Modern Optics
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Lecture - 21
Waves in guided structures and modes (Contd.)

Then will continue with the symmetric planar dielectric interface waveguide, which is very simple and common and very basic to all optical waveguides. And it is actually a very good lesson to understand the basic mechanism of wave guidance in optically Wave Guiding Structure.

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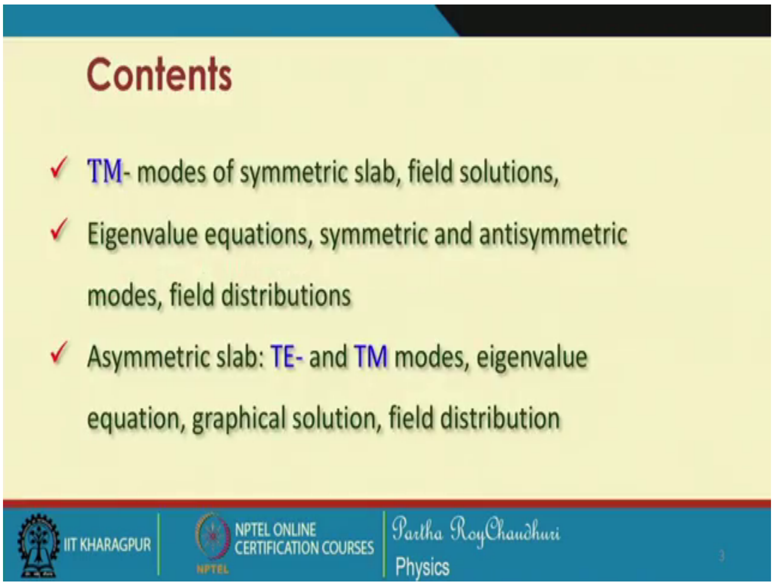

So, we will continue with the TM modes of the symmetric slab waveguide. And will look at the field solutions which will see almost similar to the those of the TE modes. And in this context will work out the eigenvalue equation the symmetric and antisymmetric modes, which will be almost similar then the field distribution. Then we will switch over to another basic and very practical optical waveguides that is; an asymmetric structure and for that we will also look at the TE and TM modes, the eigenvalue equations look for the graphical solution and then the field distribution.

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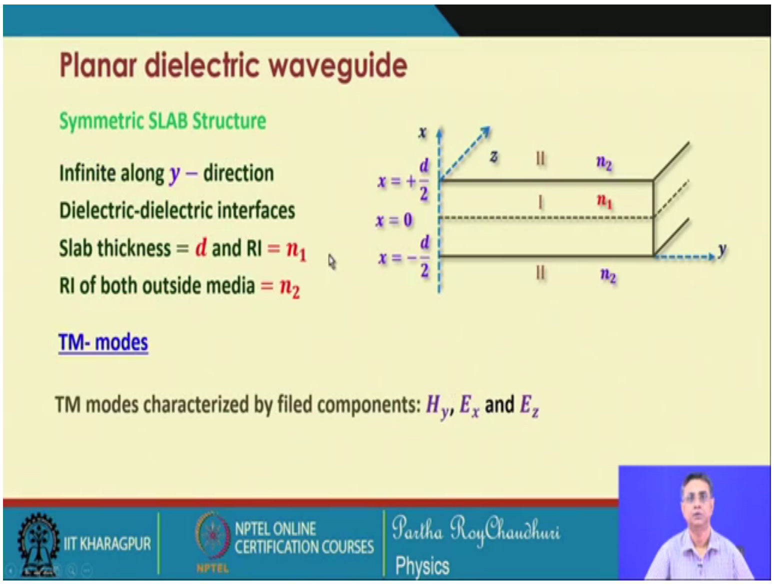

So, to do that we again consider the same a planar slab waveguide structure and where you have the same coordinate axis at the axial line that it x equal to 0 same structure, but this time will consider the TM modes. And for TM modes these are characterized by H y field, which is the tangential component of the magnetic field, and this E X and E Z will be associated with this TM mode.

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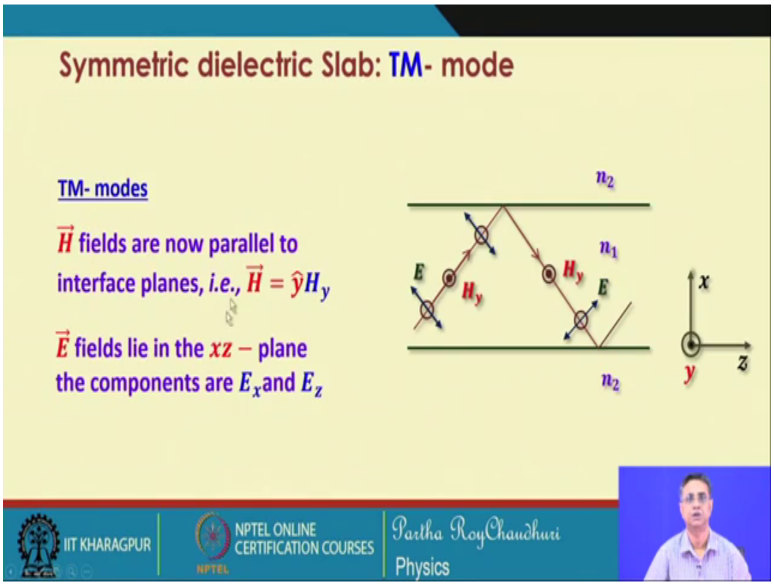

So, the TM fields if you look at the configuration that H y will be tangential to the plane of the interface will be parallel to the plane of the interface, but this time E will have 2 components E Z and E X. So, the H plane H fields are now represented by y of y cap H y.

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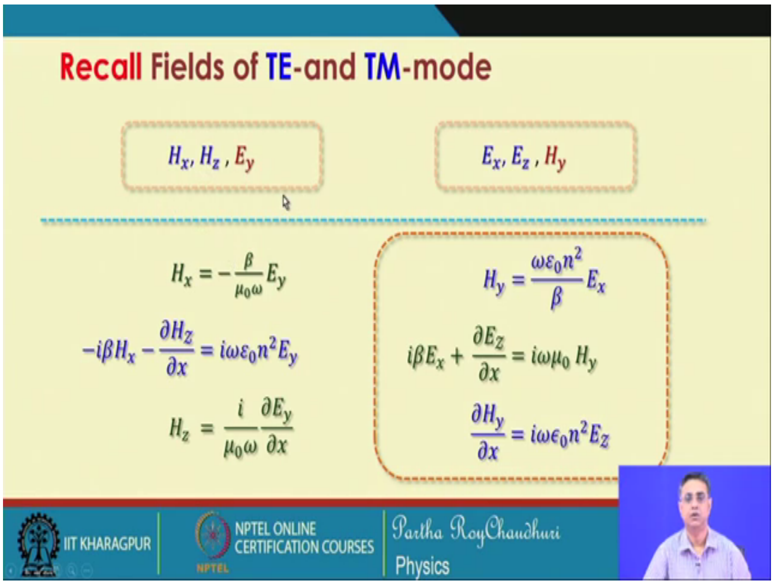

Now, let us recall the TE and TM mode and their fields associated fields. So, TE mode is characterized by H x H z and E Y, where as TM modes are characterized by this and the corresponding equations this we have seen at that H x is connected to E y, H x is connected H x and H z is connected to E ym E y is also connected to H z; that means, all the tangential components of the electric field for this mode they are intern connected to H x and H z. So, by this equation and likewise for the TM modes we have a similar relation H y, H y and H y; they are connected to E X, E Z, E X, E Z. And from there we can organize the wave equation for H y E X or E Z.

