Modern Optics
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Lecture - 24
Coupling of waves and optical couplers

In wave guide mode are specific field pattern that remain unchanged when the wave travel along the length of the wave guide, it travels with a constant phase constant which is fixed for a given mode and this specific field pattern the mode remains as it is as long as the wave guide remains uniform and there is no distortion. But if there is any distortion or any bend there will be a perturbation which will affect the propagating mode.

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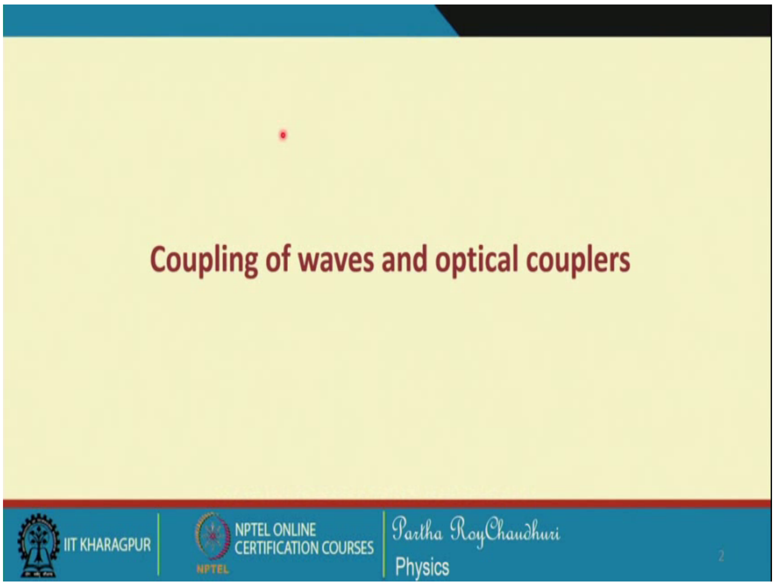

So, in today's discussion we will look at the coupling of waves and the devices made out of that that optical coupler and there they are the basic principle in the philosophy of the mode coupling that we will try to understand today.

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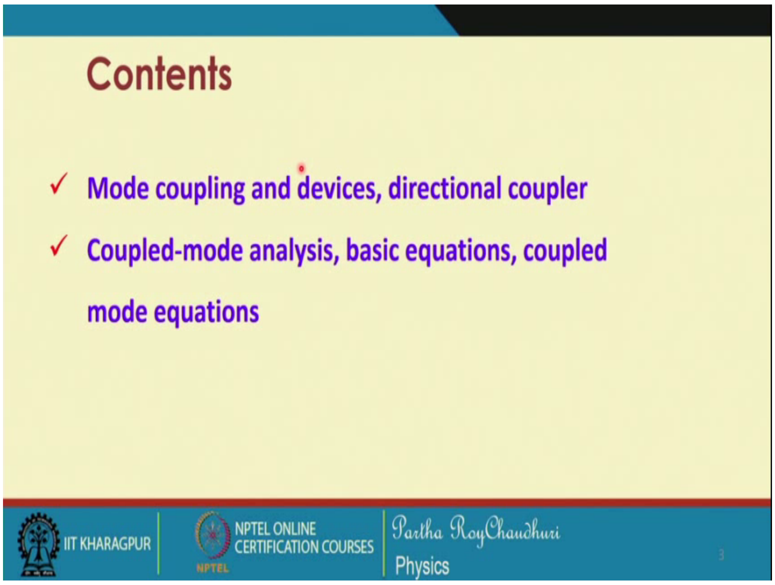

Mode coupling and devices, directional coupler then couple mode analysis, the basic equations and couple mode equations these are the point that we will be continuing for our discussion today.

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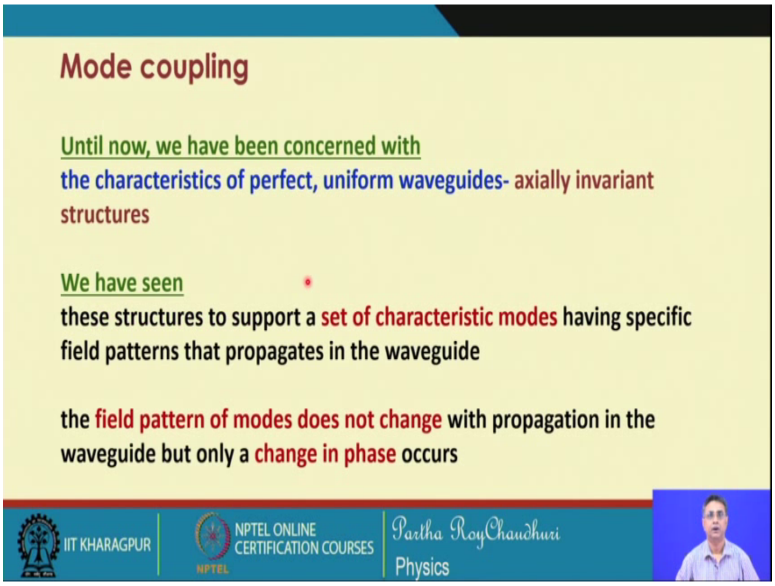

Now, let us remember that as far as a uniform axially invariant wave guide structure, the modes are modes are will travel with a phase constant and will remain undistorted along the length of the waveguide. So, this is true and the structure will support a set of characteristic modes which are the specific field patterns depending on the mode order in terms of the propagation constant and field distribution and this is all that we have been insisting that the field pattern of the modes does not change with the propagation in the waveguide. But there will be a phase change along the length of the waveguide that is the red translation of the phase, that is the one which appears as e to the power of minus i beta n z.

