Modern Optics
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Lecture - 25
Coupling of waves and optical couplers (Contd.)

So we have seen the basics governing equation for the couple mode equations, which are used for the coupling analysis of optical waveguide. And now that with that background we will continue our discussion for a very useful device that is what is called the planar directional coupler.

We will use this planar directional coupler analysis, but this can be extended to with the background with the knowledge of this analysis, we can understand the principle of other forms of the waveguides. So, that is why it is very important. And also this directional coupler is a very useful component a host of many devices in optical communication in sensor in integrated optics.

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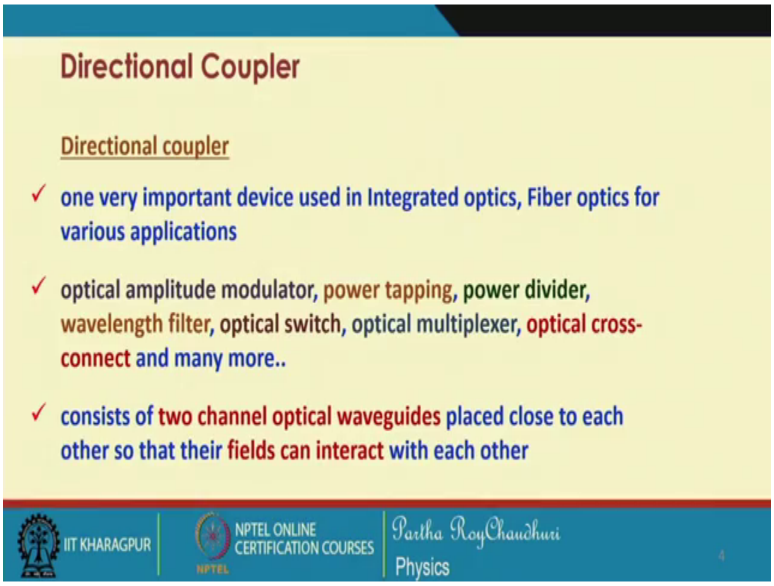

So, this is one very important device used in integrated optic, fiber optic a very useful component we will look at how this directional coupler are have include almost every part of the communication system and also a very useful device for sensor. Optical sensors, then optical many of the devices are like know very important in the in terms of this optical amplitude modulator, power tapping then power divider spliter, combiner, wavelength filter wavelength multiplexer, optical switch optical cross connect. And there are many more in fact, it is a large number of components are made out of this basic principle of directional coupler. And if you look at the structure of the waveguide is basically consists of 2 2 channel optical waveguides placed close to each other.

Ah So that there field of the individual wave can in the mode the field of the modes are individual wave guide can interact with that of the mode of the neighboring waveguide.

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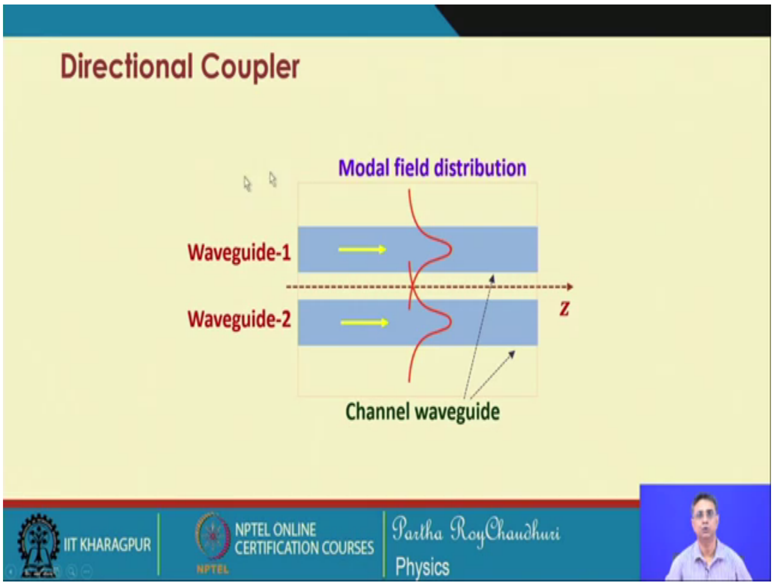

So, let us look at this configuration, you have an optical waveguide which is wave guide 1, and see that the waveguide can support assembly mode that is the fundamental mode replace like this. And you have another waveguide, which is for the time being we assume that this is identical or maybe similar. It also supports a similar fundamental mode field distribution. And now these 2 waveguides are line parallel to each other with a small gap between them. And the wave is travelling along the z direction.

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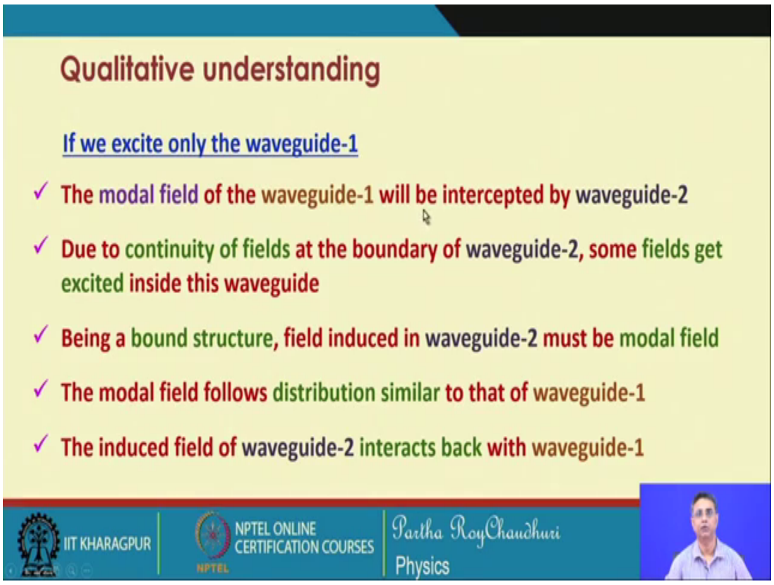

In that case, if we assume that we have exited one of the waveguide, let us suppose these waveguide will learnt field optical wave into this waveguide, then the modal field of the optical waveguide 1 will be intercepted by the waveguide 2.

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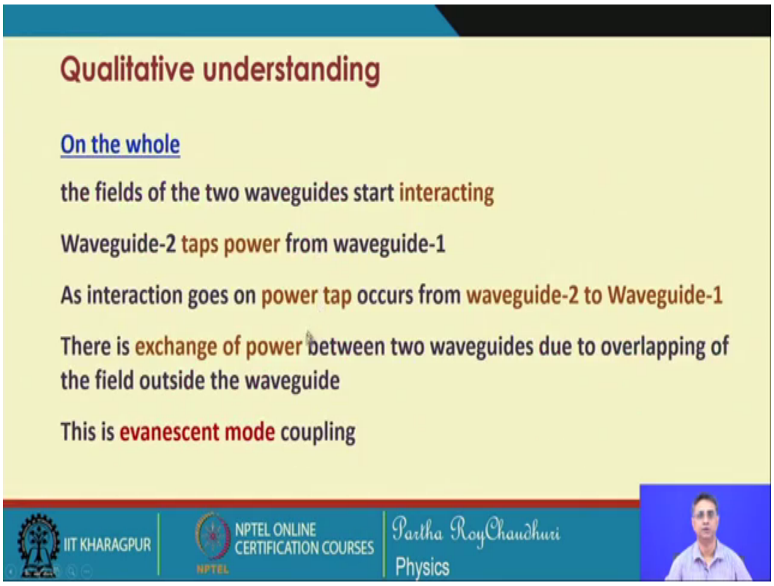

Let us look at this will be intercepted by the waveguide. You can see that this modal field is now a being intercepted by this waveguide 2 show. It hits the second waveguide therefore, this field because any field in such a waveguide structure will be continue as the tangential component of the field should be continuous across the waveguide interface.

So, this must be continuous and the continuity of the fields at the boundary of the waveguide 2 some fields get excited inside the second waveguide. And these 2 waveguides the total system is a bound structure. In a bound structure in the fields wherever it is it cannot be any arbitrary field.