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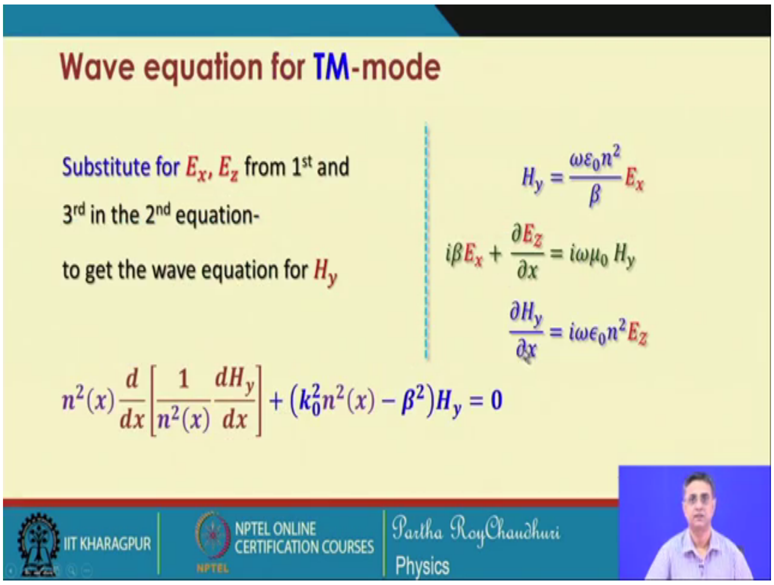

Will look at the wave equation for H y in this context, if you if you take if you substitute E X, if you substitute this value of E X in terms of H y into these equation and the value of E Z from these equation in terms of this into this equation; that means, what I am trying to tell is H y will be E E E X value will be coming here, and E Z value will be coming here. And will enrich this equation 2 which will be expressed in terms of only H y field then you get this equation, you will get this equation from here just by substituting the first and third from the second. Then this equation represents the wave equation for H y field.

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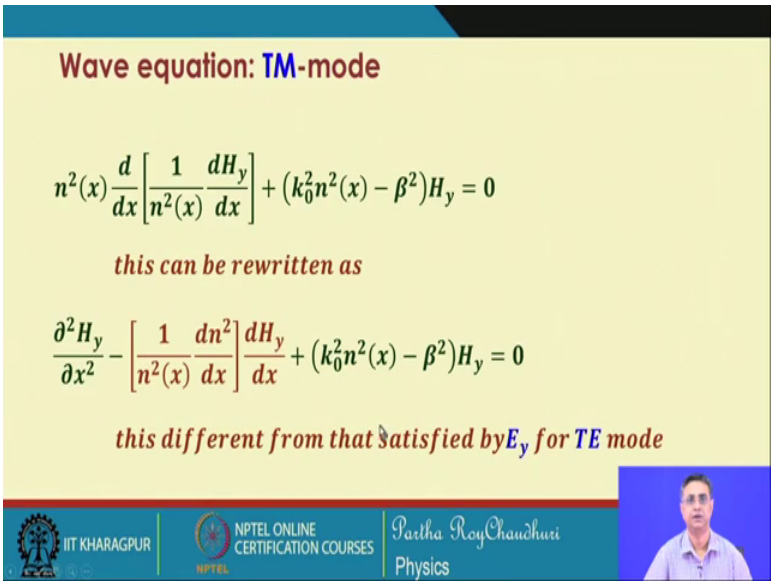

But you see there is a because of the, because this is a TM mode and you have a refractive index discontinuity at the interfaces so that is taken care by this equation. This we have discussed earlier also, so this equation can be rewritten in terms of this equation it will more simplified form. And this equation is now different from the TE mode equation. Now which was satisfied by E y mode.

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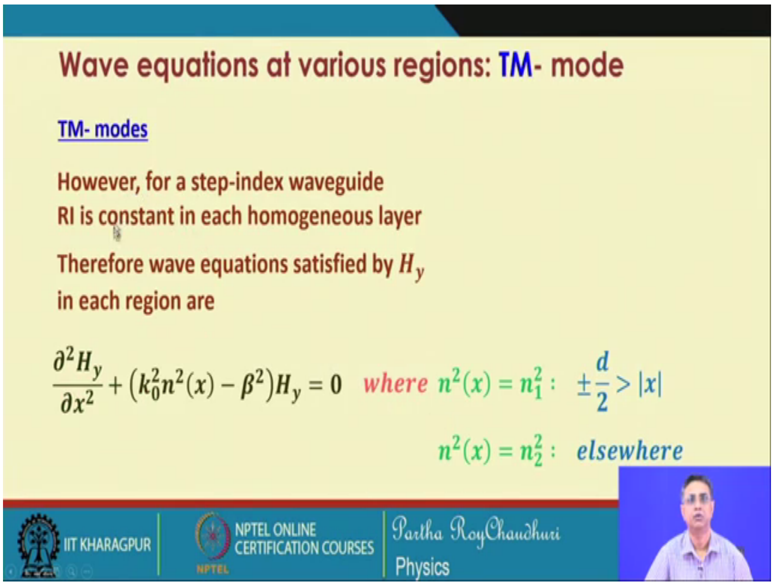

Now for TM modes because we are dealing with the layers and their piecewise homogeneous layer. So, we can remove the discontinuity this second term and we can write we can write the wave equation for the each layers. So, that is how these n square x plus minus mod x plus minus d by 2, I write the same way as we wrote in the case of TE modes.

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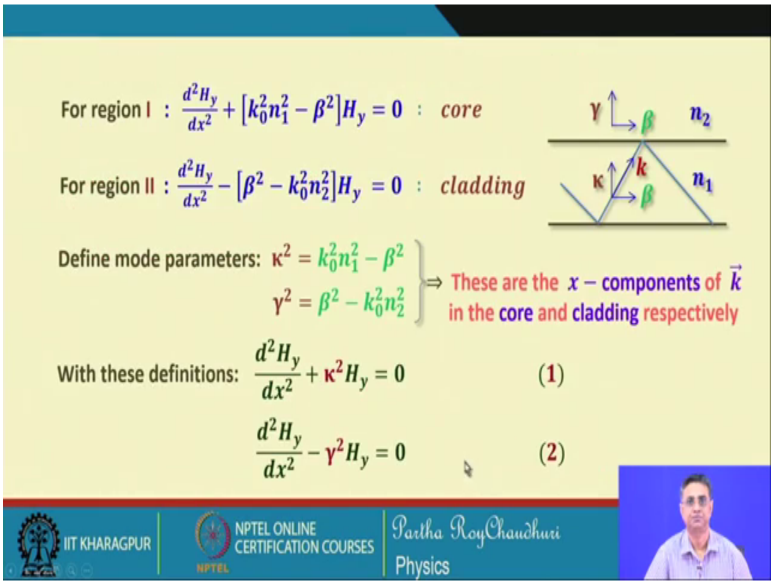

Then we have to solve these equation in the 2 regions that is the region one there is a core region and the cladding. And proceeding in the same way defining the more parameters that is kappa and gamma we can rewrite these 2 equations.