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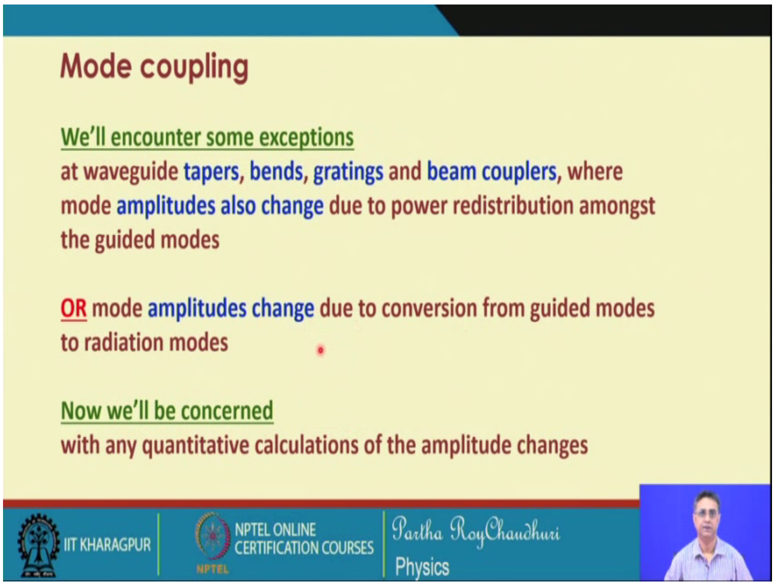

In optical waveguides we will encounter some specific situations where there will be a waveguide tapers, there will be bends, there are gratings, then there are beam couplers where the mode amplitude also change due to the power distribution among the guided mode. And, we will see that this is because of the coupling from the modes which are supported by the waveguide mode, amplitude change due to conversion from the guided modes to the radiation modes.

Now, we will be concerned with any quantitative calculation of the amplitude changes, so that is the objective of the discussion; how we can quantitatively tackle that over the conversion of the modes or coupling of the modes from one mode to another, from one guided mode to the radiation mode and so on and so forth.

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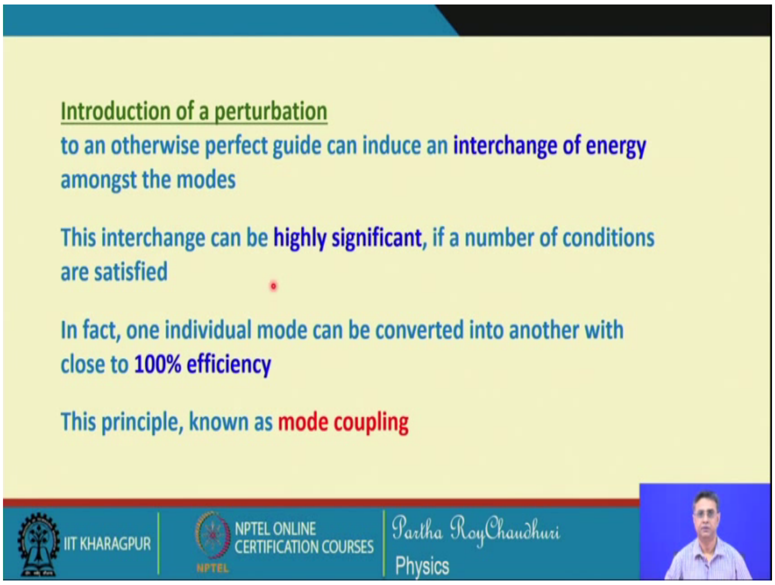

So, if we have an uniform waveguide but which is otherwise perfect, but it can induce some interchange of the energy amongst the mode. This interchange can be highly significant if the number of conditions are satisfied. In fact, it is possible that one individual mode can be converted with almost close to 100 percent of efficiency and this principle by which the energy from one mode to another mode get coupled is the mode coupling.

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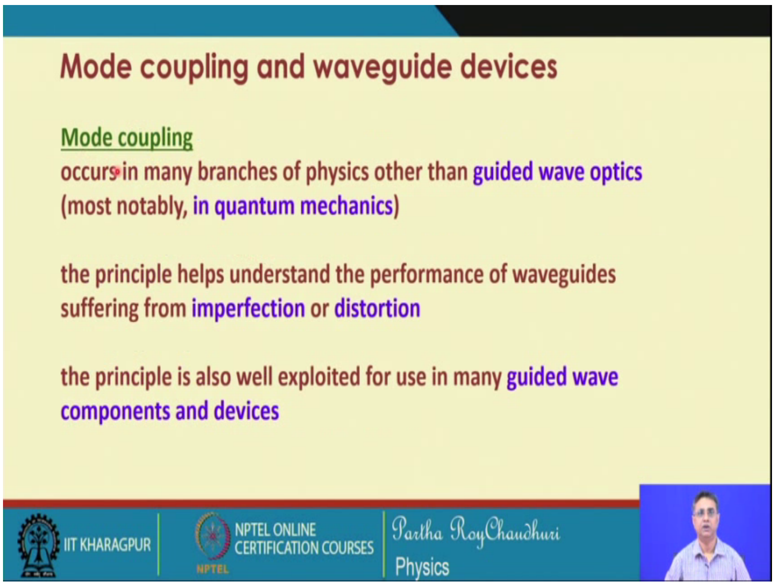

So, the mode coupling occurs in many branches of physics other than this guided wave optics and it is notably in an quantum mechanics that coupling of modes it occurs and have analyzed. That the principle help us understand the performance of wave guides suffering from imperfection or but, utilizing this drawback that there is a mode coupling because of distortion, because of imperfection, because of non uniformity along the length of the waveguide utilizing this inconvenience.

There are different branch of research that that goes into making use of this property for upward designing and developing devices the that is what is that the principle is well exploited for the use of many guided mode and components and devices so the basic principle is that the mode coupling.

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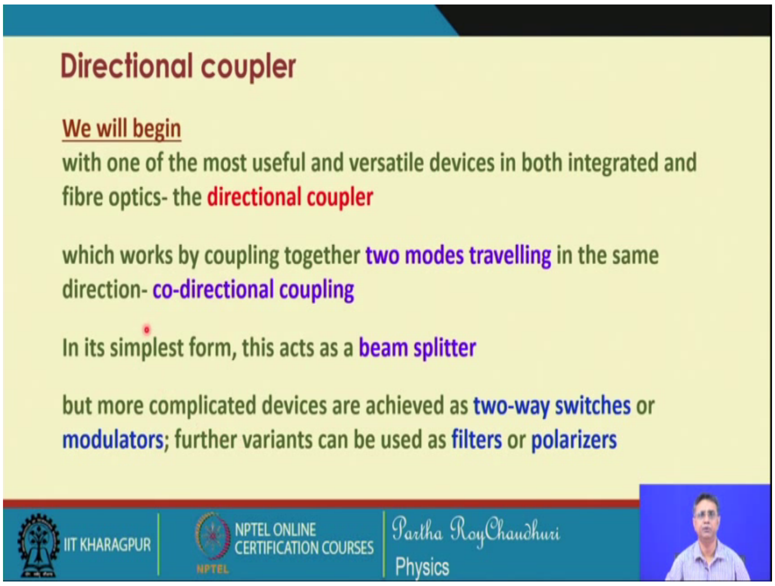

So, now we will begin with the most useful and versatile device which is very well known in both in integrated and fiber optics a directional coupler. The directional coupler works when the 2 modes up to individual wave guides they are brought together and both of the both the modes are traveling in the same direction then what is that is what we call the co directional coupling and in it is simplest form this acts as a beam splitter. Because, we will see that the beam which is there in one waveguide can be switched to the other waveguide back and forth and it can be divided into any desired ratio.

