Modern Optics
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Lecture - 20
Waves in guided structures and modes
(contd.)

We will be discussing the simplest type of optical waveguide namely the dielectric interface planers slab waveguide.

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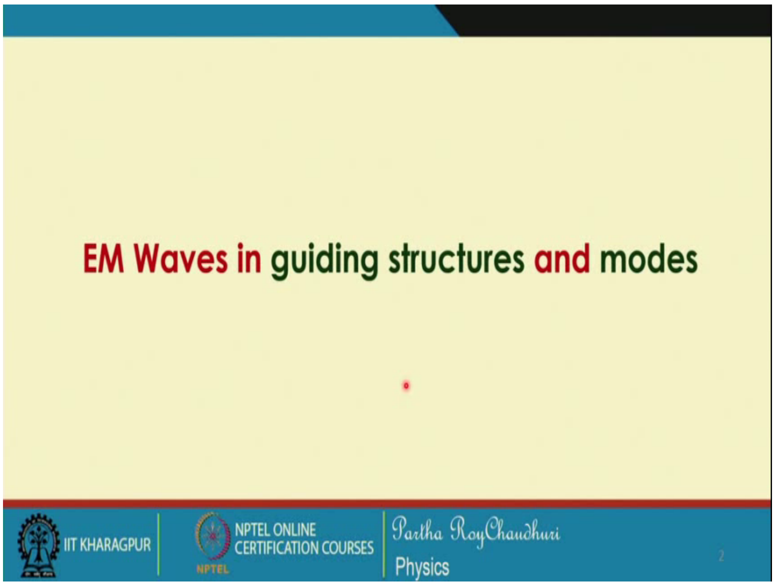

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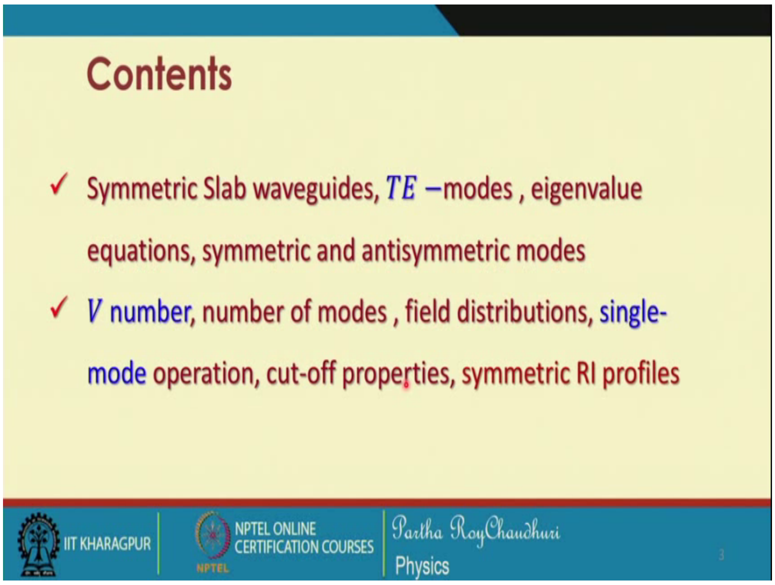

And, in the discussion we will look at the symmetric nature of the refractive index profile and the TE modes TM modes of the waveguide, then eigenvalue equation symmetric and anti symmetric modes which we will see is a consequence of the symmetric refractive index profile. Then we will define a quantity which is very relevant and very important is used for defining almost all kinds of optical waveguides, then we will look at the a number of modes supported by an optical waveguide of this kind then the field distribution across the waveguide.

Then there will be another important point that is the mode field that is the single mode operation of the waveguide, this particular property is very useful extremely useful in almost all applications then we will look at the cutoff properties of the various modes which are supported by the waveguide. Then this consequence of symmetric refractive index profile which will give rise to only symmetric and anti symmetric modes. So, with these points in mind we will discuss the planer dielectric slab waveguide.

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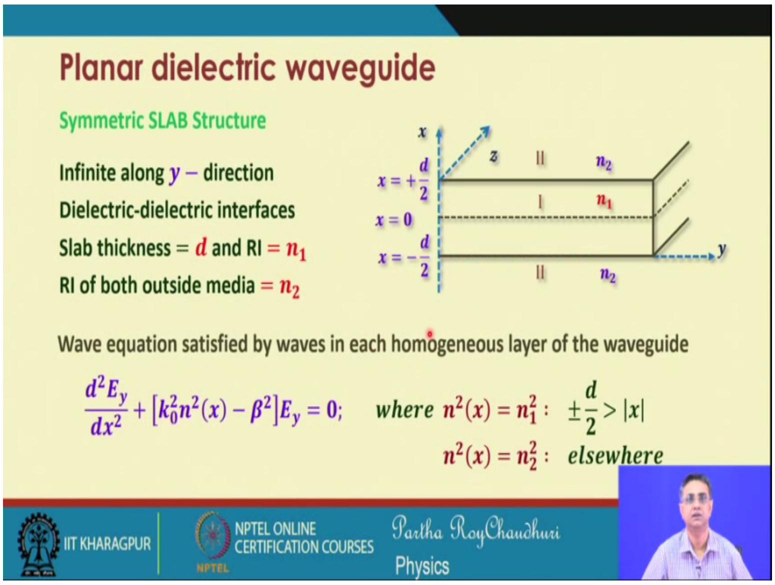

So, this is one the simple most structure where you have the refractive index n 1 which is sandwiched between the 2 same refractive index layers that is of refractive index n 2, these are the 2 interfaces so that there a there are 2 regions region 2 and region 1. Because, of the nature of the waveguide we assume that this y direction that is along this direction the waveguide is infinite and there is a refractive index profile variation only along this x direction.

But this variation is a constant it gives rise to constant homogeneous refractive indices, so there is a change in the refractive index across the interface. So, we will look at this property we assume that the thickness of the slab is d, so that and we set our coordinate system across this middle axial line such that the upper interface falls at x equal to d plus d by 2 and the lower interface at x equal to minus d by 2. So, these are the notations in parameters that will use in the determination of the waveguide modes and eigenvalue equations.

So, in this case the wave equation that is satisfied by the waves which will be propagating through this structure, in each of the homogeneous layer of the waveguide will be given by this homogenous equation. That is we consider the TE mode therefore this E y is the electric field component which is non managing and H x H z will also associate with this E y. So, for region 1 that is within this layer which will got the core and the other 2 layers surrounding this core will be called the cladding is the conventional nomenclature. Where so n square of x the refractive index varies along this direction, which is constant as long as this mode of x is less than plus minus delta 2. So, I define this layer which will have a refractive index n 1 and otherwise everywhere this is equal to n 2 square. So, the cladding refractive index is n 2 whereas the core refractive index is n 1.