But this time the equations are satisfied by H y, which will have a different boundary condition.

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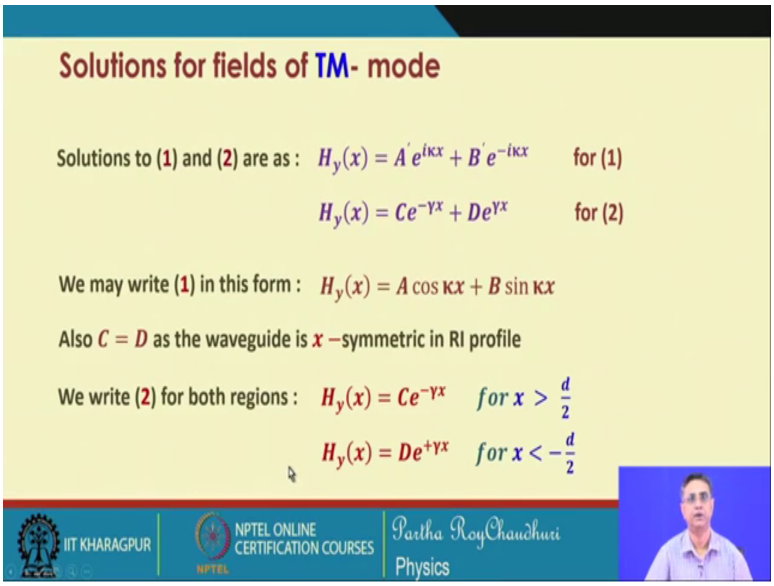

The solutions to 1 and 2 at this which is again the same as we as we have seen in the case of TE modes. So, again in the same way we can write this equation H y in terms of sin and cosine functions. C and D are also same. So everything remaining the same we write the exponentially decaying field at the cladding region in terms of e to the power of minus gamma x and e to the power of plus gamma x for the positive, and negative x regions of the cladding layer.

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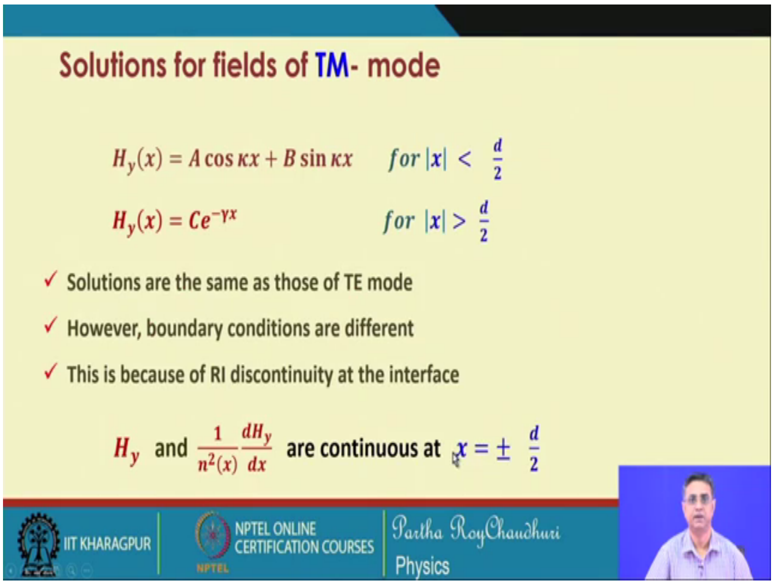

So, by doing this we have a set of field equations satisfied by the wave in the cladding and in the cladding and in the core region. So, solutions are same as those of TE modes, but the boundary conditions this time are different, because now it is not the del H y del x, but this quantity will be continuous across the interface and this quantity will be continuous across them.

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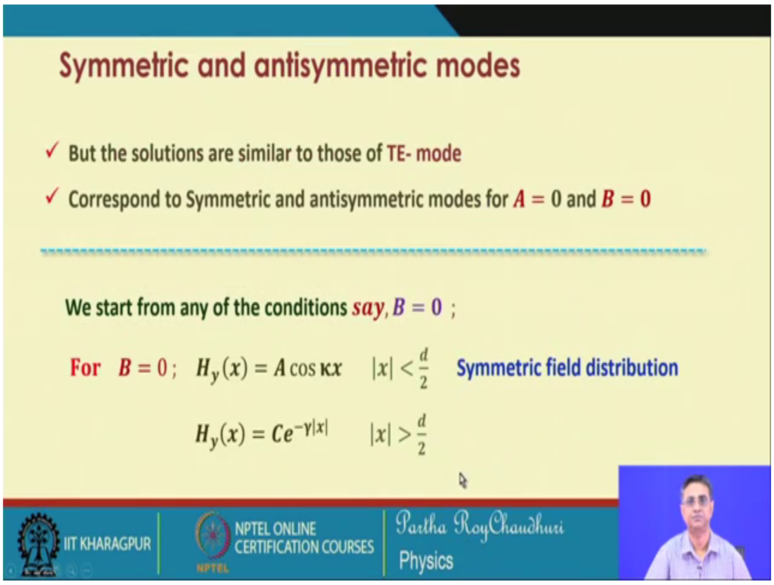

So, if we plug in these conditions into the equation, then again in the same way we will get that either A equal to 0 or B equal to 0 for these equation. A equal to 0 or B equal to 0 which will be a consequence of the boundary conditions and that will lead to the eigenvalue equation. If a equal to 0 you can see H y will be antisymmetric, but if b equal to 0 you can see that H y will be equal to will be symmetric modes that is a cosine kappa x.

So, for B equal to 0, if we move with this condition B equal to 0 that is for symmetric field distribution, this is the same as the one which we have discussed for the TE modes.

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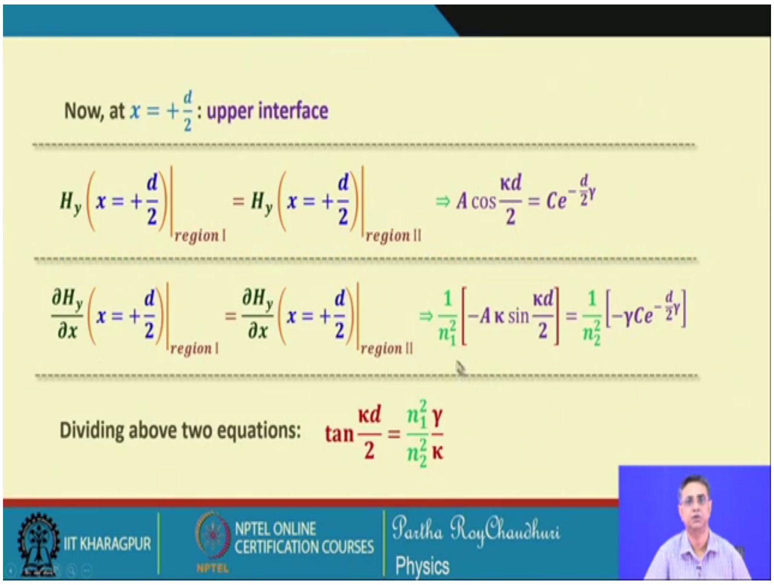

And put the boundary condition and dividing these 2 equations in the same way we end up with this condition, but this time for TM modes the difference is that it is you have n 1 square by n 2 square. Which are the refractive indices square associated with that layer. Gamma is the x component of the propagation, propagation vector for the cladding layer, and the refractive index of that layer is n 2 square. Similarly, this is the propagation constant of the x component of propagation constant for the core, which is associated with the refractive index of this.