But more complicated devices are achieved as a 2 way switches modulators and there are many other applications of this mode coupling devices in terms of the directional couplers both in integrated optics as well as in fiber optics filters and polarizer. So, we will see some of the devices they are basic principles and the mechanism.

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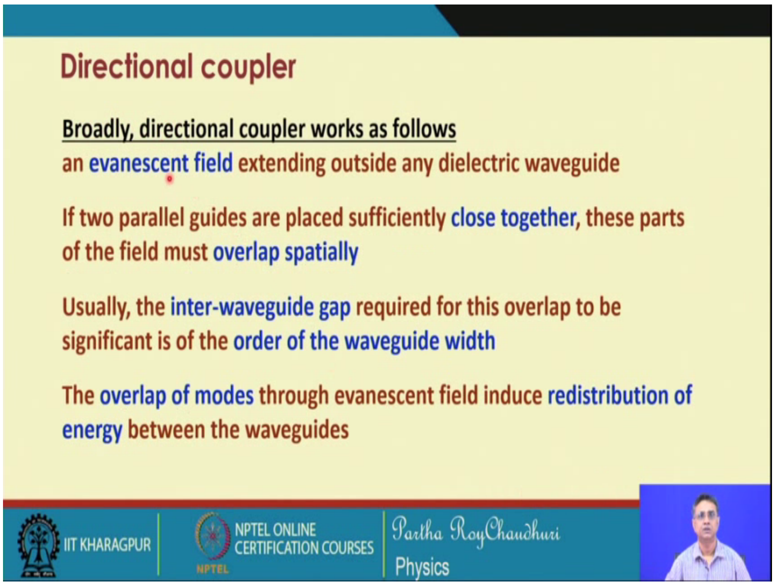

So, directional couplers works as follows there is an evanescent field evanescent tail which extend beyond the core cladding interface that we have seen in a waveguide and if 2 such wave guides are lying parallel to each other and sufficiently close then this evanescent tail of one waveguide will hit the other waveguide. Usually the inter waveguide gap required for this kind of overlapping is very small, such that significant amount of the evanescent field can hit the second whenever in waveguide. The overlap of the modes through evanescent field then influences a redistribution of the energy between the 2 waveguides.

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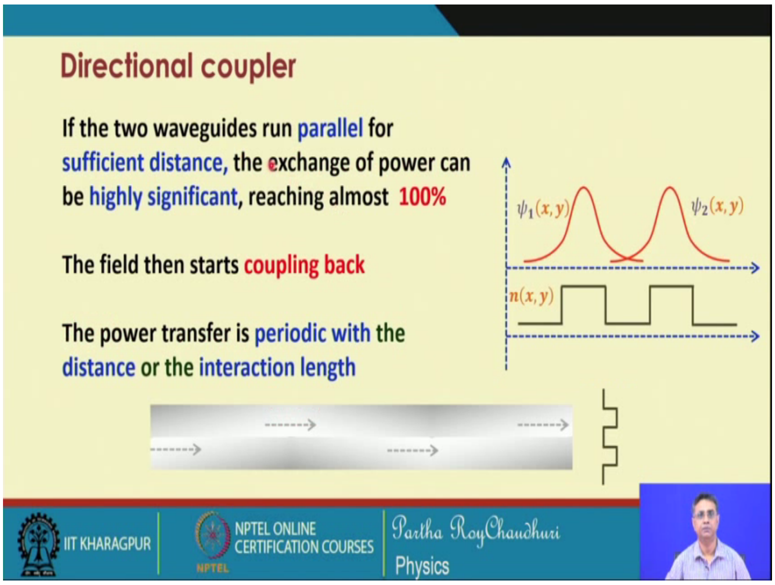

So, if we have 2 waveguide for example, this one and this one which have their individual modes like this the fundamental modes and this waveguide has their individual modes and you can see that there is an overlap of the modes through their evanescent tails.

So, and if these 2 waveguides can interact over a long length sufficient length, then it is possible that the exchange of power can be highly significant it can be 100 percent and if it is. So, then if it further goes beyond for interaction then the field can start coupling back to the first waveguide again, the power transfer is in this case periodic because this wave the power from the first waveguide goes to the second waveguide and from the second to the first waveguide and this wave it goes back and forth between the 2 waveguide. So, this is periodic with distance and over the length of the interaction.

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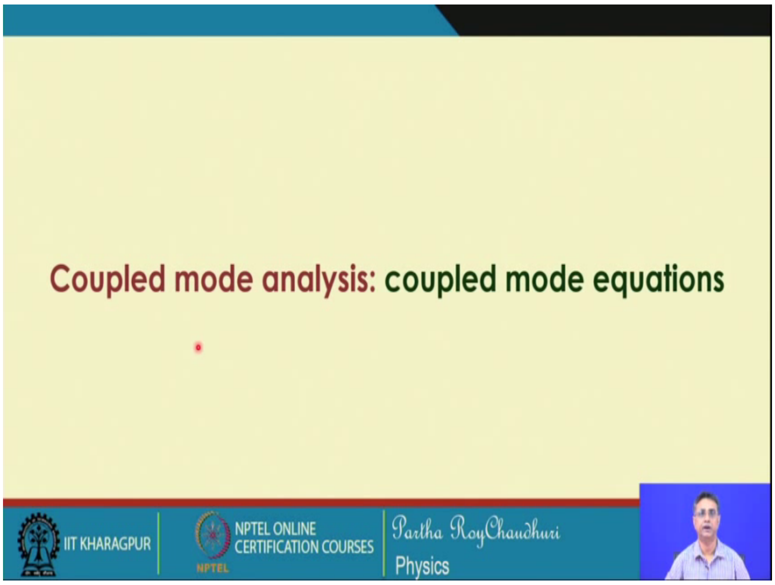

So, we will look at all these all these aspects mathematically a quantitatively through the couple mode analysis.