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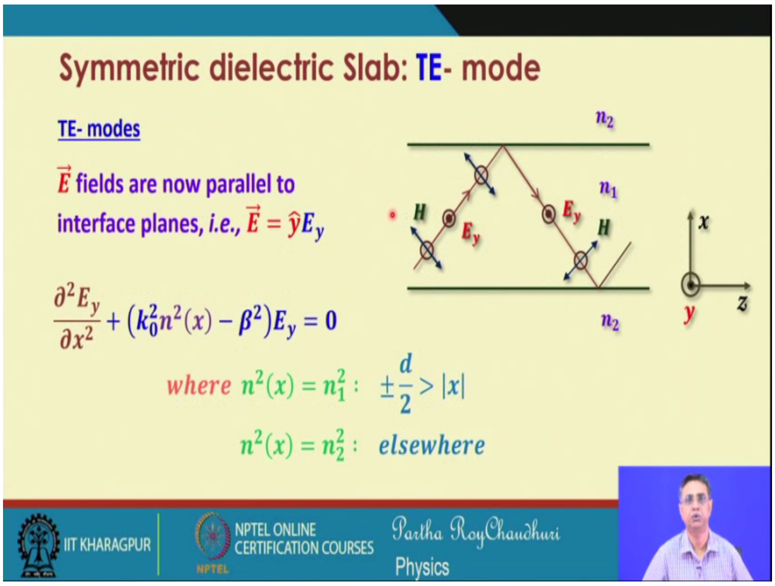

So, for TE modes once again we will look at this configuration we have E y field which are parallel to the interface planes E y and H field magnetic field components will lie in the x z plane. So, it will give rise to 2 components and this E y comes through this equation that is this homogenous equation that is del square E y by del x square plus this quantity into E y equal to 0. So, this the mathematical representation for the refractive index profile and the wave equation for this structure as long as we are concerned with the TE modes.

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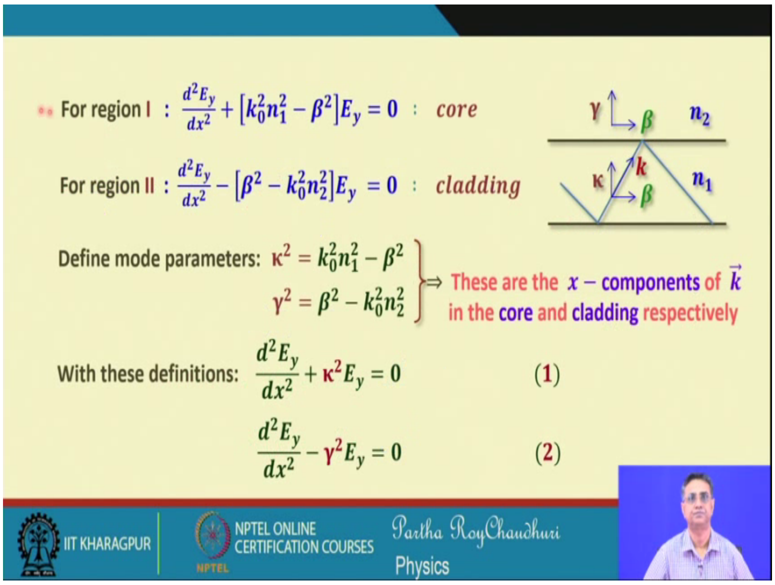

For the region one that is the one this region that is this region within the core we have this equation, because this n square of x now assumes a constant value that is n 1 square. So, for the core we have this equation and for region 2 that is for the cladding there are 2 such regions, so we have this equation and we write this slightly in a modified way we take this minus out and put it in this form. The intension is very simple because beta we will see that beta should lie somewhere between k 0 n1 and k 0 n 2. So, to make it positive beta square minus k 0 square n 1 n 2 square, this quantity will be positive which is the transverse component of the propagation constant in the cladding and we call this quantity as gamma square so I write this equation for cladding like this.

Then we define the mode parameters this we have seen these are the x components of the k vector in the core and cladding, so kappa square is written as k 0 n1 square minus beta square beta is the z component of the propagation vector beta is z and kappa will have a x component and along y direction we have assume that the waveguide is infinite. So, we get rid of the y component of this k vector, so the k x and k z these 2 constitute k k x within the core we have called kappa and outside this k within the with the within the cladding we call this is gamma and because n1 is more than beta we write in this way whereas beta is more than n 2, so we write in this way for modes to be supported.

So, we will look at this condition later with these definitions then we can write this equation that del square E y del x square plus kappa square E y equal to 0 for the first equation and for the second equation we can write this del square E y by del x square minus gamma square E y equal to 0. So, these are the 2 equations to wave equations for E y governing the propagation of the electric field in the structure for the core and the cladding.

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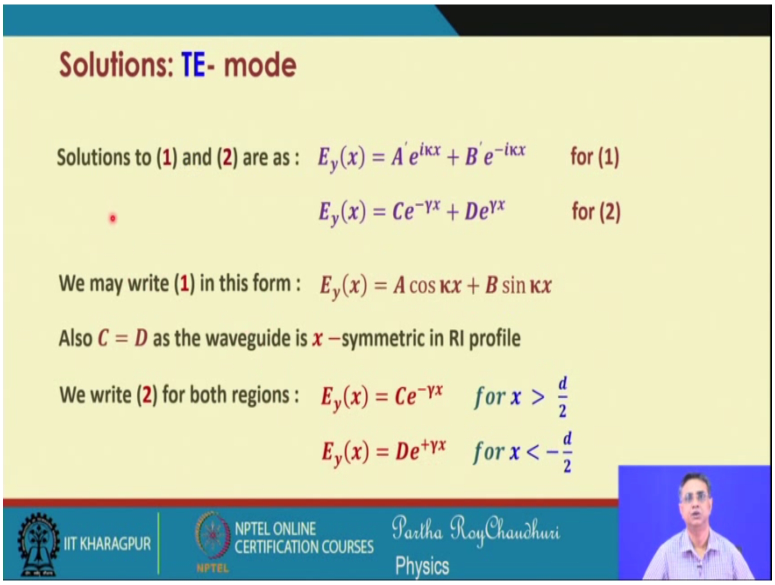

So, so the solutions to 1 and 2 the solution to this equation is very well known very well known and it appears very frequently in physics, this equation will give you an isolated solution for E y which can be written as A e to the A dash e to the power of i k x plus B dash e to the power of minus i k x. I used dash because we can we can simplify this using some different constants and for the cladding region. This equation will give you the solution that is which will be exponentially decaying solution because, you have a minus here this equation is also very well known in physics.

So, these 2 equations put together will give me the complete field solutions in the core and cladding of the symmetric refractive index dielectric waveguide. So, we may write equation 1 that is what I told just now that we can write in the sine cosine form, so this A e to the power of i k x B dash e to the power of minus i k x can be simply written in this form A cosine kappa x plus B sine kappa x. Now because of the symmetric nature of the waveguide because this side and this side they are identical, so both of them will be treated with the same constant. So, therefore C and D are same as long as the waveguide has a symmetric refractive index profile.

The variation of the field along with this direction and along this direction will associate the same constant that is what the idea is and in that case this quantity the first part will represent an exponentially decaying wave when x is positive. But this second part will be representing an exponentially decaying wave when x is negative and because C e is equal to D e. So, this will represent only the upper heart of the upper half of the interface. So, the wave which will be which will be travelling across this layer will be e to the power of minus gamma x, but the wave which will be travelling across this will be e to the power of plus gamma x because x is negative here.

Therefore with this notation and convention we write this equation E y for d x greater than d by 2 that is for upper half C e to the power gamma x and E y for the for the layer below the lower interface will be represented whenever the C and D they are identical in the case of a symmetric structure.