So, in this form it appears in the eigenvalue equation for B equal to 0 that is for the symmetric modes cosine function modes.

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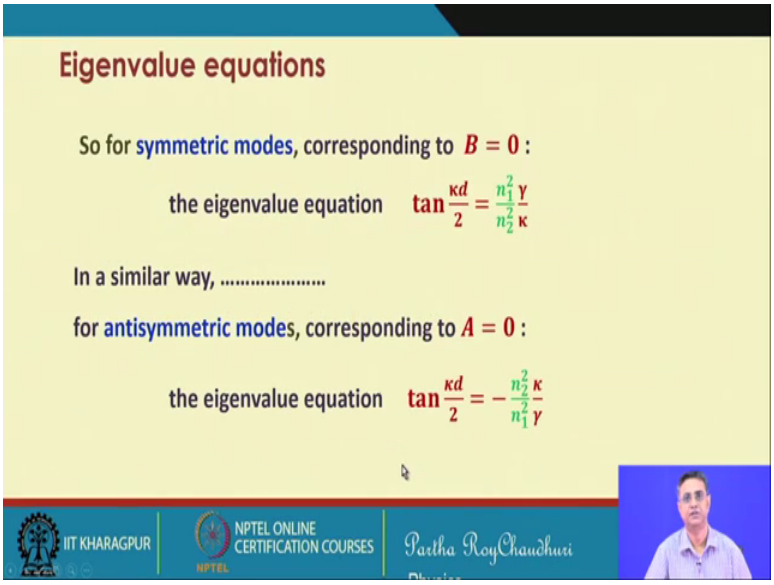

Similarly, for the antisymmetric modes that is when you put a equal to 0 will get the same equation, which is again similar to the modes of the TE modes of the structure, but there is a difference of this quantity that kappa will be associated with n 1 square, and gamma will be with n 2 square. Change this form it will appear just because of the boundary condition that is the continuity of this quantity across the interface.

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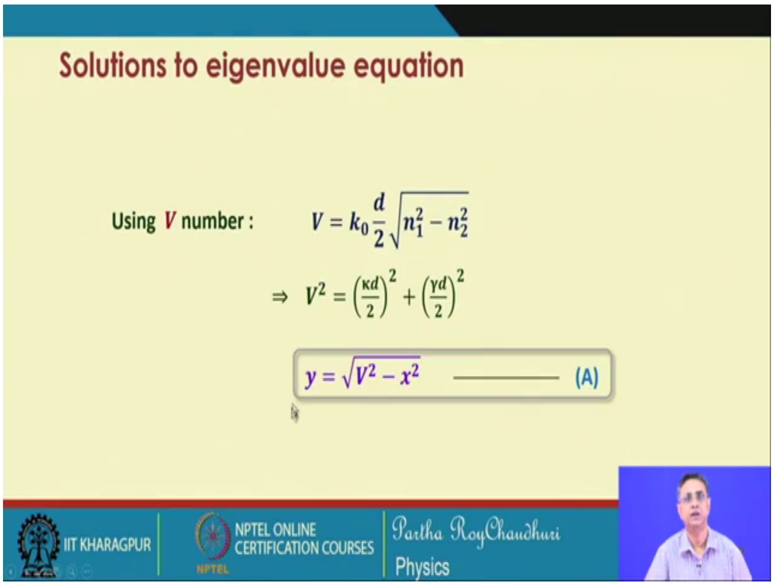

So, this is the difference which appears and now that; again the V number gives you this equation, this time y we will write this equation in terms of this y equal to V square minus x square.

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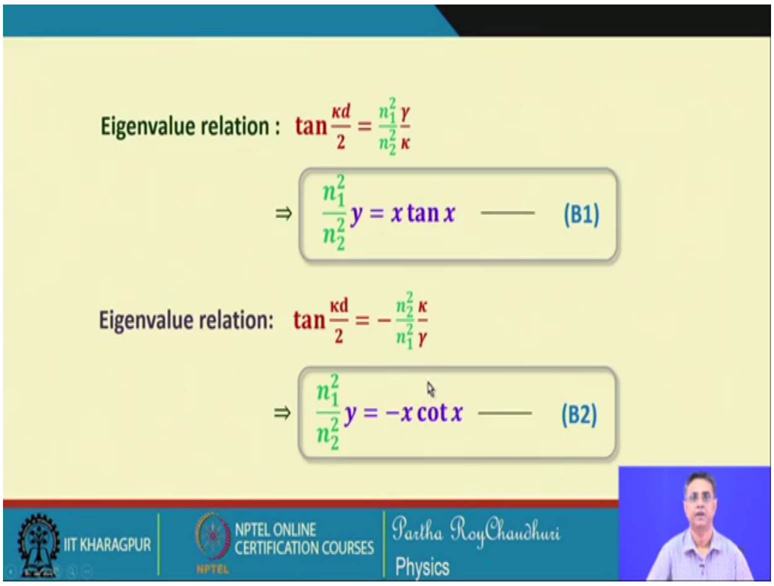

And for the eigenvalue equation for the symmetric modes we can write this equation because you see we have written this is equal to y. And so, therefore, x tan x will come in this form. And for the for the antisymmetric mode we have this equation, they are almost similar except the factor n 1 square by n 2 square which are appearing with y another left hand side.

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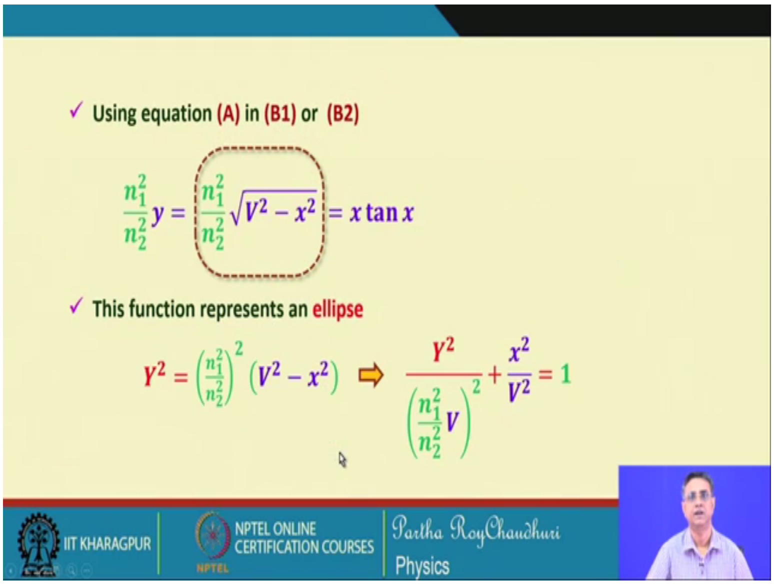

And then we can write combining this equation A with B 1 and B 2 we can get this equation we can get this equation.

So, y equal to this, B 1 equal to this so for this in place of y if I write this equation then I can write this into y, which will be equal to this and that is equal to x tan x because this is a y. So, these equation we look at this function n square n 1 square by n 2 square under root of this is equal to this function is a function of an ellipse.

