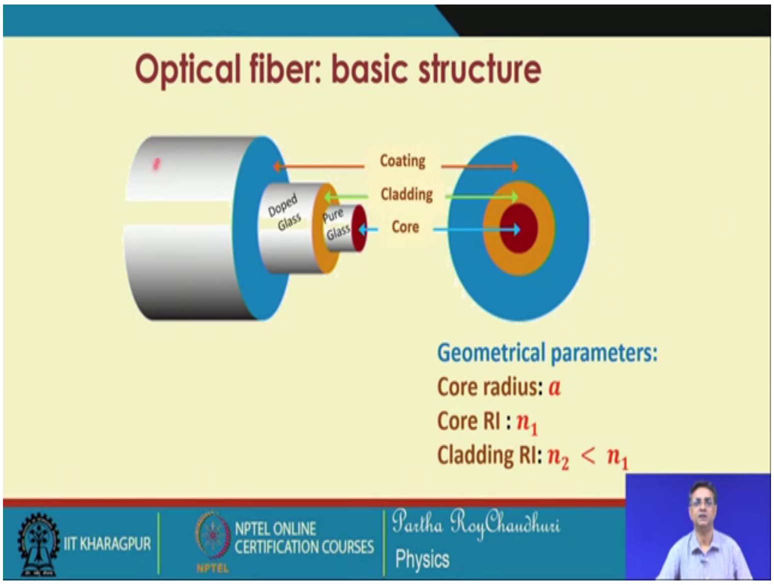
So, all of us we have seen optical fibers you see there is a laser light illuminating optical fiber green light, you can see that fiber held in hand the light is illuminated. So, basically this optical fiber is a like pipe, which is very widely used in photonics in telecommunications and sensors.

(Refer Slide Time: 02:39)



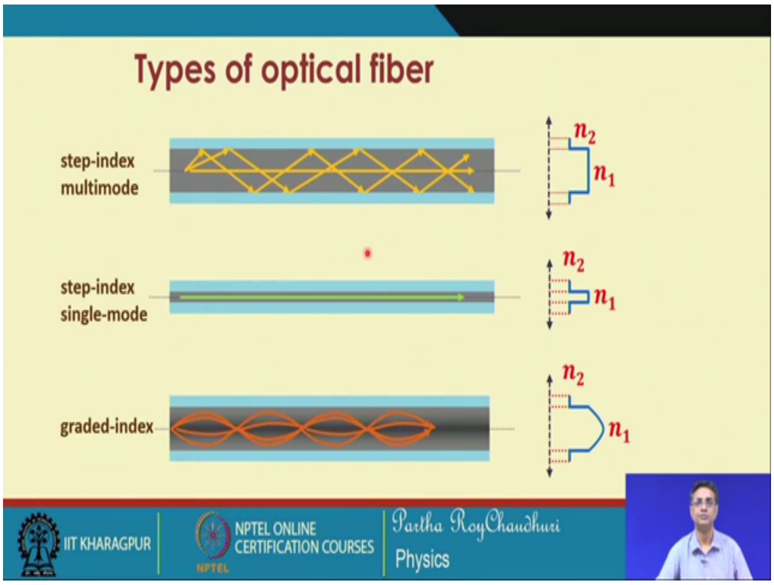
And might have seen this a mandrel of optical fibers this is a picture from corning glass USA, contains the rolled of fiber, and then this is a single fiber which is protected with the acrylic jacket and the strength member. This is a cable which contains a large number of optical fibers into groupswhich is the indication which gives the practical application of optical fiber cables. And this is the basic mechanism by which this optical fiber carries light, that is you have an acceptance cone and the light is incident it undergoes total internal reflection and it is guided through the structure.

(Refer Slide Time: 03:34)



So, the basic structure of an optical fiber consists of a pure glass then doped glass, or it may be the other way also, doped glass as a core and pure glass as the cladding, and then you have a coating which is a strength member. And we usually designate the core radius as a, core refractive index n 1 which must be greater than the cladding refractive index n 2.

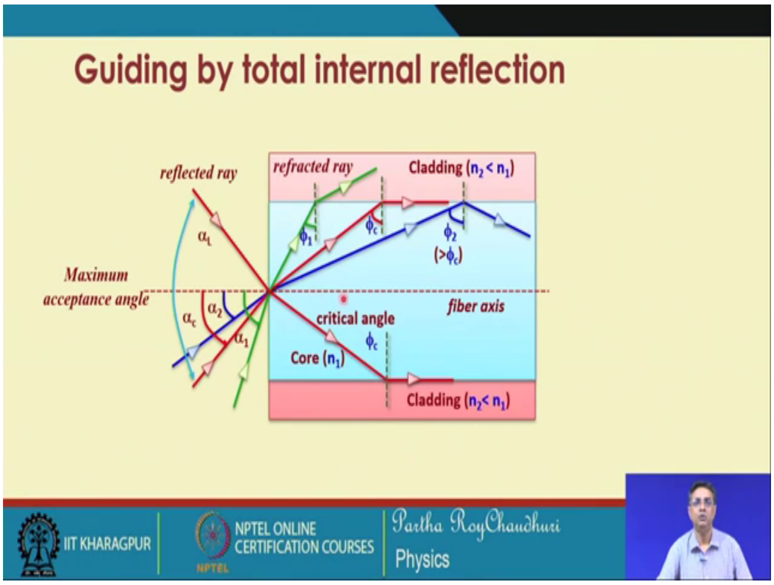
(Refer Slide Time: 04:10)



So, there are various types of optical fibers the, but most common type is the step index fiber this is one multimode fiber; which has a slightly larger core area, and you have a step index, single mode fiber the rays are almost straight which will call the weekly guiding fiber, and the core width is much smaller of the order of 9 to 10 micrometer in diameter.

Whereas, this could be 50 micrometer in diameter, graded index fiber you can see we have discussed in the graded index slab waveguide. So, the at the center the refractive index is high, the maximum value is n 1. And as you move radially outward the refractive index falls to assume a constant value of n 2 at the cladding.

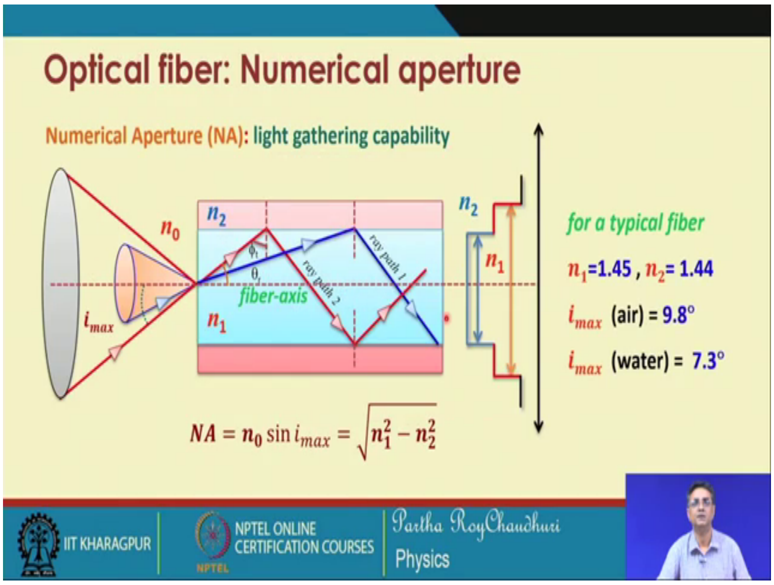
(Refer Slide Time: 05:04)



Now, we will look at how this very simple concept behind this light guiding in an optical fiber. You have several rays making different angles with the axis of the fiber. But the one who satisfies the condition of critical angle at this core cladding interface will be guided through the structure. If it is more than this critical angle, then also it is fine the light will be guided into the structure.