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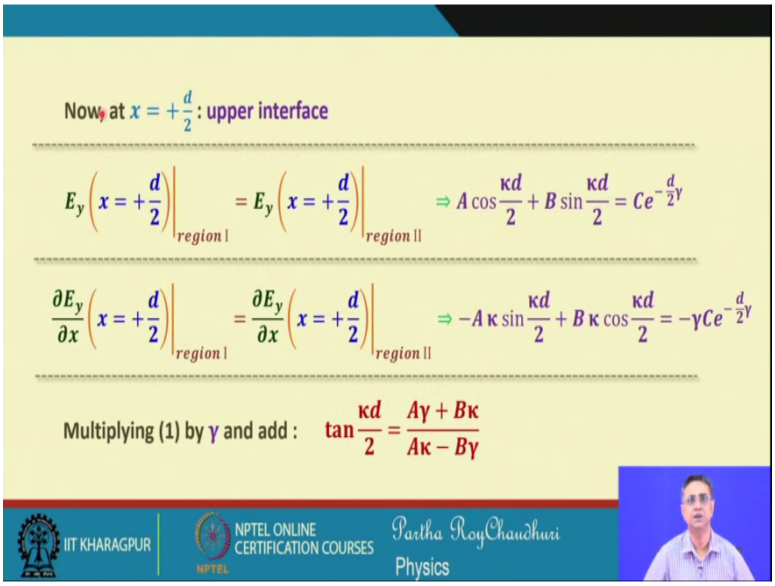

Now, we will use the boundary conditions for the continuity of the tangential component of the fields, so at x equal to plus d by 2 that is at the upper interface. At upper interface we can use this E y must be continuous across the interface layer and that gives you a cosine I just have to plug in that x equal to plus d by 2 on either side of this of this equation on either side of this equation this equation and this equation in these 2 equations we will use x equal to plus d by 2.

So, that gives you this condition and if I assume the continuity of the derivative of the field that is del E y del x across the interface, which is also a continuity condition we end up with this equation. Now, that if we just combine these 2 if we just combine these 2 equations multiplying 1 by gamma you just have to multiply one by gamma then this quantity and this quantity will be same. But with a difference of minus sign and if you then add then you will add up with this condition tan kappa d by 2 equal to A gamma plus B kappa by A kappa minus B kappa is a very very well known equation. So, I have used the continuity condition for the upper interface and that is for the field continuity and the derivative of the field continuity I have not used the lower interface, which will also give me the same boundary same equation same relation.

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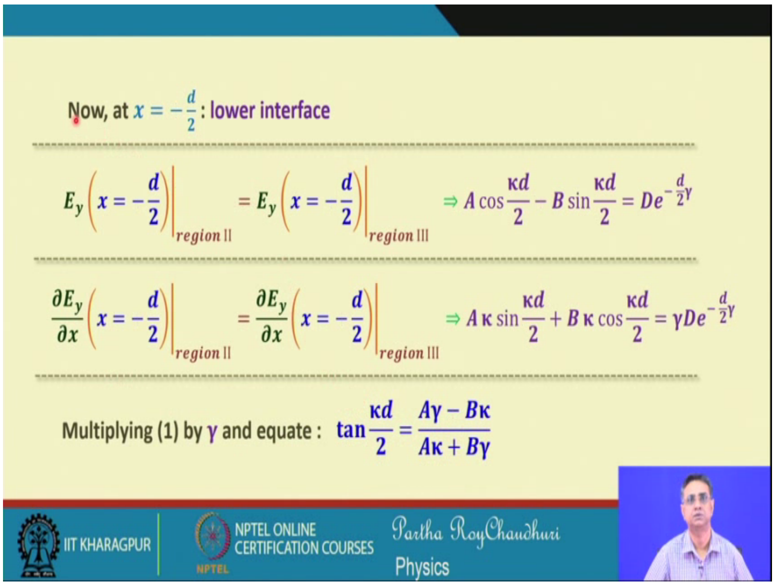

So, to see that at x equal to minus d by 2 at the lower interface that is x equal to minus d by 2, if I plug in this relation then I will end up with this equation this relation and also from this continuity of the derivative of the field will give me this condition and preceding in the same way that is multiplying 1 by gamma and then if we equate these 2 because in this case this right hand side will be equal and from there we can we can get a relation. So, there is a slight difference between this A gamma plus B kappa, whereas in this case A gamma minus B kappa and in the denominator you have plus; whereas, in the earlier case it was minus but both of them will represent tan kappa by kappa d by 2.

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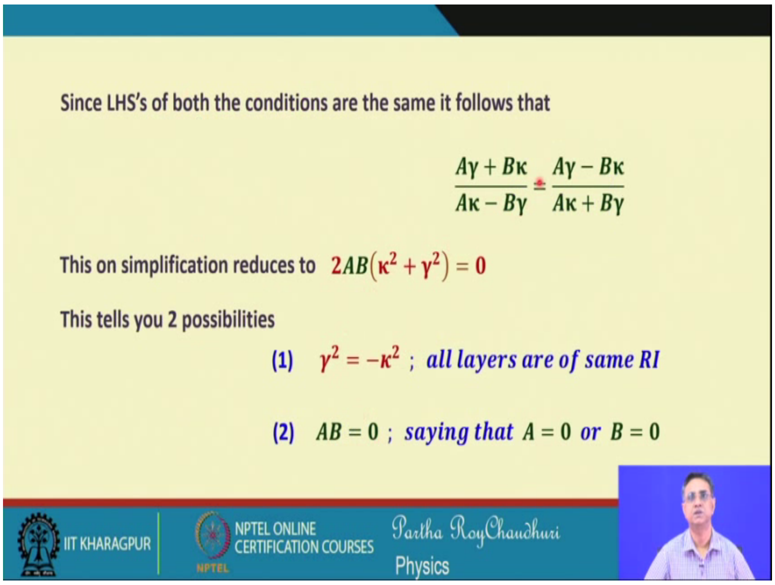

So, therefore these 2 must be equal to represent the same waveguide and same parameter same structure this eigenvalue equation must be holding good. Now if you simplify this will give you a condition that if you multiply this into this and this into cross multiplication will end up with this simplified condition twice A into B kappa square plus gamma square. It is a very simple algebraic steps to execute, then we will end up with this equation, now once we have this equation in hand it tells you 2 possibilities.

First one is that that either kappa square plus gamma square must be equal to 0, in that case gamma square equal to minus kappa square. But that means that means n1 and n 2 they are equal, if you look at the definition of kappa and gamma n1 and n 2 because beta will cancel k 0 n1 square will be equal to k 0 n 2 square. So that means n1 is equal to n 2 that means all layers are the same there is no waveguide there is no interface so that is a trivial condition. So, we will not continue with this condition we will look at this A B equal to 0 the other possibility, so that means either A equal to 0 or B equal to 0.

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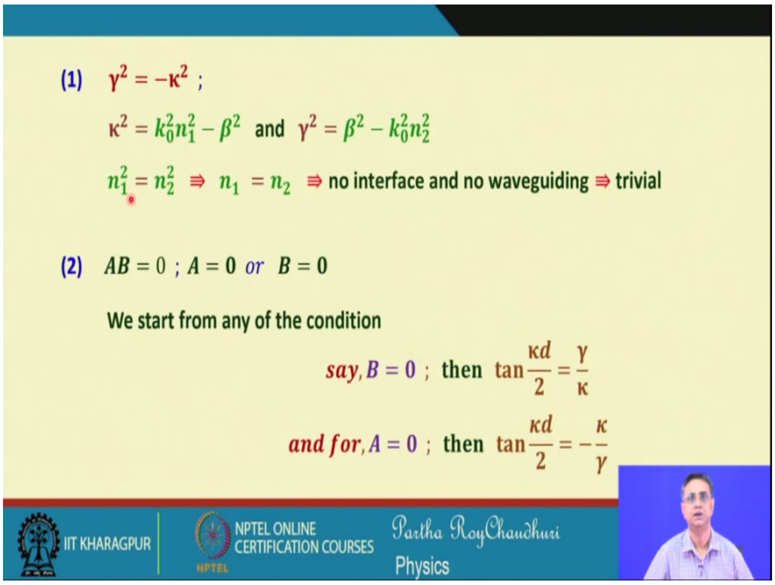

So, these 2 conditions will lead to the other possibilities, so this one I have already explained that that kappa square equal to minus gamma square will give you no interface and this is a trivial condition for this. Let us suppose we start with the condition B equal to 0, then from this equation from this equation if I put B equal to 0 B equal to 0 then you get that gamma by kappa and this gamma by kappa.