If you square then y square will be equal to square of this quantity the function represents an ellipse, y square will become equal to this. And you have the equation of ellipse of this form. So, in this case it is not a circle the intersection of the circle with the graphs that we saw in the case of TE modes, but these are the ellipse the equation of the ellipse that gives you the solution through the intersection with the graphs.

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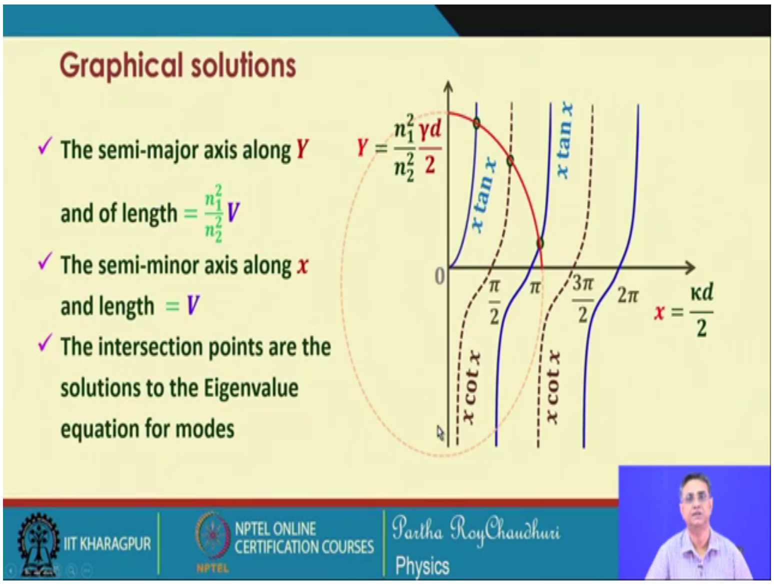

So, you can look at this ellipse it has it is y, as the y as the semi major axis along y which will be of length equal to this, and semi minor axis along this because n one is greater than n 2. So, this is more this quantity is more and this quantity just V. So, that this factor time this will be the ratio of the semi major and minor axis. And now that we have the solution points which is the case of TM mode. So, that intersection points are the solution for this.

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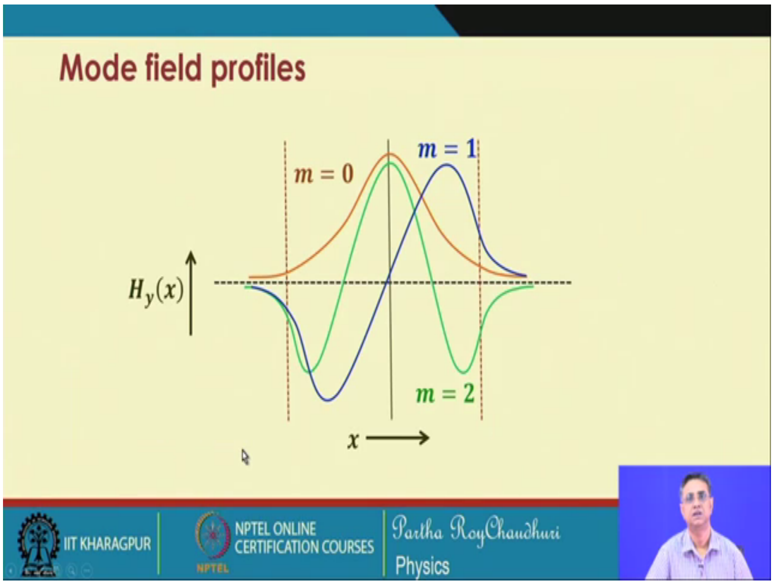

In this case we will get these guided modes for m equal to 0 this is the fundamental mode. For m equal to 1 the first order mode antisymmetric mode, and again m equal to 2 you get the symmetric mode, which are which are almost same, but the magnitude and the values are slightly different from the from the E y field. E y and H y they are related through C.

Now, this part we have discuss this planar symmetric waveguide we will take a more practical and more useful, waveguide which is the planar asymmetric slab waveguide.

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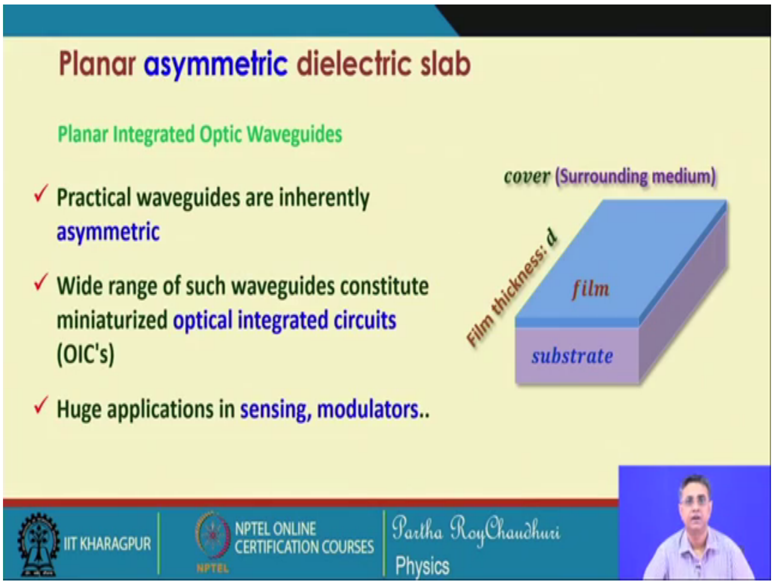

We see that this planner integrated optical waveguides optical circuits. Most of the practical waveguides are inherently asymmetric because even with the computer controlled system and with all advanced technology of fabrication the waveguides are usually to some extent asymmetric. Sometimes it is deliberately made asymmetric, and wide range of such waveguides constitute miniaturized optical integrated circuits that is what we call that OICs. And that is the most abundant most you know huge application of this waveguides. In the sensing in modulators, and in the telecommunication nodes, junctions and integrated optic devices.

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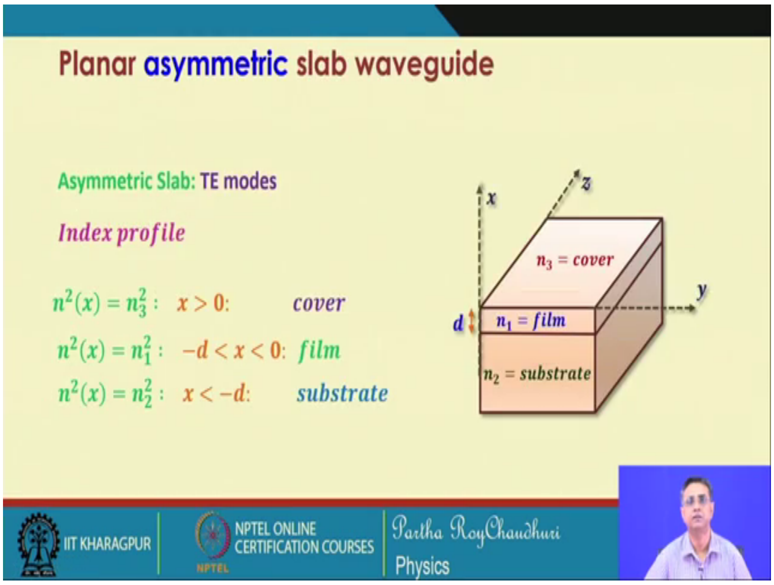

So, with that background, we look at the geometry of this. So, this is again considered that the waveguide is infinite along the wave direction. So, we have taken on the truncated view of this as if it looks like a truncated structure in the x y plane, but actually it is infinite. So, you can write that index profile in this case as this, because this will be the top layer will be their cover. The surrounding layer which is air in this case or it could be some material. The film n 1 is the guiding region and this is the cladding which is the substrate region.