But beyond this, the one which is less than the critical angle that will be reflected out. And so, this is how it defines a cone of light which will be actually guided through the structure. And the condition is n 2 must be less than n 1, that is the core reflective index is more than this.

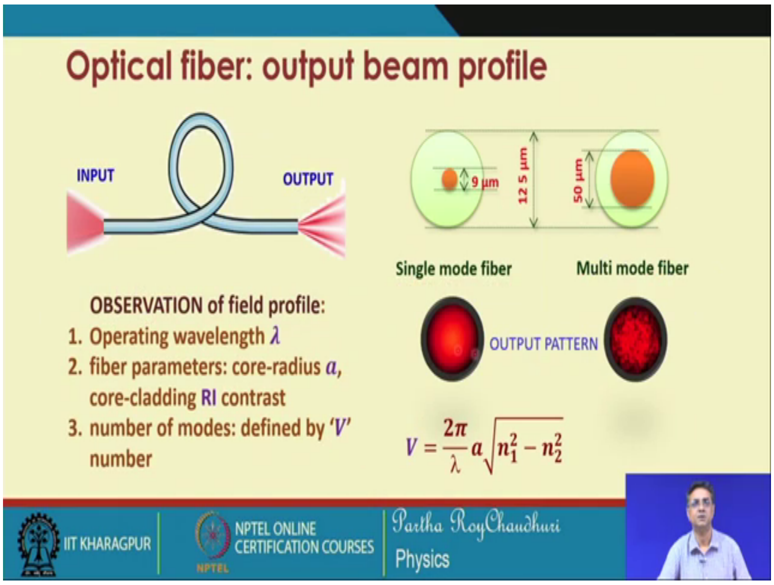
(Refer Slide Time: 05:57)



So, as I said that it defines a cone of light which will be which is known as the acceptance angle or in terms of that is called the numerical aperture; which defines that light gathering capacity of the optical fiber. And you can represent this mathematically, that is this numerical aperture which is n 1 square minus n 2 square, under root of that will be equal to n 0 sin i max. The maximum angle that is allowed for the rays to be guided into the structure that is this n 1 square minus n 2 square.

We will see so, n 1 for typical fiber it is of the order of 1.45, 1.44 slightly less. And in that case this i max for air glass this is 9.8 and for water this is 9.7, this is these are the refractive indices.

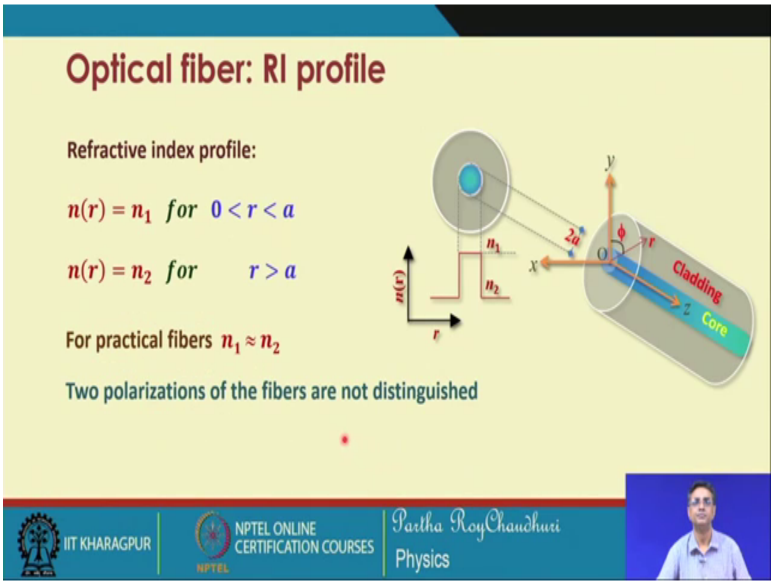
(Refer Slide Time: 06:59)



In an optical fiber as I said in the acceptance angle the light will be out and it can also accept the light through this angle. So, it moves through the fiber, and in the same way it comes out of the ; for a single mode fiber the typical core radius is of the order of 9 micrometer, and the total diameter of the fiber is 125 plus minus 1 micrometer. The this diameter is 50 micron, this gives you a speckle pattern because there are large number of modes.

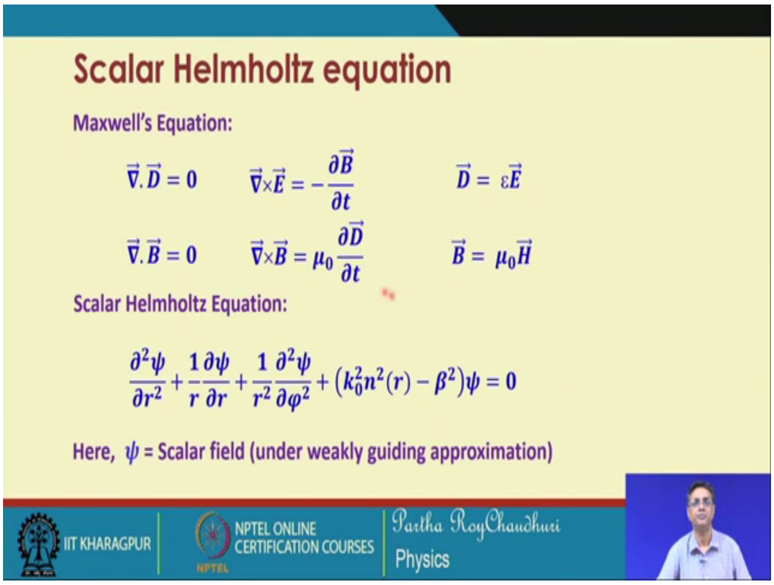
Whereas, this is a purely single mode one distribution field distribution which is being supported by this waveguide. So, operating wavelength lambda, that decides then fiber core radius a, core cladding refractive index contrast. These are the factors, that is those are going to decide, the modes number of modes and through the definition of the V number which we will discuss shortly ok.