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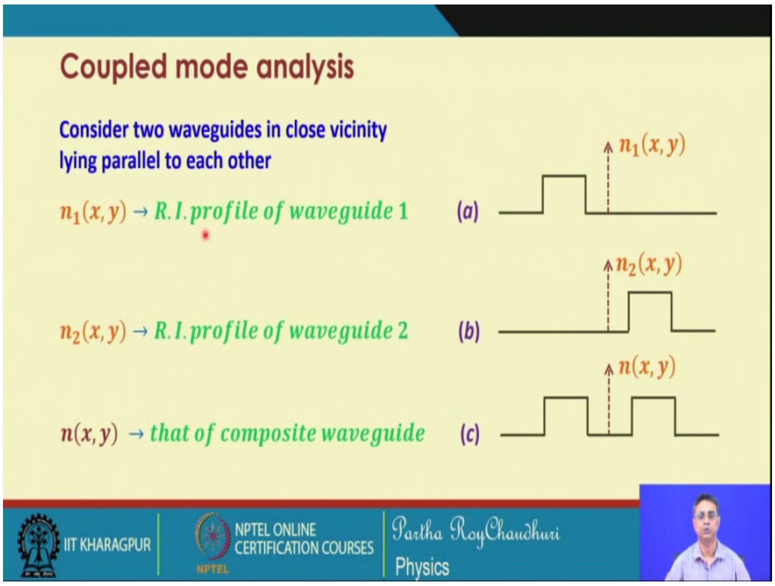

So, we will quickly go through the couple mode analysis, let us consider that you have 2 individual wave guide the refractive index profile of this is n 1 of x y and the refractive index of the second waveguide is n 2 of x y and when both of them are placed side by side, then we call this composite structure as a refractive index distribution which is n of x y. So, this is the combined waveguide when they are lying side by side.

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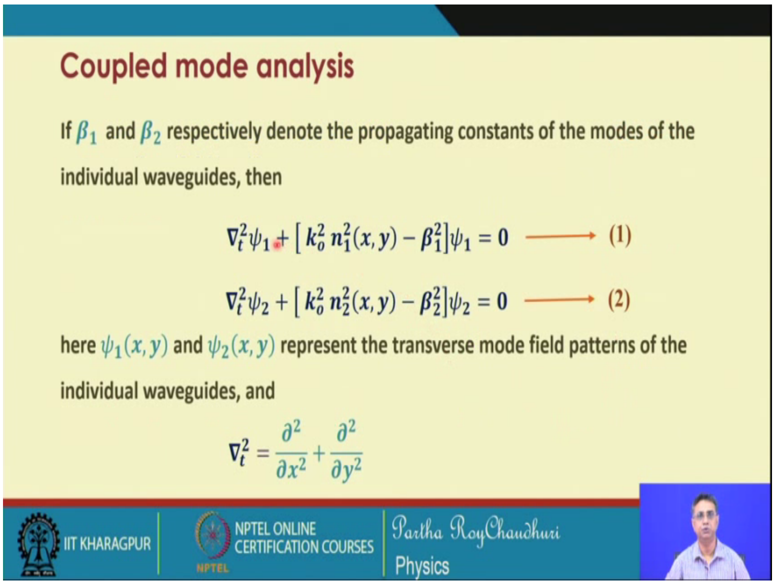

So, for the individual wave guides we have the we have the wave equation to solve this, this one which is satisfied by the mode of the waveguide first waveguide psi 1 and this is same for the second waveguide psi 2 they are corresponding to their refractive index profile n 1 square and n 2 square. So, this delta t transverse component of this del square is represented by this we have actually made this compact for this one del x and del y for this particular case.

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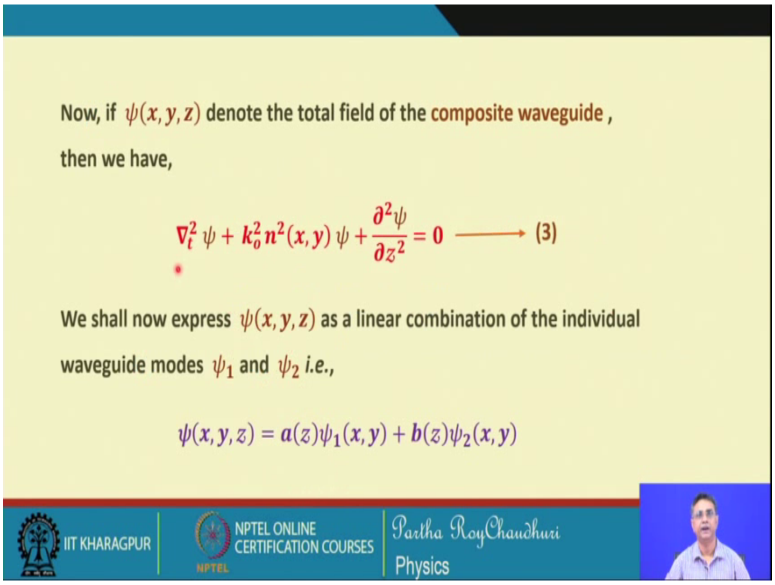

And then in this case if we look at the complete structure and look for the mode of the composite structures, the total waveguide which we will call psi x y z. Then we can represent in this form the transverse component of the Laplacian del the psi and this refractive index profile of the composite structure and this is the z component of the del square. So, now we shall now express this psi as a superposition as a linear combination of the individual waveguide modes that is psi 1 and psi 2. So, far all that we have discussed is that we have 2 individual wave guides and the modes of the wave guides are psi 1 and psi 2, now these 2 waveguides are placed side by side and we look at the complete structure the total structure the composite structure and we call the mode of this waveguide is psi.

So, psi 1 will be satisfied satisfying the wave equation for the first waveguide psi 2 will be satisfying the wave equation for the second waveguide and psi will be satisfying the composite waveguide the total wave guide and the psi must be a linear combination a superposition of the 2 individual waveguide modes. That is what is psi of x y z equal to a psi 1 plus b psi 2 a and b they are the z dependent amplitudes of this psi 1 and psi 2 the coefficient of psi 1 and psi 2.