So, I define this parameters in this way film, cover and substrate the way it appears in the waveguide.

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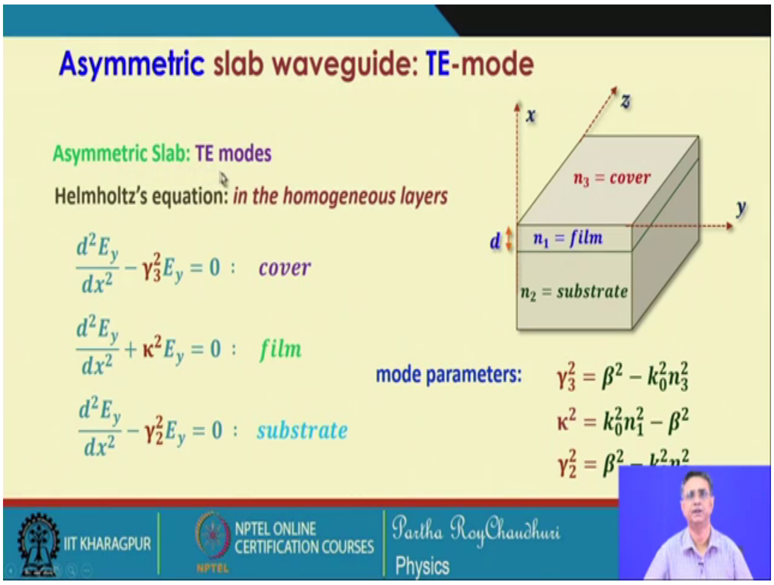

For asymmetric slab will first look at the TE modes and for TE modes we have seen these are the wave equations because in this case we define the mode parameters gamma 3 as beta 2 minus k 0 n 3, for this that the cover layer. And for the substrate you have n 2 square this quantity. So, that is that corresponds to gamma 2 square and kappa remains the same for the film. So, we have instead of 2 layers, now we have 3 different layers all 3 of them are different and we have 3 equations we have to match the conditions at the interfaces.

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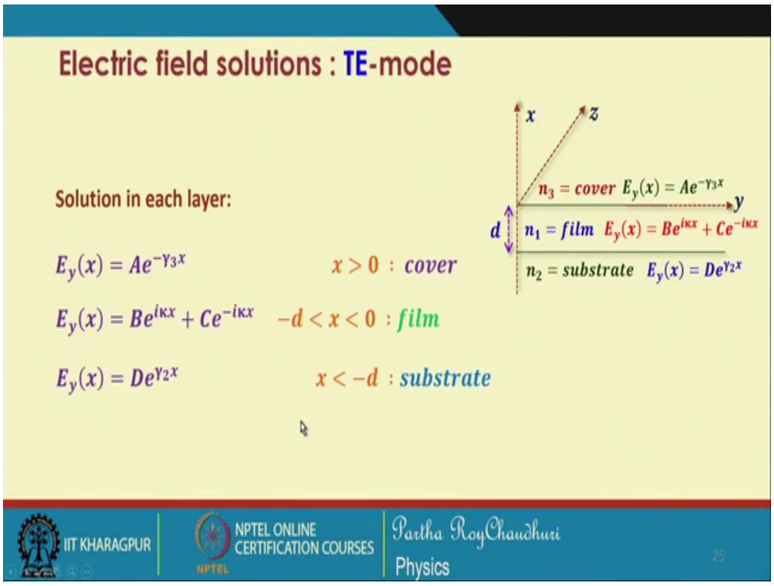

And the solution for each layer we know that the middle layer will give you an oscillator resolution, sin cosine function, where as the 2 top and bottom layers they will give you the exponentially decaying function. And these are corresponding to the gamma 3 the x component of the propagation vector at that layer.

So, this is because of the minus of this d. So, we set the coordinate system this time here at this layer, at this layer and this will be equal to x equal to minus d x equal to minus d this is x equal to 0.

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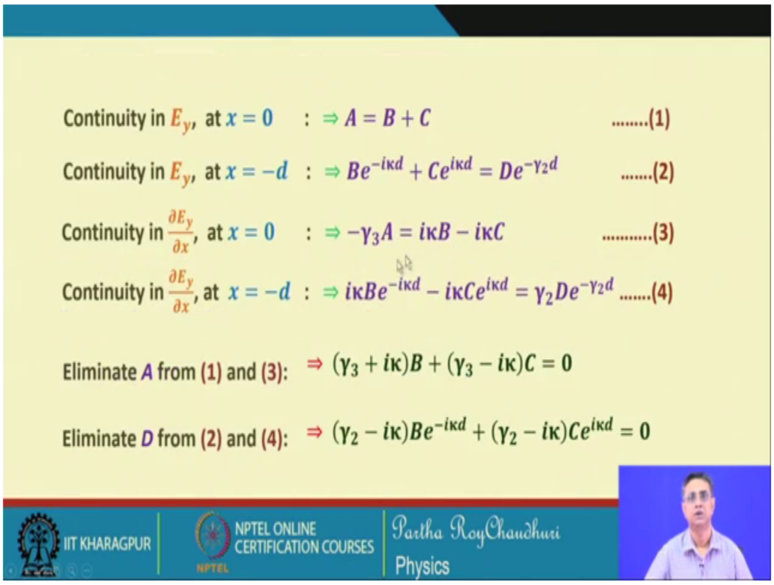

So now, we bring in the continuity conditions that at x equal to 0, you have A equal to B plus C, will look at this at x equal to 0, x is equal to 0 A equal to B plus C. So, if you connect these 2 equations for putting x equal to 0, and x equal to minus d, again if we if we write this equal to minus d, we can write that A e to the power of minus gamma 3 d so you can bring in this equation. So, this is the connection between the between the, this layer and this layer.

So, we are actually matching the continuity condition at this interface that is, at x equal to 0 and x equal to minus d both of them if you put together. Then you will look at the continuity of the derivative of the field, this is the same routine you follow for the continuity conditions E y, the tangential component and it is derivative so that gives you this equation.

Now, we have to do little bit of algebra which will give you this condition from 1 and 3, if we eliminate then and we can write this equation. And if we eliminate d from 2 and 4 then will get this equation. So, we have 2 more equations which are the outcomes of the set of 4 continuity conditions.