(Refer Slide Time: 08:01)



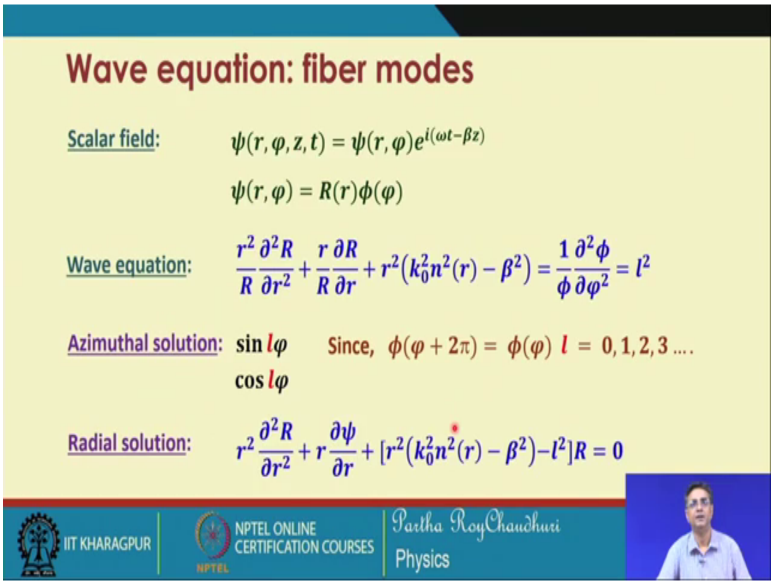
So, the refractive index profile of an optical fiber can be well described. We have seen this kind of structure. This n of r is equal to n 1 for the core; that is less than r less than a. And for cladding, r greater than a this is n 2 for practical fibers this difference is very small, but it is finite very small. And that is for the weekly guiding fibers and this because of the circular symmetry, the 2 polarizations of the fibers are not distinguishable.

(Refer Slide Time: 08:40)



So, the we start with the Maxwell's equation, and we have seen that it gives rise to that yields the wave equation, the scalar Helmholtz equation in cylindrical coordinates because the fiber structure is cylindrical. So, we write in cylindrical coordinate the scalar Helmholtz equation. And psi is the scalar field which is under the weekly writing approximation.

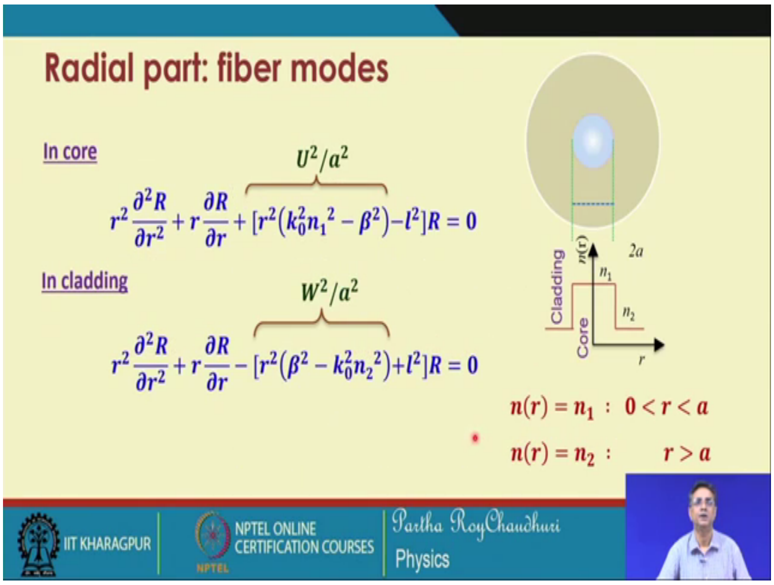
(Refer Slide Time: 09:04)



So, we assume the solution of this time, the radial part and the Azimuthal part can be written in this form. So, if you plug in this into the wave equation, this then we can write this wave equation in this form. And this gives you this gives you the Azimuthal solution if you look at this, these are the things which we have discussed earlier also.

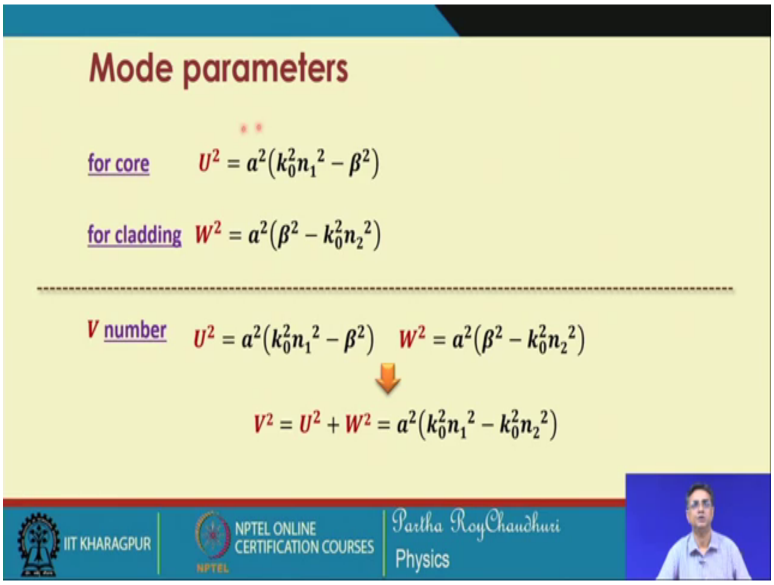
So, this will give you the Azimuthal solution which is e to the power of plus minus l phi, that corresponds to sin l phi or cosine l phi, as the solutions. And the radial solution because l square will come to the left hand side and will constitute this equationwhich will be the radial equation and this equation is well known.

(Refer Slide Time: 09:54)



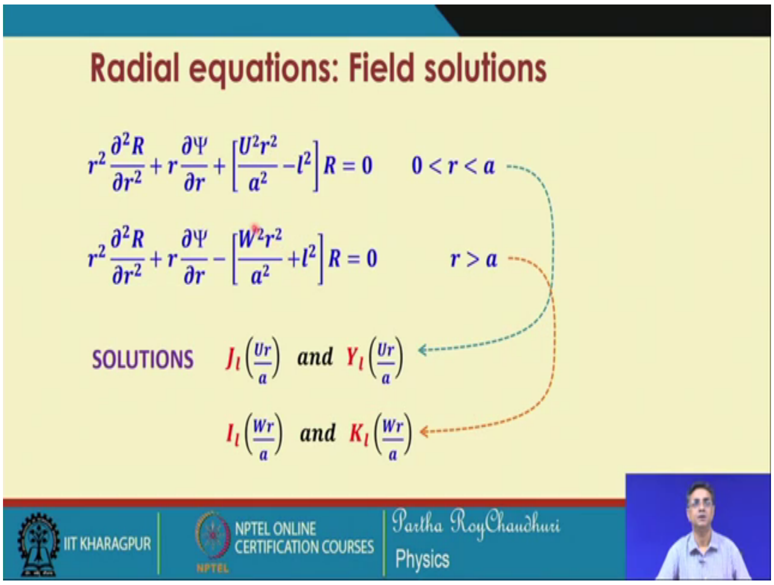
In physics this is the equation for the core because we have replaced n r square by n 1 square. Whereas, this is the for the equation for the cladding you have to separate homogeneous regions, and the wave equation will be satisfied separately in these 2 regions we will have to match the condition the continuity condition at the interfaces. And that is the same recipe that we will follow here. We will define the mode parameters this quantity U square by a square equal to this whereas, W square by a square equal to this for the cladding whereas, this is for the core right.