If I put this equation then the tangent value of this lets go to this, so tan k d by 2 will become equal to B equal to 0 you have put so you get gamma by kappa, so tan k kappa d by 2 will be equal to gamma by kappa. So that is the equation which comes out of the condition that B equal to 0. So, I put B equal to 0 I get this condition and in the similar way if I put A equal to 0 I will get this condition. In that case I will put when you put A equal to 0 you get kappa by gamma the reciprocal of the earlier case that is an with an negative sign right, so you get 2 conditions for B equal to 0 and A equal to 0.

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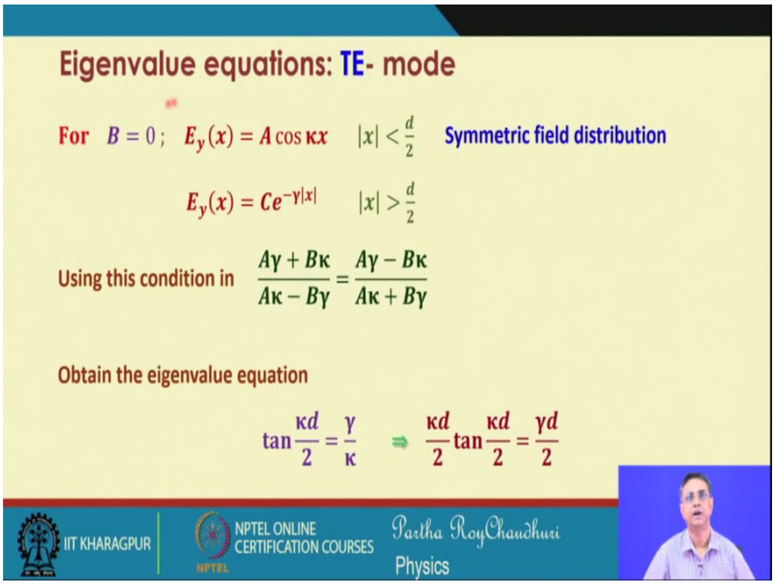

Now, eigenvalue equation so this defines the eigenvalue equations this, if I put this in the solution of E y for the core that is mode x less than d by 2 I get this field solution which tells you because it is a cosine function, so the fields are symmetric. So, this gives you the symmetric field distribution all fields will be symmetric and this is a anyway a evanescent field evanescently decaying field at the outer at the at the cladding region starting from the interface. So, using this using this condition in this equation then we get the eigenvalue equation using this condition.

We get the eigenvalue equation as this tan k d by 2 tan because in that case we have to multiply d by 2 and d by 2 numerator and denominator if you bring it to the left hand side you get k d by 2 tan kappa d by 2 equal to gamma d by 2. So, this is the eigenvalue equation corresponding to the symmetric field distribution which corresponds to the condition that B equal to 0 and let us define this V number which is a very important parameter for optical waveguides, V defined as k 0 half of the thickness this is different in different text books. This is half of the half of the thickness and this is the numerical aperture and n1 square minus n 2 square, so the core cladding refractive index square difference.

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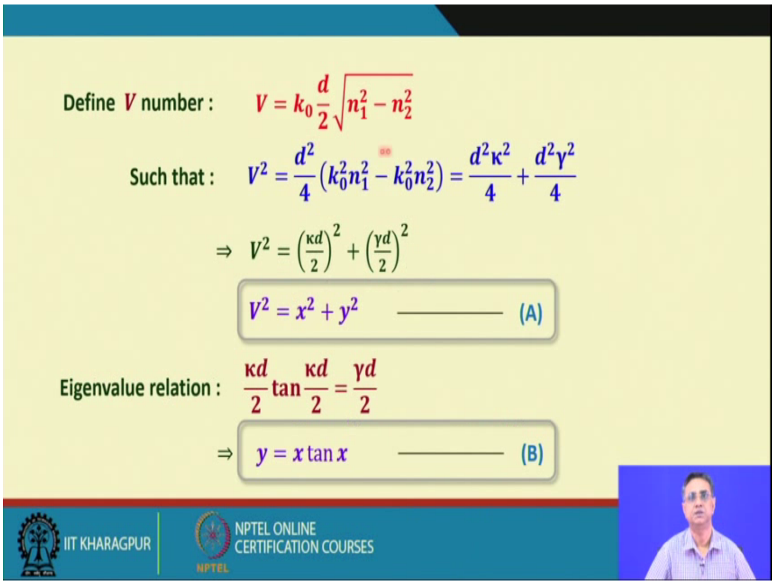

So, V square is equal to this quantity and you can see that the that you can write this equation as k d square by 4 and gamma d square by 4 because, if you add one if you subtract 1 beta square and then add 1 beta square this quantity will correspond to correspond to kappa and the other quantity will correspond to gamma. So, that is why I have written this V square is equal to this, so V square can be put into this form and you can see that V square equal to x square plus y square if you call this quantity equal to x and if you call this quantity equal to y.

So, you can write this equation which is the equation of a circle of radius V and these are the equations which are the outcome of the of the eigenvalue equation. Now, now I so this is the consequence of the definition of V number which appears in terms of the kappa and gamma and this is the eigenvalue equation.

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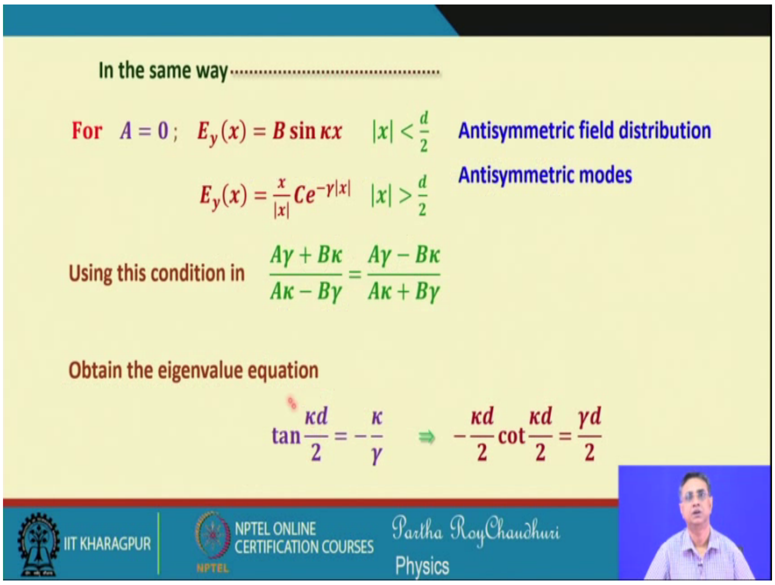

Now to look for the solution of the modes [FL] in the same way in the same way A equal to 0 e to the E y of x this will be the field solution and that corresponds to anti symmetric modes and these the evanescently decaying field will be the same. Using this condition I again from here can arrive at this condition which will be the eigenvalue equation for the anti symmetric modes. We can actually put 2 of them together in 1 compact eigenvalue equation which will represent both symmetric and anti symmetric modes.