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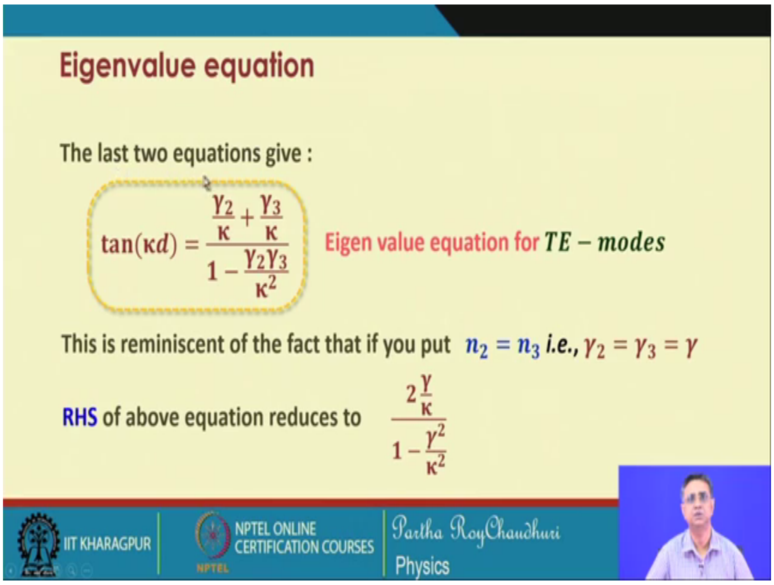

The last 2 equations that is these 2 equations; we have C we have C we have B and B will give you this condition. The simple algebra calculation will give you this. So, therefore, you get the eigenvalue equation for TE modes in this form, but TE modes you have got the eigenvalue equation is this.

So, this is actually takes care of both the symmetric and antisymmetric mode, and that we can easily show by taking this equal to tan theta one this is equal to tan theta 2. So, tan theta 1 plus tan theta 2 by 1 minus tan square theta 1 into tan square theta 2, will give you tan of theta 1 plus theta 2. So, this tan k d will be equal to tan of theta 1 plus theta 2 you can put 1 pi on either this side which will again equate and if the pi or multiples of pi, that is n pi will give you all the possible values of integral values of a plane which will correspond to different modes, which is very easy to.

So,, but this looking at this equation for antisymmetric waveguide, asymmetric waveguide we it reminds us about the fact that if you put the 2 layers the same that is top and bottom layer that is if you make n 1 n 2 and n 3 the same then it becomes gamma 2 equal to gamma 3. Let us suppose that is equal to gamma. Then the right hand side of this equation becomes twice gamma by kappa 1 minus gamma square by kappa square.

So, if you put gamma by kappa equal to tan theta.

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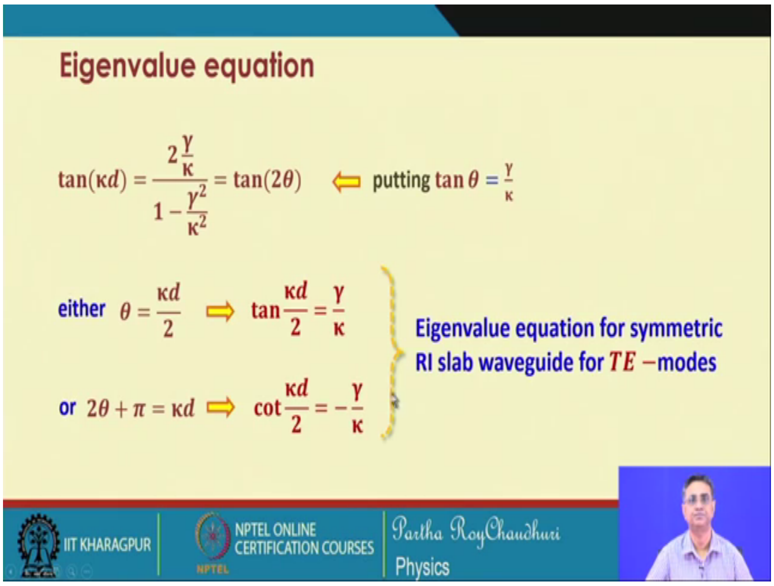

So, then it is 2 tan square theta by 1 minus tan square theta which is simply tan 2 theta by putting this, and this tan 2 theta must be equal to k d tan kappa d, which gives you a condition that theta equal to kappa d by 2 this is one possibility. Or it could be so that this kappa d is equal to twice theta plus, pi third quadrant. So, that is equal to kappa d so this gives you this condition if you divide throughout by 2 then theta will become k d minus pi by 2.

If you put m into pi by 2 that gives you all the conditions, that will also give you back this condition as well for symmetric and antisymmetric mode. So, this reminds you that you can reduce the symmetric refractive index profile waveguides eigenvalue equation from the antisymmetric mode, just by physically making the 2 layers identical ok.

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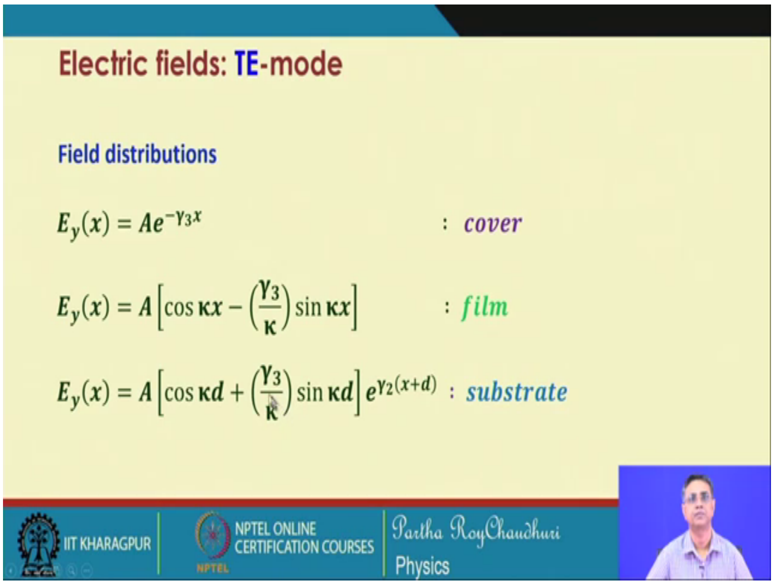

So, the electric field distribution for this waveguide will look like this, we have seen that this is evanescently decaying field in the film cover interface starting from field cover interface. And this will be that is in the cover region, this will be the oscillatory field which is the main part of the major part major fraction of the guided mode, which is confined into the film and evanescently decaying field in the substrate. So, because the field has already moved by distance of d, the exponential factor appears in this from here, x is necessarily minus x and must be greater than d.

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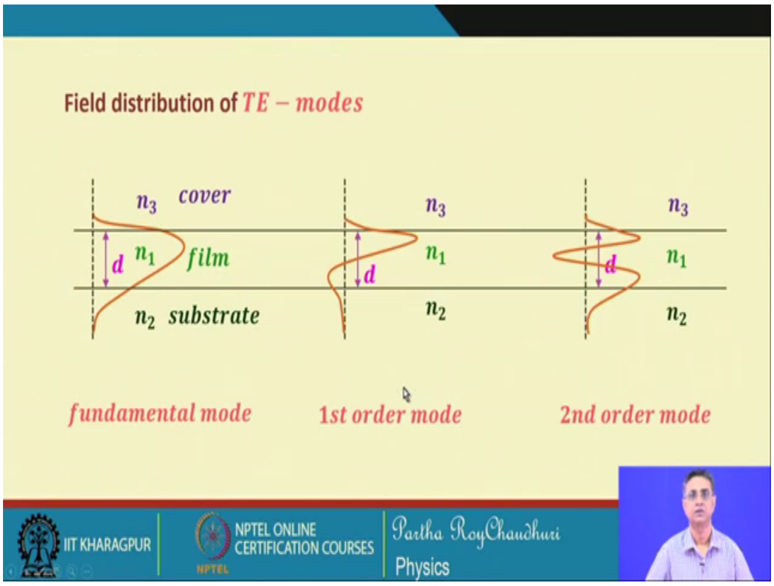

So, this is the form of the equation and if you plot this m equal to 0 that is the fundamental mode, the first order mode, the second order mode they are all consistent, but you can see the asymmetric nature of the field profile, because you have a you have a very sharp contrast in the refractive index, here for n 3 and n 1. So, the field decays very fast rapidly very fast, where as if you if because the substrate n 1 n 2 and n 1 they are slightly different. So, that field moves penetrates more into the substrate in this case and interestingly if you put n 1 equal to n 2. So, they are it becomes homogeneous and then field moves to infinity and there is no decaying nature of the field.