(Refer Slide Time: 10:32)



So, this mode parameters as I have mentioned. These are and if you add these 2 W square plus U square will yield this quantity which does not involved this beta; which is independent of beta and it defines this V number a very important parameter for an optical fiber. Using, so, by this substitution U square and W square for these quantities we can rewrite this radial equations for the core and cladding in this form.

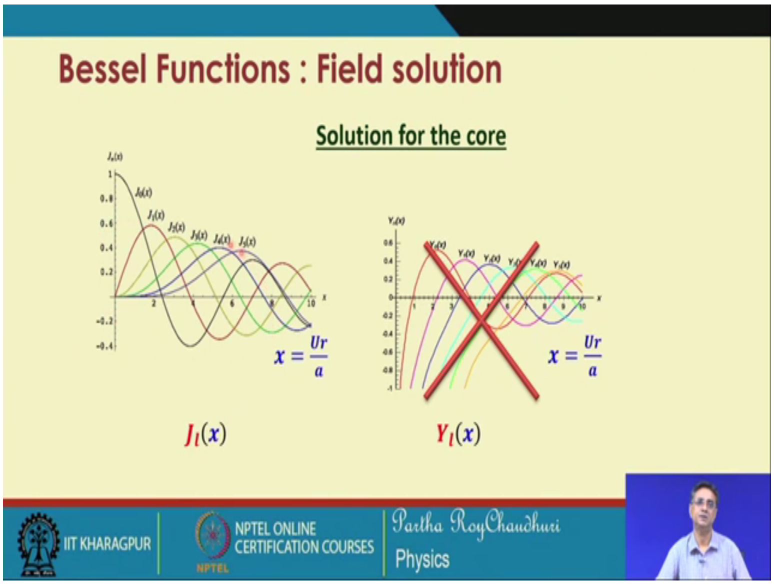
(Refer Slide Time: 11:04)



Because now we have replaced this by U square r square by a square W square r square, this is very simple and well known. And the first equation gives you this is again a very well known equation, and the solutions are the Bessel functions J l and Y l. And for this equation you have a minus and inside there is a plus. This equation gives you modified Bessel function; whose solutions are I l and k l of W r by a I l of x and W and k l of x. So, x equal to this argument of this Bessel functions ok.

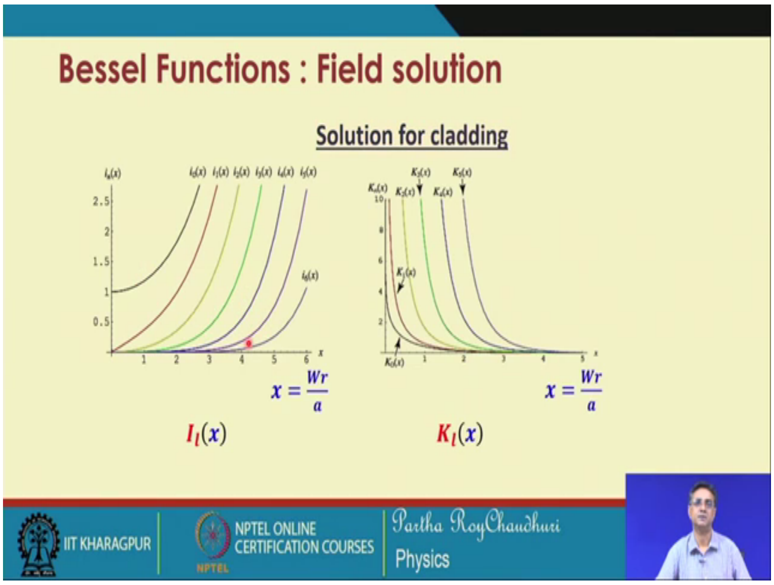
So, let us see how does this solution look like J l individually J l and Y l.

(Refer Slide Time: 11:59)



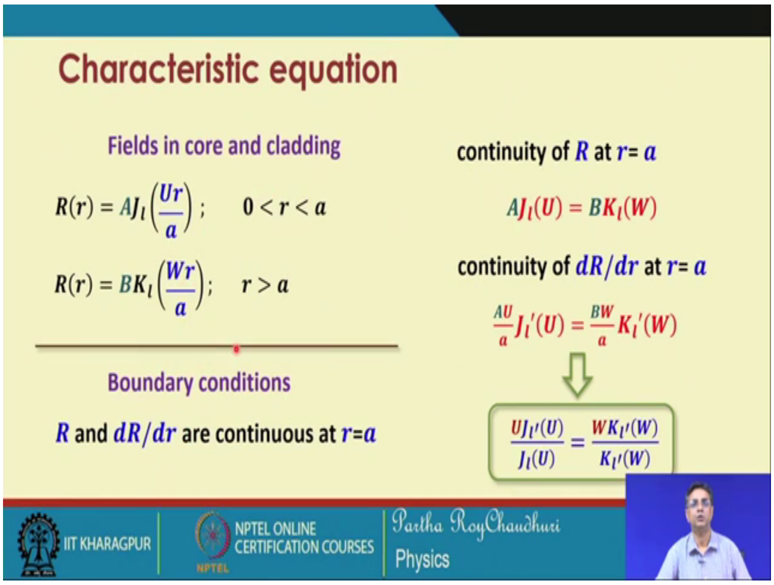
So, J l if you plot, these are this is the variation of J l as a as a function of x; whereas Y l as a function of x. From here we can make out that at r equal to 0, at r equal to 0, you can see that the field will be finite. But at r equal to 0 the field cannot be infinite. So, this solution is not physically admissible. So, we will have to discard the solution, and then for the core we will only continue with this solution that is J l of x.

(Refer Slide Time: 12:38)



And for the cladding we have the solution I l of x and k l of x they look like this. Once again we will look at this when r tends to infinity, this function I l it grows up and it becomes infinite. But the light power will be always finite at infinity it cannot be infinite. So, this solution is again not acceptable. And we will combine the core solution with this solution as the cladding. So, you have the effect we have the final solution for the core and cladding as the Bessel J l for core and for the cladding you have Bessel k l.

(Refer Slide Time: 13:14)

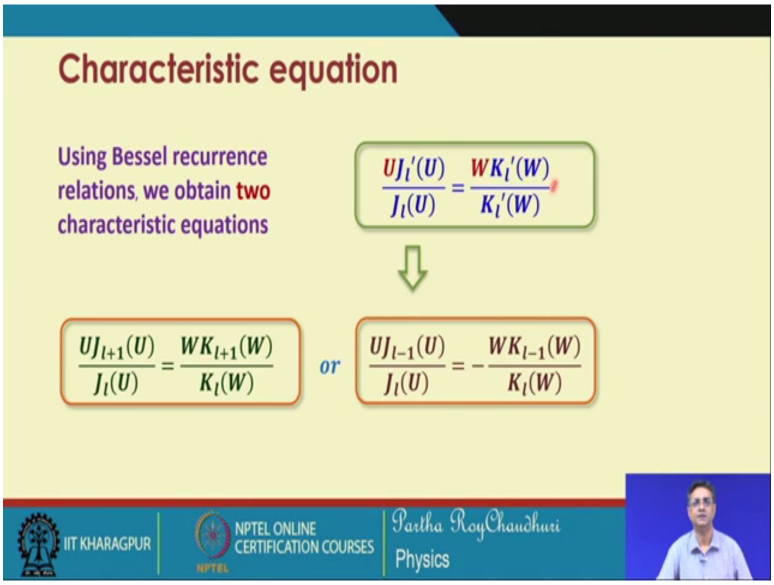


So, then we will have to match the continuity condition at the core cladding interface. And the boundary condition that is R and d R del r are continuous at r equal to a this condition. If we apply this is true as long as this refractive index contrast is very slow, very low that is n 1 is almost same approximately equal to n 2, but slightly less, in that case we can we can apply this boundary condition.