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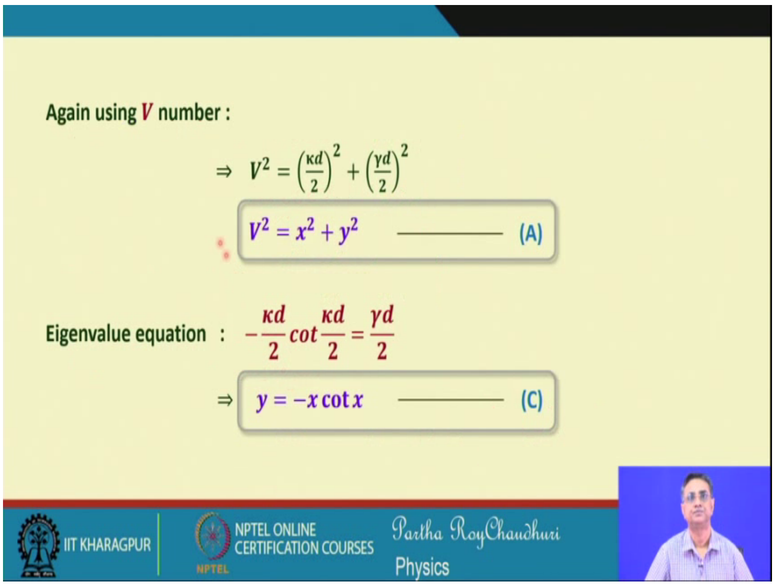

Now, again using V number for the anti symmetric modes we can write this equation which is the same and for the eigenvalue equation now this time it becomes y equal to minus x cot of x cotangent of x. So, we have 2 eigenvalue equations and one equation of a circle.

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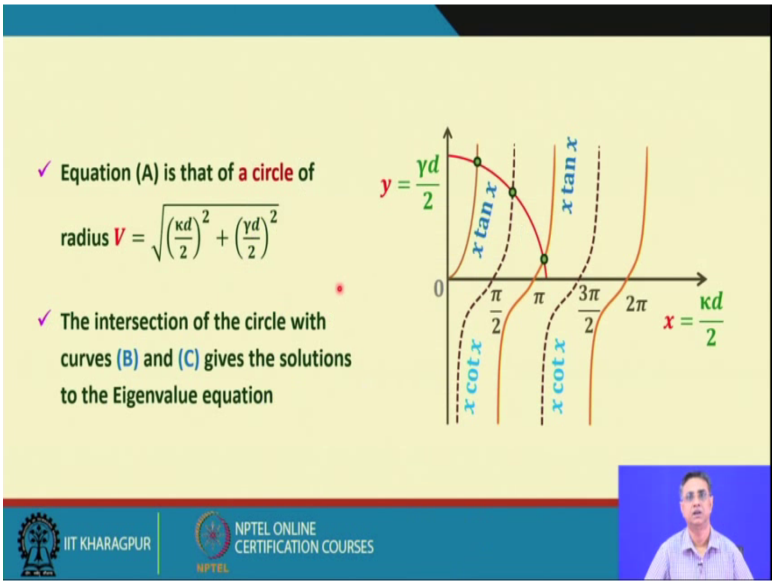

So, we plot them we plot them because we are looking for the solution of the modes, so this is your x tan x this is your x cotangent x alternatively they are appearing and now I draw a circle of radius V which is equal to this which is equal to this. So, at this point it will be gamma d by 2 and at this point it will be kappa d by 2. So, this radius the point of intersection of the circle with the curves B and C gives the solution to the eigenvalue equation.

So, you can see that this there will be one solution here for the value of V which is less than pi by 2, here this will be less than pi again it is close to pi so on and so forth for this value of V and if you increase the value of V we will get we will get more number of solutions. So, it is the V that is going to decide V number we going to decide how many modes will be supported in the structure.

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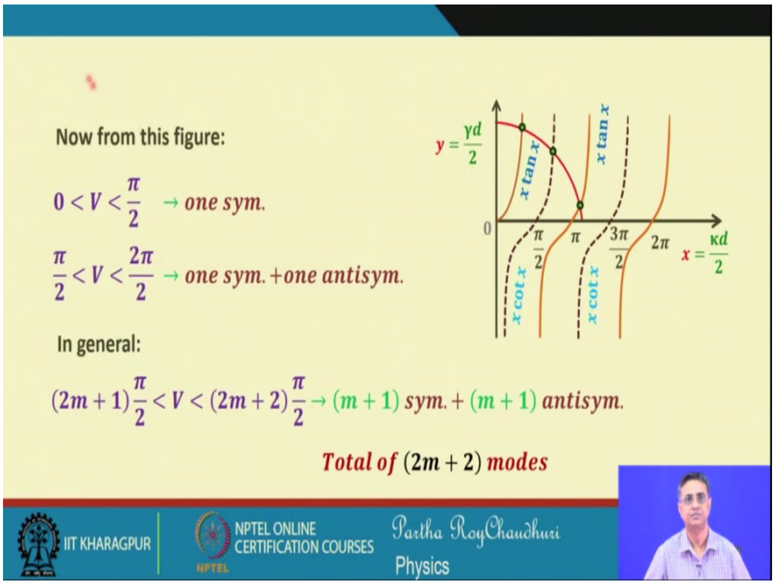

Now from this figure you can see when the value of V is less than pi by 2 is less than pi by 2 because this line less than pi by 2. So, you have only one symmetric mode, but if the value of V is between pi by 2 and pi between pi by 2 and pi then you have this point of course, this encloses this point. So, you have one symmetric mode the value which lies between pi by 2 and pi somewhere here then you can have one symmetric one symmetric mode and one anti symmetric mode. So, there will be 2 modes and in this way if you proceed for the values of V which lies between twice m plus 1 pi by 2 and twice m plus 2 pi by 2 that is within a spread of pi by 2 between these 2 positions.

Then you will have m plus one symmetric mode and m plus one anti symmetric mode modes. So, this is simply just by induction of these 2 initial facts we can we can arrive at the conclusion that the total number of modes will be supported that is equal to twice m plus 2, provided that the value of V lies between this is twice m plus 2. So, this is the total number of modes which will be supported, we will see how the modes look like now equation A is so total number of modes we have supported.

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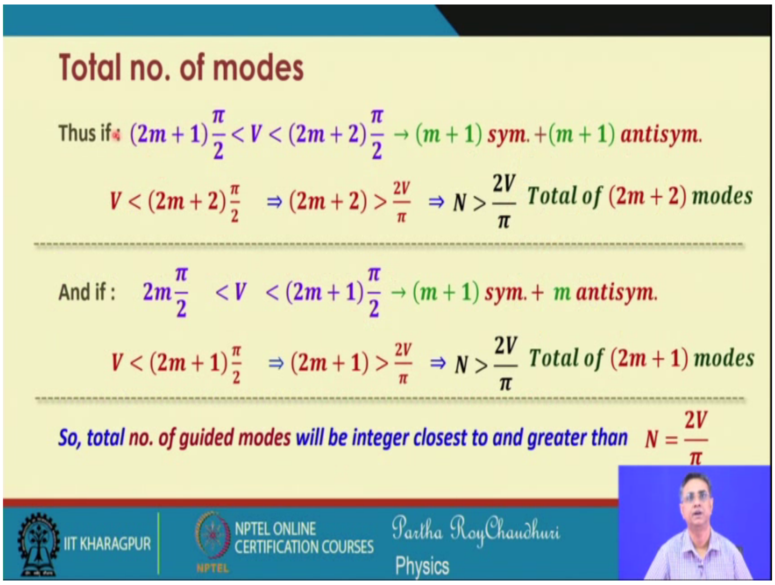

Now, thus this condition gives you twice m plus 2 modes and if this V lies between twice m pi by 2 and twice m plus 1 pi by 2 between them, then the total number of modes in the same way will be twice m plus. That means whether you are you are at the symmetric mode or you are at the anti symmetric mode as the last point of intersection. So, in these way you can generalize the condition that the total number of modes supported, in the in the waveguide will be a integer which is closest to or greater than this twice V by pi. So, this is a very beautiful finding and it is used for determining the properties of the waveguide when we know the V number.