So, this is how the modes are excited in the asymmetric waveguide. Wherever there is a refractive index contrast the electromagnetic waves, they react and you get a sharp change in the field profile, when there is a very high contrast of the interface.

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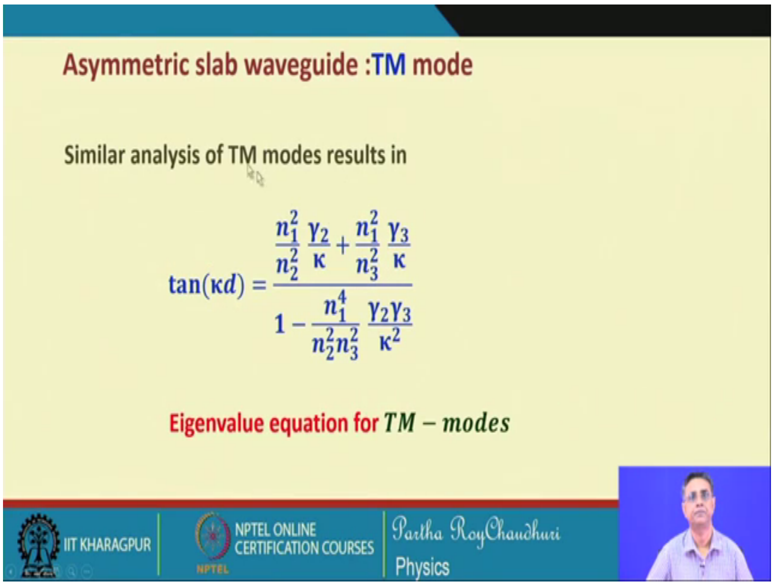

For TM mode analysis, we can we can carry out in the same way, and for H y fields we will get this equation which will be again attached with this n 1 square by n 2 square.

So, but this is the general form of the TM mode for asymmetric waveguides for 3 layers in gamma 2 and gamma 3. They will represent the bottom layer and the top layer for the cladding region.

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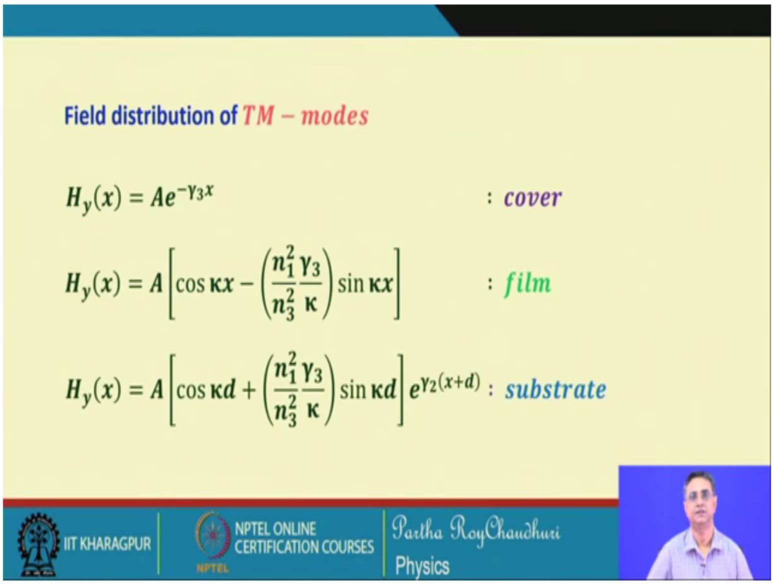

The field distribution for the TM modes are also similar, but this is in terms of H y and you have this evanescently decaying field in the cover region, oscillatory field in the film region, which is represented by this equation all taken care of. And in the substrate you again have a evanescently decaying field, but this gives you the strength of the field at the at the at the second interface, the amplitude of the film comes from this quantity. The field distribution again across the waveguide this is for H y field also that is TM mode, you have a sharply decaying evanescent field across the film cover interface, and it happens for all other higher order modes.

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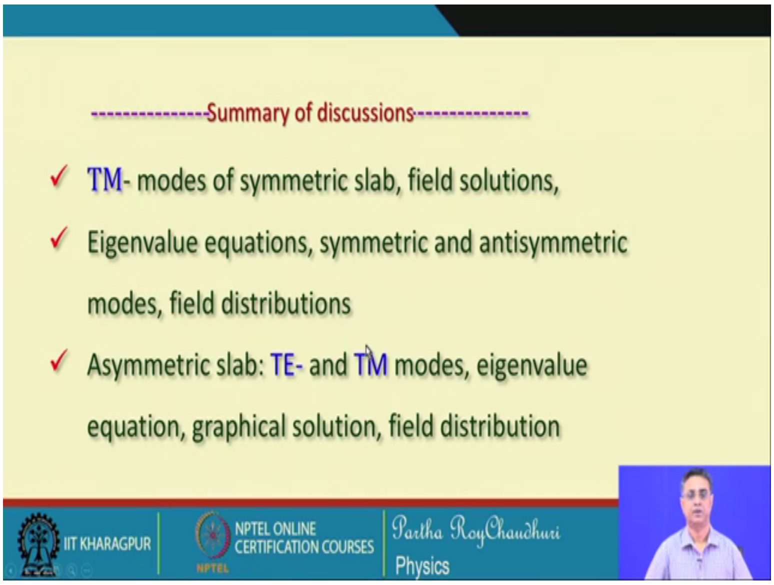

So, we conclude by saying that we have discussed this symmetric, very simple form of the dielectric waveguide which is the basic. And it is very important to know this as a basic foundation to understand the modes of optical waveguides. Of course, with this background you can switch over to higher order and completes geometry of the, but the rule is the law the physics is the same, only there will be some complexities in the boundary condition, multiple layers, writing the equation, but the basic physics is the same.

So, we discussed TE and TM modes for the symmetric slab waveguides altogether here. And also we tried to depict the field solutions the field the profile field distribution in the process, we evaluated the eigenvalue equations symmetric and antisymmetric mode fields. Then we discussed more practical optical waveguides that is the asymmetric slab waveguide, which are very common in integrated optic devices, miniature devices and interferometers modulators and sensors; very randomly use this is.

So, this is more relevant more practical. And in that case how the TE and TM modes and their eigenvalues are take the form that we have discussed. And we have also tried to look at how the modes can be solved in this particular case. So, we have taken up the graphical solution. So, unlike the circle this time V number will give you an ellipse. And the intersection of the ellipse with the tangent and cotangent kappa’s will give you the solutions the mode solutions. Then also we have seen the field distribution for the various modes for TE and TM fields across the waveguide.

Thank you very much.