And this continuity at r equal to a will give you this you just simply put that r equal to a, then a a cancels you will get this equation. Del r d R d r at r equal to a will give you this condition; where this J l prime and k l prime are the differentiation with respect to r. So, you again have another condition. So, these are the 2 conditions if I combine you can eliminate B and A from these 2, then we can write this equation which takes care of the continuity both the continuity conditions.

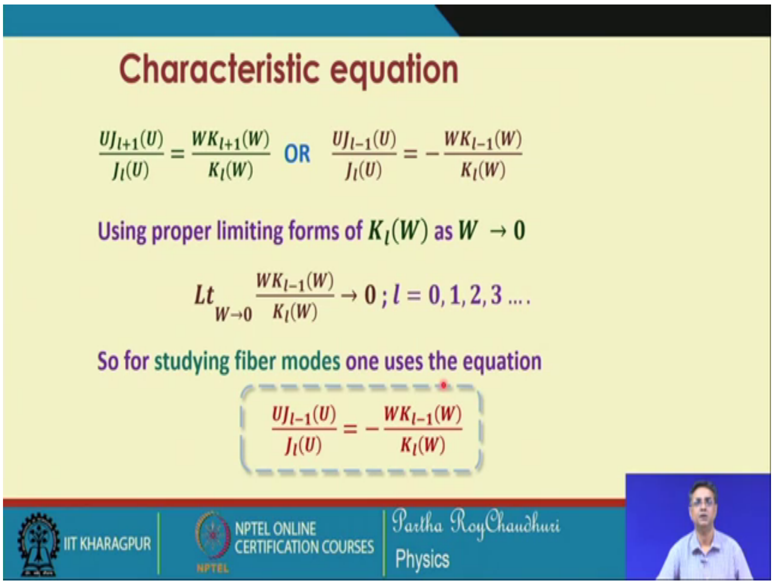
And this is the dispersion equation. This is the characteristic equation for the modes of the structure, and it it includes the fiber parameters and wavelengths in terms in U and W ok.

(Refer Slide Time: 14:49)



So, this we find this characteristic equation is of this kind. Now J l prime and K l prime these are the first order differentiation of J l and K l. If I use the recurrence relation, then it will take the form 2 forms that is J l prime can be expressed as J l plus 1. And it can also be expressed as J l minus 1. And you have a set of relationship for J l dash to J l plus 1 and J l minus 1. Using this we can reduce this form to these 2 forms. But once again this equation is not valid, this equation will be valid for the physical solutions.

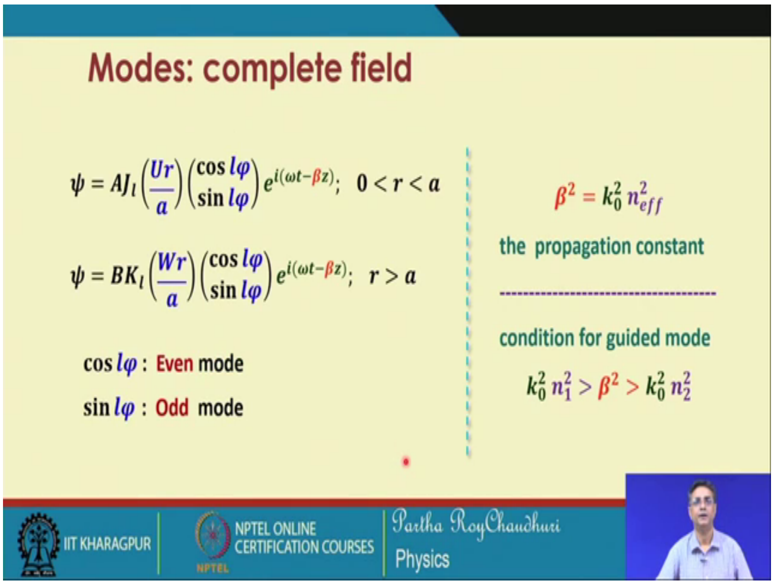
(Refer Slide Time: 15:40)



Because if I take this equation, so these are the 2 solutions 2 dispersion equations.

So, using the proper limiting forms of K l, if you take W tends to 0, then this quantity also tends to 0, and l equal to 0, 1, 2, 3. So, this quantity is the only possible because as W tends to 0, then this quantity has to be 0. So, for studying fiber modes one uses the equation of this form and then we can write the complete field solution in this form.

(Refer Slide Time: 16:25)



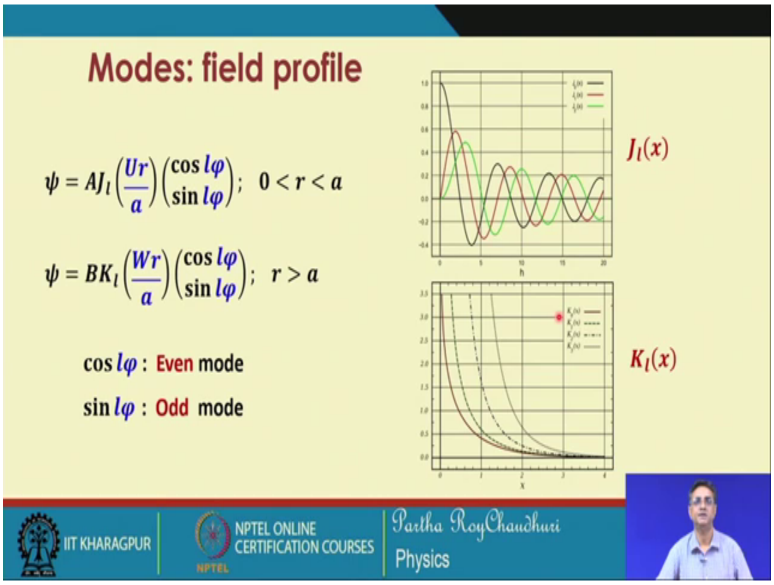
That is psi of A j l U r by a, then e to the power of plus minus I l phi, e to the power of i omega t minus beta z. Beta is the propagation constant which will come from the solution, from the transcendent solution of this transcendental equation. You have to plot the left hand side and you have to also plot the right hand side, and look at the intersection of the 2 graphs those are the solutions.

So, it can be done numerically as well as it can be done by graphical method also. So, this I get these complete solutions field solutions for the core as well as for the cladding. And this beta takes care of the continuity at the core cladding interface we will take an example. So, this cosine l phi will correspond to the even modes whereas, sin l phi is called correspond to the odd modes.