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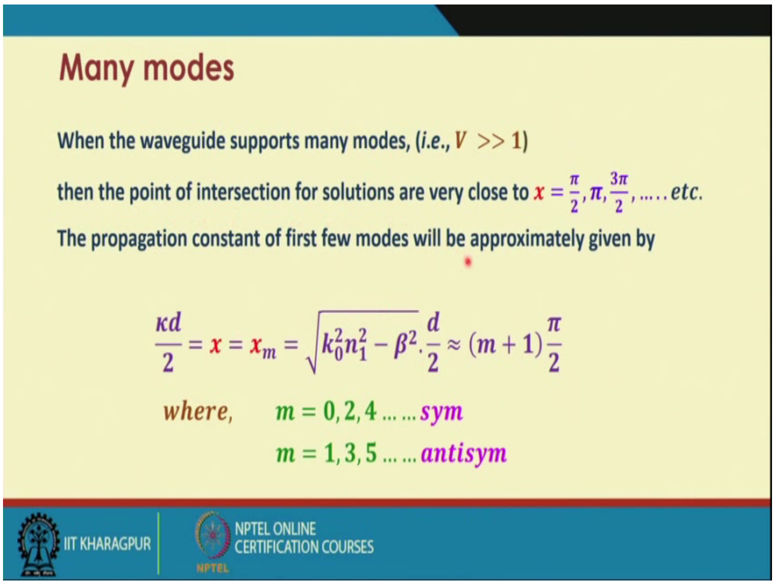

So, it tells you that this V number gives you many modes that is k d by 2 equal to x this approximately equals to this quantity. So, the point of intersections are the solution so this is a rough way of estimating the propagation constants for the various modes because, this at the cutoff at this point x equal to pi by 2 these are very close points you can just have a look at it this is very close to pi this is also very close to pi by 2. But this will be even more closer when you approach for large number of modes, so therefore, a an empirical way of finding the mode parameters the mode the mode propagation constants by using this relation ok.

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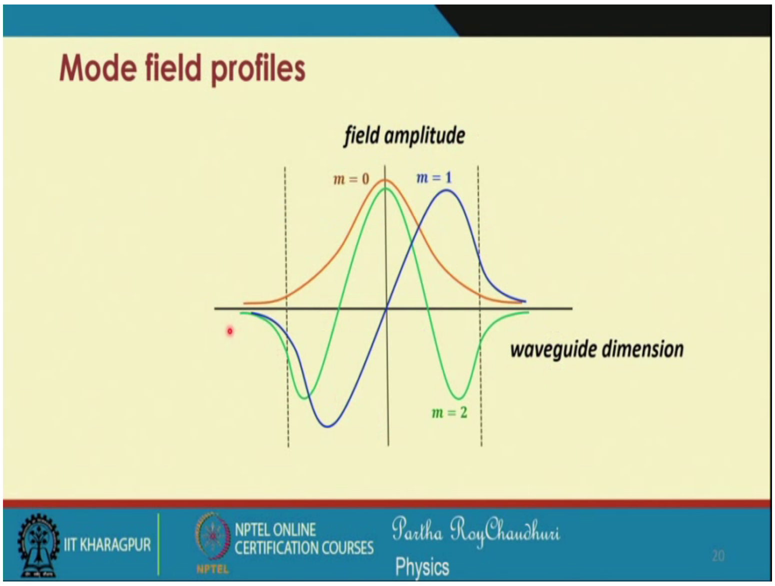

So, these are the mode field distribution you can see for m equal to 0 you have this symmetric mode which is called the fundamental mode m equal to 0, for m equal to 1 you have the anti symmetric mode and for m equal to 2 you have again the symmetric mode. So, these are the modes which will be supported in the structure.

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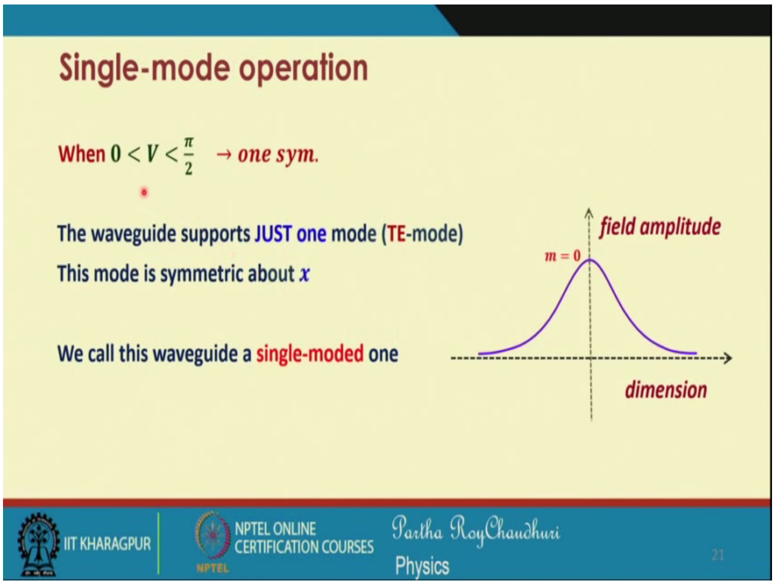

When V is less than 0 V is greater than 0, but less than 2 you have one symmetric mode. So, for this regime of operation when I choose the waveguide parameters in such a way including the wavelength, such that the V number falls between 0 and pi by 2 then the structure we support only one mode and that is the fundamental mode such a waveguide is called a single mode waveguide at that operating wavelength and you see that this mode will be symmetric about x. All the even modes that is n m equal to 0 2 4 etcetera will be all symmetric modes, so we have a and we call this wave waveguide as a single mode waveguide which is very relevant and very important in the study of in the in the in the signs of optical waveguide.

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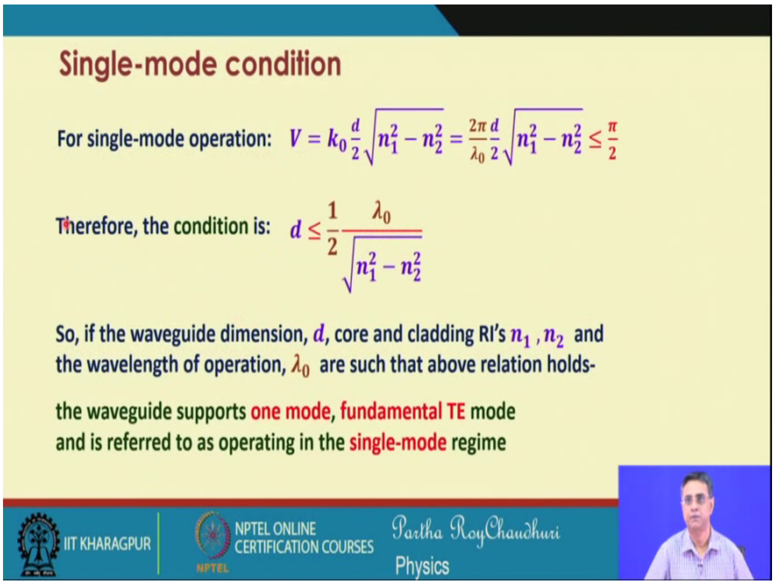

For single mode operation V equal to V equal to this quantity because we have defined this, so therefore the condition d is equal to half of lambda 0 n1 square minus n 2 square. So, if the web guide dimension that is core refractive index and lambda 0 are such that they satisfy this condition, then you automatically take care of the single mode nature of the waveguide the waveguide supports one fundamental TE modes and it is referred to as the single mode regime of operation.