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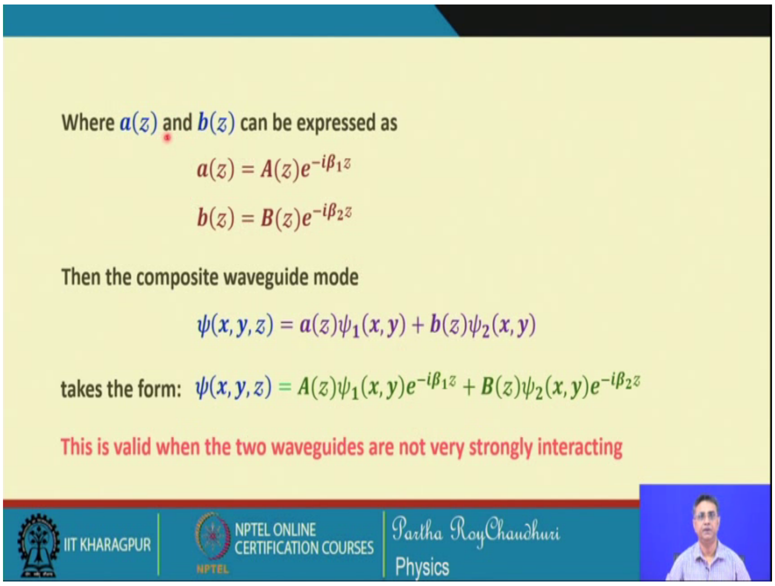

Now, this a and b these are z dependent and this can be written in this form because, it is only this along the z that the wave has a phase change. So, you can write a z capital A z e to the power of I beta z and similarly for B, then the composite waveguide mode can be expressed in this form that psi of x y z equal to A psi 1, then the mode of the individual wave guide and we write the mode of the individual wave guide. This is valid when the 2 waveguides are not very strongly interacting, so they are weakly interacting but there is a there is an interaction.

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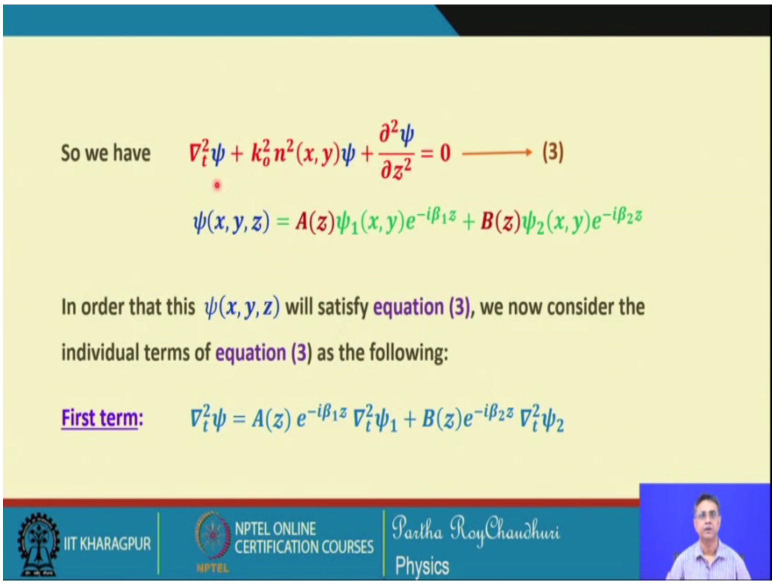

So, we will substitute this in the wave equation for the complete wave guide for the total structure and this will be the, this is the mode psi of x y z for which we will write this. In order that this psi x y z will satisfy the equation 3 this equation we will now consider this equation term by term because, otherwise will become too long an expression. So, the first term that is del square of del t square of psi we can write in this form because of this and the third term that is this one del square psi by del z square which is only z dependent.

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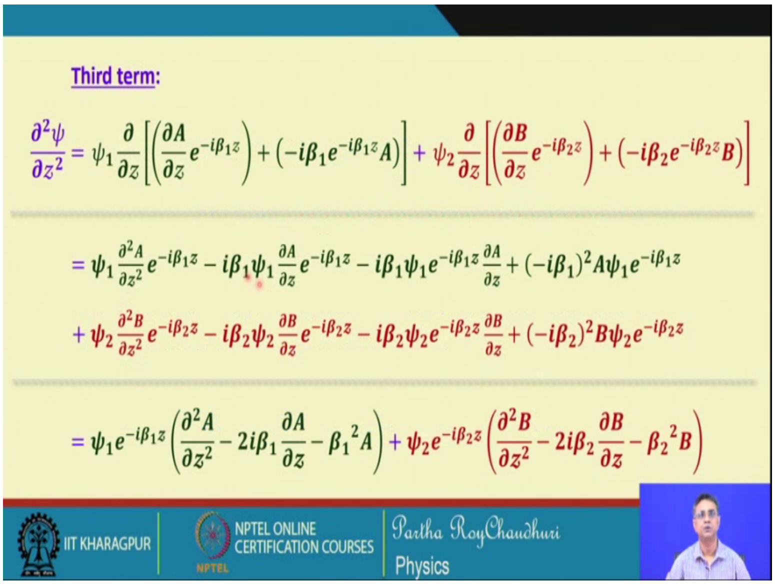

So, this will give you more number of terms that is if I take psi del square psi del z square, then this will be the first part will be this is because of the psi 1 and similarly this is because of the psi 2 will have to do the derivative of this. So, psi 1 will give you these 4 component and the psi 2 del, del z of this expression will give you this 4 component 4 terms in this expression of del square psi del z square and if we now segregate, if we now put them together so psi 1 e to the power of i beta 1 z is a common factor appearing everywhere in the in the terms for psi 1. So, we take them out psi 1 and this quantity so all that remains inside is this quantity and similarly for psi 2 we have this quantity right.

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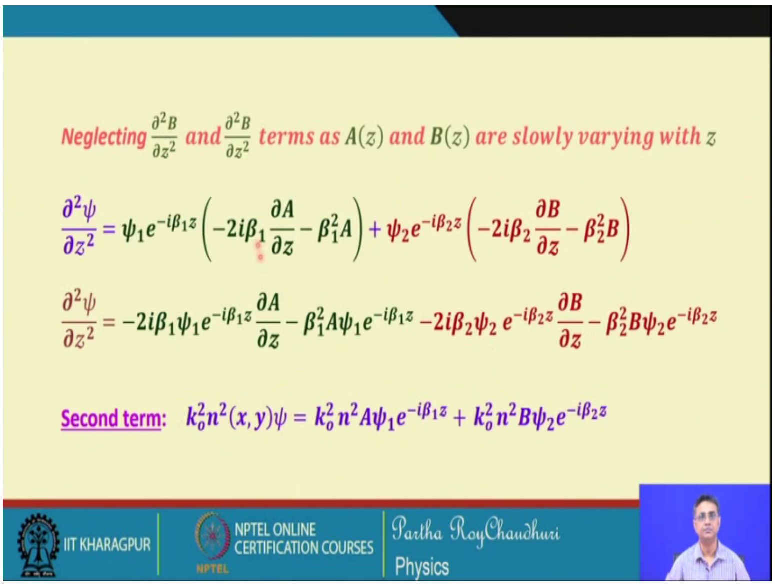

But this del square B del z square that is the change in the change of amplitude of the B and A are very small as long as the wave guides are slowly interacting. So, this will be A so this will be A and this will be so del square A del z square and del square B del z square these terms are second or derivative will be very small and therefore we can neglect this because the wave guides are a weakly interacting. So, in that case del square psi del z square will be equal to this we have neglected this term we have neglected this term and del square psi del z square can be written in the reduced form that is equal to this quantity.