Then the propagation constant beta, we can also write as beta square equal to K 0 square. N effective square is n effective is the effective propagation effective refractive index seen by the wave which is traveling along the axis of the fiber, fiber where this n 1 and n 2 this n effective lies between n 1 and n 2. So, K 0 n 1 square must be greater than beta square that is K 0 n effective square, and that must be greater than K 0 n 2 square.

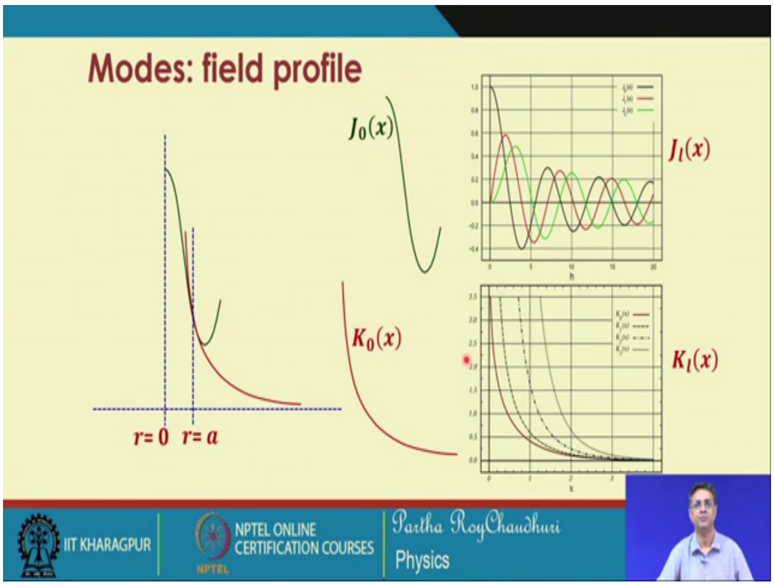
So, that is this propagation constant square must be somewhere between K 0 n 1 square and K 0 n 2 square. So, in effective must be within n 1 and n 2 must be greater than n 2, but less than n 1. At n 2 it gives you the cutoff of the mode we will see that also.

(Refer Slide Time: 18:26)



Let us look at the field profile. You have this as the solution for the core; you have this Bessel k function as a solution for the cladding. Then the core solution you have this Bessel j function and this is Bessel function put. Together we get the solution for the modes.

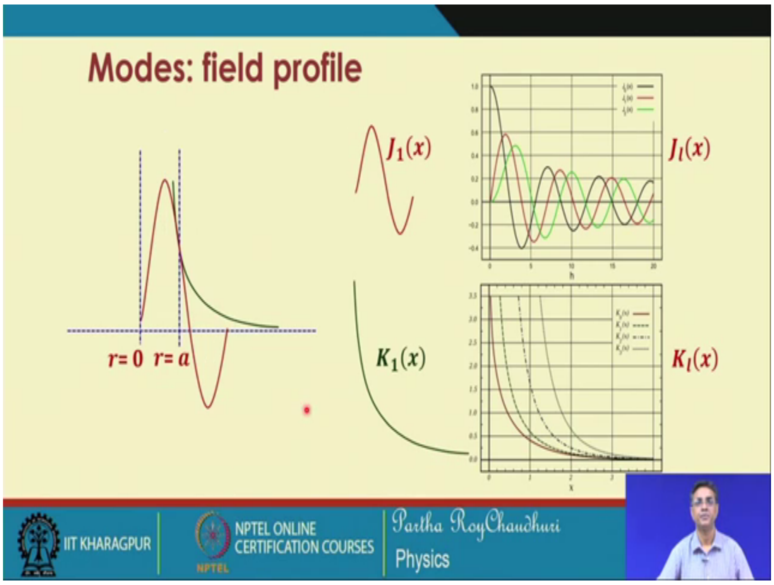
(Refer Slide Time: 18:48)



Ok Let us take one example if I pick up l equal to 0, then you see that the variation of the mode J 0 is like this. You take pick up this J 0 value, and the K 0 for l equal to 0 is this one, this graph.

So, if you take these 2 and put together at the interface r equal to a they must be matching. So, you get a continuous curve which starts from here and moves down to this. I just put these 2 things together, you get the continuity at r equal to a. So, that gives you the fundamental l equal to 0 mode.

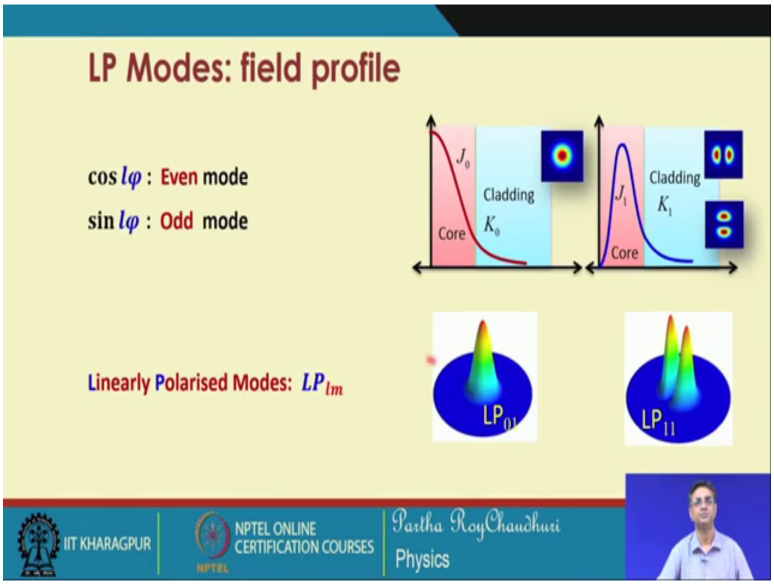
(Refer Slide Time: 19:28)



You can see this for l for the second example that is if I take J 1, this is the Bessel J 1 function, this red graph the second graph. And this is the Bessel K 1 function second graph.

Now, again we pick them up and put together at the core cladding interface that is at r equal to 0, I match this, then you get the mode which will be this and then exponentially decaying at the cladding. So, this defines the first order modes. So, these are 2 very beautiful examples and we can actually carry on with other examples other number of modes mode orders.

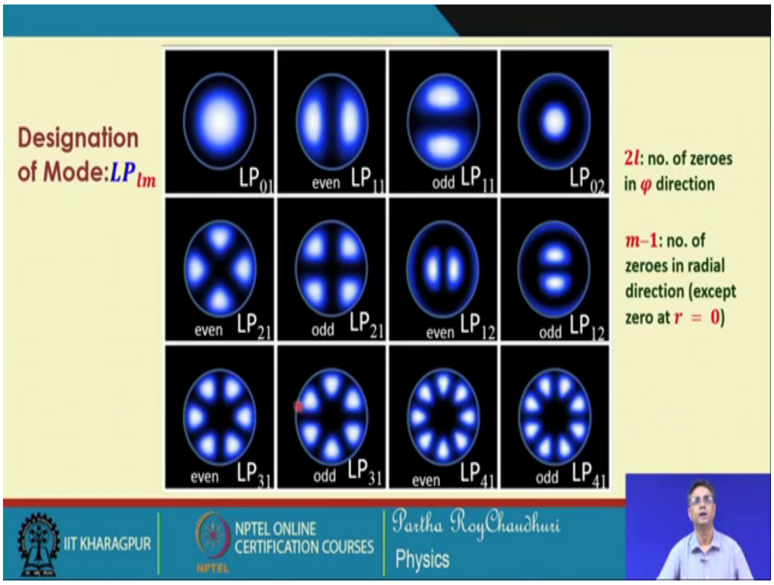
(Refer Slide Time: 20:07)



So, the fundamental mode J 0 and K 0 we will define this mode picture, the intensity that is the amplitude of the field will be maximum at the center, and will be falling and finally, decaying at the cladding.