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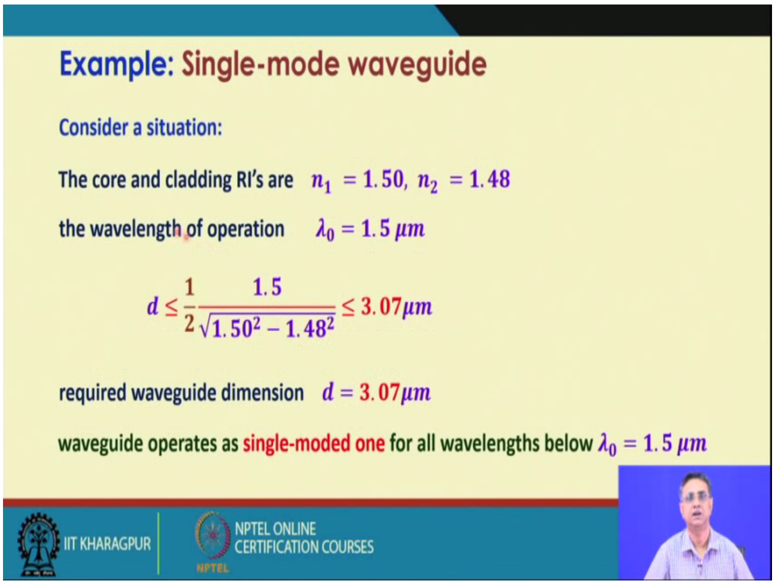

Let us take an example the core and cladding refractive indices are n1 equal to this, which are the typical values for glass and planar optical waveguides and the wavelength of operation is it is equal to 1.5 micrometer. If you plug in this values in the in the relation here then you get the value of d which is very close to 3.07 micrometer. If the waveguide dimension is less than this wavelength of operation is this or below this and the refractive indices of the core and cladding are these, then this waveguide will be a single mode one for all wavelengths which are below lambda 0 equal to 1.5.

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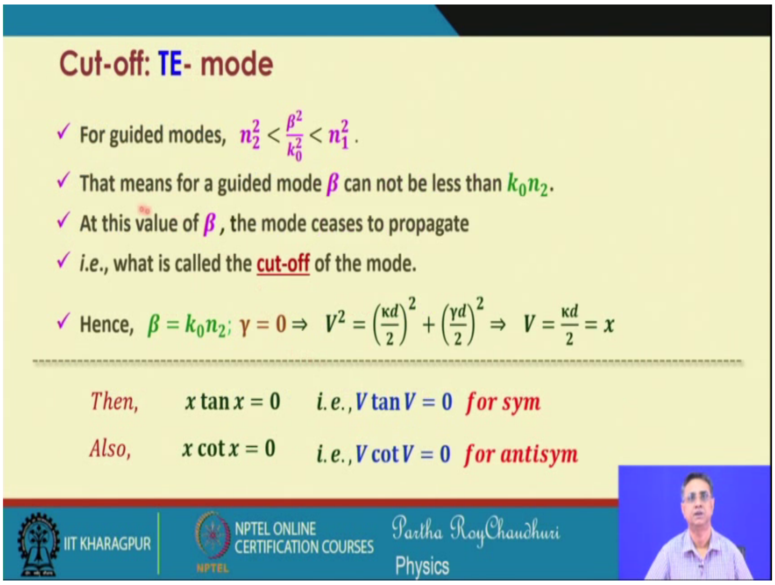

Then the cutoff property for guided modes you have this property because, anyway the propagation constant has to lie between these for every mode. Now if they are critically equal that is k 0 n 2 is equal to B 2 beta 2 then that is what we call the cutoff so that gives you the condition. So, if gamma that is k 0 n 2 minus beta 2 square will become equal to 0 that is equal to gamma equal to 0, because of this condition so that tells you V equal to kappa d by 2 which is equal to x. So, x tan x equal to 0 from the eigenvalue equation and x cot x will be equal to 0 these are the conditions for symmetric and anti symmetric modes and V end up with this condition that tells you for V c V c will be equal to 0, V c will be equal to m pi to get this condition equal to 0.

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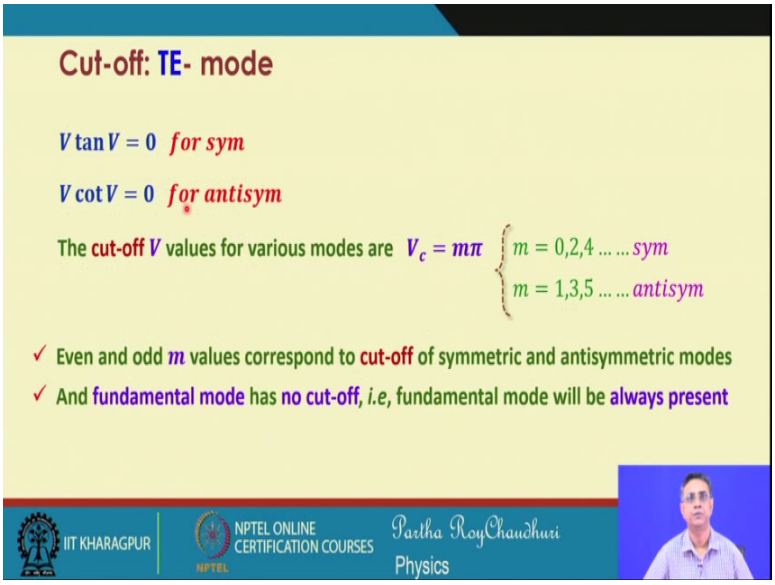

So, so that is so I just roughly take the value of m equal to 1 2 3 that gives me the value of V c, the cutoff V value at which and below which those modes will be supported. Fundamental mode has no cutoff because, you whenever there is some excitation it is always the fundamental mode and you can see from this equation also that there is no cutoff for the fundamental mode it is always present in the waveguide.

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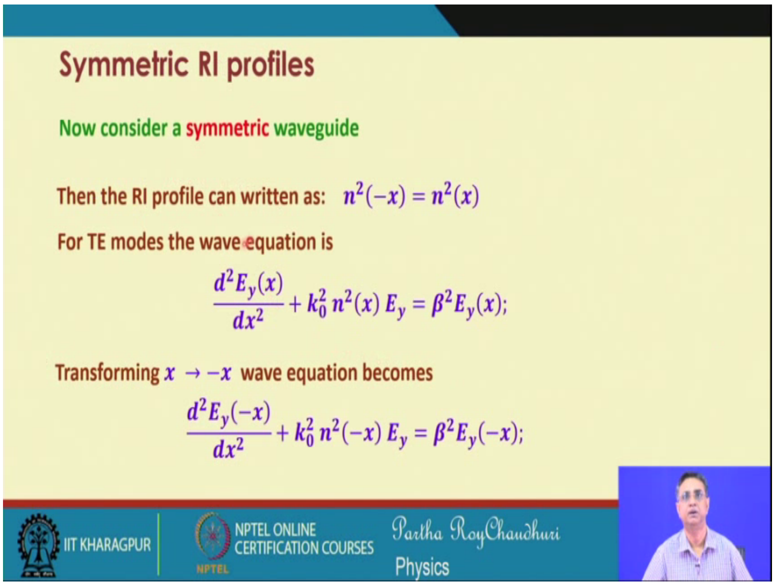

Now, there is one issue one important point that if you consider a symmetric refractive index profile of the waveguide you can write this mathematically like this and for TE modes you can write down this equation which is an eigenvalue equation. If you transform x to minus x I can write this equation like this, you can see that both the equations I mean both the both the e E of y E of E y of x and E y of minus x they satisfy the same equation.