It cannot be any arbitrary field the field induced in the waveguide 2 must be a modal field of the system. And that is the main point that the field which is induced into the second waveguide because of the intersection of it is evanescent tail with the second waveguide must be a modal field the system. The modal field follows a distribution which is similar to that of the waveguide 1. And the induced field of the waveguide 2 interact that with the waveguide 1.

So, it so happened that if this field can interact with this waveguide, the field which will be induced here that can also intern interact with this waveguide the first waveguide. And this process goes on as long as these 2 waveguides are interacting and line parallel to each other. So, on the whole if you look at the total structure as a whole, it happens that the fields of the 2 waveguide is start interacting with each other.

Waveguide 2 will tap optical energy from waveguide 1. Because the waves are travelling along the z, z direction for the traveling of the wave along the z direction it needs power. And therefore, it will tap the power tap occurs from waveguide 2 to waveguide 1. Know it occurs from waveguide 1 to waveguide 2, an intern as the interaction goes on, then the similar process will occur between the waveguide 2 and waveguide 1.

So, there is an exchange of power between the 2 waveguide due to overlapping of the field outside the guiding region. That is outside the core of the 2 waveguide. So, this is called evanescent tail, then because evanescent conjugate the entire process is due to the interaction of the wave through their evanescent tail.

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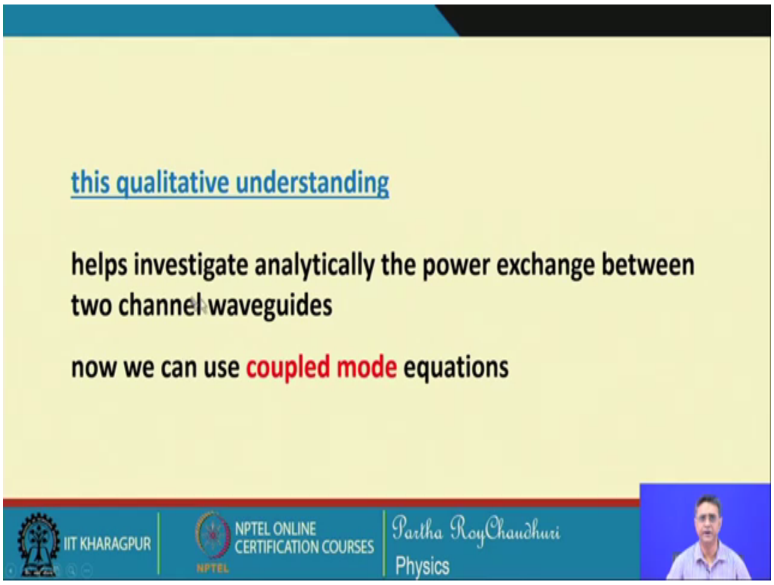

Now, this quantitative this qualitative discussion understanding helps us in investigate how the power exchange between the 2 waveguide step. Plus, and we will look at the exact formulation and setup couple mode equations to draw this conclusion. So now, we begin this analysis of directional coupler.

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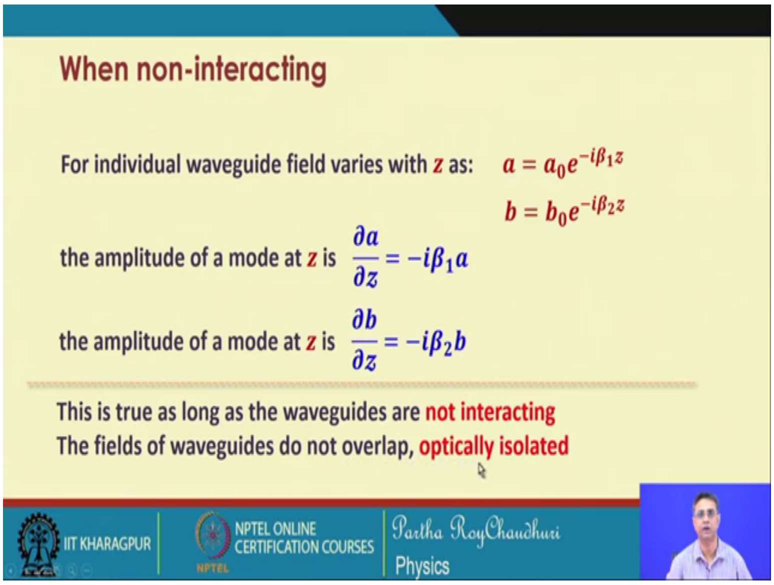

So, for individual waveguide the fields varies the fields the fields vary as z as a equal to a 0 e to the power of minus i beta 1, this is for the first waveguide and for the second waveguide b equal to b 0 e to the power of minus i beta 2. So, these are the 2 individual independent waveguide mode we are talking about. The amplitude of the mode z other mode at z is d a d z. So, as long as this is an individual independent waveguide the amplitude variation of the mode will be like this. And for the second waveguide the amplitude variation will be like this.

This thing is true as long as these 2 waveguides are isolated optically isolate and they are not interacting as if they are well separated, the fields of waveguide do not overlap and their optically isolated.

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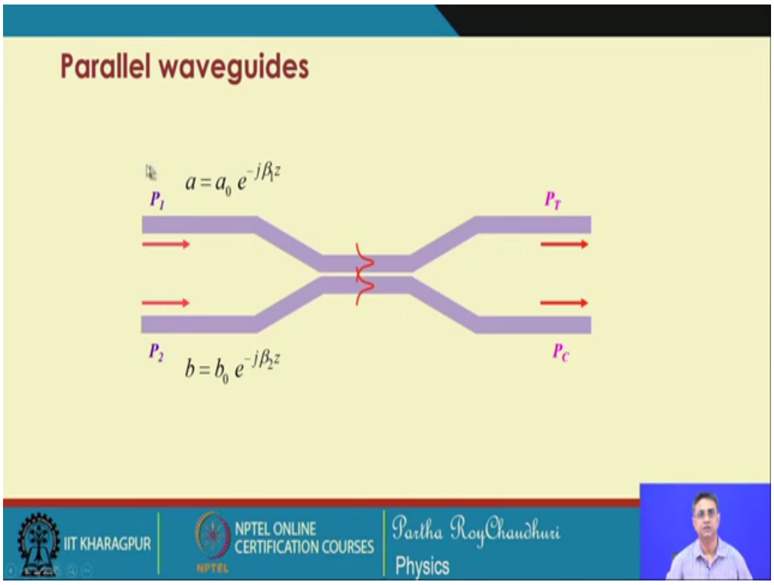

Now that so this is the situation that we are discussing when we are pleased to waveguides are quite apart, but when they are close to each other, the fields will interact with each other. And this is the structure the schematic of this directional coupler structure. You have an input port, here you have another input port, here you can use any of these 2 as an input or you can use both of them. Then this 2 waveguides who are very close to each other at this region will interact, and there will be a redistribution of power at this point which will give rise to the wave along this waveguide, which will call the transmitted or the throughput waveguide and this one which will be called the couple waveguide.

This will be called couple waveguide if this is the input waveguide then this is a transmitted of throughput and this is a couple, but if this is the input waveguide when this will be called the coupled waveguide, and this will be called the throughput or transmitted port.

So, this is by convention because it is a directional coupler. So, and bidirectional so both of these, any one of these or both of these can be used as the input and this side can also be used as input so they are also sometimes called bidirectional coupler.