Here the intensity at the center will be 0, then it will grow up and again will be falling as in the cladding. But there will be 0's, because this function we will see that l equal to 1 will give you 1 0 at the center, and this kind of modes, these the solutions will give you the polarization which is along one of the fibers principle access. This these kind of all the these kind of modes are called linearly polarized modes. We can see when we go into the details of the polarization of the vector modes of this.

(Refer Slide Time: 21:01)

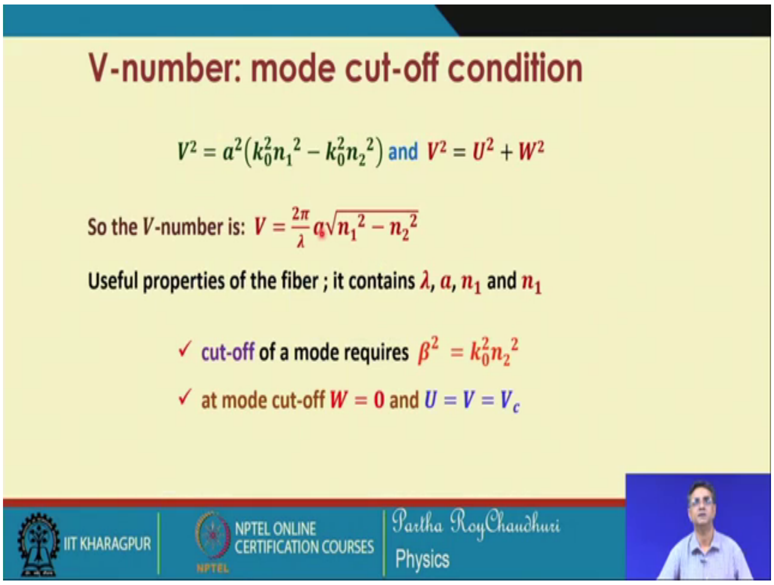


Now the various modes LP 0 1, LP 1 1, LP 1 1 odd and even it can have 2 parities. It may have an orientation like this or it can be oriented in this way. And so on we have LP 1 2; so, basically this mode designation comes from LP l m; where l is the Bessel order and m is the number of 0's of the Bessel function in the acceleration.

So, this m minus 1 number of 0s in the radial direction will give you this mode pattern, and twice l is the number of 0s in the phi direction. In the phi direction if you take this 4 l equal to 4. So, it will become 8 and number of zeros ones so 1 minus 1 so, there will be one only 1 0. So, that is how you can actually constitute all the modes theoretically, numerically. And if you look at the mode pattern from the of the light coming out of the of an optical fiber they are also similar. But usually when you have a multimode fiber, all the lower order modes will be coexisting.

So, it is difficult to view visualize the higher order modes separately

(Refer Slide Time: 22:23)



But there are methods and mechanisms to view that. Now we have an important parameter as I have mentioned that is what we call this V number, which is defined through this. Actually, it can be defined through the numerical aperture because V equal to K 0 that is twice pi by lambda a n 1 square minus n 2 square. We have seen that n 1 square minus n 2 square under root of that is the numerical aperture of the fiber.

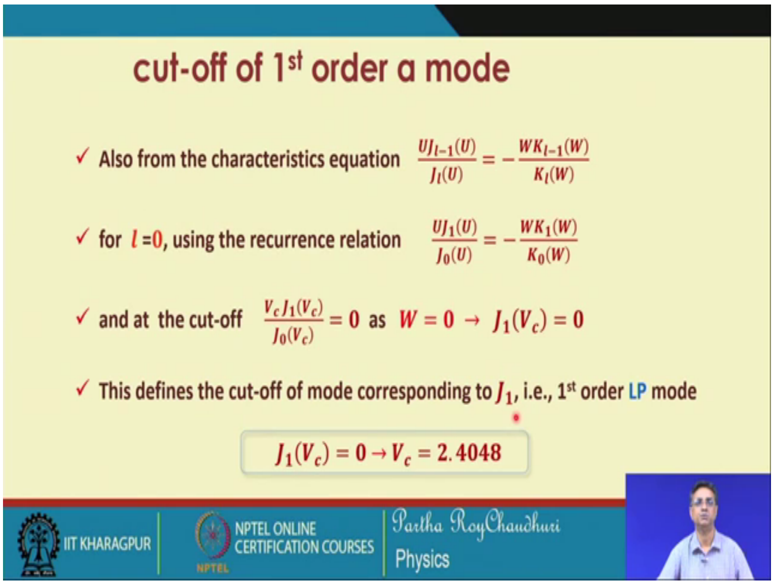
So, with numerical aperture also this V number is defined. So, whenever we look for a fiber we look at this V number coated for this fiber; where at the operating wavelength the numerical aperture is given, and also the core radius is mentioned. So, this is a very important and very useful property of the fiber, because once it is coated it contains lambda a n 1 and this should be n 2 it should be n 2.

So, n 1 n 2 lambda a all are included with this number. And it also from here we can also look at the cut off properties, cut off of a mode requires that the you see we have seen for modes to be guided in the structure. This value of beta should be somewhere between n 1 and n 2 K 0 n 1 and K 0 n 2.

So, the maximum value of beta is K 0 n 1should be practically less than K 0 n 1. And the minimum value should be slightly more than K 0 n 2. But if this beta is equal to K 0 n 2, if it so happens then you can see that looking at the definition of W this W becomes 0. Because W is beta square minus K 0 square n 2 square which is equal to 0. As a result, if W equal to 0 then V and U they are equal.

So, u so this under this condition, we call that, but that is what we will call the cutoff condition, U equal to V and you call this V as V c where there is no W, W e made equal to 0 at this point.