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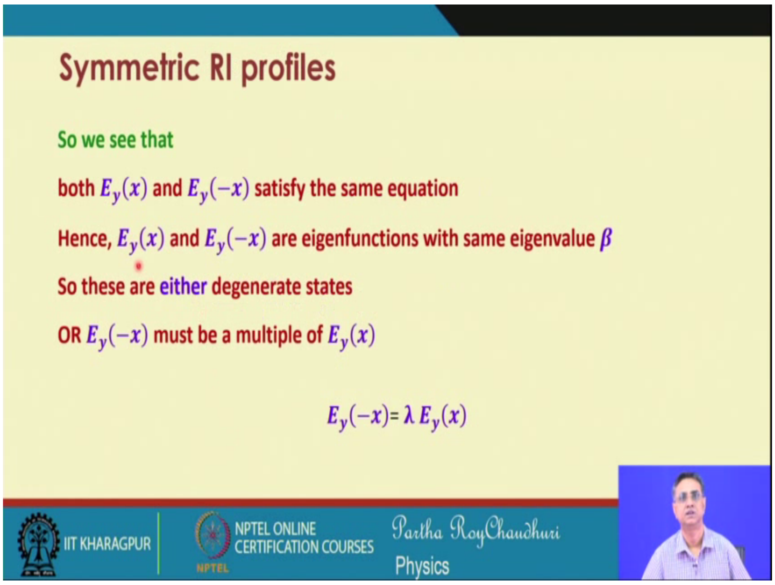

So, both E y and E y of minus x satisfy the same equation hence these 2 are the Eigen functions with the Eigen value. So, this means that these 2 are the degenerate steps or it could be that this must be a multiple of E y. So, this possibility of degenerate sets can also be shown to be the identically the same consequence. But if we consider E y of minus x equal to lambda times this, this will this then again you transform this x to minus x that is E y of x will become lambda into E y.

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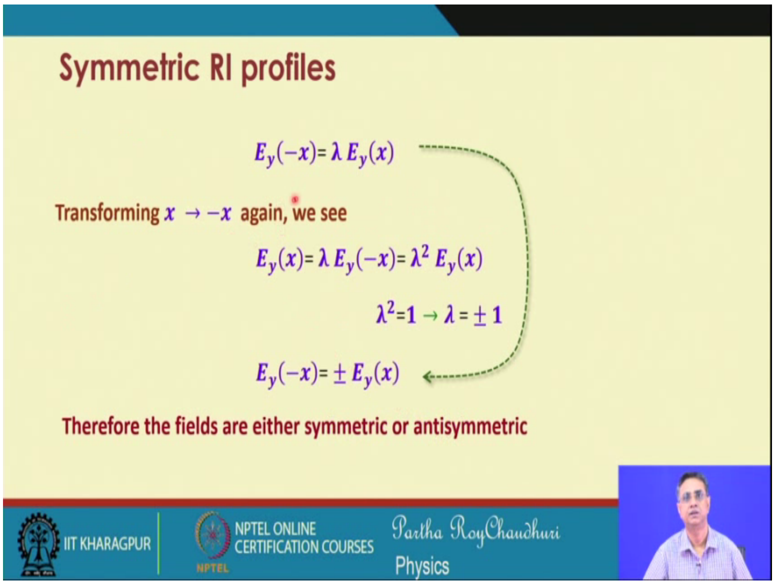

So, I put minus x again put back x equal to minus x which gives you that lambda square equal to 1, so that is lambda equal to plus minus 1. That means, E y of minus x is equal to plus minus this is this is very clearly the condition of the symmetric nature of the symmetric and anti symmetric nature of the mode profile. So, therefore if the fields are either symmetric or anti symmetric provided that the refractive index profile is of symmetric nature.

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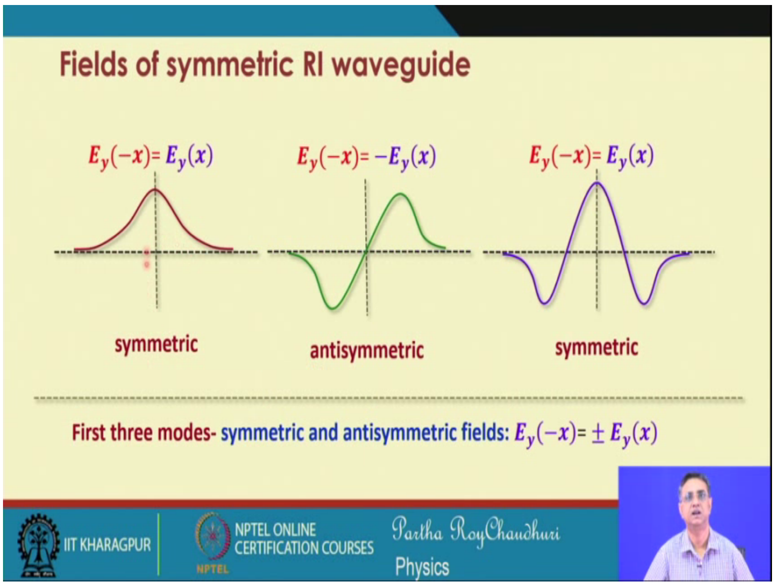

So, you can see that E y of minus x equal to E y of x is a symmetric field, but E y of minus x equal to minus E y of x then it is anti symmetric and so on. So, these are the these are the consequence of a symmetric refractive index profile.

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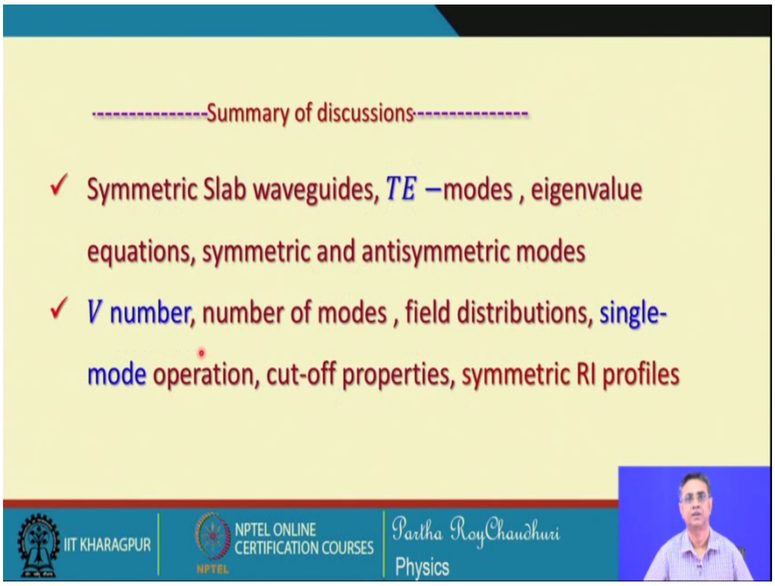

Now, I conclude here by saying that the we discussed this symmetric slab waveguides for the TE modes we will take up the TM modes next. We discussed the eigenvalue equations and then symmetric and anti symmetric modes even and odd modes which are supported by this waveguide, we have try to depict the mode pictures also mode profiles. Discussed the V number then from the V number how we can evaluate you can estimate the number of modes that will be supported, field distribution of the modes then very very particular thing about the optical waveguides is the single mode operation for this slab waveguide that also we have discussed.

Then we discussed the cutoff properties symmetric refractive index profile and the consequence of that in terms of symmetric and anti symmetric field profiles.

Thank you.