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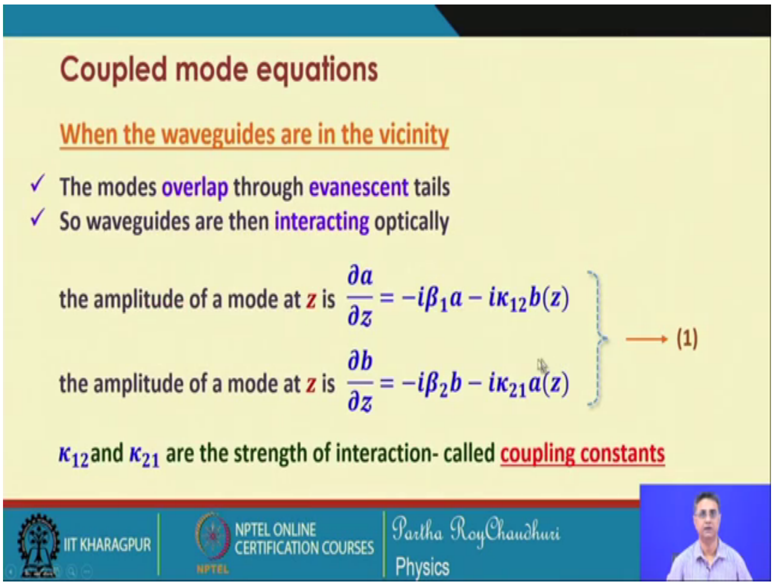

When the waveguide have in the vicinity, that is when we will look at the wave guide, when they are closely space that is there optical interacting the most overlap through the evanescent tails we have seen that. So, the waveguides are then interacting optically and the result is that instead of d a del a del z is equal to minus i beta 1 a. You have one more time sitting here because this is the interaction term. This is the other the radiation in the wave of variation in the amplitude of a is a function of the amplitude of the wave in the waveguide 2. So, this is the coupling term, similarly for the waveguide 2 you have the coupling term, which is due to the waveguide 1. So, these are the couple mode equation which we have derived and we have seen how they have come from.

Now, this k 1 2 and k 2 1 they will represent the strength of interaction. If it is large then more will be the interaction if it is small, then more will be the less will be the interaction. So, they are call the coupling constant k 1 2 and k 2 1.

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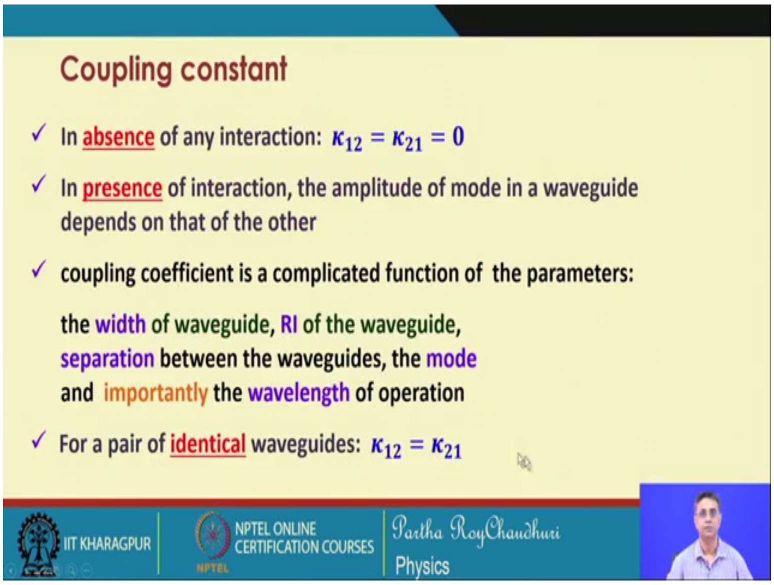

The coupling constant you can see that in absence of any coupling if there well separated these 2 quantities will become 0, k 1 2 and k 2 1 they will become 0 because there is no interaction, but in presence of interaction. The amplitude of mode in a waveguide depends on few other parameters involved in this configuration. Whatever depends on that of the other, the amplitude of the field in one waveguide depends on itself as well as the field on the other waveguide.

The make out from here the amplitude read dependent amplitude depends on the amplitude of this waveguide and also that of the other waveguide through the coupling constant. The coupling coefficient this k 1 2 or k 2 1 is a very complicated function of 2 parameters we are depend on the width of the waveguide. The refractive index profile of the 2 waveguide involved their separation which is very important. If it is well separated, then there is no interaction coupling coefficient drastically reduces become 0.

And the kind of mode we are interacting it also depends on the kind of mode; whether their fundamental mode, whether their first order mode they are interacting. So, that is that is going to decide the coupling coefficient as well. And very importantly it is also a parameter that depends on the wavelength of operation. And this principle that a coupling coefficient depends on the wavelength of operation is utilized in wave many interesting devices like wavelength division multiplexer, wavelength division couplers and so on and so forth.

For a pair of identical waveguide let us suppose the constituting the, thesetwo waveguides their identical; in that case k 1 2 and k 2 1 will be identical. And therefore, this k 1 2 equal to k 2 1 we can write equal to kappa.

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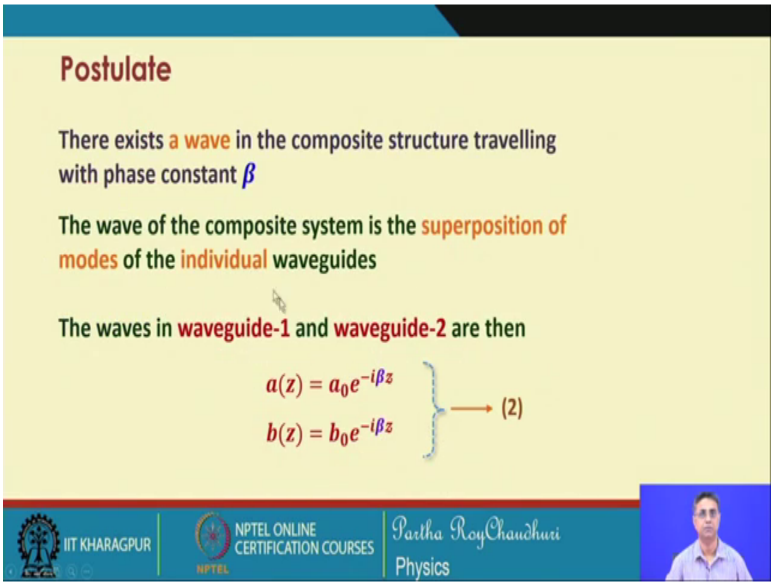

So, one coupling coefficient that will represent the entire system now, we make a general understanding and postulate that if we think of the total structured the composite waveguide, then there will be a wave which is travelling through this waveguide structure with a phase constant beta. And this wave of this composite structure the total structure has to come from the waves of the individual waveguide that is the modes of the individual waveguide.

And therefore, this composite waveguide must be there must be drawn out of the combination of the modes of the individual waveguide. The waves in the waveguide 1 and 2 are then should be should be expressed in terms of the wave of the composite structure I called this wave of the composite structure as having a phase constant beta.

So, just the z dependent amplitude of the first waveguide should be represented by this and where the amplitude is a 0, and for the second waveguide the z dependent amplitude will have a similar distribution, that is b 0 e to the power of. So, this b beta is now the wave which is travelling along the total structure with a phase constant beta so this is the same beta.

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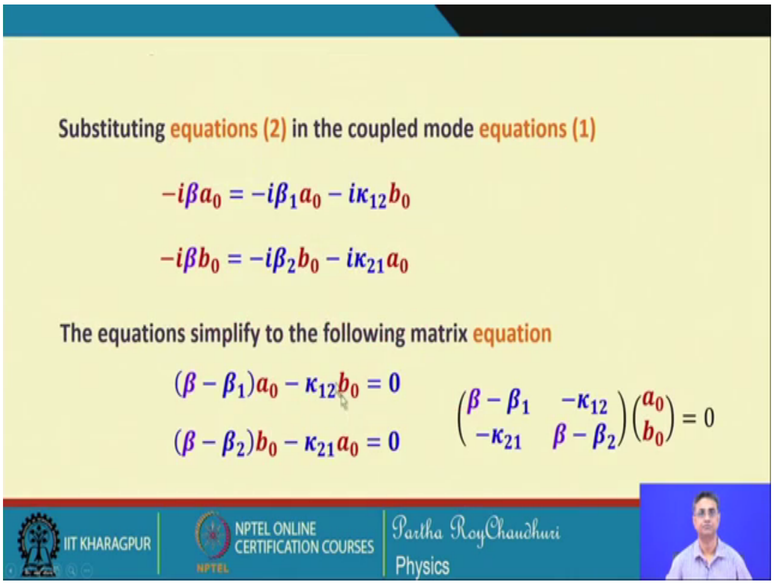

So, if I now substitute this assumption that there is a wave of that total system, and we substitute this wave back into the wave equation the couple mode equation then we can write this couple mode equation in this form. So, we get from the couple mode equation i beta (Refer Time: 14:40) del a del z will be give i beta and this was already i beta 1.