Now, we will look at the second term of this of this equation that is k 0 square n square of x y z which is very straightforward and we can write this just multiply this n square with the component terms for psi 1 and psi 2, now we can put all the 3 terms together.

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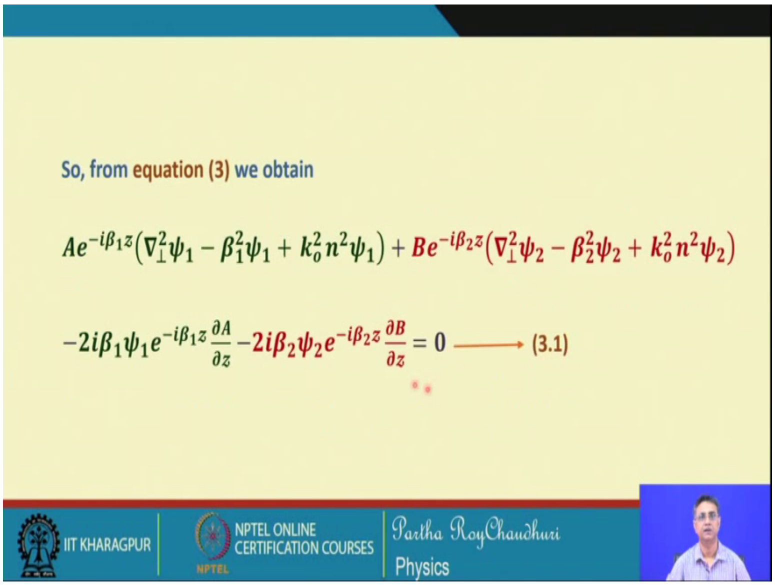

And from equation 3 then we can write this equal to Ae to the power of minus which is the common factor for the first 3 term and B e to the power of i beta 2 z is again a common factor for the were the last 3 term and if we put them in the bracket you can remember that this quantity is again the wave equation for the first wave guide and this is again the wave equation for the second wave guide.

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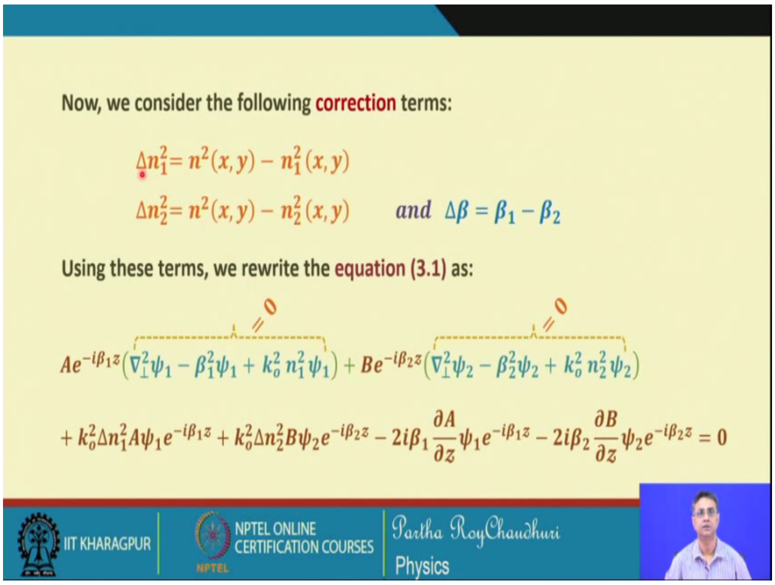

So, now we also make a correction term that the region where you have the only one wave guide, but looking at the composite structure which is represented by this n square. So, this is this is the correction which is introduced to this quantity to get this equation. So, we can treat this delta n 1 square as a correction to this quantity and similarly delta n 2 square is a correction to this composite wave guide structure. So, using these term we can rewrite this equation and also we will use delta B which is the difference between the propagation constants of the 2 individual wave guide mode.

So, we can write this equation and we have seen that this quantity is nothing but the wave equation that is satisfied by the first wave guide, similarly this equation is the wave equation satisfied by the first wave guide so they will be vanishing. And therefore, we have this equation 3.1 that is the last form of the wave equation for the composite wave guide.

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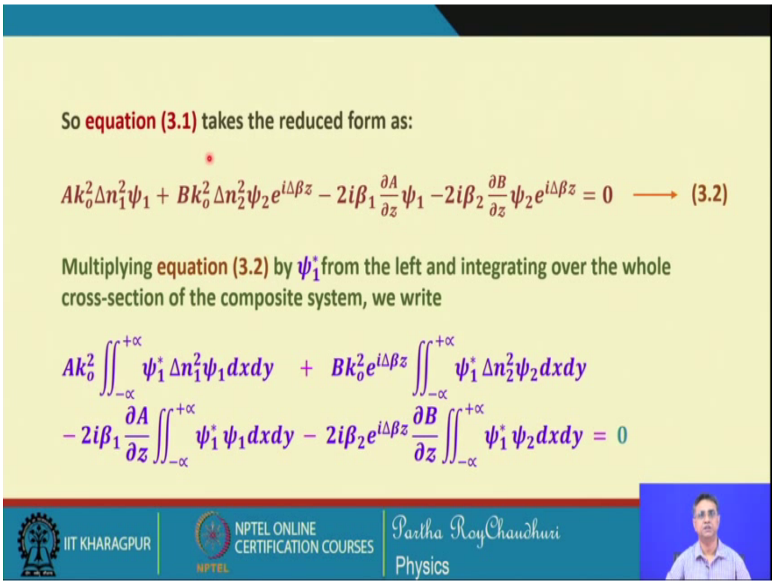

We can write this equation we contain term belonging to psi 1 and terms belonging to psi 2 in the form of psi 1 and del del A del z. So, now you multiply this equation from the left by psi 1 star and then integrate over the entire cross section of the total system that is the complete wave guide. Then you can write this equation in this form you have multiplied with psi 1 star from the left hand side and with delta n square between them which is the correction term.