(Refer Slide Time: 24:38)



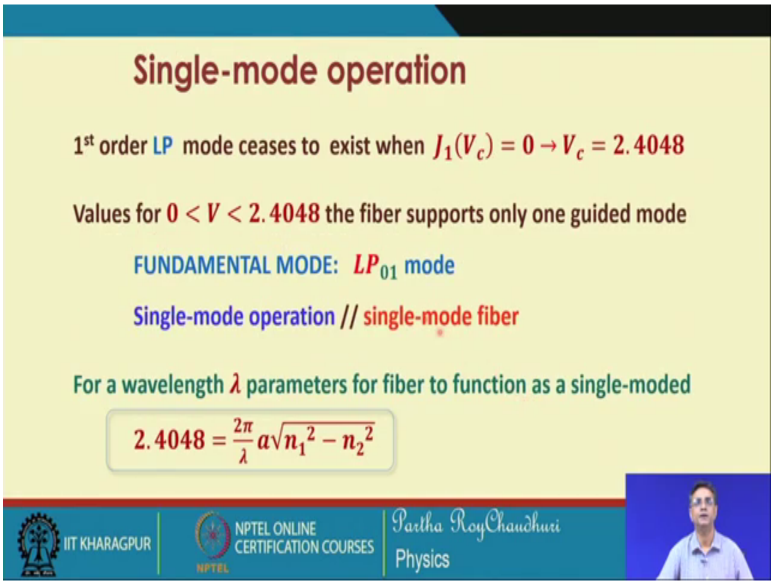
So, cut off of the first order mode is you can see that from the characteristic equation, the one which we have discussed, if you put l equal to 0, then using this Bessel recurrence relation l minus 1 and this J l equal to 0 will correspond to J 1. So, we can write down this equation from here to here. Now look at this equation at the cutoff this, because W equal to 0. So, W 0 will make the right hand side equal to 0 and because J 0 of U cannot be 0. So, U J 1 U must be equal to 0. So, that tells you when W equal to 0 J 1 of U that is now U is equal to V c which is equal to 0.

So, it actually defines the 0 of the Bessel function J 1. So, at this point, the value of the 0 value of V c for which the Bessel function J 1 will have a 0 we will define the cutoff property. So, this defines the cutoff of the mode corresponding to this. That is the first order LP mode, LP 1 1 mode will be cut off for the value which is for the value of V c, which is coming from this condition, but V c J 1 of V c is equal to 0 when V c is equal to this value that is V cutoff value is equal to this.

This is a very well-known number in fiber optical fiber system. And J 1 function if you want to see this is your J 1 function; you can see this is J 1 function which will have the first 0 here at this point. So, this point corresponds to x equal to 2.4048 at this point the mode will be cutoff.

So, this defines this cutoff property.

(Refer Slide Time: 26:46)

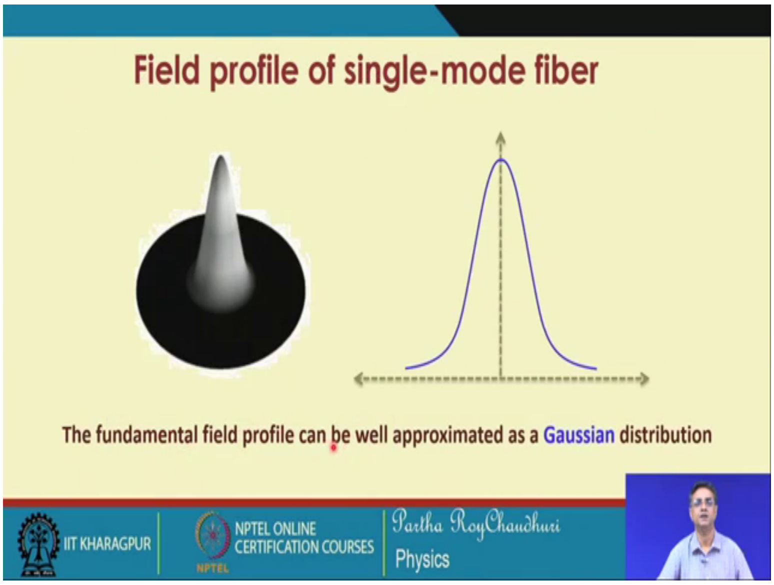


So, first order LP mode ceases to exist when V c is equal to this. But V c we can have this number equal to this by knowing by putting the values of a lambda n 1 and n 2. So, for a wavelength lambda, parameters for the fiber to function as a single moded one is to come from this at this equation that is 2.4048 equal to the V number.

So, from here given n 1 n 2 and lambda we can decide what will be the value of a, that is the core radius. Or given the core radius knowing n 1 and n 2 of the core cladding structure, we can define the limit up to which this fiber will remain single moded, that is only one mode that is LP 0 1 mode will be supported the fundamental mode. And no other modes the fundamental mode, you have seen. So, this is the fundamental mode LP 0 1 mode, and this mode will cease to exist at the value of V which is equal to the 2.4048.

So, at this point the first order mode will disappear.

(Refer Slide Time: 28:06)



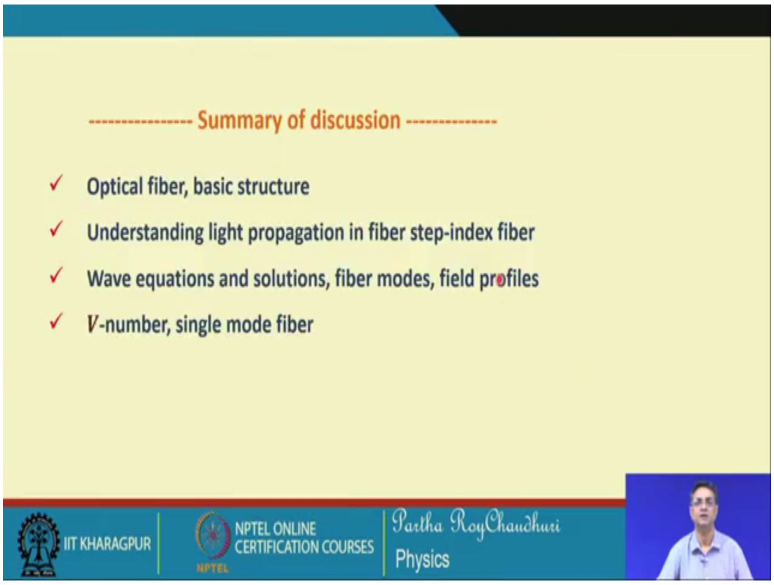
And the fundamental mode profile you have seen it looks like this, you have the highest intensity at the center. And as you as you move away from the core to the cladding across the interface the field vanishes. And it has a transverse mode profile which is like this. But it reminds you about a Gaussian distribution. And we will see that for a value of V which is between 1.8 and 2.5.

This field distribution can be well approximated with a Gaussian field distribution. And it has a number of advantages that is instead of describing the field distribution with 2 functions that is J l and k l in combination with the matching conditions that is for a given propagation constant beta. You can then represent the fundamental mode by one function which is a simple Gaussian function r square by a square, r square by sigma square or W square.

This W square will define the spot size, that is the intensity will fall to 1 upon e times the maximum intensity. And that actually helps a lot in several occasions in fiber mode matching splice loss then, but coupling calculation and many other occasions. When it is the mode field, the propagation constant these are the 2 important parameters of any waveguide that actually details that details almost everything almost all guided properties of an optical waveguide ok.

So now we stop at this point and will continue this optical fiber as a waveguide and the coupling characteristics of this optical fiber. With this background we will discuss the coupling characteristics of the optical fibers, and how these fiber couplers are useful in telecommunication and photonics that we will be discussing in the next occasion.

(Refer Slide Time: 30:27)



So, we have discussed these basics of optical fibers the mode properties wave equations and solutions fiber modes. We will continue with fiber coupler and other coupling mechanisms existing in optics.

Thank you very much.