So, I can rewrite in this from, the equations simplify to the following matrix because you can put it in the form of a matrix which will give you the coefficient matrix for a 0 and b 0, that is in terms of b beta b one k 2 1 and k 1 2 in this form. Now for further solution for a 0 and b 0, this non trivial solution this determinant of this matrix will be put equal to 0.

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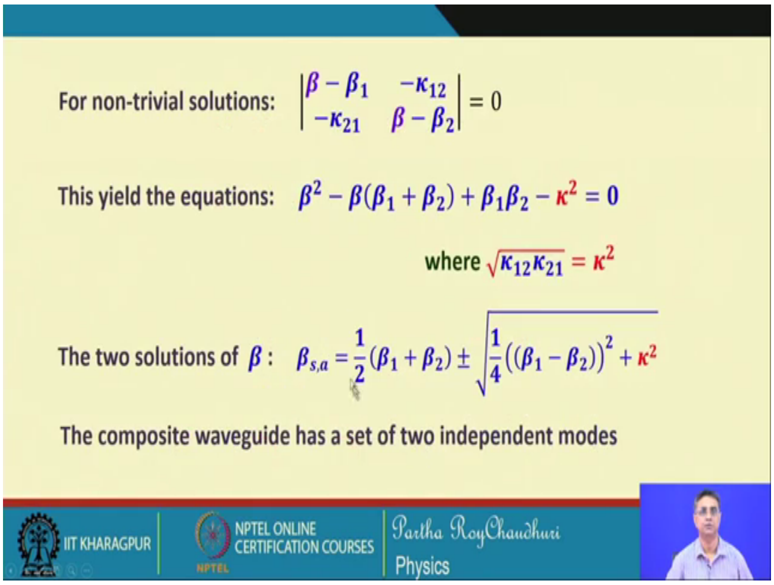

Which will give you an equation quadratic equation of beta square because you are looking for a beta, we have assume that there is a beta there is a total wave for the composite system, which is propagating with a phase constant with a propagation constant beta. And we are looking for this beta in terms of the amplitudes and propagation constant of the individual waveguide.

So, look at this beta square beta has come up in the form of a quadratic equation. Beta square equal to beta into beta 1 plus beta 2 this and this gives you this solution for beta s a because it gives you 2 solutions for beta 1 is beta s and beta a this suffix beta s and beta a are very usual to represent that; s means symmetric a means antisymmetric. So, you have 2 values of beta half of beta 1 plus beta 2 and 1 4th of the difference that is delta beta square plus k square, where is kappa square is under root of this k kappa 1 2 into kappa 2 1.

So, this composite waveguide has a set of 2 independent mode, because we started with the assumption that; the z dependent amplitude for the first waveguide and for the second waveguide are drawn out of one wave which has a propagation constant beta, but it turns out that the total system has 2 mode; that is one symmetric and one antisymmetric, these 2 modes will have 2 propagation 2 different propagation constant, as a has a solution of this quadratic equation, which are beta s and beta a and are given by this equation.

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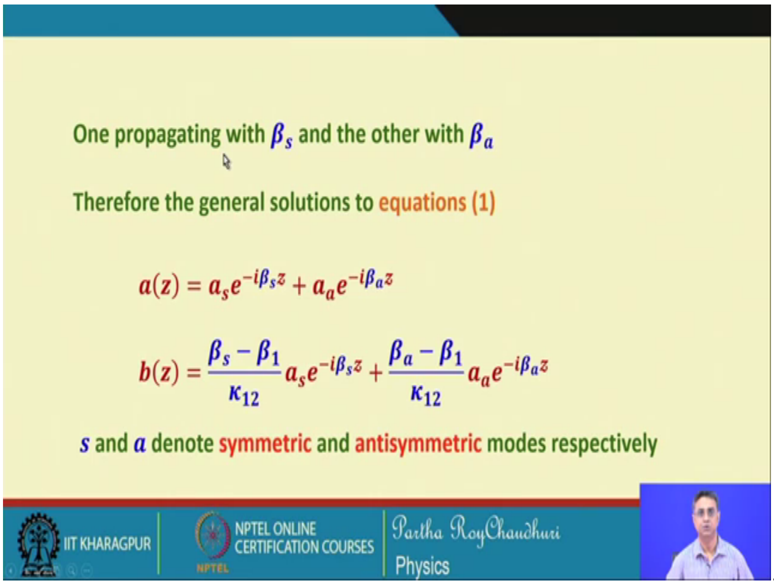

So, one of these 2 waves of the total system the composite system is propagating with a propagation constant beta s, and the other one is with the propagation constant beta a.

So, this beta s is to represent that symmetric mode and this one 2 represent the antisymmetric mode therefore, we have a general solution to equation one that is, the assumption that is a z and b z how it depends on the amplitude of a and b, b is a interaction a by itself. So, a z and b z we can write in this form, having known the values beta s beta 1 k 1 and k 2 so you can represent in this equation. This s and a as I have mentioned earlier are to represent symmetric and antisymmetric mode of the total system of the composite waveguide.

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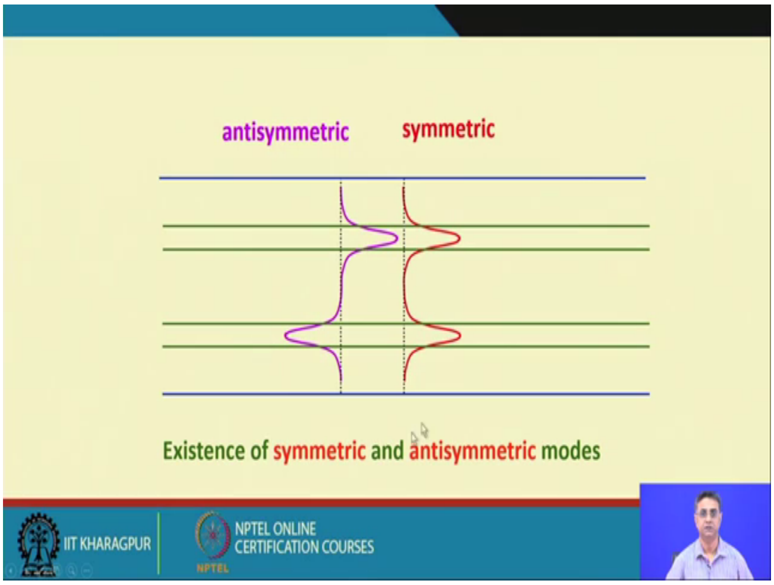

So, this beta s that is symmetric wave will have filled profile like this. You know this is the even mode and this will be antisymmetric which we call the odd mode.

So, these are the total system you have 2 waveguide and the modes of the total system the 2 waves to other mode of the total system can be thought of there is one symmetric mode and there is one antisymmetric mode. So, these are the 2 mode which belong to the total system that is that.