So, you can write the first quantity like this the second quantity will be like this delta n 2 square will be there and because it is psi 2. So, psi 1 star delta n 2 square psi 2 d y d x and for these 2 quantities we can write these 2 equation. So, this is the form of this equation when you have taken the integration over the whole square to see.

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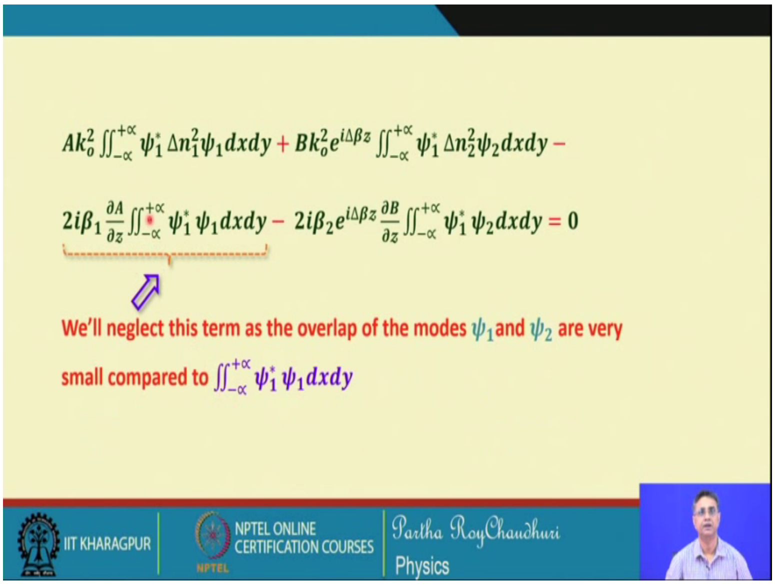

That how this wave equations look like Ak square in integration from minus infinity to plus infinity, this quantity will see that this represents the coupling constant this also and this also. So, we will we will neglect this term because it is psi 1 star this under normalization this is the same mode this is the same mode and similarly for this term also not this term psi 1 star ok. So, similarly we will neglect this term for the overlap of the mode psi 1, so the it represents the overlap of the mode with itself. So which is insignificant which has no meaning so this term we can neglect and then this term can be represented by del A del z is equal to this quantity.

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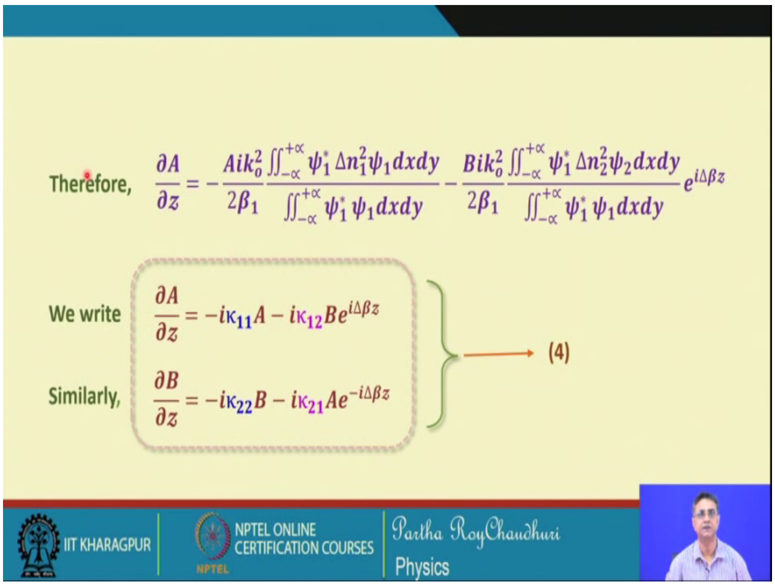

So, we can write this equation that del A del z this part is equal to i kappa 11 A minus i kappa 1 to B e to the power of I delta beta z and similarly we can also write for the second part that is delta B. So, del B del z will be equal to i kappa 22 B minus i kappa 21 A e to the power of minus beta z, so these are the form of the wave equation the coupled wave equations.

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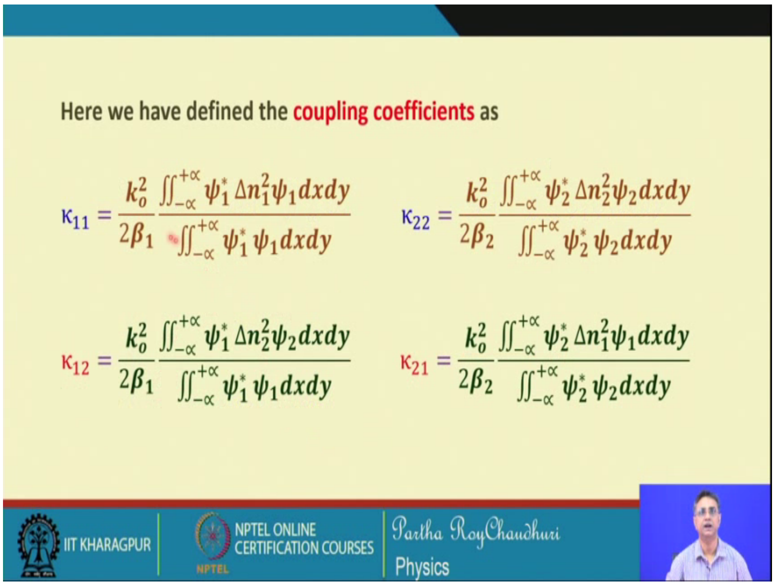

And you can see that we have defined the coupling coefficient that is that is kappa 11 kappa 12 kappa 22 kappa 21 this coupling coefficients. We have used for this expression that is k 0 square by twice beta 1 which will give you the coupling coefficient this kappa 11 that is k 1 star k delta n square psi 1 and kappa 22 which will include this psi 2 delta n 2 square psi 2 and kappa 12 kappa 21 are defined in this wave.