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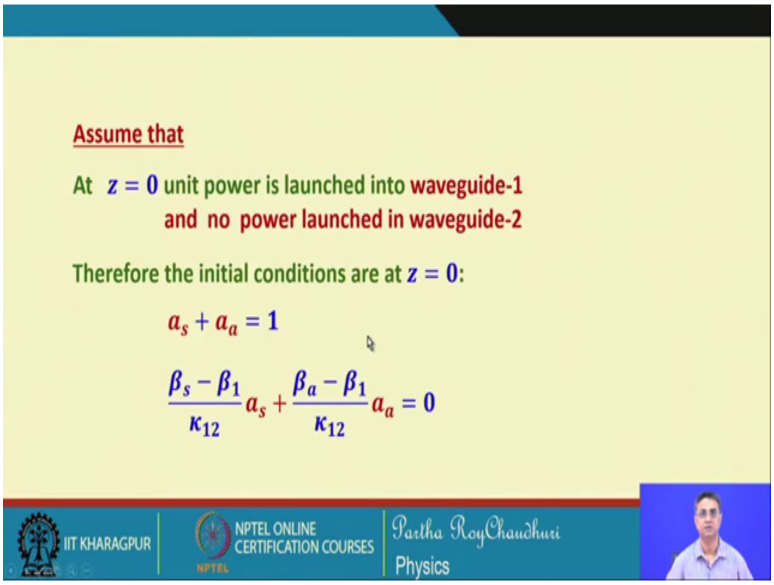

And we can assume that we can put the boundary condition, that have z equal to 0, if we at z equal to 0. If we launch unit power into one of the waveguide, I just launch unit power into one of the wave guide then these 2 amplitudes will be added up because the phase difference is 0 at z equal to 0, both of them will have z equal to 0 phase difference between them is 0. So, they will be added up and these 2 we will be cancelling each other.

So, it is like as good as telling that there is a wave which is confined only into one waveguide. So, the superposition of these 2 waveguide at z equal to 0 will be equivalent to the wave at the first waveguide that is an input in the first waveguide. The input unit power is launched into waveguide 1 and no power launching to waveguide 2 under that condition then we can write a s plus a, a equal to 1 at z equal to 0, because you have e to the power of i beta s z e to the power of minus i beta a z both z 5 0. So, you get this one for this is a total unit amplitude at z equal to 0.

And for the second waveguide this will be put equal to 0 because this is the amplitude in the second waveguide, this is in the first waveguide.

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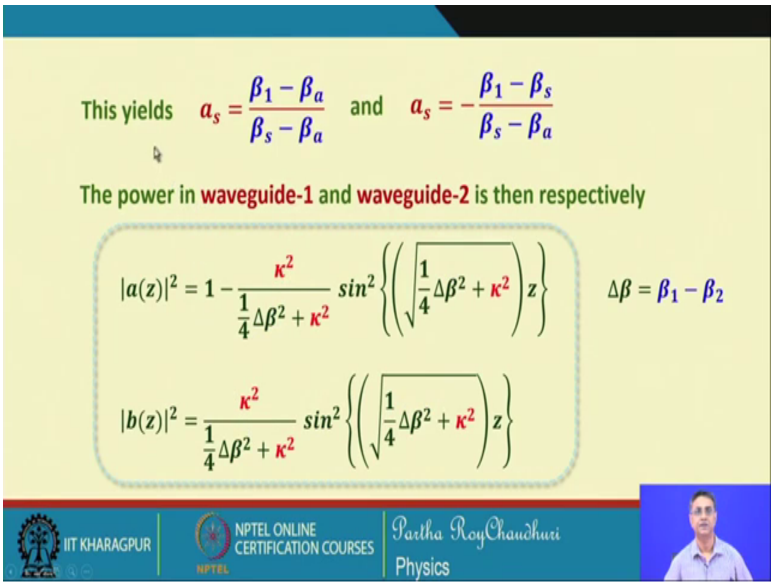

So this fields that a s is equal to this, because if you if you solve these 2 if you eliminate the value can calculate the value of a s and a a by solving these 2 equation. You can see that a s equal to this and this should be a a, where is a mistake it should be a a will be equal to this. Therefore, knowing this fact that we can now put them back into a z and b z equation, which we have assumed then you can write this amplitude that is the power which is the square of the amplitude in the first waveguide can be represented by this equation.

And so this is the power transfer equation and b z square the power in the second waveguide which can be represented by this equation. If I substitute this a s and a a in back into the equation for the power of the individual waveguide. In this equation a z and b z in place of a I will substitute that; in place of a s I will substitute the terms containing that beta 1 beta s beta a and k 1. So, this quantity if I, so I will get that pattern for equation. And this is the must general form of power transfer for weakly the interacting waveguides and the form of directional couplers. And this a z square and b z square at the powers in waveguide a and waveguide b are waveguide 1 or waveguide 2. So, this equation is very useful and this is very general when you consider two identical waveguides then you can.

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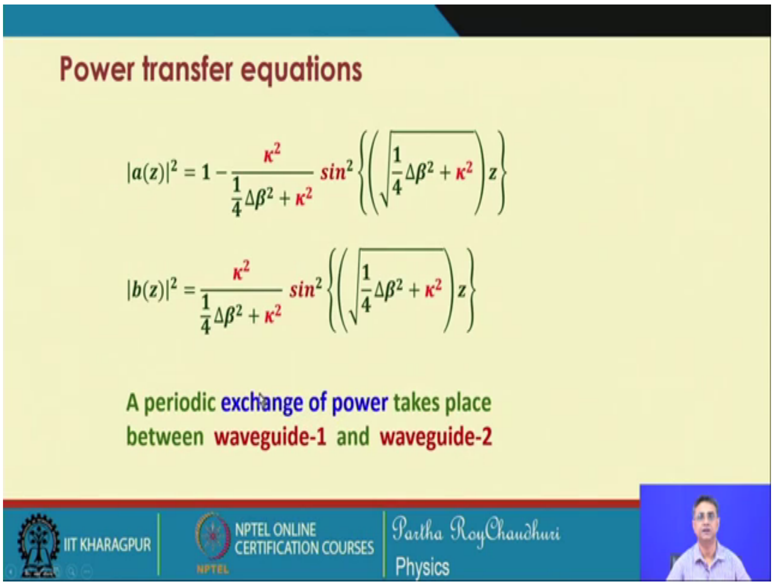

So, we write this equation in this form a z square and b z square. You can see that there is a the periodic exchange of power between the 2 waveguide, that is the periodicity will be twice pi is k twice pi by lambda this quantity.

So, 1 upon pi by this quantity will be the periodicity of the power transfer, which will be periodicity along the z direction between the waveguide 1 and 2.

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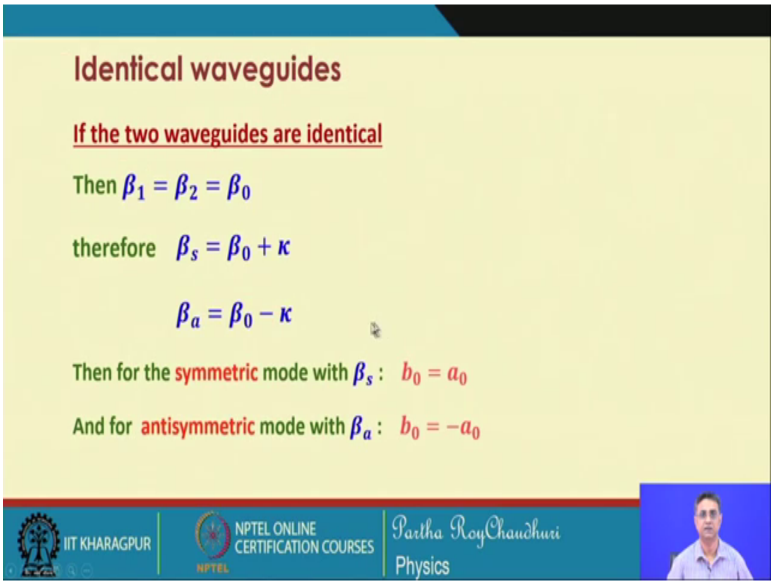

So, it is obvious from this that there is a periodic exchange of power between the 2 waveguide. If we now considered that these the 2 waveguides, which are forming this directional coupler they are identical. In that case this the modes of the individual waveguides will be the same and there will be represented by the same propagation constant, that is beta 1 equal to beta 2 is equal to beta naught. Let us suppose in that case then beta s will be equal to beta 0 plus.

So, this will become kappa and beta s will be equal to beta 0 minus kappa. If you go back and look at the equation for kappa 1 1 and kappa 2 1 so, beta s and beta a are now having this reduced from. So, then for the symmetric mode beta s will become b 0 equal to b a. So, you can look at the governing equation for b a and b 0, and for antisymmetric mode you see that there is a minus sign. So, they are pi by phase out of phase in the second waveguide, and that is what as I have shown here if we completely this plus this will be minus for the second waveguide where as in this case both of them are the plus amplitude.

So, beta 0 equal to minus beta a whereas, beta 0 equal to. So, this is for the symmetric mode and this is for the antisymmetric mode. This means a very interesting to see that is beta 0 is the part of the amplitude of the field in waveguide 2. And beta a 0 sorry, b 0 this is a 0 is the field in the waveguide 1.

So, they are now identical and they are of the same phase, that is why in a symmetric mode you have this and you have this. Both of them are having the same amplitude distribution as well as they are having the same phase. Whereas for the antisymmetric mode you are in the b 0 equal to minus a 0; that means, the in the waveguide a this amplitude is positive, but in the waveguide, b this amplitude is negative. Because of this minus sign, but they are having the same amplitude because b 0 the magnitude of b 0 and a 0 they are same. So, that very nicely explain this even and odd modes that is a symmetric and antisymmetric mode as an outcome of this couple mode equation calculation for the identical waveguide.

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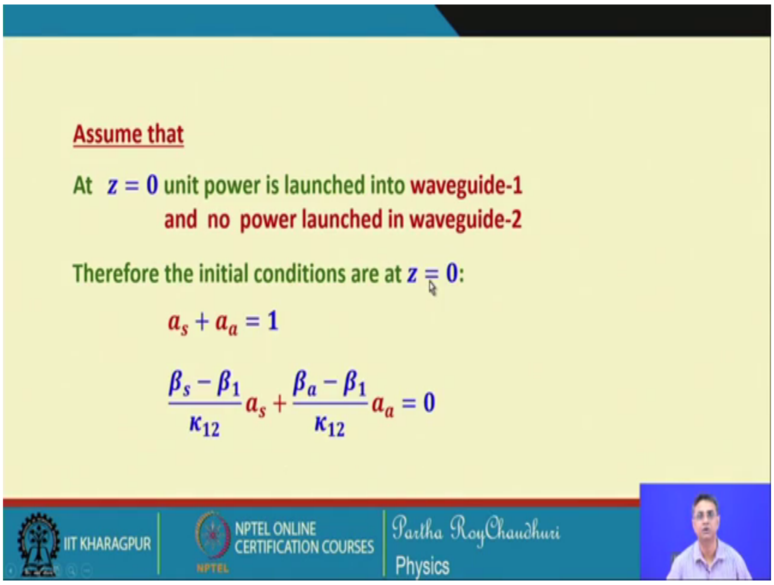

Now we have seen that if these 2 waveguides are identical and if I substitute this beta s and beta a by beta 0 plus k, and beta 0 minus k back into the this equation, into this equation, then we can write this expression for the a z and b z in this form, it requires a simple algebra calculation manipulation.

So, you can so that the amplitude in a is a sinusoidal function sin not kappa z. Amplitude is b will also sinusoidal function, but is a phase difference of pi by 2 initial phase pi by 2 with this the power in waveguide 1 and waveguide 2 is then is, then the mod square of a z and b z, which gives you cosine square kappa z and b z square will give you the sin square kappa z. So, these are the 2 power transfer equation which comes from this a z and b z calculation. Now coupling length in the power transfer equation, if you look at this or the power transfer equation for non-identical waveguides that is the general, waveguide we have 2 waveguides or dissimilar, then this is the equation governing equation for the power transfer.

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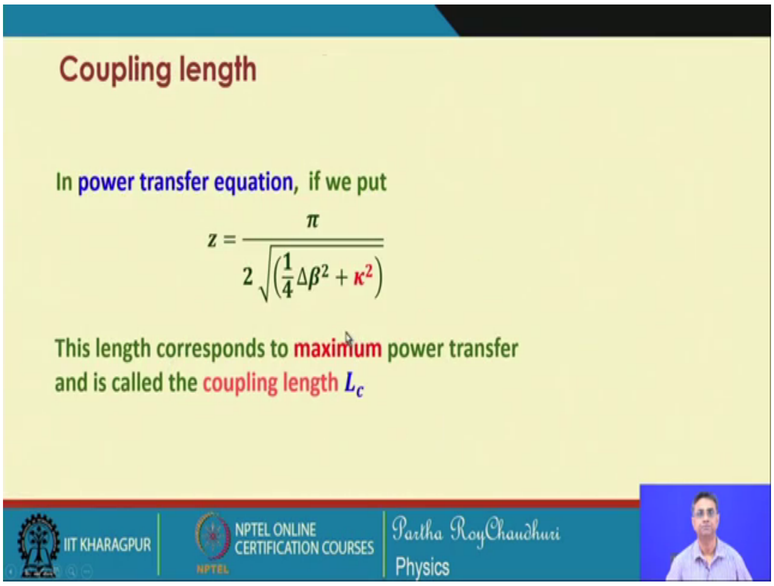

So, here if we put that z equal to pi by 2, and under root of this quantity. At this distance the length of the interaction between the 2 waves between the 2 waveguides, you can see if you put the value of z into that into that equation; that the length corresponds to maximum power transfer because this will give a sin function equal to 1 and cos function will become equal to 0. And is this length will be called the coupling length.

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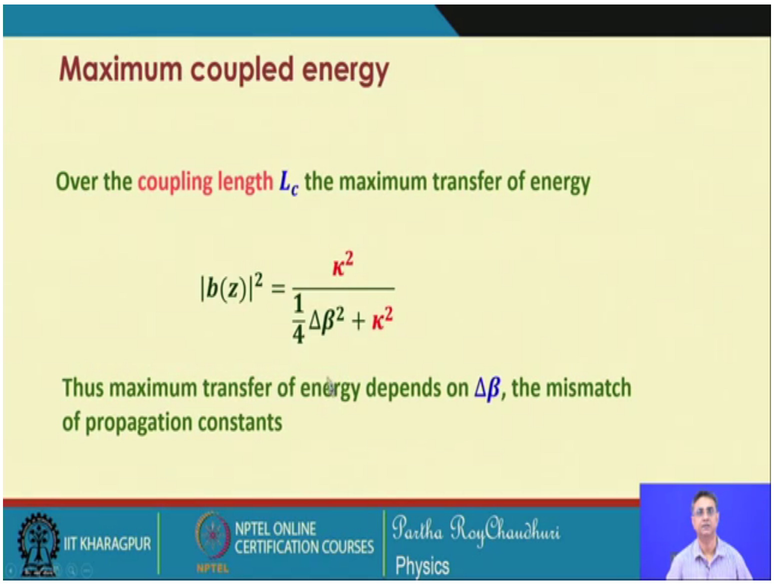

So, over the coupling length the maximum transfer of energy that is b z square will be equal to this for non-identical system, but if they are identical delta beta will be 0. So, this is equal to 1. So, that is maximum transfer of energy will take place from a z square and a z square at that moment will become equal to 0 and this will become equal to 1.

So, thus the maximum transfer of energy depends on delta beta. If delta beta is there is a mismatch between the 2 waveguide, then power transfer is not maximum.

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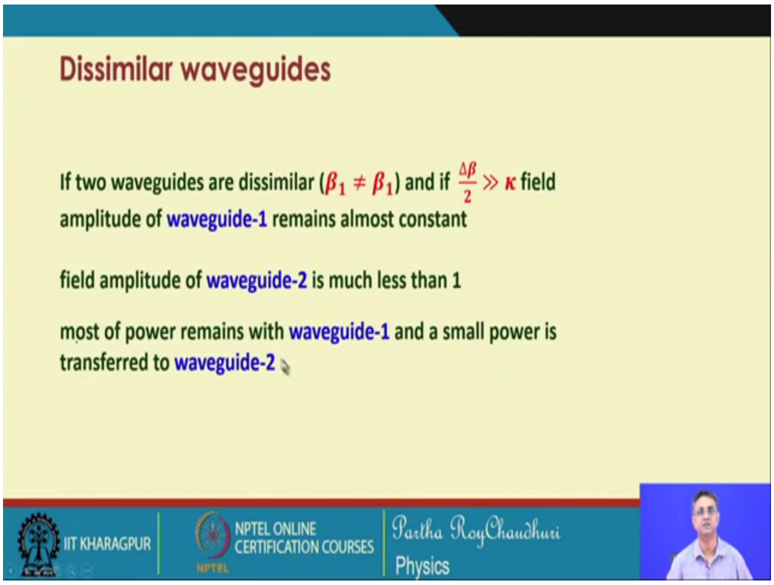

It is it is not 100 percent the field amplitude of wave guide 2 is much less than 1.