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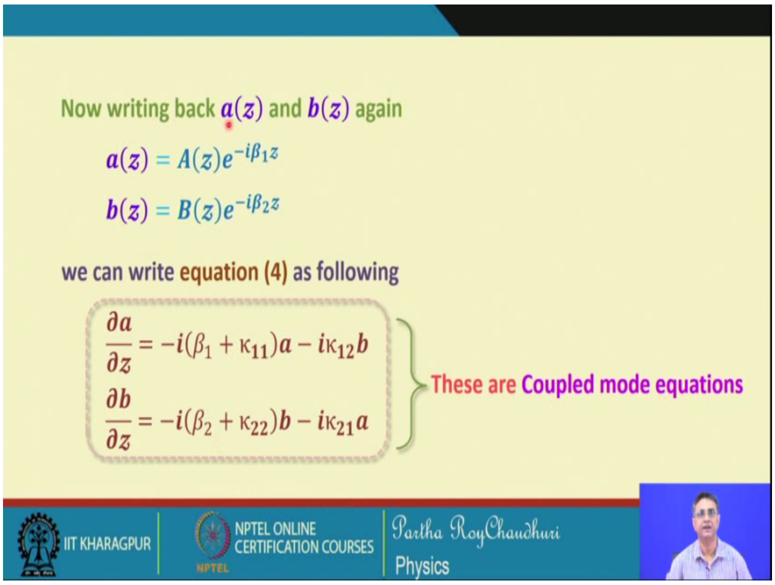

Now, writing back A z and B z again a z equal to this A z e to the power of minus i beta 1 z and B z equal to B e to the power of minus i beta 2 z. So, we can write this equation the pair of equations in this form and this is what we call the couple mode equation, it tells you that the amplitude field amplitude of the wave guide in A how it varies with z. It appears that the variation is related to the amplitude in the waveguide A itself and the interaction of the amplitude in the waveguide B through the coefficient K 12 and similarly the amplitude variation in the wave guide B along the length of the of the of that the system that is z.

In that case it is the related to the amplitude of the field in the waveguide amplitude of the mode in the waveguide to itself and the interaction with the amplitude of the wave in the waveguide 1. So, these are the couple mode equations very famous and well known and used in many occasions in physics when the coupling of waves are concerned and from this expression we can see that K 11 and K 22 these quantities are the corrections to the propagation constants of the individual wave guide.

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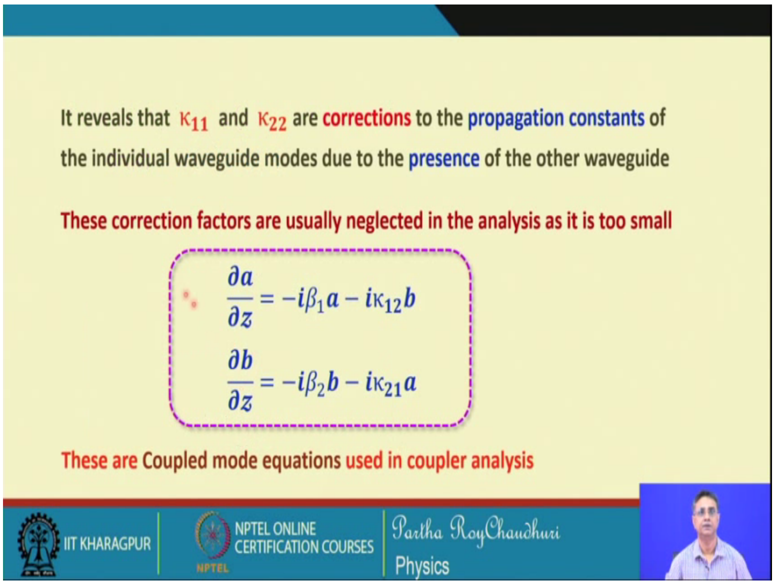

So, when we if I if I take del a del z for the waveguide individual waveguide which is represented by some a into e to the power of i beta 1 z. So, if it is not non-interacting then I write this del a del z equal to minus a minus i beta one into a, but in this case it has appeared in association with another term that is K 11. So, this is only the correction to the propagation constant of the mode of the waveguide 1 and similarly this is a correction to the propagation constant of the mode of the waveguide 2, so these are only small correction terms which can be which can be neglected.

Then for weakly interacting wave guide and then we can we can after neglecting the correction terms we can write the couple mode equations. In this form del a del z equal to minus i beta 1 a minus i kappa 12 b and similarly for del b del z that is the variation in the amplitude of the field in wave guide 2 will be given by these 2 equation. These are the well known couple mode equation which is used in the analysis of wave guide couplers.

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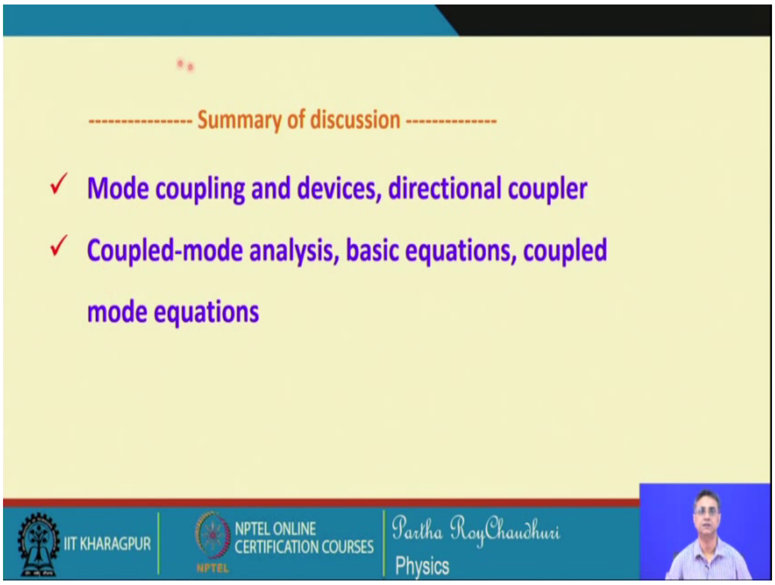

So, in this discussion we have we have brought in the coupling of the waves, when the wave guides are otherwise non-uniform or they are interacting with the neighboring wave guide there are devices made out of this principle that we have discussed and we will be discussing more about this. Then we talked about the directional coupler how the wave guides 2 wave guides can we will look at more details of the directional couplers in the next part. We then for couple mode equations we discussed the basic equations and we actually brought in finally the couple mode equations.

Thank you.