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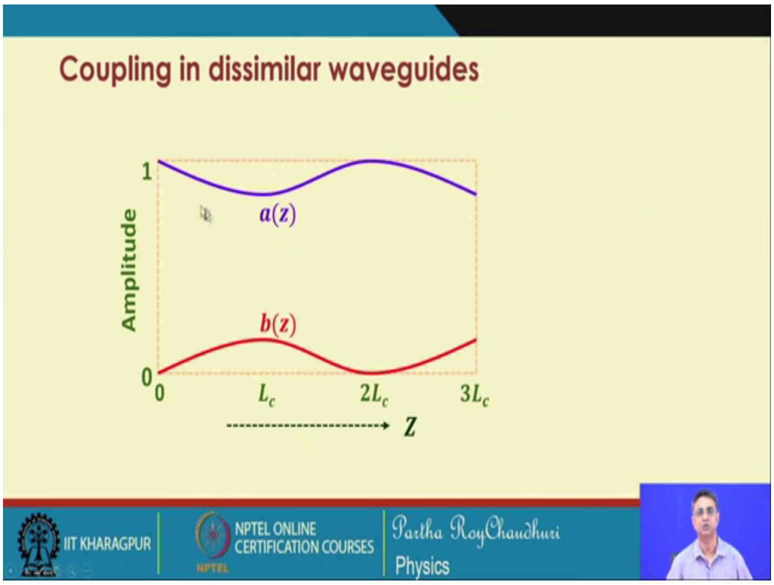

This we can so with a plot computed the plot of the amplitude transfer for waveguide a this is how it varies there is a small change because of the interaction. And in waveguide only a small part of the wave is coupled over the introduction. So, this is the coupling length over reach the maximum transfer of power takes place. This is the cross back state when again the power goes back into the first waveguide and so on.

So, this is a periodic exchange of power between the 2 waveguide, and the 2 constituent waveguides are dissimilar they are not identical.

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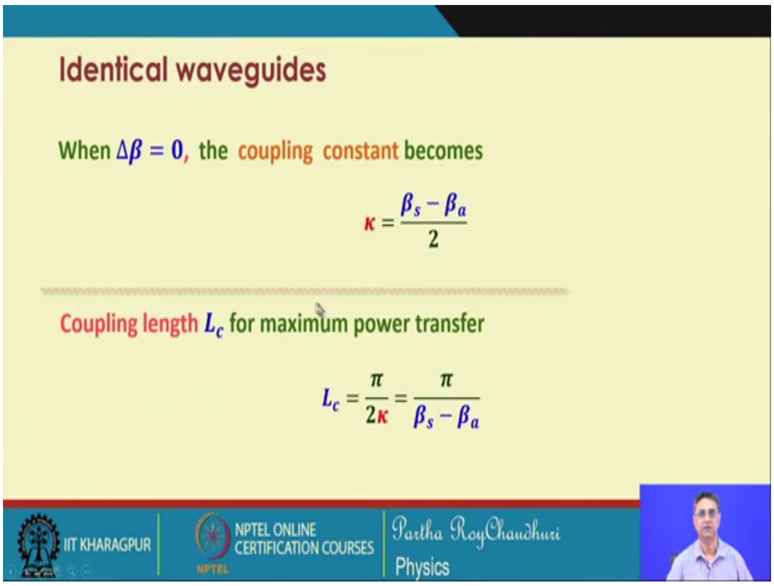

But it beta delta beta equal to 0, we have this is the coupling constant coupling coefficient. And the length that defines the coupling length, now comes out to be pi by beta s minus. So, this is the difference in the symmetric and antisymetric mode that gives you the coupling length.

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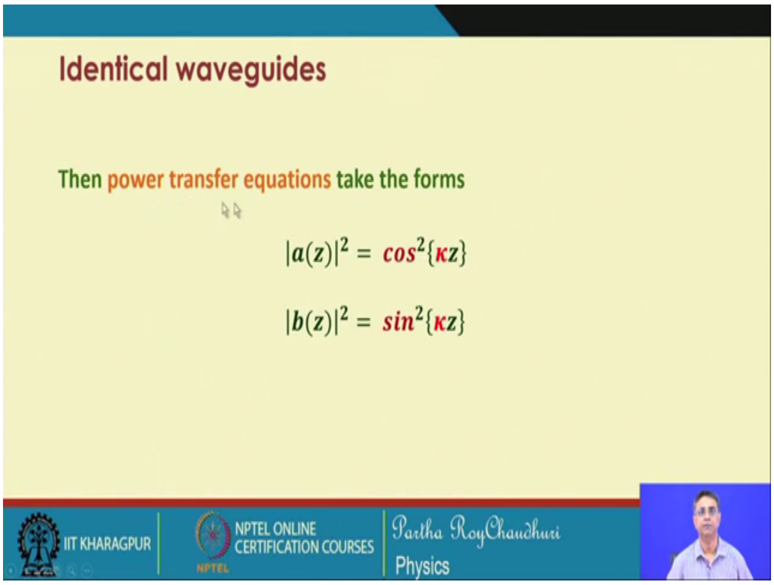

And then the power transfer equation comes out like this is a z square equal to cosine square kappa z; and which is very well known for identical waveguide coupler. And is the foundation for many devices for understanding many interesting coupler devices.

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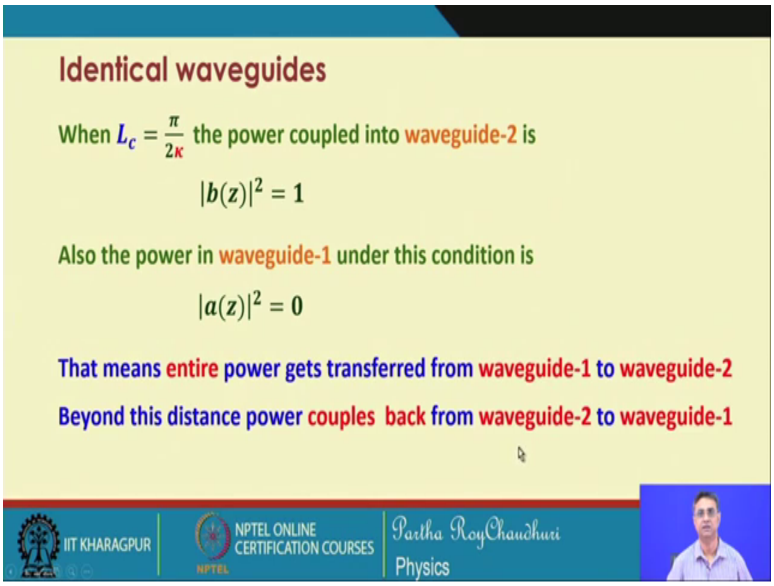

Where L c, if I put back you get that b z square equal to 1 into back into this equation if you putL c for z; then you will get beta square equal to b z square 1 a z square equal to 0.

That means the complete transfer of power has taken place to waveguide 2 from waveguide 1 where the power is now 0. So, this means entire power gets transferred from waveguide 1 to waveguide 2. And if it so happened then beyond this there will be again transfer of power from waveguide 2 to waveguide 1, which is called the cross back state or the couple the coupling back state.

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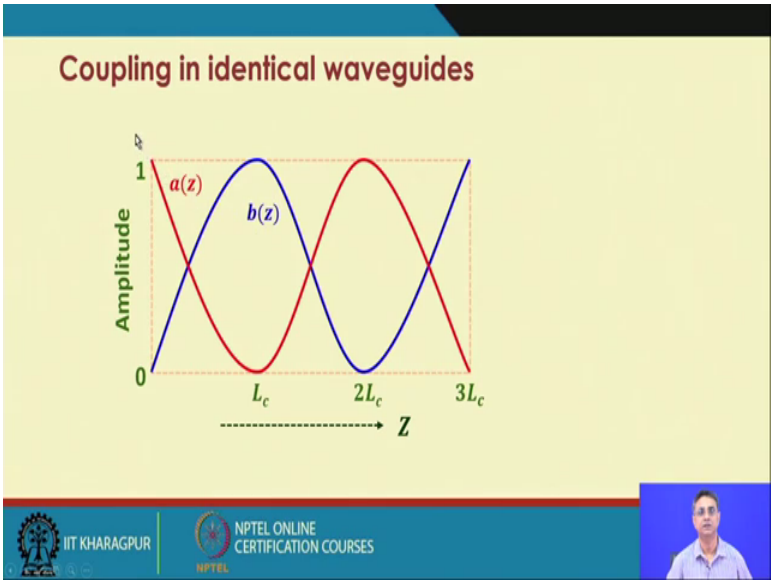

And then we can repeat this power variation in the 2 wave guide, where you can see that there has been a complete transfer of power; taken place between the 2 waveguide and this is the L c, the coupling length and you can also define the length overreach the power in the 2 waveguides are exactly 50 50. And that is what you call a 3 d B directional coupler, who will see some interesting applications of this 3 d b coupler in the following sections.

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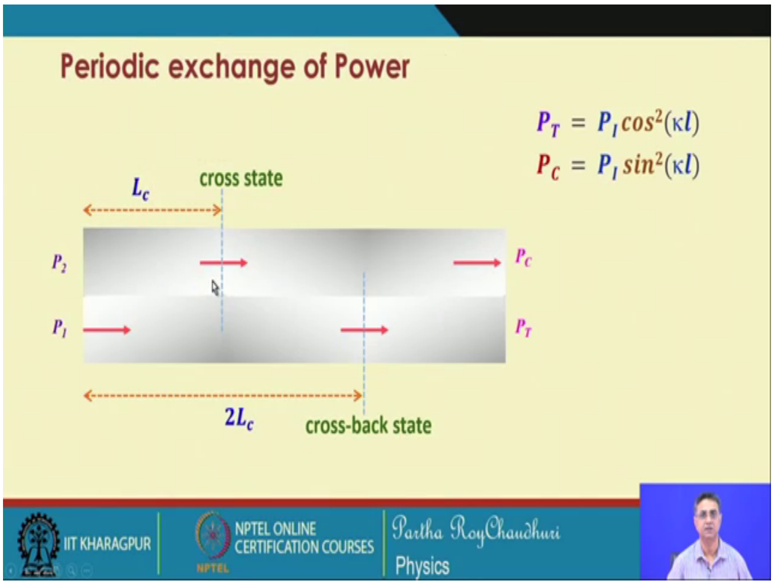

So, we have a visual favor of how this coupling takes place we have an input power in the waveguide 1. And it goes to waveguide 2 and then it again comes back to waveguide 1 and again goes to. So, in this wave the periodic exchanger power takes place back and forth between the 2 waveguide. This state is called cross state because we have entire curve which 2, the second waveguide this is cross back state where the power is restored back into the first waveguide.

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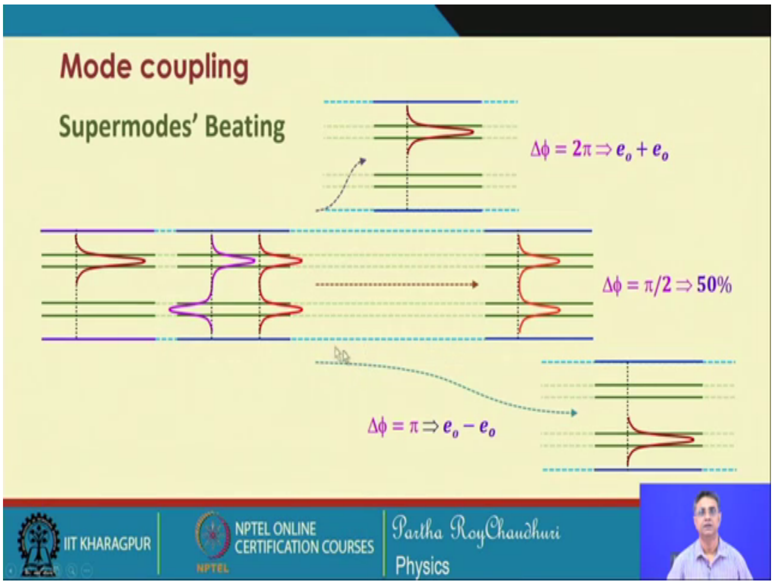

So this is the supermode beating pictures very interesting to understand, I talked about this that if you have a so this is the total the 2 modes of the composite structure which are the odd odd mode and the even mode. The superposition of these 2 modes at z equal to 0 gives you just add them up because there is no phase difference between them. So, these 2 amplitudes will be added up to get a power in the first waveguide, which can be assumed to be the input in the first waveguide. And as it propagates down the total the 2 modes there is a the 2 modes are characterize by 2 different propagation constant. Therefore, they will develop a phase difference as the travel down.

When the phase difference is pi that is; the phase difference is pi then you have to subtract this mode from this mode which will now add these 2, but subtract this 2 will make it 0, this will be added up we where the entire field a into the second waveguide, but if the phase difference is 0 you have at the input then you have the entire power in the first waveguide. But there is a position which is in between these 2 states where the phase difference is just pi by 2, then you will get half of the power in this waveguide and half of the power in this waveguide.

So, this is what you call that 50 percent coupling state of the 2 waveguide. So, this is the beating of the 2 supermode, you have the 2 normal modes of the structure the even mode than odd mode, which is like the modes of a coupled pendulum. So, when there in phase you have one system to oscillate, when there out of phase the other system to oscillate. When they are in between then both of them are oscillating with 50 percent of their power energy. So, this is a very interesting part that by considering the supermodes of the structure, you can explain the coupling of the modes of the composite structure.

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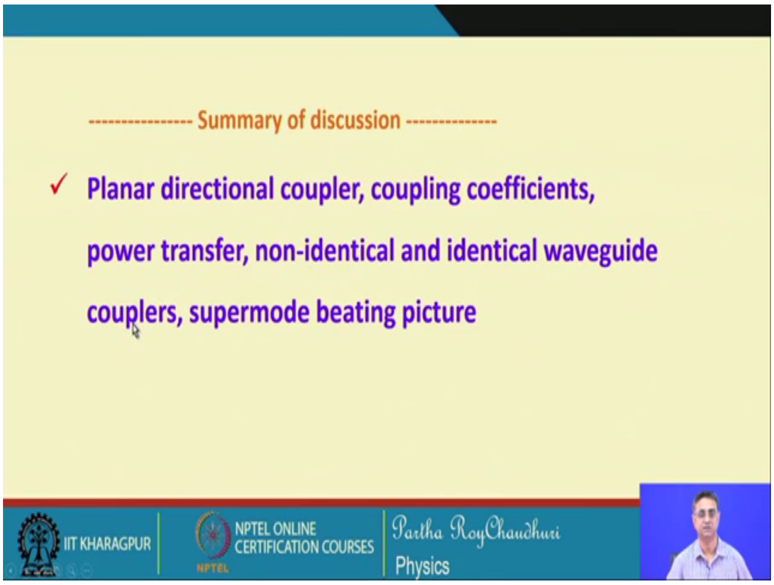

So, in this discussion we have considered this planar directional coupler, and in terms of the coupling coefficient we have also looked at the power transfer equation for the general wave guide, then the 2 waveguides are non-identical. We also looked at the 2 waveguide when their power transfer when the 2 waveguides are identical and finally, we looked at the configuration visually through the supermode beating picture.

Thank you.